MODEL VALIDATION FOR ROBUST CONTROL AND CONTROLLER VALIDATION IN A PREDICTION ERROR FRAMEWORK

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Abstract:
This paper presents a coherent framework for model validation for control and for controller validation (for stability and for performance) in the context where the validated model uncertainty sets are obtained by prediction error identification methods. Thus, these uncertainty sets are parametrized transfer function sets, with parameters lying in ellipsoidal regions in parameter space. Our results cover two distinct aspects: (1) *Control-oriented model validation results,* where we show that a measure of size of the validated model set is connected to the size of the model-based controller set that robustly stabilizes the model set, leading to validation design guidelines. (2) *Controller validation results,* where we present necessary and sufficient conditions for a controller to stabilize all models, or to achieve a given level of performance for all models, in such uncertainty set.

Keywords: identification, validation, robust stability, robust performance

1. INTRODUCTION

This paper presents in summary form the key ideas and results that we have developed over the last few years on *model validation for control* and *controller validation*, in the prediction error identification framework. The full presentation of these results, as well as the proofs, can be found in the comprehensive paper (Gevers *et al.*, 2000), and in the more technical supporting papers (Gevers *et al.*, 1999*a*), (Bombois *et al.*, 1999*b*), (Bombois *et al.*, 1999*c*), (Gevers *et al.*, 1999*b*), (Bombois *et al.*, 1999*a*), (Bombois *et al.*, 2000*b*), (Bombois *et al.*, 2000*a*). Our results on validation establish a robust stability and robust performance theory for model uncertainty sets produced by prediction error identification and validation, which we call PE uncertainty sets for ease of reference. In the case of full order models, these validated PE model uncertainty sets are defined by parametrized transfer function sets whose parameter vectors lie in ellipsoids in parameter space. In the case of restricted complexity models, they are obtained by a stochastic embedding technique (see (Goodwin *et al.*, 1992)) and are made up of ellipsoids at every frequency in the space of transfer functions (Bombois *et al.*, 2000*a*). These

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PE uncertainty sets are not the classical uncertainty sets used in robust control analysis and design, such as additive, multiplicative, coprime factor uncertainty sets, etc. Our robust stability and robust performance results take the form of necessary and sufficient conditions for a controller to stabilize all models in such PE sets, or to achieve some given level of performance with all models in such PE sets, without the need to first approximate or overbound these sets by one of the more standard uncertainty sets of robust control theory.

In addition to these *controller validation* results, we also present results on *model validation for control* that impact on validation design. To put it simply, we define a 'control-oriented measure of size' for the validated PE uncertainty sets mentioned above, that is directly connected, via a robust stability result, to the size of the sets of model-based controllers that stabilize all models in the validated PE model sets. This measure of size is called the worst case ν-gap between the validated model and the validated model set. We show how this worst case gap is affected by the design of the validation experiment that delivers the validated PE set. Thus, we now have a complete connection all the way from the validation experiment that defines a PE uncertainty set to the set of controllers that are guaranteed to stabilize all models in this PE uncertainty set. This allows one to formulate 'validation for control design guidelines'.

Because of the space limitations imposed by the format of a Conference paper, we shall not attempt here to describe the state of the art in model validation, model validation for control, and controller validation.[1] We shall also not review the abundant literature that already exists on these very broad and important questions. In the full paper (Gevers *et al.*, 2000) we have attempted to perform this review of the literature and we have also put our results in perspective with respect to the existing literature.

This paper is organized as follows. In Section 2 we introduce the prediction error framework for validation, and we present our results on model set validation for robust control, while Section 3 covers controller validation in this prediction error framework, both for stability and for performance. For reasons of space, our results are presented here without proof; we refer the reader to the appropriate references for the proofs and for further details.

[1] We just want to repeat, as we have done in our previous work, that these are three different issues, each requiring a different formulation and hence different solutions.

2. MODEL SET VALIDATION FOR CONTROL

We consider that input-output data y and u are generated from a Linear Time Invariant "*true system*":

$$\mathcal{S} : y(t) = G_0(z)u(t) + v(t), \qquad (1)$$

where $G_0(z)$ is a linear time-invariant causal operator, u is a measured input signal, and v is zero mean stationary noise.

We consider the situation where one is allowed to perform experiments on the true system in order to contruct a model set to which the true system is guaranteed to belong, at some a priori fixed probability level α, say 95%. The experiment delivers a data record of N input-output data: $Z^N = \{y(1), u(1), \ldots, y(N), u(N)\}$. The construction of the model set is achieved, in our framework, by a Prediction Error (PE) identification experiment. For the sake of simplicity and brevity, we shall consider here that this PE identification is performed using a model set of the same structure as that of the true system, leading to an unbiased estimate $G(z, \hat{\delta})$ of $G_0(z)$. This may appear like a severe restriction. However, as shown in (Gevers *et al.*, 2000), the results that follow apply without any modification to the case where a low order model is used, and where the set validation is performed by a prediction error identification step applied to an unbiased model of the Model Error Model (see e.g. (Ljung, 1997)). One can even entirely remove the need for the identification of an unbiased model (of the true system or of the Model Error Model) by the use of stochastic embedding techniques à la Goodwin et al. (Goodwin *et al.*, 1992), as we have recently shown in (Bombois *et al.*, 2000*a*). In such case, the parametrization of the validated PE uncertainty set is somewhat different from the one we present below, but the methodology presented here remains unchanged and all the results and conclusions of this paper apply.

Thus, using N input-output data, collected in open or in closed loop, and a model set $\mathcal{M}$ containing $\mathcal{S}$, we estimate a model $G(z, \hat{\delta})$ and a covariance matrix P_δ of $\hat{\delta}$ using classical PE identification (Ljung, 1999). We have shown in (Bombois *et al.*, 1999*d*) and (Bombois *et al.*, 2000*b*) that, whether the validation is performed in open loop or in closed loop, by identification of a full order model for G_0 or by identification of an unbiased estimate of a Model Error Model, the validated model sets can all be described in the following generic structure that we call the **generic PE model uncertainty set**.

Proposition 2.1. The model sets resulting from prediction error validation, which contain the true system $G_0 = G(z, \delta_0)$ at a prescribed probability level, can all be described in the following generic form called the generic PE model uncertainty set:

$$\mathcal{D} \;=\; \left\{ G(z,\delta) \mid G(z,\delta) = \frac{e + Z_N\delta}{1+Z_D\delta} \right.$$
$$\left. \text{and } \delta \in U = \{\delta \mid (\delta - \hat{\delta})^T R(\delta-\hat{\delta}) < 1\} \right\} \quad (2)$$

where

- $\delta \in \mathbf{R}^{k\times 1}$ is a real parameter vector, and $\hat{\delta}$ is the parameter estimate resulting from the identification/validation step.
- $R \in \mathbf{R}^{k\times k}$ is a symmetric positive definite matrix, proportional to the inverse of the covariance matrix P_δ of $\hat{\delta}$.
- $Z_N(z)$ and $Z_D(z)$ are row vectors of size k of known transfer functions.
- $e(z)$ is a known transfer function with a delay equal to the delay of G_0.

In the special case where the validation is performed in open loop, or in closed loop using a direct identification method, the PE uncertainty set $\mathcal{D}$ reduces to the more familiar set:

$$\{G(z,\delta) \mid G(z,\delta) = \frac{b_1 z^{-1} + \ldots + b_m z^{-m}}{1 + a_1 z^{-1} + \ldots + a_n z^{-n}}$$
$$= \frac{Z_2\delta}{1+Z_1\delta} \; \textit{and } \delta \in U \Big\} \quad (3)$$

with

- $\delta^T = [a_1 \; \ldots \; a_n \; b_1 \; \ldots \; b_m] \in \mathbf{R}^{k\times 1}, \; k \triangleq n+m$
- $Z_1(z) = [z^{-1} \; z^{-2} \; \ldots \; z^{-n} \; 0 \; \ldots \; 0] \in \mathbf{C}^{1\times k}$
- $Z_2(z) = [0 \; \ldots \; 0 \; z^{-1} \; z^{-2} \; \ldots \; z^{-m}] \in \mathbf{C}^{1\times k}$

Comments

- Observe that the PE model uncertainty sets are very different from the classical uncertainty sets that are used in mainstream robust control design procedures, such as additive, multiplicative, feedback, coprime factor uncertainty sets, etc.
- The validated model set $\mathcal{D}$ depends very much on the experimental conditions under which the validation has been performed. This is perhaps not so apparent in the definition (2) of $\mathcal{D}$ via the parameter covariance matrix P_δ which defines R. However, let us recall that the covariance of the transfer function estimate $G(z, \hat{\delta})$ is a function of the input spectrum and, in a closed loop experiment, of the controller and the reference spectrum; see e.g. (Ljung, 1999). Thus, two different validation data sets will yield two different validated regions $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$, both of which contain the true G_0 with probability α.

In the study of *model validation for control* it is therefore important to examine whether one PE validated region, $\mathcal{D}^{(1)}$, is better tuned for robust control design than another one, $\mathcal{D}^{(2)}$.

We shall from now on assume that the control design is based on some nominal model G_{mod}, and that this nominal model is any validated model, i.e. any model in $\mathcal{D}$. An obvious option would be to use the center of the validated set, i.e. $G(z, \hat{\delta})$, as the model used for control design. However, one may often prefer to use a low order model for control design, since this results in a lower order controller. For the theory that follows, any validated model can be used for control design. [2]

In order to address the question of whether a PE uncertainty set is tuned for robust control design, we have defined a 'measure of size' of a validated PE set $\mathcal{D}$ that can be connected, via a robust stability theorem, to the size of the model-based controller set that robustly stabilizes all models in $\mathcal{D}$. This measure is called the *worst-case ν-gap* (and its frequency by frequency version, the worst-case chordal distance) between some model G_{mod} and all models in a validated PE model set $\mathcal{D}$. In order to define these worst-case measures, we first recall the definitions of chordal distance and of ν-gap between two transfer functions, introduced by Vinnicombe (Vinnicombe, 1993). To keep things simple we consider scalar transfer functions only.

Definition 2.1. (Vinnicombe, 1993) The ν-gap metric between two transfer functions G_1 and G_2 is defined as

$$\delta_\nu(G_1, G_2) = \max_\omega \kappa\left(G_1(e^{j\omega}), G_2(e^{j\omega})\right) \quad (4)$$

if $W(G_1, G_2) = 0$, and 1 otherwise, where $\kappa\left(G_1(e^{j\omega}), G_2(e^{j\omega})\right)$

$$\triangleq \frac{|G_1(e^{j\omega}) - G_2(e^{j\omega})|}{\sqrt{1+|G_1(e^{j\omega})|^2}\sqrt{1+|G_2(e^{j\omega})|^2}} \quad (5)$$

and where $W(G_1, G_2) = wno(1+G_1^* G_2) + \eta(G_2) - \tilde{\eta}(G_1)$.

Here $\eta(G)$ (resp. $\tilde{\eta}(G)$) denotes the number of poles of G in the complement of the closed (resp. open) unit disc, while $wno(G)$ denotes the winding number about the origin of $G(z)$ as z follows the unit circle indented into the exterior of the unit disc around any unit circle pole and zero of $G(z)$. The function $\kappa(G_1(e^{j\omega}), G_2(e^{j\omega}))$ is the chordal distance between the projections of $G_1(e^{j\omega})$ and $G_2(e^{j\omega})$ onto the Riemann sphere of unit diameter with South Pole at the origin of the complex plane.

[2] In fact, our theory applies even to the case when the model used for control design lies outside the PE validated region $\mathcal{D}$.

Consider now a closed loop system with transfer function matrix

$$T(G,C) = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} \dfrac{GC}{1+GC} & \dfrac{G}{1+GC} \\ \dfrac{C}{1+GC} & \dfrac{1}{1+GC} \end{pmatrix} \quad (6)$$

Definition 2.2. (Vinnicombe, 1993) The **generalized stability margin** of this closed loop system is defined as

$$b_{GC} = \|T(G,C)\|_\infty^{-1} = \min_\omega \kappa(G(e^{j\omega}), -\frac{1}{C(e^{j\omega})}) \quad (7)$$

if (G,C) is stable, and 0 otherwise.

Thus, the generalized stability margin of a closed loop system $[G\ C]$ is measured by the least chordal distance between the projections on the Riemann sphere of G and of the inverse of $-C$.

The main interest of the ν-gap metric and the chordal distance is their use in robust stability. Here we recall a result that is most useful for our 'validation for control' analysis.

Proposition 2.2. (Vinnicombe, 1993) Let C stabilize a model G_{mod}. Then C stabilizes all G such that $\delta_\nu(G_{mod}, G) < 1$ and such that $\forall\omega$

$$\kappa\left(G_{mod}(e^{j\omega}), G(e^{j\omega})\right) < \kappa(G_{mod}(e^{j\omega}), -\frac{1}{C(e^{j\omega})}).$$

In particular, C stabilizes all models in the set :

$$\mathcal{G} = \{G : \delta_\nu(G, G_{mod}) < b_{G_{mod}C}\} \quad (8)$$

We now build on these robust stability results to connect validated PE uncertainty sets to sets of robustly stabilizing controllers. To do this, we have introduced two new notions that extend definitions of distance between two models to definitions of **worst case distance** between a model and all models in a PE model set.

Definition 2.3. Consider a PE uncertainty set $\mathcal{D}$ of the form (2) and a model G_{mod}. The 'worst case chordal distance' at frequency ω between G_{mod} and $\mathcal{D}$ is defined as

$$\kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}) = \sup_{G_\mathcal{D} \in \mathcal{D}} \kappa(G_{mod}(e^{j\omega}), G_\mathcal{D}(e^{j\omega}))$$

The 'worst case ν-gap' between G_{mod} and $\mathcal{D}$ is defined as

$$\delta_{WC}(G_{mod}, \mathcal{D}) = \sup_{G_\mathcal{D} \in \mathcal{D}.} \delta_\nu(G_{mod}, G_\mathcal{D}) \quad (9)$$

An alternative characterization of the worst case ν-gap is as follows: see (Bombois *et al.*, 1999*c*).

Lemma 2.1. If $W(G_{mod}, G_\mathcal{D}) = 0$ for one plant $G_\mathcal{D} \in \mathcal{D}$, then $\delta_{WC}(G_{mod}, \mathcal{D})$ can also be expressed as

$$\delta_{WC}(G_{mod}, \mathcal{D}) = \sup_\omega \kappa_{WC}\ (G_{mod}(e^{j\omega}), \mathcal{D})).\square$$

We have shown in (Bombois *et al.*, 1999*a*) that the worst-case chordal distance can be computed as the optimal value of a convex optimization problem involving Linear Matrix Inequality (LMI) constraints.

Having extended the concepts of chordal distance and of ν-gap between plants to those of worst-case chordal distance and worst-case ν-gap between a model and a validated PE set, we can now also extend the stability results of Proposition 2.2 to the context of our validated PE sets.

Theorem 2.1. Consider a PE uncertainty set $\mathcal{D}$ containing the true G_0 with probability α, a model G_{mod} with $\delta_{WC}(G_{mod}, \mathcal{D}) < 1$, and let C be a controller that stabilizes G_{mod}. Then C stabilizes all models in $\mathcal{D}$ if $\forall\omega$

$$\kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}) < \kappa(G_{mod}(e^{j\omega}), -\frac{1}{C(e^{j\omega})})$$

and, a fortiori, if $\delta_{WC}(G_{mod}, \mathcal{D}) < b_{G_{mod}C}$. $\square$

Observe that the left hand side depends on the validated set, while the right hand side depends on the controller; both quantities are known. Both of these stability conditions are sufficient conditions only, and that the latter is a conservative (Min-Max) version of the former. Necessary and sufficient conditions for the stabilization of all models in a validated PE model set by a controller C will be given in Section 3. The main use of Theorem 2.1 is therefore not so much for checking the stability of a particular controller, but rather it allows us to link the validation experiment (through the measure of size $\kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D})$ of the validated set) with the set of robustly stabilizing controllers for $\mathcal{D}$. Indeed, for each validated PE uncertainty set $\mathcal{D}$ we can now define, by the use of Theorem 2.1, the corresponding set of G_{mod}-based controllers that robustly stabilize all models in the set $\mathcal{D}$ as[3] :

$$\mathcal{C}(G_{mod}, \mathcal{D}) = \{C \mid \kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}) \quad (10)$$
$$< \kappa\left(G_{mod}(e^{j\omega}), -\frac{1}{C(e^{j\omega})}\right) \ \forall\omega\}$$

We then have the following result (Gevers *et al.*, 2000).

Theorem 2.2. Consider two different validated PE sets $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$, obtained from two different validation experiments, both containing the model G_{mod}. If for all ω

$$\kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}^{(1)}) < \kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D}^{(2)}),$$

then $\mathcal{C}(G_{mod}, \mathcal{D}^{(2)}) \subset \mathcal{C}(G_{mod}, \mathcal{D}^{(1)})$. $\square$

[3] Observe that there may be other controllers, outside the set $\mathcal{C}(G_{mod}, \mathcal{D})$, that stabilize all models in $\mathcal{D}$, because the stability condition used to define this set is only a sufficient condition.

Theorem 2.2 establishes a clear link between a measure of size defined on validated PE model sets and the sets of stabilizing controllers for all models in these validated model sets. A more compact measure of size is $\delta_{WC}(G_{mod}, \mathcal{D})$ (i.e. just one number to characterize the size of $\mathcal{D}$) for which a corresponding but more conservative link can be established with a set of stabilizing controllers: see (Bombois *et al.*, 1999*c*), (Gevers *et al.*, 2000). Thus, $\kappa_{WC}(G_{mod}(e^{j\omega}), \mathcal{D})$ and $\delta_{WC}(G_{mod}, \mathcal{D})$ are two measures of the ability of a validated PE model set to be robustly stabilized by a large set of controllers. It makes sense therefore, in identification for robust control, to choose those experimental conditions for validation that lead to a validated set with the smallest possible worst-case chordal distance, or the smallest possible worst-case ν-gap. Thus, our results give the designer a handle on what to aim for in terms of "control-oriented validated sets". In a Prediction Error context, we believe that these results are the first that give substance to the concept of *control-oriented model validation.*

3. CONTROLLER VALIDATION FOR STABILITY AND PERFORMANCE

In this section, we present *necessary and sufficient conditions* for some given controller $C(z)$ to stabilize all models in a PE uncertainty set, as well as necessary and sufficient conditions for this controller to achieve a specified level of performance with all models in such PE uncertainty set. In other words, we develop a robust stability analysis (for both stability and performance) for uncertainty sets of the generic PE form described in (2).

Robust stability

For our PE uncertainty sets, necessary and sufficient conditions for robust stability have been obtained by showing that the set of feedback connections of C with all models in $\mathcal{D}$ can be reformulated, using an LFT framework, as a set of feedback connections $[M_{\mathcal{D}}(z)\ \ \phi]$, where $M_{\mathcal{D}}(z)$ is fixed and where the uncertainty part ϕ is a real vector, linearly related to the parameter vector δ that defines $\mathcal{D}$: see (2). We have then shown that the (real) stability radius linked with the set of loops $[M_{\mathcal{D}}(z)\ \ \phi]$ can be computed exactly. The main robust stability result is as follows (Bombois *et al.*, 2000*b*).

Theorem 3.1. Consider a generic PE uncertainty set of the form (2) and a controller $C(z) = X(z)/Y(z)$ that stabilizes $G(\hat{\delta})$. Then all models in $\mathcal{D}$ are stabilized by $C(z)$ if and only if $\max_{\Omega} \mu_{\phi}(M_{\mathcal{D}}(e^{j\omega})) \leq 1$, where

- $M_{\mathcal{D}}(z) = \dfrac{-(Z_D + \frac{X(Z_N - eZ_D)}{Y+eX})T^{-1}}{1+(Z_D + \frac{X(Z_N - eZ_D)}{Y+eX})\hat{\delta}}$
- T is defined as $\mathcal{D} : R = T^T T$.
- $\phi = T(\delta - \hat{\delta})$, whereby $\delta \in U \Leftrightarrow |\phi|_2 < 1$
- $\mu_{\phi}(M_{\mathcal{D}}(e^{j\omega}))$ is the *stability radius* of the loop $[M_{\mathcal{D}}(z)\ \ \phi]$. For a real vector ϕ, μ_{ϕ} is computed as:

$$\sqrt{|Re(M)|_2^2 - \frac{(Re(M)Im(M)^T)^2}{|Im(M)|_2^2}} \quad \text{if } Im(M) \neq 0$$
$$|M|_2 \quad \text{if } Im(M) = 0.$$

The necessary and sufficient conditions for controller validation depend critically on the uncertainty set $\mathcal{D}$, hence the importance of the validation design. Of course, this observation applies to all robust control methodologies. What distinguishes our approach from most others is that, in mainstream robust control theory, the uncertainty sets are assumed a priori, while here they are the result of experiments, and our analysis of the previous sections has given us at least some handle on how we can shape these uncertainty sets towards robust control design.

Robust performance

Most commonly used performance criteria are derived from some norm of a frequency weighted version of the transfer matrix $T(G, C)$ of the closed-loop system $[G\ C]$ defined in (6). Thus we shall start from the following definition.

Definition 3.1. The performance of a closed loop system (G, C) is defined by the following frequency function:

$$J(G, C, W_l, W_r, \omega) = \sigma_{\max}\left(W_l T(G(e^{j\omega}), C(e^{j\omega}))W_r\right), \quad (11)$$

where $W_l(z)$ and $W_r(z)$ are diagonal frequency weights that allow one to define specific performance levels, and where $\sigma_{\max}(A)$ denotes the largest singular value of A. □

The frequency function J defines a template. Most commonly used performance functions are functions of J, or special cases of J. The worst case performance over a validated PE set is then defined as follows.

Definition 3.2. The worst case performance achieved by a controller $C(z)$ at a frequency ω over all models in a validated PE uncertainty set $\mathcal{D}$ is defined as

$$J_{WC}(\mathcal{D}, C, W_l, W_r, \omega) \quad (12)$$
$$= \max_{G(z,\delta)\in\mathcal{D}} \sigma_{\max}\left(W_l T(G(e^{j\omega}, \delta), C(e^{j\omega}))W_r\right).$$

The following theorem, established in (Bombois *et al.*, 2000*b*), gives a procedure for the computation of the criterion $J_{WC}(\mathcal{D}, C, W_l, W_r, \omega)$ at the frequency ω. A crucial feature that makes this computation possible is the rank one property of the matrix $T(G, C)$.

Theorem 3.2. Consider a PE uncertainty region $\mathcal{D}$ defined by (2) and a robustly stabilizing controller $C = \frac{X}{Y}$. Then, at frequency ω, the criterion function $J_{WC}(\mathcal{D}, C, W_l, W_r, \omega)$ is obtained as

$$J_{WC}(\mathcal{D}, C, W_l, W_r, \omega) = \sqrt{\gamma_{opt}}, \qquad (13)$$

where γ_{opt} is the optimal value of γ for the following standard convex optimization problem involving LMI constraints:

$$\begin{array}{ll} \textit{minimize} & \gamma \\ \textit{over} & \gamma,\ \tau \\ \textit{subject to} & \tau \geq 0 \quad \textit{and} \end{array}$$

$$\begin{pmatrix} Re(a_{11}) & Re(a_{12}) \\ Re(a_{12}^*) & Re(a_{22}) \end{pmatrix} - \tau \begin{pmatrix} R & -R\hat{\delta} \\ (-R\hat{\delta})^T & \hat{\delta}^T R \hat{\delta} - 1 \end{pmatrix} < 0$$

where

$$\begin{aligned} a_{11} &= (Z_N^* W_{l1}^* W_{l1} Z_N + Z_D^* W_{l2}^* W_{l2} Z_D) - \gamma(Q Z_1^* Z_1); \\ a_{12} &= Z_N^* W_{l1}^* W_{l1} e + W_{l2}^* W_{l2} Z_D^* - \gamma(Q Z_1^*(Y + eX)); \\ a_{22} &= e^* W_{l1}^* W_{l1} e + W_{l2}^* W_{l2} - \gamma(Q(Y + eX)^*(Y + eX)); \\ Q &= 1/(X^* W_{r1}^* W_{r1} X + Y^* W_{r2}^* W_{r2} Y); \\ Z_1 &= X Z_N + Y Z_D. \end{aligned}$$

4. CONCLUSIONS

Using Prediction Error identification methods, we have developed a coherent framework for *model validation for robust control* and for *controller validation.* Our model validation procedure is nothing but an identification experiment with a full order model,[4] which leads to a validated PE model set made up of parametrized transfer functions, whose parameters lie in ellipsoidal regions. The true system belongs to this set with some probability, say α. The sets resulting from such PE validation method are non-standard in mainstream robust control theory. We have therefore developed robust analysis tools, that have led to necessary and sufficient conditions for controller validation with PE uncertainty sets, both for stability and for worst-case performance. This is one major contribution of our work.

Another major focus has been to highlight the design aspects of our model set validation procedure, and how they impact on the set of robustly stabilizing controllers. In order to establish a connection all the way from the design of the validation step to the specification of a set of robustly stabilizing controllers, we have defined a measure of the size of the PE validated sets, and shown that it is connected to the size of the controller sets that are guaranteed to robustly stabilize all models in these PE validated sets. Even though these results are based on sufficient conditions for stability only, they provide us with guidelines for *model validation for robust control.*

In the full paper (Gevers *et al.*, 2000), our results and techniques are illustrated on two real-life applications: a flexible transmission system that has been used as benchmark for control design, and a ferrosilicon production process.

[4] We have recently shown in (Bombois *et al.*, 2000*a*) that biased models can also be handled.

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