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CLOSED-LOOP OR OPEN-LOOP MODELS IN IDENTIFICATION FOR CONTROL ?[1]

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Abstract

In this paper we derive a generic indirect approach of closed-loop identification in order to compare estimated model of the closed-loop transfer function versus reconstructed estimated model of the open-loop plant in terms of asymptotic bias and variance considerations. We show the impact of the sensitivity function on both estimates.

1 Introduction

Schemes for system identification, based on closed-loop experiments, naturally arise when the ultimate objective of the identification is to use the model for control design ('identification for control').

Consider the settings shown in Figure 1, where P is a linear plant to be identified, K_y is a known linear controller and $v(t)$ is an output disturbance signal. It is assumed that the controller internally stabilizes the unknown plant. The different approaches in closed-loop identification are obtained by different parameterizations of the input-output dynamics and the noise models. The classification made here is similar to the one in [SS89],

- The direct approach: Ignore the feedback and identify an estimate of the open-loop plant using measurements of the input $u(t)$ and the output $y(t)$.
- The indirect approach: Identify an estimate of the closed-loop transfer function using measurements of the reference signal $r(t)$ and the output $y(t)$ and use this estimate to reconstruct the open-loop model with the knowledge of the controller.

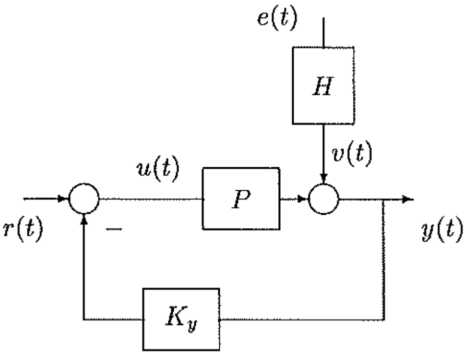

Figure 1: The internally stable feedback loop formed by true system and a stabilizing controller.

- The joint input-output approach: Identify both transfer functions from $r(t)$ to $y(t)$ and $r(t)$ to $u(t)$ and use them to compute an estimate of the open-loop model.

A detailed comparison of these three groups is found in [FL98, Van97], where the focus is on the analysis of the statistical properties of these approaches. All these approaches have in common the same asymptotic variance expressions of the resulting transfer function estimates [GLV97], and if asymptotic bias expressions are considered, use can be made of a filter (equivalently a noise model) that links the different approaches [FL98]. As examples, the indirect approaches are direct approaches that use a specific noise model (filter) that contains a regulator [Ega97], while a joint input-output approach can be formulated as a 'multivariable' indirect approach. Under these considerations, we will limit our study to a generic indirect method that proceeds as follows. The closed-loop transfer functions from $r(t)$ and $e(t)$ to $y(t)$ are estimated using measurements of the reference $r(t)$ and the output $y(t)$; it is an open-loop problem. Then, the model of the open-loop plant is reconstructed using the knowledge of the controller. This generic indirect

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method allows us to present the well known asymptotic expressions of the variance and the bias of both models.

We show that in terms of asymptotic bias considerations, the estimated model of the closed-loop transfer function must be preferred to the reconstructed model of the open-loop plant, when both estimates of input-output and noise dynamics are needed for the design of a new controller. In terms of variance considerations, we show the role of the sensitivity function in the variance distribution, whose impact on the reconstructed model of the open-loop plant and the estimate of the closed-loop transfer function is entirely different. We show that the reconstructed model must be preferred if the controller present during the identification experiment is close to its optimum value. The discussion is illustrated by an example.

2 Some preliminaries

We consider that the task is to design a controller for some true system described by

$$G : y(t) = P(q)u(t) + H(q)e(t) \tag{1}$$

where $P(q)$ and $H(q)$ are scalar rational transfer function operators and $H(q)$ stable and minimum phase, q^{-1} is the backward shift operator and $e(t)$ is white noise of zero mean and variance σ^2. The system is assumed to be controlled by some initial controller

$$K : u(t) = r(t) - K_y(q)y(t). \tag{2}$$

The reference signal $r(t)$ is assumed independent of the noise $e(t)$. The internally stable feedback loop formed by the true system (1) and the controller (2) is depicted in Figure 1, while the equations that describe the closed-loop system are

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = \begin{bmatrix} T_{yr}(q) & T_{ye}(q) \\ S_P(q) & T_{ue}(q) \end{bmatrix} \begin{bmatrix} r(t) \\ e(t) \end{bmatrix} \tag{3}$$

where $S_P(q) \triangleq \frac{1}{1+P(q)K_y(q)}$ is called the *sensitivity transfer function* of the feedback system, $T_{yr}(q) \triangleq P(q)S_P(q)$, $T_{ye}(q) \triangleq H(q)S_P(q)$ and $T_{ue} \triangleq -K_y(q)T_{ye}(q)$. We also consider the closed-loop system driven by the same external signals $r(t)$ and $e(t)$ as in (3), formed by the same controller $K_y(q)$ and the following model

$$\hat{G} : y(t) = \hat{P}(q)u(t) + \hat{v}(t) \tag{4}$$

where $\hat{v}(t) \triangleq \hat{H}(q)e(t)$. Here, the model may be considered as a nominal model, typically simpler than the true system, that is selected for the design of the controller $K_y(q)$ or as the to be identified model. The closed-loop expressions for the output $y(t)$ and the input $\hat{u}(t)$ are

$$\begin{bmatrix} \hat{y}(t) \\ \hat{u}(t) \end{bmatrix} = \begin{bmatrix} \hat{T}_{yr}(q) & \hat{T}_{ye}(q) \\ \hat{S}_{\hat{P}}(q) & \hat{T}_{ue}(q) \end{bmatrix} \begin{bmatrix} r(t) \\ e(t) \end{bmatrix}, \tag{5}$$

where $\hat{S}_{\hat{P}}(q) = \frac{1}{1+\hat{P}(q)K_y(q)}$ is the *nominal sensitivity transfer function*, $\hat{T}_{yr}(q) = \hat{P}(q)\hat{S}_{\hat{P}}(q)$, $\hat{T}_{ye}(q) = \hat{H}(q)\hat{S}_{\hat{P}}(q)$ and $\hat{T}_{ue} \triangleq -K_y(q)\hat{T}_{ye}(q)$.

3 The prediction error method

We provide a brief review of the classical prediction error method of system identification with some of the most important analysis results. A model of the plant is to be identified using a finite set of data $Z^N = \{y(t), u(t), t = 1, 2, ..., N\}$ collected on the closed-loop system (3). The data collection can be done in open-loop ($K_y = 0$) or in closed-loop. In prediction error identification one typically considers a general model structure of the form

$$y(t, \theta) = \hat{P}(q, \theta)u(t) + \hat{H}(q, \theta)e(t), \tag{6}$$

parameterized in terms of a parameter vector θ. Specifically, the corresponding one-step ahead prediction error is,

$$\epsilon(t, \theta) = L(q, \theta)\{y(t) - \hat{P}(q, \theta)u(t)\}, \tag{7}$$

where $L(q, \theta)$ explicitly plays the role of the inverse noise model $\hat{H}(q, \theta)$ for the model structure (6), but can be extended to any stable transfer function without loss of generality, possibly parameterized by the vector θ. The parameter estimates are usually obtained by the minimization of a least-squares criterion function of the form,

$$J_{id}^N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \epsilon(t, \theta)^2. \tag{8}$$

The parameter estimate is then defined as

$$\theta_N = \underset{\theta \in D_\theta}{argmin}\, J_{id}^N(\theta). \tag{9}$$

where D_θ is a predefined set of admissible values. In turn, it produces estimated input-output and noise models $\hat{P} = \hat{P}(\theta_N)$ and $\hat{H} = \hat{H}(\theta_N)$, respectively. Under reasonable conditions on the data and the model structure (see [Lju87]), θ_N converges as $N \to \infty$ to

$$\theta^* = \underset{\theta \in D_\theta}{argmin}\, J_{id}(\theta), \tag{10}$$

where

$$J_{id}(\theta) = \lim_{N\to\infty} EJ_{id}^N(\theta) = E[\epsilon(t, \theta)]^2. \tag{11}$$

The Parseval identity allows one to obtain an expression for the frequency distribution of the asymptotic model error. Consider the prediction errors in (7). Then:

$$J_{id} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\omega \mid L(\theta) \mid^2 \{\mid P - \hat{P}(\theta) \mid^2 \phi_u + \phi_v\} \tag{12}$$

where ϕ_u, ϕ_v are the spectra of $r(t)$ and $v(t)$, respectively. When the system G is not in the model set, the estimated transfer function has a *biased* frequency response. However, the bias term can be affected by manipulation of the filter $L(\theta)$. This shows the importance of selecting a filter, or equivalently a particular noise model in (6).

Asymptotic variance expressions for the estimated transfer functions $\hat{P}$ and $\hat{H}$ are available through the standard expressions for open-loop identification [Lju87]:

$$cov\, \hat{P}(e^{i\omega}) \simeq \frac{n}{N} \frac{\phi_v(e^{i\omega})}{\phi_u(e^{i\omega})}$$
$$cov\, \hat{H}(e^{i\omega}) \simeq \frac{n}{N} \frac{\phi_v(e^{i\omega})}{\sigma^2} \quad (13)$$

where n is the size of the vector θ, and n, N are supposed to tend to infinity at appropriate rates. These expressions say that the noise-to-signal ratios determine how well $P(e^{i\omega})$ and $H(e^{i\omega})$ can be estimated for each frequency. Note that a novel high order expression for the estimated transfer function $\hat{P}$ is derived in [HN98].

4 The indirect approach

As mentioned, one motivation for performing closed-loop identification is when the models are used for the purpose of designing new controllers with higher performance. In this section we focus on the indirect method because we shall compare the statistical properties of the estimates of T_{yr} and P. We first consider the identification of the closed-loop transfer functions using measurements of the reference signal $r(t)$ and the output $y(t)$. Recall that the first step is an open-loop identification experiment: the equations (6)-(12) are valid but with a model structure (6) that takes the form:

$$y(t, \theta) = \hat{T}_{yr}(q, \theta) r(t) + \hat{T}_{ye}(q, \theta) e(t). \quad (14)$$

The parameter estimates are obtained by the minimization of the least-squares criterion function (8) with a set of data $Z^N = \{y(t), r(t)\}$ collected on the closed-loop system. Here $\epsilon(t, \theta)$ are the prediction errors, which for the model (14) can be expressed as

$$\epsilon(t, \theta) = L(q, \theta)\{y(t) - \hat{T}_{yr}(q, \theta) r(t)\}. \quad (15)$$

Let θ_N be the estimated parameter vector that defines the following estimated models of the closed-loop transfer functions $\hat{T}_{yr} = \hat{T}_{yr}(\theta_N)$ and $\hat{T}_{ye} = \hat{T}_{ye}(\theta_N)$.

In all indirect methods, a parameterization of $\hat{T}_{yr}$ and $\hat{T}_{ye}$ is needed in order to relate the closed-loop to the open-loop parameters. In principle, we have to invert the following algebraic relations

$$\hat{T}_{yr}(q) = \frac{\hat{P}(q)}{1 + \hat{P}(q) K_y(q)}$$
$$\hat{T}_{ye}(q) = \frac{\hat{H}(q)}{1 + \hat{P}(q) K_y(q)}, \quad (16)$$

to recover $\hat{P}$ and $\hat{H}$ from $\hat{T}_{yr}$ and $\hat{T}_{ye}$. The equations can be solved in many ways, using the knowledge of the controller. In order to compare the bias and variance distributions of the closed-loop and open-loop estimates, it is natural to consider the following (Taylor-made) parameterizations

$$\hat{T}_{yr}(q, \theta) = \frac{\hat{P}(q, \theta)}{1 + \hat{P}(q, \theta) K_y}$$
$$\hat{T}_{ye}(q, \theta) = \frac{\hat{H}(q, \theta)}{1 + \hat{P}(q, \theta) K_y}, \quad (17)$$

since the prediction error method allows arbitrary parameterizations. It is important to realize that *as long as the parameterizations describe the same set of P (and H), the resulting transfer function $\hat{P}$ (and $\hat{H}$) will be the same, regardless of the parameterization* [Lju97]. The message is that a parameterization as in (17) is inherent in all indirect methods, but the way it is computed does not affect the statistical properties of the asymptotically estimated transfer functions.

It then follows easily that the one step-ahead prediction errors are given by

$$\epsilon(t, \theta) = L(\theta)\{\frac{P - \hat{P}(\theta)}{1 + \hat{P}(\theta) K_y} S_P r(t) + S_P H e(t)\}. \quad (18)$$

Here the prediction errors are function of the sensitivity function S_P. The impact of the sensitivity function is illustrated in two following Sections.

4.1 Bias expressions

Consider the prediction errors in (15) and denote $v_{cl}(t) = T_{ye}(q) e(t) = S_P(q) v(t)$. It follows that the asymptotic expression for the cost criterion (11) becomes:

$$J_{id} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\omega \mid L(\theta) \mid^2 \{\mid T_{yr} - \hat{T}_{yr}(\theta) \mid^2 \phi_r + \phi_{v_{cl}}\} \quad (19)$$

where $\phi_r, \phi_{v_{cl}}$ are the spectra of $r(t)$ and $v_{cl}(t)$, respectively. If the goal of the identification is to find the open-loop models, we can also consider the expression (18) for the prediction errors and formulate the expression of the cost criterion (11) as

$$J_{id} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\omega \mid L(\theta) \mid^2 \{\mid \frac{P - \hat{P}(\theta)}{1 + \hat{P}(\theta) K_y} \mid^2 \mid S_P \mid^2 \phi_r + \phi_{v_{cl}}\}. \quad (20)$$

Since (19) is an open-loop expression, the filter $L(\theta)$ can be the inverse of the parameterized closed-loop noise model $\hat{T}_{ye}(q, \theta)$, some fixed filter chosen by the user in order to shape the bias frequency distribution or a combination of these. Thus, the bias expression for $\hat{T}_{yr}$ can explicitly be tuned to the frequency regions which are of interest for the intended application of the model. Moreover, if $L(q,\theta) = \hat{T}_{ye}(q,\theta)^{-1}$, the model $[\hat{T}_{yr}(q,\theta)\ \hat{T}_{ye}(q,\theta)]$ can be identified consistently (the true plant G is in the model set).

The main heuristic motivation for performing closed-loop identification is the presence of the sensitivity function S_P as a filter in (20). The sensitivity function emphasizes the frequency region where S_P is large and one expects to obtain an accurate model fit in those frequency regions: typically around the cross-over frequency where the sensitivity reaches its maximum value and in high frequency where the sensitivity is close to one. Moreover, a poor model fit occurs in low frequency where S_P is low. The filter L must be fixed in order to provide a consistent estimate of P for the situation that P is in the model set because incorporation of a parameterized noise model in L will result in a bias distribution that becomes dependent on the identified noise model as well as on the spectrum ϕ_v. Moreover, in the presence of bias there exists no simple choice of a fixed filter L that can flexibly tune the expression (20).

Thus in identification for control and in terms of bias considerations, the identification of models of the closed-loop transfer functions must be preferred to reconstructed models of the true open-loop transfer functions for its flexibility and if both input-output and noise transfer functions have to be identified for the design of a controller.

4.2 Variance expressions

In this section, we present the asymptotic variance of the resulting open-loop transfer function estimates when they are obtained as function of the closed-loop estimates, for the situation where the true system is in the model set, and both plant model and noise model are estimated. Since all closed-loop identification approaches give the same asymptotic variance in the case of linear feedback, see [GLV97], we present the result for the generic indirect method presented previously.

Variance expressions for the estimated closed-loop transfer functions $\hat{T}_{yr}$ and $\hat{T}_{ye}$ are available through the standard expressions for open-loop identification [Lju87]:

$$\begin{aligned} cov\, \hat{T}_{yr}(e^{i\omega}) &\simeq \frac{n}{N}\frac{\phi_{v_{cl}}}{\phi_r} = \frac{n}{N} \mid S_P(e^{i\omega}) \mid^2 \frac{\phi_v}{\phi_r} \\ cov\, \hat{T}_{ye}(e^{i\omega}) &\simeq \frac{n}{N}\frac{\phi_{v_{cl}}}{\sigma^2} = \frac{n}{N} \mid S_P(e^{i\omega}) \mid^2 \frac{\phi_v}{\sigma^2}, \end{aligned} \qquad (21)$$

where n is the size of the vector θ, and n, N are supposed to tend to infinity at appropriate rates. The feedback introduces the sensitivity function S_P. Thus for a given noise to signal ratio $\frac{\phi_v}{\phi_r}$, the sensitivity function will contribute to variance reduction at frequencies where $\mid S_P \mid< 1$, i.e. in low frequency. Thus one typically expects to obtain an accurate model fit of the closed-loop transfer functions in low frequencies than in high frequencies.

For obtaining variance expressions of the reconstructed estimates $\hat{P}$ and $\hat{H}$, use can be made of first order approximations, leading to

$$\begin{aligned} \hat{P} &= \frac{\hat{T}_{yr}}{1-\hat{T}_{yr}K_y} \simeq \frac{T_{yr}}{1-T_{yr}K_y} + \frac{\hat{T}_{yr}-T_{yr}}{(1-T_{yr}K_y)^2} \\ \hat{H} &= \frac{\hat{T}_{ye}}{1-\hat{T}_{yr}K_y} \simeq \frac{T_{ye}}{1-T_{yr}K_y} + \frac{\hat{T}_{ye}-T_{ye}}{(1-T_{yr}K_y)^2}. \end{aligned} \qquad (22)$$

According to the definitions of T_{yr} and T_{ye} in (3), we obtain the following result:

$$\begin{aligned} cov\, \hat{P}(e^{i\omega}) &\simeq \frac{1}{\mid S_P(e^{i\omega}) \mid^4} cov\, \hat{T}_{yr}(e^{i\omega}) \\ cov\, \hat{H}(e^{i\omega}) &\simeq \frac{1}{\mid S_P(e^{i\omega}) \mid^4} cov\, \hat{T}_{ye}(e^{i\omega}). \end{aligned} \qquad (23)$$

If we combine equations (21) and (23), we obtain the well known result

$$\begin{aligned} cov\, \hat{P}(e^{i\omega}) &\simeq \frac{n}{N}\frac{1}{\mid S_P(e^{i\omega}) \mid^2}\frac{\phi_v}{\phi_r} \\ cov\, \hat{H}(e^{i\omega}) &\simeq \frac{n}{N}\frac{1}{\mid S_P(e^{i\omega}) \mid^2}\frac{\phi_v}{\sigma^2}. \end{aligned} \qquad (24)$$

In closed-loop identification the role of the sensitivity function (and the feedback) is essential. Compare the two couples of equations (21) and (24): the sensitivity function contributes to variance reduction for the estimates $\hat{T}_{yr}$ and $\hat{T}_{ye}$ where the sensitivity function $\mid S_P \mid< 1$, and has the opposite effect on the reconstructed estimates $\hat{P}$ and $\hat{H}$: it contributes to variance increase at these frequencies.

- The sensitivity function contributes to variance and bias reductions for the reconstructed estimates $\hat{P}$ and $\hat{H}$ in the same frequency regions: see (20) and (24), the errors are minimal around the cross-over frequency and in high frequency.
- Most of the control designs have a sensitivity reduction objective. If the nominal closed-loop system constructed based on the nominal model is far from giving optimal performance, one can choose either reconstructed estimates of the open-loop plant or estimates of the closed-loop transfer functions to improve the controller. In that case however, the sensitivity function only affects the reconstructed estimates in a narrow low-frequency band that is not essential for the design of the controller. On the other hand, if the controller is close to its optimum, the sensitivity function is maximum around the cross-over frequency and may greatly affect the estimates $\hat{T}_{yr}$ and $\hat{T}_{ye}$.

- For a non-minimum-phase plant the sensitivity function is constrained by the unstable zeroes: as S_P is pushed down to one frequency range, it pops up somewhere else (the water-bed effect). The role of the sensitivity function is therefore emphasized.

- In terms of the errors on $P - \hat{P}$, the sensitivity function acts in the same way on bias and variance errors.

Thus, in identification for control and in terms of variance considerations, it is preferable to use the reconstructed models when the ultimate objective is to design a new controller, especially if the controller is close to its optimum.

5 An example

We have developed a simulation that illustrates the impact of the sensitivity function on estimates $\hat{T}_{yr}$ and $\hat{P}$.

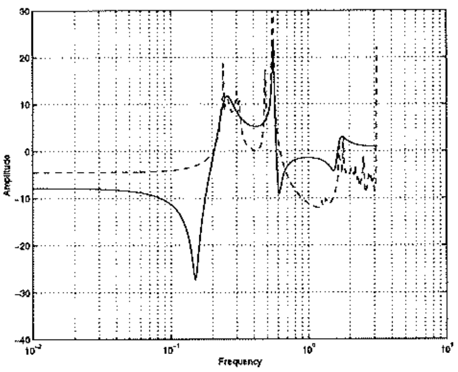

Figure 2: Bode diagrams of the true sensitivity function S_p (solid), and the experimental expectation of the covariance distribution of $\hat{T}_{yr}$ (dotted) obtained by averaging the 100 Monte Carlo runs.

Assume that the true system G is described by the following OE model:

$$y(t) = \frac{B(q)}{A(q)}u(t) + e(t), \tag{25}$$

of order 6, $e(t)$ is white noise with variance $\sigma^2 = 1$. The poles are at $q = -0.0030 \pm j0.9119$, $0.8082 \pm j0.5605$ and $0.9837 \pm j0.1491$, the zeros at $q = 1.1865 \pm j0.7993$, $0.4624 \pm j0.2838$ and 0.4307, and the DC gain is 0.4897. This is a stable and non-minimum phase plant with single delay. The sampling period used in the simulation is $T_s = 1$. The excitation signal $r(t)$ used in the identification experiment is taken to be white noise with power spectrum $\phi_r = 10$, and 1000 samples are computed. The stabilizing controller is a proportional controller $u(t) = k(r(t) - y(t))$ with $k = 3$, close to the maximum gain that guarantees closed-loop stability. Since the plant is non minimum phase, and the controller is close to its maximum value, we expect to obtain large variations in the sensitivity function S_P: indeed, see in Figure 2 the amplitude Bode plot of the sensitivity function. The advantage of using a proportional controller is that the order of the closed-loop T_{yr} and open-loop P transfer functions are the same: the reconstruction (16) is therefore easy and the consistency of the reconstructed model is guaranteed.

The first step of the example is to perform an 'open-loop' identification experiment that estimates T_{yr} from the data r and y, with a full order OE model structure, of order 6 in order to analyze the effect of the sole variance errors. The experiment was run 100 times, in order to get conclusions that would not depend on a particular noise realization. In Figure 2, we have compared the Bode plots of the true sensitivity function S_p and the experimental expectation of the covariance distribution of the model $\hat{T}_{yr}$ obtained by averaging 100 Monte Carlo runs. Observe the two curves in 2: the covariance distribution curve follows the variations of the sensitivity function curve, especially above 0.2 rad/s. In Figure 3 we have compared the Bode plots of the true closed-loop transfer function T_{yr} and the experimental expected model $\hat{T}_{yr}$ obtained by averaging 100 Monte Carlo runs. The two curves in Figure 3 are close to one another where the sensitivity function is low. This clearly shows the impact of the sensitivity functions as expected in theory.

In the second step of the design, we perform the reconstructions of the expected open-loop model $\hat{P}$ as well as the experimental expectation of the covariance distribution of $\hat{P}$ via (15). The two reconstructions are again obtained by averaging the 100 Monte Carlo runs. Ob-

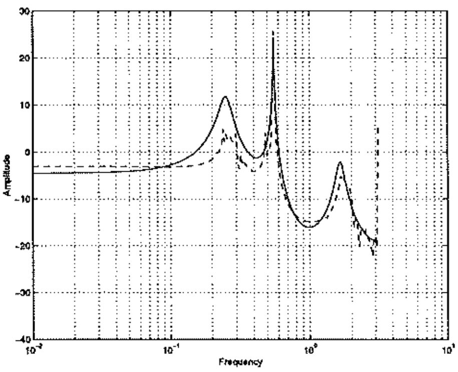

Figure 3: Bode diagrams of the true input-output closed-loop dynamics T_{yr} (solid) and the experimental expectation of the estimated transfer function $\hat{T}_{yr}$ (dotted).

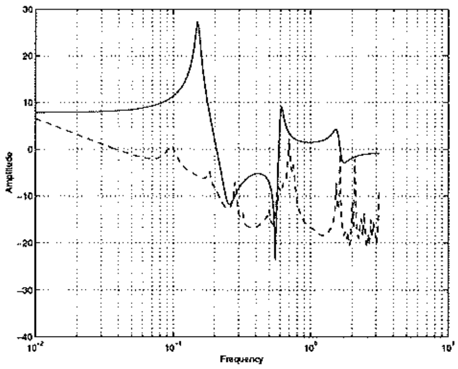

Figure 4: Bode diagrams of the inverse of the true sensitivity function S_p (solid), and the experimental expectation of the covariance distribution of the reconstructed model $\hat{P}$ (dotted) obtained by averaging the 100 Monte Carlo runs.

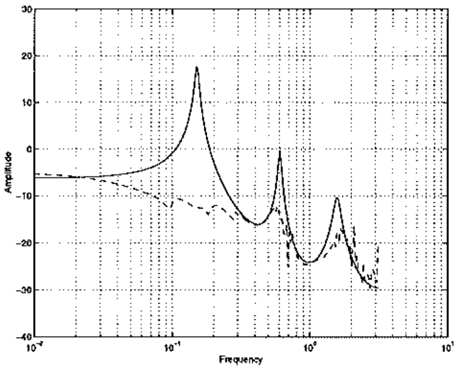

Figure 5: Bode diagrams of the true input-output closed-loop dynamics P (solid) and the experimental expectations of the reconstructed transfer function $\hat{P}$ (dotted).

serve the Figure 4: the experimental expectation of the covariance distribution of the reconstructed estimate $\hat{P}$ is large (resp. low) where the inverse of the sensitivity function is large (resp. low). In Figure 5, the Amplitude Bode plot of the expected open-loop model $\hat{P}$ is displayed. The model $\hat{P}$ can not capture the three resonances of the true plant because the inverse of the sensitivity function reaches its minimum value at these three frequencies. This clearly shows that since the role of the feedback is to reject some frequencies, the reconstructed open-loop model can not capture these frequencies.

6 Conclusions

In identification for control, the ultimate objective is to use the estimates for control design. In this paper we have compared control designs based on open-loop transfer function estimates versus closed-loop estimates. In terms of bias considerations, we have shown that models of the closed-loop transfer functions must be preferred to reconstructed open-loop models when both estimates of P and H are needed for the design of a new controller. In terms of variance considerations, we have shown the role of the sensitivity function in the variance distribution, whose impact on open-loop and closed-loop estimates is entirely different. We have presented a simulation that reveals the importance of the sensitivity function in the variance distribution for both estimates of T_{yr} and P.

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