A Practical Application of Recent Results in Model and Controller Validation to a Ferrosilicon Production Process

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Abstract

This paper presents the application of our recently developed theory on model validation for control and controller validation in a Prediction Error framework to a realistic industrial case study. The industrial application, taken from [1], concerns the control of the silicon concentration in a ferrosilicon production process. Our case study produces findings about the design of the validation experiment (validation in open or closed loop). It also illustrates the respective merits of the tools developed, respectively, for control-oriented model validation and for the validation of a particular controller.

1 Introduction

Our recent theoretical work on model validation for control and on controller validation [2, 3, 4, 5, 6, 7, 8, 9, 10] has allowed us to connect classical prediction error (PE) identification and robust control. We have developed tools in order to test if a given controller stabilises (and achieves a desired level of performance with) all systems in an uncertainty region obtained via a PE identification experiment. Such an uncertainty set is nonstandard in mainstream robust control and could until now only be dealt with by embedding it in a larger standard set (such as an additive, multiplicative or coprime-factor uncertainty set), which yields more conservative results. An alternative is to use some existing methods that directly deliver standard uncertainty sets. Our tools give necessary and sufficient conditions for the stabilisation of an uncertainty set, delivered by PE identification, by a given controller and for assessing the worst-case performance of this controller over that set [8].

In addition to these results on controller validation, we have also developed a framework for control-oriented system identification and model validation. In [11] it was shown that closed-loop identification generically delivers nominal models for which the corresponding model-based controllers have better performance. More recently, we have studied the role of the experimental conditions in model validation for control [4, 10]. In [12, 10] we proposed a procedure for closed-loop validation that extended the Model Error Model approach proposed by Ljung [13, 14] for open-loop validation and we illustrated how closed-loop validation yields uncertainty sets that are better tuned for control design. The Model Error Model approach for validation used in [13, 14] or [12] requires an approximation to go from covariance information in parameter space to covariance information in transfer function space. We have since shown in [3] that we can avoid this approximation altogether and define PE uncertainty sets exactly on the basis of parameter confidence regions. By defining a measure of size of these uncertainty sets (namely the worst-case ν-gap between the nominal model and all validated models) we can now relate sets of stabilising controllers to sets of validated models (i.e. PE uncertainty sets): see [5, 6].

In [9] we have presented the global framework of our new validation theory based on Prediction Error methods, ranging from model validation for control (an experiment design problem) to controller validation (a verification problem). The purpose of the present paper is to illustrate these results on a realistic industrial process control problem, namely the regulation of a ferrosilicon production process studied by Ingason and Jonsson in [1]. We shall very briefly summarise the main results of [9] in Section 2, and then present the application in Section 3. We refer the reader to [9] for a complete presentation of the theoretical results as well as their discussion.

2 Model and controller validation procedure

We shall consider that input-output data are generated from a Linear Time Invariant, Single-Input-Single-Output (SISO) "*true system*":

$$\mathcal{S}:\ y(t) = G_0(z)\,u(t) + v(t) \qquad (1)$$

where $G_0(z)$ is an unknown strictly proper causal rational transfer function (the argument z is the shift operator; we

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shall often omit it to simplify the notations); $u(t)$ is the control input signal, $y(t)$ is the measured output signal, and $v(t)$ is a process disturbance signal.

We consider the situation where somebody has delivered to us a model $\bar{G}(z)$ for $G_0(z)$. We make no assumption about the computation of this model; in particular, we do not assume that it was obtained by prediction error identification.

2.1 Model validation procedure

Our prediction error model and controller validation procedure consists of building an uncertainty set $\mathcal{D}$ of parametric models, to which $G_0(z)$ is guaranteed to belong at some probability level p, by a step of PE identification using an unbiased model structure. Therefore, N input-output data are collected on the actual system:

$$Z^N = \{y(1), u(1), \ldots, y(N), u(N)\}. \tag{2}$$

These data are used to identify an *unbiased* estimate $G(z, \hat{\theta})$ of the system $G_0(z)$. The identification delivers $\hat{\theta}$ and an estimate P_θ of $\mathrm{cov}(\hat{\theta})$. The confidence ellipsoid of probability p around $\hat{\theta}$ is given by

$$\mathcal{U} = \left\{\theta \mid \left(\theta - \hat{\theta}\right)^T P_\theta^{-1} \left(\theta - \hat{\theta}\right) < \alpha\right\}, \tag{3}$$

where α is defined as $\Pr\left(\chi^2(n) < \alpha\right) = p$, with $\chi^2(n)$ a χ^2-distributed random variable with $n = \dim\theta$ degrees of freedom and p the chosen probability level, typically 0.95 or 0.99. This parametric uncertainty region $\mathcal{U}$ defines the uncertainty region $\mathcal{D}$ in the space of transfer functions:

$$\mathcal{D} = \{G(z, \theta) \,|\, \theta \in \mathcal{U}\}, \tag{4}$$

to which $G_0(z)$ belongs with probability p. The model $\bar{G}$ is called **validated** with the uncertainty region $\mathcal{D}$ if $\bar{G} \in \mathcal{D}$ or, equivalently, if $\exists\, \theta^* \in \mathcal{U} : G(z, \theta^*) = \bar{G}(z)$.

The validation experiment, which is nothing but a PE identification experiment aimed at constructing an uncertainty set, can be performed in open loop or in closed loop. Instead of identifying a full-order estimate for G_0, the validation step can also be performed by identifying a full-order estimate for the model error $\partial G = G_0 - \bar{G}$, following the approach initially proposed by Ljung [13, 14, **?**]. We have shown in [8, 9] that the resulting uncertainty region $\mathcal{D}$ always has the generic form

$$\mathcal{D} = \left\{G(z, \theta) \mid G(z, \theta) = \frac{e + Z_N\theta}{1 + Z_D\theta} \text{ and } \theta \in \mathcal{U}\right\} \tag{5}$$

where $Z_N(z)$ and $Z_D(z)$ are row vectors of size n of known transfer functions, and $e(z)$ is a known transfer function with a delay equal to the delay of G_0.

2.2 Model set validation for control: a design problem

The validated set $\mathcal{D}$ depends very much on the experimental conditions used for the validation experiment: open-loop or closed-loop setup, choice of signal spectra, etc. Thus, different validation experiments yield different validated sets $\mathcal{D}^{(i)}$, each containing the true G_0 with probability p, say. It is therefore useful to possess a measure of quality of a validated model set that is connected to the size of a controller set that stabilises all models in the validated model set. To this end, we have defined the **worst-case chordal distance (at frequency** ω**)** $\kappa_{WC}(\bar{G}(e^{j\omega}), \mathcal{D})$ and the **worst-case** ν**-gap** $\delta_{WC}(\bar{G}, \mathcal{D})$ between the nominal model $\bar{G}$ and all models in $\mathcal{D}$. They are defined as follows [3, 5, 6, 9]:

$$\kappa_{WC}(\bar{G}(e^{j\omega}), \mathcal{D}) \triangleq \sup_{G \in \mathcal{D}} \kappa(\bar{G}(e^{j\omega}), G(e^{j\omega})) \tag{6}$$

and

$$\delta_{WC}(\bar{G}, \mathcal{D}) \triangleq \sup_{G \in \mathcal{D}} \delta_\nu(\bar{G}, G) = \max_\omega \kappa_{WC}(\bar{G}(e^{j\omega}), \mathcal{D}), \tag{7}$$

where $\kappa(\bar{G}(e^{j\omega}), G(e^{j\omega}))$ and $\delta_\nu(\bar{G}, G)$ are, respectively, the chordal distance at frequency ω and the ν-gap between $\bar{G}(e^{j\omega})$ and $G(e^{j\omega})$, as defined by Vinnicombe in [15].

We have developed Linear Matrix Inequality (LMI)-based optimisation tools to compute the worst-case chordal distance and the worst-case ν-gap: see [3] or [9].

The following theorem gives sufficient conditions for robust stabilisation of all plants in a PE uncertainty set by a given controller.

Theorem 1 *[3, 9] Consider a PE uncertainty set $\mathcal{D}$ containing the true G_0 with probability p, with $\delta_{WC}(\bar{G}, \mathcal{D}) < 1$, and let C be a controller (typically computed from $\bar{G}$) that stabilises $\bar{G}$. Then C stabilises all models in $\mathcal{D}$ (and it therefore stabilises G_0 with probability p) if*

$$\kappa_{WC}(\bar{G}(e^{j\omega}), \mathcal{D}) < \kappa\left(\bar{G}(e^{j\omega}), -\frac{1}{C(e^{j\omega})}\right) \quad \forall\omega. \tag{8}$$

In particular (Min-Max version), C stabilises all models in $\mathcal{D}$ (and it therefore stabilises G_0 with probability p) if

$$\delta_{WC}(\bar{G}, \mathcal{D}) < b_{\bar{G},C} \triangleq \min_\omega \kappa\left(\bar{G}(e^{j\omega}), -\frac{1}{C(e^{j\omega})}\right), \tag{9}$$

where $b_{\bar{G},C}$ is the generalised stability margin of the nominal closed-loop system $(\bar{G}, C)$.

The worst-case ν-gap is essentially a control-oriented measure of the size of $\mathcal{D}$ [5, 6]. In other words, it is a tool for validating the experimental design under which the validation is carried out. If two PE uncertainty sets $\mathcal{D}^{(1)}$ and $\mathcal{D}^{(2)}$ obtained from two different validation experiments are such that $\delta_{WC}(\bar{G}, \mathcal{D}^{(1)}) < \delta_{WC}(\bar{G}, \mathcal{D}^{(2)})$, then the set of $\bar{G}$-based controllers that robustly stabilise $\mathcal{D}^{(2)}$ is contained within the set of controllers that robustly stabilise $\mathcal{D}^{(1)}$ [6, 9]. Hence, the smaller $\delta_{WC}(\bar{G}, \mathcal{D})$, the larger the set of controllers that stabilise $\mathcal{D}$. It is therefore important to design the validation experiment in such a way that the obtained $\mathcal{D}$ has a worst-case ν-gap (with respect to $\bar{G}$) that is as small as possible. The role of the experimental conditions on the properties of the validated PE uncertainty set $\mathcal{D}$ is discussed in some detail in [4]. Here we just stress that, since a major property of the ν-gap is that its resolution is largest around the crossover frequency of the plant, we expect that closed-loop validation experiments will deliver PE uncertainty sets with smaller worst-case ν-gap than open-loop validation experiments. This will be illustrated in the application described in this paper.

2.3 Controller validation for stability: a verification problem

The stability conditions of Theorem 1 are only sufficient and may be quite conservative and lead to reject a controller that, in fact, robustly stabilises $\mathcal{D}$, as our application example will show. The following theorem (only valid in the SISO case) gives a necessary and sufficient condition for a given controller $C(z)$ to stabilise all models in a PE uncertainty set $\mathcal{D}$.

Theorem 2 *[8] Consider a generic PE uncertainty set $\mathcal{D}$ of the form (5) and a controller $C(z) = X(z)/Y(z)$ that stabilises the center of that set, $G(z, \hat{\theta})$. Then all models in $\mathcal{D}$ are stabilised by $C(z)$ if and only if*

$$\max_{\omega} \mu(M_{\mathcal{D}}(e^{j\omega})) \leq 1, \tag{10}$$

where

- $M_{\mathcal{D}}(z) \triangleq \frac{-\left(Z_D + \frac{X(Z_N - eZ_D)}{Y+eX}\right)T^{-1}}{1+\left(Z_D + \frac{X(Z_N - eZ_D)}{Y+eX}\right)\hat{\theta}}$;
- $T^T T \triangleq \frac{P_\theta^{-1}}{\alpha}$;
- $\phi \triangleq T(\theta - \hat{\theta})$, *whereby* $\theta \in \mathcal{U} \Leftrightarrow \|\phi\|_2 < 1$;
- $\mu(M_{\mathcal{D}}(e^{j\omega}))$ *is called the* stability radius *of the loop* $[M_{\mathcal{D}}(z) \quad \phi]$. *For a real vector ϕ it is computed as follows:*

$$\begin{aligned} &\mu(M(e^{j\omega})) \\ &= \sqrt{\|Re(M)\|_2^2 - \frac{(Re(M)Im(M)^T)^2}{\|Im(M)\|_2^2}} \\ &\quad \textit{if } Im(M) \neq 0 \\ &= \|M\|_2 \quad \textit{if } Im(M) = 0. \end{aligned}$$

2.4 Controller validation for performance: a verification problem

When designing a controller for a plant G_0, one does generally not only aim at stabilising this plant: there is most often a performance issue. The following criterion is a possible definition of the worst-case performance a controller will achieve over a validated PE uncertainty set:

$$J_{WC}(\mathcal{D}, C, W_l, W_r, \omega) = \max_{G \in \mathcal{D}} \sigma_{\max} \left(\overbrace{\begin{bmatrix} W_{l1} & 0 \\ 0 & W_{l2} \end{bmatrix}}^{W_l(e^{j\omega})} \times T(G(e^{j\omega}), C(e^{j\omega})) \times \overbrace{\begin{bmatrix} W_{r1} & 0 \\ 0 & W_{r2} \end{bmatrix}}^{W_r(e^{j\omega})} \right) \tag{11}$$

where

$$T(G, C) = \begin{bmatrix} \frac{GC}{1+GC} & \frac{G}{1+GC} \\ \frac{C}{1+GC} & \frac{1}{1+GC} \end{bmatrix}, \tag{12}$$

$W_{l1}(z), W_{l2}(z)$ and $W_{r1}(z), W_{r2}(z)$ are frequency weights that allow one to define specific performance levels, and where $\sigma_{\max}(A)$ denotes the largest singular value of A. Any function that is derived from J_{WC} can of course also be handled, such as $\|J_{WC}\|_\infty$, for example. We have shown in [8] that J_{WC} can be computed by solving an optimisation problem involving LMI constraints.

3 Application to a ferrosilicon production process

We now present an application that is representative of industrial process control applications, in which the control objective is the rejection of stochastic disturbances.

3.1 Problem setting

The plant model and the controllers used in this simulation example are taken from a paper by Ingason and Jonsson [1]. Ferrosilicon is a two-phase mixture of the chemical compound $FeSi_2$ and the element silicon. The balance between silicon and iron is regulated around 76% of the total weight in silicon, 22% in iron and 2% in aluminium by adjusting the input of raw materials to the furnace. Those are charged batchwise to the top of the furnace, each batch consisting of a fixed amount of quartz (SiO_2) and a variable quantity of coal/coke (C) and iron oxide (Fe_2O_3). The quantity of coal/coke which is burned in the furnace does not influence the silicon ratio in the mixture, hence the control input is the amount of iron oxide.

The authors of [1] have obtained the following ARX model for the process:

$$y(t) + ay(t-1) = bu(t-1) + d + e(t) \tag{13}$$

where the sampling period is one day, $y(t)$ is the percentage of silicon in the mixture that must be regulated around 76%, $u(t)$ is the quantity of iron oxide in the raw materials (expressed in kilogrammes), d is a constant and $e(t)$ is a stochastic disturbance. The nominal values of the parameters and their standard deviations are:

$$\begin{aligned} a &= -0.44, \quad b = -0.0028, \quad d = 46.1, \\ \sigma_a &= 0.07, \quad \sigma_b = 0.001, \quad \sigma_d = 5.6. \end{aligned} \tag{14}$$

Here, for the sake of illustrating our theory, we make the assumption that the true system is

$$G_0(z) = \frac{b_0 z^{-1}}{1 + a_0 z^{-1}} = \frac{-0.0032z^{-1}}{1 - 0.34z^{-1}}, \tag{15}$$

$$H_0(z) = \frac{1}{1 + a_0 z^{-1}} = \frac{1}{1 - 0.34z^{-1}} \tag{16}$$

while the nominal model is the one obtained by Ingason and Jonsson [1]:

$$\bar{G}(z) = \frac{bz^{-1}}{1 + az^{-1}} = \frac{-0.0028z^{-1}}{1 - 0.44z^{-1}}, \tag{17}$$

$$\bar{H}(z) = \frac{1}{1 + az^{-1}} = \frac{1}{1 - 0.44z^{-1}}. \tag{18}$$

This model was used by the authors of [1] to compute a GPC controller minimising the following cost function:

$$J_{GPC} = E\left(\sum_{j=1}^{2} (y(t+j) - r(t+j))^2 + \sum_{j=1}^{2} (\Delta u(t+j-1))^2\right), \tag{19}$$

where $\Delta(z) = 1 - z^{-1}$ and λ is a tuning parameter. The resulting controller is a two-degree-of-freedom controller

$C_\lambda(z)$:

$$\begin{aligned} u(t) &= C_\lambda(z) \begin{bmatrix} r(t) \\ y(t) \end{bmatrix} \\ &= \begin{bmatrix} C^r_\lambda(z) & C^y_\lambda(z) \end{bmatrix} \begin{bmatrix} r(t) \\ y(t) \end{bmatrix} \end{aligned} \quad (20)$$

where

$$C^r_\lambda(z) = \frac{b^3 + 2b\lambda - ab\lambda}{(b^4 + 3b^2\lambda + a^2b^2\lambda + \lambda^2 - 2ab^2\lambda) - (b^2\lambda + \lambda^2)z^{-1}}, \quad (21)$$

$$C^y_\lambda(z) = \frac{ab^3 + ab\lambda - a^2b\lambda + a^3b\lambda}{(b^4 + 3b^2\lambda + a^2b^2\lambda + \lambda^2 - 2ab^2\lambda) - (b^2\lambda + \lambda^2)z^{-1}}. \quad (22)$$

Experiments made by the authors of [1] showed that, for a constant reference $r(t) = 76$ and for $\lambda = 0.0007$, a control input variance $\sigma_u^2 = 20$ was obtained. This results from a white noise variance $\sigma_e^2 = 0.078$.

In this example, we shall compare open-loop and closed-loop experiments for the validation of the GPC controller $C_\lambda(z)$ with $\lambda = 0.0007$, which we simply denote $C(z)$.

3.2 Open-loop validation experiment

The "true plant" model (G_0, H_0) was excited with $u(t)$ chosen as a PRBS with variance $\sigma^2_{u_{ol}} = 20$, which is the maximum input variance admissible for this process. $e(t)$ was chosen as a Gaussian white noise sequence with variance $\sigma_e^2 = 0.078$, which corresponds to the noise acting on the real process. The variance of the output was then $\sigma^2_{y_{ol}} = 0.0884$. 300 input-output data samples were collected for validation, corresponding approximately to one year of measurements. These data were used to identify an ARX model with exact structure

$$G_{ol}(z, \theta_{ol}) = \frac{\theta_2 z^{-1}}{1 + \theta_1 z^{-1}}, \quad (23)$$

$$H_{ol}(z, \theta_{ol}) = \frac{1}{1 + \theta_1 z^{-1}}. \quad (24)$$

We found

$$\hat{\theta}_{ol} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} = \begin{bmatrix} -0.3763 \\ -0.0073 \end{bmatrix}, \quad (25)$$

$$P_{\theta_{ol}} = \begin{bmatrix} 2.8131 \times 10^{-3} & -1.2784 \times 10^{-5} \\ -1.2784 \times 10^{-5} & 1.4887 \times 10^{-5} \end{bmatrix}, \quad (26)$$

and the nominal model is validated with respect to the 95% uncertainty region $\mathcal{D}_{ol}$ around $G_{ol}(z, \hat{\theta}_{ol})$:

$$\left(\begin{bmatrix} a \\ b \end{bmatrix} - \hat{\theta}_{ol} \right)^T P^{-1}_{\theta_{ol}} \left(\begin{bmatrix} a \\ b \end{bmatrix} - \hat{\theta}_{ol} \right) = 2.6151 < \alpha = 5.99. \quad (27)$$

(For $p = 0.95$ and $n = 2$ free parameters, we have $\alpha = 5.99$ in $\Pr\left(\chi^2(n) < \alpha\right) = p$.) Observe that $\mathcal{D}_{ol}$, whose probability of containing the true plant G_0 is $p = 95\%$, effectively contains it:

$$\left(\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} - \hat{\theta}_{ol} \right)^T P^{-1}_{\theta_{ol}} \left(\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} - \hat{\theta}_{ol} \right) = 1.6744 < \alpha = 5.99. \quad (28)$$

For this open-loop uncertainty region $\mathcal{D}_{ol}$, $\kappa_{WC}(\bar{G}, \mathcal{D}_{ol})$ is as shown in Figure 1 and its maximum value over frequency is

$$\delta_{WC}\left(\bar{G}, \mathcal{D}_{ol}\right) = 0.0225. \quad (29)$$

3.3 Closed-loop validation experiment

The closed-loop validation experiment was performed with a sub-optimal controller $C_{sub}(z)$ obtained by setting $\lambda = 0.001$ in (21), (22). Since this value of λ reduces the input variance (compared to the optimal one $\lambda = 0.0007$), it was possible to add a PRBS signal to the reference $r(t)$ without violating the constraint $\sigma_u^2 \leq 20$. The variance of $r(t)$ was chosen as $\sigma_r^2 = 0.014$, $e(t)$ having the same properties as in open-loop validation. The input and output variances were then, respectively, $\sigma^2_{u_{cl}} = 20$ and $\sigma^2_{y_{cl}} = 0.0880$. Observe that the input variance is the same as in open loop, and that the output variance is very close to that of the open-loop experiment. Again, 300 input-output data samples were collected and used to identify an ARX model with the same structure as in open-loop validation (23), using a direct prediction error method. We found

$$\hat{\theta}_{cl} = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} = \begin{bmatrix} -0.3575 \\ -0.0067 \end{bmatrix}, \quad (30)$$

$$P_{\theta_{cl}} = \begin{bmatrix} 2.8323 \times 10^{-3} & -8.7845 \times 10^{-6} \\ -8.7845 \times 10^{-6} & 6.2416 \times 10^{-6} \end{bmatrix}. \quad (31)$$

As in open-loop validation, the nominal model is validated with respect to the 95% uncertainty region $\mathcal{D}_{cl}$ around $G_{cl}(z, \hat{\theta}_{cl})$:

$$\left(\begin{bmatrix} a \\ b \end{bmatrix} - \hat{\theta}_{cl} \right)^T P^{-1}_{\theta_{cl}} \left(\begin{bmatrix} a \\ b \end{bmatrix} - \hat{\theta}_{cl} \right) = 4.5575 < \alpha = 5.99. \quad (32)$$

One can also verify that $\mathcal{D}_{cl}$ effectively contains the true G_0:

$$\left(\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} - \hat{\theta}_{cl} \right)^T P^{-1}_{\theta_{cl}} \left(\begin{bmatrix} a_0 \\ b_0 \end{bmatrix} - \hat{\theta}_{cl} \right) = 2.1611 < \alpha = 5.99. \quad (33)$$

3.4 Model set validation for control: comparison of the validated sets

Observe that the nominal model $\bar{G}$ is closer to the center of the validated set $\mathcal{D}_{ol}$ than to the center of the validated set $\mathcal{D}_{cl}$, when measured by the normalised distance in *parameter space* (compare (27) with (32)). However, when measured in the control-oriented measures κ_{WC} or δ_{WC}, which are computed in *transfer function space*, the conclusions are reversed. Figure 1 shows the worst-case chordal distance between $\bar{G}$ and, respectively, all models in $\mathcal{D}_{ol}$ and all models in $\mathcal{D}_{cl}$. Clearly $\kappa_{WC}(\bar{G}, \mathcal{D}_{cl})$ is smaller than $\kappa_{WC}(\bar{G}, \mathcal{D}_{ol})$ at all frequencies, and

$$\delta_{WC}\left(\bar{G}, \mathcal{D}_{cl}\right) = 0.0156 \quad (34)$$

is smaller than $\delta_{WC}\left(\bar{G}, \mathcal{D}_{ol}\right) = 0.0225$. We observe that the open-loop validation procedure does not guarantee that the GPC controller C stabilises G_0, since

$$\kappa_{WC}\left(\bar{G}, \mathcal{D}_{ol}\right) > \kappa\left(\bar{G}, \frac{-1}{C}\right) \quad (35)$$

at some frequencies, while the closed-loop validation procedure does guarantee such stability with G_0, since

$$\kappa_{WC}\left(\bar{G},\, \mathcal{D}_{cl}\right) < \kappa\left(\bar{G},\, \frac{-1}{C}\right) \tag{36}$$

at all frequencies. The Min-Max version of this stability test yields the same conclusion, since

$$\underbrace{\delta_{WC}\left(\bar{G},\, \mathcal{D}_{ol}\right)}_{0.0225} > \underbrace{b_{\bar{G}\,C}}_{0.0169} > \underbrace{\delta_{WC}\left(\bar{G},\, \mathcal{D}_{cl}\right)}_{0.0156}. \tag{37}$$

We conclude from this analysis that the uncertainty region $\mathcal{D}_{cl}$ delivered by the closed-loop experiment should be preferred over $\mathcal{D}_{ol}$ delivered by the open-loop experiment, since it leads to the largest set of stabilising controllers: see [5, 6].

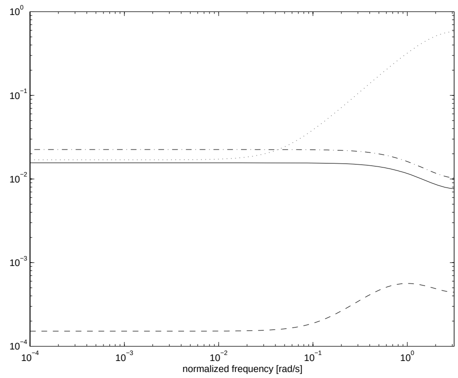

Figure 1: $\kappa_{WC}(\bar{G}, \mathcal{D}_{ol})$ $(-\cdot)$, $\kappa_{WC}(\bar{G}, \mathcal{D}_{cl})$ (—), $\kappa(\bar{G}, G_0)$ $(--)$, $\kappa\left(\bar{G}, -\frac{1}{C}\right)$ $(\cdots)$

3.5 Controller validation for stability: necessary and sufficient condition

Starting from the open-loop and closed-loop PE uncertainty sets $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$, we built the dynamic matrices $M_{\mathcal{D}_{ol}}\left(e^{j\omega}\right)$ and $M_{\mathcal{D}_{cl}}\left(e^{j\omega}\right)$ with respect to the candidate controller C and we computed their stability radii. Their respective values are

$$\max_{\omega}\, \mu(M_{\mathcal{D}_{ol}}(e^{j\omega})) = 0.6572 < 1, \tag{38}$$

$$\max_{\omega}\, \mu(M_{\mathcal{D}_{cl}}(e^{j\omega})) = 0.2111 < 1. \tag{39}$$

According to this test, C stabilises all systems contained in both uncertainty sets $\mathcal{D}_{ol}$ and $\mathcal{D}_{cl}$, hence both the open-loop and the closed-loop approach validate the controller. This shows that the necessary and sufficient condition for closed-loop stability is much less conservative than the sufficient condition based on the chordal distance or the ν-gap stability results.

3.6 Controller validation for performance: necessary and sufficient condition

Since the control objective is to reject the noise $v(t) = H_0(z)e(t)$ which is essentially located at low frequencies ($H_0(e^{j\omega})$ is a first order low-pass filter; see the thick dotted line in Figure 2), a performance specification in the frequency domain is that the sensitivity function $S_0 = \frac{1}{1+G_0 C}$ be low at low frequencies, since it filters $v(t)$ in closed loop. However, since $S_0(e^{j\omega})$ cannot be made arbitrarily low at all frequencies, a tradeoff must be made between reducing the noise at low frequencies and amplifying it at high frequencies. Actually, the ideal situation would be that $|H_0(e^{j\omega})S_0(e^{j\omega})|$ be uniform over frequency, i.e. that the noise be whitened by the closed-loop system dynamics.

A possible performance criterion is thus

$$J_{WC}^{(1)}(\mathcal{D}, C, \omega) = \max_{G(e^{j\omega},\theta)\in\mathcal{D}} \left| \frac{1}{1 + G(e^{j\omega},\theta)C(e^{j\omega})} \right|, \tag{40}$$

obtained by setting $W_{l1} = W_{r1} = 0$ and $W_{l2} = W_{r2} = 1$ in (11). The controller is validated if $J_{WC}^{(1)}$ remains at all frequencies below some template, which is high-pass since we want the worst-case sensitivity to be small in the frequency range where the noise is large. The amplitude Bode diagrams of the worst-case and achieved sensitivity functions are depicted in Figure 2.

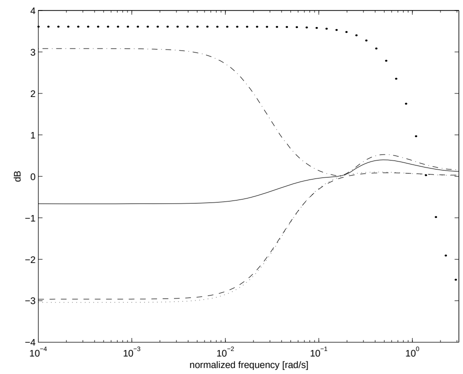

Figure 2: $J_{WC}^{(1)}(\mathcal{D}_{ol}, C)$ $(-\cdot)$, $J_{WC}^{(1)}(\mathcal{D}_{cl}, C)$ (—), $|S(G_0, C)|$ $(--)$, $|S(\bar{G}, C)|$ $(\cdots)$, $|H_0|$ $(\cdot)$

Clearly, for most reasonable templates the controller is validated by the closed-loop validation experiment yielding $\mathcal{D}_{cl}$, but not by the open-loop experiment yielding $\mathcal{D}_{ol}$.

Now observe that a noise model $H(z, \hat{\theta}) = \frac{1}{1+\hat{\theta}_1 z^{-1}}$, sharing its parameter with $G(z, \hat{\theta})$, is also identified during each validation experiment, due to the particular ARX model structure we use. As a result, each θ in $\mathcal{U}_{ol}$ or $\mathcal{U}_{cl}$ defines not only a plant model $G(\theta)$ in $\mathcal{D}_{ol}$ or $\mathcal{D}_{cl}$, but also a noise model $H(\theta)$ associated with this plant model. An alternative performance criterion[1] could thus be

$$J_{WC}^{(2)}(\mathcal{U}, C, \omega) = \max_{\theta\in\mathcal{U}} \left| \frac{H(e^{j\omega},\theta)}{1 + G(e^{j\omega},\theta)C(e^{j\omega})} \right|. \tag{41}$$

In this case, the distribution of $J_{WC}^{(2)}$ over frequency should be as close as possible to a uniform distribution, and its amplitude should be as low as possible. As shown in Figure 3, the worst-case performance attached to the closed-loop uncertainty region is closer to the performance achieved by the controller on the actual system, and to the uniform distribution we would like to obtain, than the worst-case performance attached to the open-loop uncertainty region.

[1] This performance criterion is not one of the form (11). However, it only requires a slight modification of the LMI-based algorithm used to compute (11).

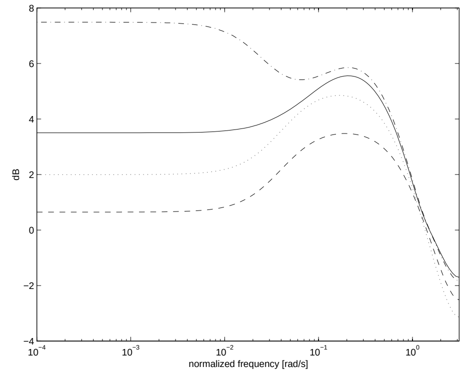

Figure 3: $J_{WC}^{(2)}(\mathcal{U}_{ol}, C)$ $(-\cdot)$, $J_{WC}^{(2)}(\mathcal{U}_{cl}, C)$ (—), $|H_0 S(G_0, C)|$ $(--)$, $|\tilde{H} S(\tilde{G}, C)|$ $(\cdots)$

The results we have obtained in this subsection confirm that the PE uncertainty set $\mathcal{D}_{cl}$ obtained via a closed-loop experiment is to be preferred to $\mathcal{D}_{ol}$ obtained via an open-loop experiment, as the worst-case ν-gap test of Subsection 3.4 already indicated, when the purpose is to validate a given candidate controller. Let us recall once again that the choice of the experimental conditions used for the validation are entirely up to the designer. Thus, the designer is free to choose either $\mathcal{D}_{ol}$ or $\mathcal{D}_{cl}$ as his/her uncertainty region since both are constructed from real data.

4 Conclusions

We have illustrated our recent results in model validation for control and controller validation for stability and performance by means of a numerical example representative of standard industrial control problems. This case study has delivered two important findings. The first one concerns an experiment design issue, while the second one concerns the usefulness of the tools we have developed:

• Closed-loop validation experiments yield uncertainty regions that are better tuned for control design than open-loop validation experiments. This result, related to the construction of uncertainty regions using variance error expressions, is the counterpart of the well-established fact that the nominal model is better tuned for control design if the identification of this model is carried out in closed loop than in open loop. This result applies for performance considerations as well as for stability considerations.

• The worst-case extension of the ν-gap metric provides us with a convenient way to assess the size of an uncertainty set by a single number: the worst-case ν-gap between the nominal model and this set. This measure is a control-oriented measure because of the higher resolution of the ν-gap around the crossover frequency of the plant. However, the stability theorem attached to this measure (or to its frequency-by-frequency version) may be quite conservative and may lead to reject a stabilising controller. Therefore, whereas these worst-case measures of size of a validated region are useful for validation design purposes (one should opt for an experiment that delivers a validated region with the smallest possible worst-case measure), for the validation of a candidate controller with respect to a given uncertainty region it may be required to apply the necessary and sufficient conditions for controller validation. We believe that our validation procedure will prove to be a very powerful tool in the near future, because it establishes, for the first time, a connection between robust control and uncertainty sets obtained by standard prediction error identification without embedding these sets in standard ones, which would systematically yield more conservative control designs.

References

[1] H. Ingason and G. Jonsson, "Control of the silicon ratio in ferrosilicon production," *Control Engineering Practice*, vol. 6, pp. 1015–1020, 1998.

[2] X. Bombois, M. Gevers, and G. Scorletti, "Controller validation for a validated model set," in *CD-ROM Proc. of the 5th European Control Conference*, (Karlsruhe, Germany), 1999. Paper 869.

[3] X. Bombois, M. Gevers, and G. Scorletti, "Controller validation based on an identified model," in *Proc. of the 38th IEEE Conference on Decision and Control*, (Phoenix, Arizona, USA), pp. 2816–2821, December 1999.

[4] M. Gevers, X. Bombois, B. Codrons, F. De Bruyne, and G. Scorletti, "The role of experimental conditions in model validation for control," in *Robustness in Identification and Control* (A. Garulli, A. Tesi, and A. Vicino, eds.), vol. 245 of *Lecture Notes in Control and Information Sciences*, pp. 72–86, London, UK: Springer-Verlag, 1999.

[5] X. Bombois, M. Gevers, and G. Scorletti, "A measure of robust stability for an identified set of parametrized transfer functions," 2000. Accepted for publication in *IEEE Trans. on Automatic Control*.

[6] X. Bombois, M. Gevers, and G. Scorletti, "A measure of robust stability for an uncertainty set obtained by prediction error identification," 2000. Accepted for presentation at *the 39th Conference on Decision and Control*, Sydney, Australia, December 2000.

[7] M. Gevers, X. Bombois, B. Codrons, F. De Bruyne, and G. Scorletti, "Model validation for robust control and controller validation in a prediction error framework," in *CD-ROM Proc. of the IFAC System Identification Symposium (SYSID 2000)*, (Santa Barbara, California, USA), June 2000. Paper WeAM1-1.

[8] X. Bombois, M. Gevers, G. Scorletti, and B. Anderson, "Controller validation for stability and performance based on an uncertainty region designed from an identified model," in *CD-ROM Proc. of the IFAC System Identification Symposium (SYSID 2000)*, (Santa Barbara, California, USA), June 2000. Paper WePM1-6; also submitted to *Automatica*.

[9] M. Gevers, X. Bombois, B. Codrons, G. Scorletti, and B. Anderson, "Model validation for control and controller validation: a prediction error identification approach," 2000. Submitted to *Automatica*.

[10] B. Codrons, *Experiment Design Issues in Modelling for Control*. PhD thesis, Université Catholique de Louvain, Louvain-la-Neuve, Belgium, 2000.

[11] H. Hjalmarsson, M. Gevers, and F. De Bruyne, "For model-based control design, closed-loop identification gives better performance," *Automatica*, vol. 32, pp. 1659–1673, 1996.

[12] M. Gevers, B. Codrons, and F. De Bruyne, "Model validation in closed loop," in *Proc. of the 1999 American Control Conference*, (San Diego, California, USA), pp. 326–330, June 1999.

[13] L. Ljung and L. Guo, "The role of model validation for assessing the size of the unmodelled dynamics," *IEEE Trans. on Automatic Control*, vol. AC-42, pp. 1230–1239, September 1997.

[14] L. Ljung, "Identification for control – what is there to learn?," in *Workshop on Learning, Control and Hybrid Systems*, (Bangalore, India), 1998.

[15] G. Vinnicombe, "Frequency domain uncertainty and the graph topology," *IEEE Trans. on Automatic Control*, vol. AC-38, pp. 1371–1383, 1993.