AN APPROACH TO FOLLOW THE VARIATION
OF THE BASE FLOW OF SMALL WATERSHEDS

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ABSTRACT

The term "base flow" means the contribution of the aquifer flow which sustains the river flow during droughts and permits pumping activities the year round.

This paper proposes a way to obtain rather rapidly a good approximation of the behaviour of the base flow of a watershed only after a few months of measurements.

After some considerations about data collecting, the hydrological assumptions which have lead to a simple base flow model are explained. The mathematical procedure of identification of the parameters of the model is then described and applied to three subbasins of the Draye basin.

INTRODUCTION

The term "base flow" means the contribution of the aquifer flow which sustains the river flow during droughts and permits pumping activities the year round.

Base flow has been treated in many ways which consist mostly in a qualitative or mathematical analysis of an important set of data, extending over a period of several years.

This paper proposes a way to obtain rather rapidly a good approximation of the behaviour of the base flow of a watershed, only after a few months of measurements, in order to follow its evolution and to detect the periods of discharge or the influence of recharge or pumping of the aquifer.
The accent is put on two aspects of the problem: on the one hand, the field equipment necessary to get reliable data and on the other hand, the modelisation of the base flow process and the identification of the parameters with the help of the collected data.

1. DATA COLLECTING

1.1. Measuring stations

The first objective in such a study is to collect rapidly good measurements of river flow. As the amplitudes of the variations of base flow are much smaller than those implied in runoff phenomena which are usually considered, it is important to dispose of a reliable system of measurement in order to follow with precision the evolution of the base flow. In our case, the size of the outlet of the total studied basin (650 km²) would have required expensive equipment to get such measurements of an average flow discharge of ± 2 m³/sec. Therefore, it was easier to equip subbasins of average flow discharge < 0.3 m³/sec with control sections like sharp or broad crested weirs, insuring a good accuracy and reducing greatly the work necessary to establish the curve relating the measured water level to the outflow.

Water level measurements are collected with the help of the LGR telemeter system working through the public telephone line (PERSOONS and al., 1974).

Concerning the identification of the parameters characterising the base flow, working on subbasins rather than on a global watershed presents an advantage: it allows to approach more easily and more precisely the main feature of the base flow which is to characterize at once the relationship between the river and the aquifer. Indeed, a study concerning a too great area would probably include several types of aquifer river relationships which would complicate the studied phenomenon and reduce the efficiency of the work.

1.2. Selection of the data

In order to avoid the complexity of the runoff process, the study will only use the information from periods free of any storm or resulting runoff.

As in our country, precipitation free periods exceeding one week are unusual, the information over the base flow must be obtained from recession curves of the type of those illustrated in figures 4, 5 and 6. Each curve starts at the presumed end of the surface runoff, theoretically represented by the point of inflection of the falling limb of the last storm hydrograph and stops at the beginning of the surface runoff due to the following storm.
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The time step chosen in this study is the day. In dry periods, the telemesasurement system provides three measurements a day, that is to say each every 8 hours.

In order to respect the required accuracy, the variation with temperature of the electronic of the system of measurement as well as sewage discharge must be taken into account.

2. MODELISATION OF THE BASE FLOW PROCESS

2.1. Description of the model

The purpose of the work is to characterize the law of evolution of the base flow of a river, which is supposed to be constant during one identification period.

The proposed model "MOBASE" uses the informations obtained from the recession curves available during such a period.

It is assumed that each recession curve is composed of two components:
- the first is the part of the flow discharge due to the fast released groundwater storage which is supposed to be more or less refilled after each storm event;
- the second is the much more stable contribution of the aquifer flow which provides the actual base flow of the river.

We shall represent each recession curve by a sum of two exponentials, one rapid and the other slow, such as:

\[ Q_N = Q_{R,N} + Q_{S,N} = Q_{R,0} e^{-\alpha R N} + Q_{L,0} e^{-\alpha L N} \]  

(1)

where \( Q_N \) is the measured flow at day \( N \) of the recession
\( Q_{R,N} \) is the rapid component at day \( N \)
\( Q_{S,N} \) is the slow component at day \( N \)
\( \alpha_R \) and \( \alpha_L \) are the corresponding recession coefficients
\( Q_{R,0} \) and \( Q_{L,0} \) are the initial values of the two components for the considered recession curve.

Equation (1) can be written in another form:

\[ Q_N = K_R Q_{R,N-1} + K_L Q_{S,N-1} \]  

(2)
\[ K_R = e^{-Q_R} \]
\[ K_L = e^{-Q_L} \]

At this step arises the problem of the number of data which are necessary in an identification procedure. Taking into account the uncertainties inherent to field measurements, a minimum of three recession curves will be used in order to allow a successful identification of the parameters. It seems plausible to assess that, during the corresponding period (one month on the average), the two components of the flow are supposed to follow a constant law: same decrease for the fast component and constant evolution for the base flow: recharge or discharge characterized by a uniform time constant. Therefore, we shall estimate a mean value of the recession coefficient \( K_R \) for the considered curves and, by difference, compute the values of \( Q_L \) for each recession curve. It is then possible to adjust a mean exponential curve, passing through the computed \( Q_L \) segments of each recession curve and which reflects the evolution of the base flow at that moment.

2.2. The identification procedure

The procedure is iterative. One iteration proceeds as follows:

a. Estimation of \( K_R \)

The problem consists in evaluating the time constant characterizing the fast component of a sum of two exponentials in the case of very different time constants.

Therefore, a high-pass filter procedure is applied to the data. The filtered series is defined as follows:

\[ y_n = q_n - K_L q_{n-1} \]  \hspace{1cm} (3)

Using equations (1) and (2), we can write:

\[ y_n = q_R n + q_L - K_L (q_R_{n-1} + q_L_{n-1}) \]

\[ = K_R q_R_{n-1} - K_L K_R q_R_{n-2} \]  \hspace{1cm} (3')

\[ = K_R (q_R_{n-1} - K_L q_{n-2}) \]
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According to equation (3), this can be written:

\[ Y_N = F_R Y_{N-1} \]  \hspace{1cm} (4)

Using an estimate of \( K_L \), the series \( Y_N \) can be computed and the evaluation of \( K_R \) can be made by a mean square method, the cost function to minimize being:

\[ F_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} \left( \frac{1}{n_i} \sum_{j=a_i+1}^{b_i} (y_{ij} - K_R y_{ij-1})^2 \right) \]

where \( n_1 \) is the number of terms of the sum
\( i \) is the number of the considered recession curves
\( (i = 1, 2, 3 \text{ or more}) \)
and the \((a_i, b_i + 1, \ldots, b_i)\) are the measurement days for the curve \( i \).

The parameter \( K_R \) is given by the linear relation:

\[ \frac{\partial F_1}{\partial K_R} = 0 \]

b. Estimation of QR

The estimated value of \( K_R \), which is a mean value for the recession curves considered, is used to compute the series of QR values in the following manner:

\[ Q_{R_{a_i+1}} = Q_{a_i+1} - K_L Q_{a_i} \]

\[ Q_{R_j} = Q_{R_{j-1}} \quad a_i < j < b_i \]

for the \( i \)-th recession curve.

c. Estimation of QL

By difference, it is possible to compute for each recession curve the QL values, given by the following relation:

\[ Q_{L_i} = Q_j - Q_{R_j} \quad a_i < j < b_i \]
d. Estimation of $K_L$

Our purpose is to determine the optimal exponential characterizing the evolution of the base flow.

The two parameters involved are:

- the parameter $K_L$, characterizing the rate of variation of the base flow
- the initial value $Q_{L0}$ of the exponential.

The estimation of these two parameters will be executed through the minimization of the following cost function:

$$F_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} \left( \sum_{j=1}^{b_1} (Q_{i,j} - K_L Q_{L0})^2 \right)$$

where $n_2$ is the total number of measurement days.

The minimization is made by a steepest descent method.

e. Iterative work

Using these new estimates of $K_L$ and $Q_{L0}$, a new iteration can be performed to obtain better estimates of $K_L$, $K_L$ and $Q_{L0}$.

The iterative procedure ends when the convergence of the KL values is satisfactory as shown in the MORAS flow chart illustrated in figure 1.

2.3. Tests of the procedure

a. Test data

In order to check the performance of the model, test data have been used (figure 2). With a set of 10 data values, the algorithm converges already at the 3rd iteration.

b. Application to the field data

Data of three small rivers were tested:

1. The river Pisselet at Din-Valmont
   subbasin area = 7.42 km$^2$
   outlet equipped with a Venturi flume.
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$KL_m, QL_{U,m}$

$m = 0$

Initial condition
$KL_0 = 1$

$m = m + 1$

Data

Estimation of $KL_m$

Computation of the serie $QR_m$

Computation of $Q_{0,m}$ and of the serie $QL_m$

Estimation of $KL_m$

$|KL_m - KL_{m-1}| < K$

Print
- $m$
- $Q_{U,m}$
- $KL_m$
- the serie $QR_m$
- the serie $QL_m$

YES

NO

END

Figure 1: Flow chart of the identification procedure of NOBASE
Figure 2: Test of the identification procedure of the two exponential components.

Figure 3: The Dyle Basin and the tested subbasins.

C : test serie 0
KL = 10/150
C1 : QL serie - m = 1
KL1 = 10/120
C2 : QL serie - m = 2
KL2 = 10/151
2. The river Orne at Blamont
subbasin area = 36 km²
outlet equipped with a rectangular sharp crested weir.

3. The river Orne at Cortil-Noirmont
subbasin area = 20 km²
outlet equipped with a broad crested weir.

The localisation of these basins is represented in figure 3. As already mentioned in 1.2, rough field data should be used with care. Tests were therefore realized on rough and on smoothed data. It appears that the estimation of the parameters is very sensitive to the deviation of the measurements from the smoothed curve. This is due to the small number of data available for a test. The following tests have thus been performed on smoothed data:

**Test 1. Pisselet river (figure 4)**

<table>
<thead>
<tr>
<th>Iteration (m)</th>
<th>KL</th>
<th>KR</th>
<th>PI</th>
<th>Q₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9529</td>
<td>0.4531</td>
<td>0.6835</td>
<td>25.5</td>
</tr>
<tr>
<td>2</td>
<td>0.9503</td>
<td>0.2683</td>
<td>0.3445</td>
<td>27.1</td>
</tr>
<tr>
<td>3</td>
<td>0.9493</td>
<td>0.2544</td>
<td>0.3258</td>
<td>30.7</td>
</tr>
<tr>
<td>4</td>
<td>0.9495</td>
<td>0.2490</td>
<td>0.3186</td>
<td>31.2</td>
</tr>
</tbody>
</table>

**Test 2. Orne river at Blenmont (figure 5)**

<table>
<thead>
<tr>
<th>Iteration (m)</th>
<th>KL</th>
<th>KR</th>
<th>PI</th>
<th>Q₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9885</td>
<td>0.4309</td>
<td>1.5606</td>
<td>208.5</td>
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<tr>
<td>2</td>
<td>0.9878</td>
<td>0.3293</td>
<td>0.7318</td>
<td>212.</td>
</tr>
<tr>
<td>3</td>
<td>0.9875</td>
<td>0.3231</td>
<td>0.7312</td>
<td>214.</td>
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</tbody>
</table>

...
Test 3. Orne river at Cortil-Noirmont (figure 6)

<table>
<thead>
<tr>
<th>Iteration (m)</th>
<th>KL</th>
<th>KR</th>
<th>F1</th>
<th>Q₀</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.6617</td>
<td>0.6843</td>
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<tr>
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<td>0.5769</td>
<td>0.6495</td>
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<tr>
<td>3</td>
<td>0.9878</td>
<td>0.5822</td>
<td>0.6345</td>
<td>122.2</td>
</tr>
</tbody>
</table>

c. Discussion of the results

1. Verification of the hypothesis

The identification method is based on the assumption that each recession curve is composed of two processes, one slow and the other rapid, both following an exponential law. The rapid convergence of the algorithm of identification as well as the regular decrease of the cost function $F_1$ at each iteration indicate that the hydrological hypothesis are valid.

2. Reliability of the computed values of KL (figures 4, 5 and 6)

The parameter KL is the most important in this study because it characterizes the aquifer-river relation during the considered period. It is obvious that more tests should be realized in order to ascertain the reliability of the results; but the following observations are favourable:

- as expected, the values of KL computed for the two measurement points on the river Orne are identical, since their respective watersheds are overlapping;
- the 70 l/s base flow at Cortil-Noirmont measured on day 150 corresponds very well to the computed base-flow obtained by extrapolation of the computed curve during the period - days 106 to 128 - under the assumption of a constant discharge of the aquifer;
- the three tests done during the period - days 165 to 180 - show the general decrease of the rate of discharge at these three points as illustrated by the identified values of KL, respectively of 0.9710, 0.9980, 0.9929.
Figure 4, 5 and 6: Application of NOBASE to the measurements
CONCLUSION

The two hydrological assumptions, namely the two component flow discharge and the constant aquifer discharge made during runoff-free periods, have led to the model NOBASE, which characterizes the river base flow.

Its mathematical performances were tested on test data. When applied to smoothed field data, the procedure gives reliable results, which confirm these hypothesis.

This method, developed for humid regions, seems more efficient than any graphical procedure. It improves the knowledge of the river base flow behaviour even when field data are scarce during a short period of study involving at least three recession curves.

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