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On Iterative Feedback Tuning for non-minimum phase plants *

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Abstract

The Iterative Feedback Tuning (IFT) is a data-based method for the tuning of restricted-complexity controllers. In the classical formulation, the IFT aims at minimizing a certain model-reference criterion in which the reference-model is chosen by the user. This minimization is based on signal information only. In this paper we formulate a new criterion for the IFT method. In the new criterion some freedom is given to the reference-model in order to let it reproduce the features of the unknown plant (i.e. the delay and non-minimum phase zeros) which the controller should not attempt to change. It is shown that using the new criterion corresponds to giving more emphasis to the placement of the closed loop poles.

1 Introduction

In this paper, we consider the data-based tuning of a parameterized controller $C(z, \rho)$ for a plant (possibly non-minimum phase) whose transfer function $P(z)$ in unknown. We indicate by $T(z, \rho)$ the I/O transfer function of the closed-loop system formed by the feedback connection of $C(z, \rho)$ and $P(z)$:

$$T(z, \rho) = \frac{P(z)C(z, \rho)}{1 + P(z)C(z, \rho)} .$$

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We concentrate on the model-reference criterion:

$$J^{\mathrm{MR}}(\rho) = \frac{1}{N}\sum_{t=1}^{N}\Big(y(t,\rho) - \bar{M}(z)\,r(t)\Big)^2 \qquad y(t,\rho) = T(z,\rho)r(t) \tag{1}$$

in which $r(t)$ is a certain reference-signal and $\bar{M}(z)$ is a desired reference-model chosen by the user. A practically important control design problem is that of finding the controller parameter vector ρ^* such that $\rho^* = \arg\min_\rho J^{\mathrm{MR}}(\rho)$ on the basis of input-output data $\{u(t),\, y(t)\}$ collected on the system, without knowledge of $P(z)$.

The Iterative Feedback Tuning (IFT) method gives a solution to such a problem. The IFT is based on the fact that an estimate of the gradient of $J^{\mathrm{MR}}(\rho)$ can be obtained from data collected from experiments on $T(z,\rho)$. The cost-function $J^{\mathrm{MR}}(\rho)$ can then be minimized through a gradient-based iterative minimization scheme, in which a sequence of controllers $C(z,\rho_i)$ are computed and applied to the plant. The reader is referred to [3, 4, 5] for a presentation of the method and the description of successful applications.

One of the problems in the application of the IFT method is the choice of an adequate reference model $\bar{M}(z)$ in (1). In the application of IFT it is assumed that one has no knowledge or only partial knowledge of the plant and, therefore, one cannot know in advance if a certain reference model is achievable (even approximately) or not. This is particularly crucial when $P(z)$ is non-minimum phase. In such case, one should put the non-minimum phase zeros of the plant in the reference model; otherwise the controller attempts to achieve an unstable pole-zero cancellation in order to reduce the phase lag. In practice, with restricted complexity controllers, simulations have shown that this reduces to the cancellation of the fixed integrator in the controller. In model-based control, one would choose a reference model $\bar{M}(z)$ that has the same delay and non-minimum phase zeros as the plant. However, the IFT method has been designed for the case where no model of the plant is available.

The purpose of this paper is to deal with this issue. It is addressed by introducing a new IFT criterion. The new criterion is still a model reference criterion but with some free parameters in the reference model. The free parameters correspond to the position of the zeros of the reference model and are tuned together with the parameters of the controller. In this way, the data obtained during the iterative procedure are used to tune the reference model towards one that is compatible with the requirements of the plant. The criterion also contains a second term which aims at pulling the adjustable reference model towards a desired reference model while remaining compatible with the plant. This second term can be viewed as a regularization term. The minimization of the proposed criterion can be performed through a simple modification of the original IFT procedure.

A first attempt to give some freedom to the design criterion of IFT has been proposed in [7, 8]. In [7, 8], the idea was to put - in the case of a step reference signal - a zero-weighting time mask on the first t_0 instants of the closed-loop response. In this way, the emphasis was put on achieving a small settling time without forcing the closed-loop response to follow a particular pre-imposed transient response which was possibly not achievable. The criterion proposed in [7, 8] took the form:

$$J_{t_0}^{\text{Masked}}(\rho) = \frac{1}{N} \sum_{t=t_0}^{N} \left(y_t(\rho) - K\,\text{step}(t)\right)^2$$

where $\text{step}(t)$ is the unit step starting at time $t = 0$ and K is the new desired reference level. Interestingly enough, we will show that the masked-IFT comes out as a particular case of the adjustable criterion proposed in our contribution.

The paper is organized as follows. The proposed criterion is introduced and motivated in Section 2. Some technical results, for the case of a step reference signal, are collected in Section 3. The effectiveness of the proposed criterion is shown with simulations in Section 4. The conclusions in Section 5 end the paper. The Appendix contains all the technical proofs.

2 The proposed criterion

We assume that, as in the classical model reference setting, a desired reference model $\bar{M}(z)$ is given. However, since the plant is unknown, we do not know if $\bar{M}(z)$ is achievable, in particular $\bar{M}(z)$ may not contain the delay and possible non-minimum phase zeros of $P(z)$. As already stated, the idea is to give some freedom to the reference model in order to overcome this issue. An adjustable reference model $M(z, \eta)$ is then defined. The tunable parameter vector η in $M(z, \eta)$ defines the position of the zeros. Throughout the paper, we assume that the controller $C(z, \rho)$ has a fixed integral action so that we know that the static gain of the closed-loop is 1; we then fix the static gain of $M(z, \eta)$ to 1. Moreover, for simplicity we assume that the reference models $\bar{M}(z)$ and $M(z, \eta)$ have all their poles in a, $|a| < 1$. It then makes sense to express all their transfer functions as combinations of Laguerre basis functions with poles in a.

It is well known that, as long as $T(z,\rho)$ is stable, it can be expressed as a Laguerre expansion with infinite elements, i.e.:

$$T(z,\rho) = \sum_{k=1}^{\infty} g_k(\rho) L_k(z,a) \qquad L_k(z,a) = \frac{K}{z-a}\left(\frac{1-a\,z}{z-a}\right)^{k-1} \qquad K = \sqrt{1-a^2} \tag{2}$$

the convergence rate of the expansion depending on a. The reader is referred to ([2, 9, 10]) for the general theory; moreover, of particular interest is the paper ([11]) in which the Laguerre expansion is treated in connection with control criteria.
The adjustable reference model $M(z,\eta)$ is then given by

$$M(z,\eta) = \sum_{k=1}^{n} \eta_k\, L_k(z,a) \quad \text{subject to} \quad M(1,\eta) = 1\,,$$

and we assume that the desired reference model belongs to that family, i.e. $\bar{M}(z) = M(z,\bar{\eta})$ for some $\bar{\eta} = [\bar{\eta}_1\,, \bar{\eta}_2\,, \dots\,, \bar{\eta}_n]^T$.
The design criterion $J_\lambda(\eta,\rho)$ is defined as the following modification of the criterion (1):

$$J_\lambda(\eta,\rho) = \frac{1-\lambda}{N}\sum_{t=1}^{N}\left(y(t,\rho) - M(z,\eta)r(t)\right)^2 + \frac{\lambda}{N}\sum_{t=1}^{N}\Big(y(t,\rho) - \bar{M}(z)r(t)\Big)^2 . \tag{3}$$

The criterion $J_\lambda(\eta,\rho)$ is minimized with respect to η and ρ:

$$(\eta^*_\lambda, \rho^*_\lambda) = \arg\min_{\eta,\rho} J_\lambda(\eta,\rho) \quad \text{subject to} \quad M(1,\eta) = 1\,. \tag{4}$$

This defines the optimal controller $C(z,\rho^*)$. The parameter λ expresses the relative importance of the desired response $\bar{M}(z)\,r(t)$. The minimization of $J_\lambda(\eta,\rho)$ can be performed through a suitable modification of the IFT procedure for the classical model-reference criterion. The same number of experiments is required. As usual, at each step one can obtain an estimate of the gradient and a positive definite approximation of the Hessian. We shall not elaborate on these modifications, since they are straightforward. Instead, the focus of this paper is to motivate the adoption of this new criterion $J_\lambda(\eta,\rho)$ and to show how well it copes with the problem of mismatch between a desired reference model $\bar{M}(z)$ and the plant $P(z)$, in terms of delay and non-minimum phase zeros

In order to motivate $J_\lambda(\eta,\rho)$, let us first consider $J_0(\eta,\rho)$ (i.e. $J_\lambda(\eta,\rho)$ for $\lambda = 0$). Since the zeros of the adjustable reference-model $M(z,\eta)$ are completely free, one should expect that the controller tuned according to $J_0(\eta,\rho)$ spends all its degrees of freedom on the placement of the poles of the closed-loop transfer function. We could refer to this strategy as approximate pole-placement control, since, due to the restricted complexity of the controller, the poles will not be precisely in a. Then, since we have from (3) that:

$$J_\lambda(\eta,\rho) = (1-\lambda)\, J_0(\eta,\rho) + \lambda\, J^{\mathrm{MR}}(\rho)\,, \tag{5}$$

we conclude that $J_\lambda(\eta,\rho)$ realizes a compromise between such (approximate) pole-placement strategy and the classical model-reference criterion.
In order to put such intuitive reasoning on a more formal basis, we define $\bar{J}_\lambda(\rho)$ as:

$$\bar{J}_\lambda(\rho) = \min_\eta J_\lambda(\eta,\rho) \quad \text{subject to} \quad M(1,\eta) = 1\,. \tag{6}$$

Observe that the optimal controller $C(z,\rho^*_\lambda)$ defined by (4) is, equivalently, defined by

$$\rho^*_\lambda = \arg\min_\rho \bar{J}_\lambda(\rho)\,.$$

The criterion $\bar{J}_\lambda(\rho)$ can also be written as:

$$\bar{J}_\lambda(\rho) = (1-\lambda)\,\bar{J}_0(\rho) + \lambda\, J^{\mathrm{MR}}(\rho)\,.$$

Let us introduce the notation:

$$\tilde{g}(\rho) = [\, g_{n+1}(\rho) \;\ldots\ldots\,]\,.$$

We shall refer to $\tilde{g}(\rho)$ as the vector of the coefficients of the tail of the Laguerre expansion of $T(z,\rho)$. Then, we have the following proposition for the expression of $\bar{J}_0(\rho)$.

Proposition 1
Let $T(z,\rho)$ have static gain 1 (i.e. $C(z,\rho)$ contains a fixed integrator), and let both $\bar{M}(z)$ and $M(z,\eta)$ have static gain 1. Then the criterion $\bar{J}_0(\rho)$ takes the form:

$$\bar{J}_0(\rho) = \tilde{g}(\rho)^T\, Q_0\, \tilde{g}(\rho)$$

in which Q_0 depends on the pole location a and the reference signal $r(t)$.
Proof: see Appendix.

Proposition 1 states that tuning the controller according to $\arg\min_{\eta,\rho} J_0(\eta,\rho)$ corresponds to minimizing a quadratic cost function constructed with the coefficients of the tail of the Laguerre expansion of $T(z,\rho)$. The non-negative definite weighting matrix Q_0 depends on the particular choice of the reference signal (in Section 3 we illustrate the case of a step reference signal). The connection with (approximate) pole-placement lays in the observation that minimizing $\bar{J}_0(\rho)$ with respect to ρ corresponds to minimizing some norm of the tail of the Laguerre expansion of $T(z,\rho)$. It is well known from Laguerre function theory (see [1, 10]) that if the coefficients of the Laguerre expansion of $T(z,\rho)$ converge to zero quickly, this means that the poles of $T(z,\rho)$ are close to a.
In the general case where $\lambda \neq 0$, the optimal controller will again attempt to put all the poles of $T(z,\rho)$ near a. Since

$$\bar{J}_\lambda(\rho) = (1-\lambda)\,\tilde{g}(\rho)^T\, Q_0\, \tilde{g}(\rho) + \frac{\lambda}{N}\sum_{t=1}^{N}\Big(y(t,\rho) - \bar{M}(z)r(t)\Big)^2\,,$$

the optimal controller will result from a compromise between making the tail of $T(z,\rho)$ small (in the $\| \; \|_{Q_0}$ norm) and making $T(z,\rho)$ close to $\bar{M}(z)$, both of which correspond to putting all poles close to a.
Thus, in terms of the location of the closed-loop poles, the new criterion is consistent with the pole-placement objective of the traditional criterion $J^{\mathrm{MR}}(\rho)$. The key difference is in the handling of delay and/or non-minimum phase zeros. If $P(z)$ has non-minimum phase zeros which have not been put in $\bar{M}(z)$, then the adjustable model reference parameter vector η^* will reflect this and, for small enough λ, the optimal controller will be essentially determined by the minimization of $\bar{J}_0(\rho)$.

In Section 3 we develop the result of Proposition 1 for a step reference signal. In Section 4 we illustrate the use of the modified IFT criterion with some simulation examples.

3 Some results for the step reference signal

In this section, we assume that the reference signal $r(t)$ is a step. The aim of the section is to analyze the behaviour for this particular case. This will allow us to establish a connection with the masked-IFT developed in [7, 8]. First we collect our assumptions as follows.

Assumption A
The reference signal $r(t)$ is a step applied at time 0 when the closed-loop system is at rest. The controller $C(z,\rho)$ includes a fixed integral action (i.e. $T(z,\rho)$ has gain 1). Both the desired reference model $\bar{M}(z)$ and the parameterized reference model $M(z,\eta)$ have gain 1. $\square$

We also introduce the notation:

$$\begin{aligned}
\bar{g}(\rho) &= [\, g_2(\rho) \, \ldots \, g_n(\rho) \,]^T \\
\bar{m} &= [\, \bar{\eta}_2 \, \ldots \, \bar{\eta}_n \,]^T \, .
\end{aligned}$$

Since we have assumed that $T(z,\rho)$ has gain 1, the coefficient $g_1(\rho)$ is determined for any given $[\, \bar{g}(\rho)^T \;\; \tilde{g}(\rho)^T \,]^T$, and the same holds for $\bar{\eta}_1$ and $\bar{m}$.
The next proposition concerns the relationship between the step response of a system and the coefficients of its Laguerre expansion.

Proposition 2
Let $T(z)$ be a stable transfer function with gain 1 and let $T(z) = \sum_{k=1}^{\infty} g_k \, L_k(z,a)$ be its

Laguerre expansion as in (2). Define $g = [\,g_2\ g_3 \ldots.\,]$ and $q_i(t)$ as:

$$q_i(t) = \frac{K\ (1-a^2)}{1-a}\left(\frac{1-az}{z-a}\right)^{i-1}\frac{z}{(z-a)^2}\,\delta(t)\quad i = 1, 2, \ldots.$$

Let V be the upper triangular matrix of infinite dimension

$$V = \begin{bmatrix} 1 & 1 & 1 & \cdot \\ 0 & 1 & 1 & \cdot \\ 0 & 0 & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}. \tag{7}$$

Then the following expression holds for the step response of $T(z)$:

$$T(z)\text{step}(t) = \frac{1-a}{K}\frac{K}{z-a}\,\text{step}(t) - [q_1(t)\,q_2(t)\ \ldots]\,V\,g\,.$$

□

Proof: see [6].

Using the above result, we can write the expressions of $\bar{J}_0(\rho)$ and $J^{\mathrm{MR}}(\rho)$ in terms of the coefficients of the Laguerre expansion of $T(z,\rho)$ for the case of a step reference signal. In the following proposition, we give the asymptotic expressions of $\bar{J}_0(\rho)$ and $J^{\mathrm{MR}}(\rho)$ as $N \to \infty$. Even though these are asymptotic results, such expressions approximate $\bar{J}_0(\rho)$ and $J^{\mathrm{MR}}(\rho)$ very well as long as N is greater than the settling time of the step response of $T(z,\rho)$ (this is discussed in [6]). Since it is reasonable to choose an N in $J^{\mathrm{MR}}(\rho)$ that satisfies such a condition, we are justified in considering the following expressions to be of interest.

The expression of $J^{\mathrm{MR}}(\rho)$ is given in the following proposition.

Proposition 3

Let Assumption A hold. Define α and β as

$$\alpha = \frac{1+a^2}{(1-a)^2} \qquad \beta = \frac{a}{(1-a)^2}\,.$$

Then the following holds:

$$\lim_{N\to\infty} N \cdot J^{\mathrm{MR}}(\rho) = \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix}^T V^T\,\bar{Q}_{\mathrm{MR}}\,V \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix} \tag{8}$$

$$\bar{Q}_{\mathrm{MR}} = \begin{bmatrix} \alpha & \beta & 0 & 0 & \cdot \\ \beta & \alpha & \beta & 0 & \cdot \\ 0 & \beta & \alpha & \beta & \cdot \\ 0 & 0 & \beta & \alpha & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

with V defined as in (7). □

Proof: see the Appendix.

We now examine the expression of $\bar{J}_0(\rho)$ for a step reference signal. The first step is to find the minimum point of $J_0(\eta,\rho)$ with respect to η. This is given below.

Proposition 4
Let Assumption A hold. Define $\hat{\eta}(\rho)$ as

$$\hat{\eta}(\rho) = \arg\min_{\eta} J_0(\eta,\rho) \quad \text{subject to} \quad M(1,\eta) = 1\,.$$

Then, in the limit as $N \to \infty$, $\hat{\eta}(\rho)$ satisfies:

$$\hat{\eta}(\rho) - \begin{bmatrix} g_1(\rho) \\ \tilde{g}(\rho) \end{bmatrix} = \left(\frac{1+a}{1-a^{2n}} \sum_{k=n+1}^{\infty} g_k(\rho) \right) \left[\begin{bmatrix} (-a)^{n-1} \\ \vdots \\ (-a)^1 \\ (-a)^0 \end{bmatrix} + (-a)^n \begin{bmatrix} (-a)^0 \\ (-a)^1 \\ \vdots \\ (-a)^{n-1} \end{bmatrix} \right] . \tag{9}$$

□

Proof: see [6].

The criterion $\bar{J}_0(\rho)$ is then obtained by substituting $\hat{\eta}(\rho)$ (as given by (9)) in the expression of $J_0(\eta,\rho)$. For the case where the pole of the Laguerre expansion is set at $a = 0$ we have the following interesting result. Notice that this corresponds to the classical impulse response representation, i.e. $T(z,\rho) = \sum_{k=1}^{\infty} g_k(\rho) z^{-k}$.

Proposition 5
Let Assumption A hold. Let $a = 0$, then the following holds:

$$\bar{J}_0(\rho) = J_{t_0=n}^{\text{Masked}}(\rho).$$

□

Proof: see the Appendix.

The above result puts the criterion of masked-IFT in the framework of this paper. Following from the discussion of Section 2, it comes out that the use of masked IFT corresponds to trying to place all poles of the closed-loop system at the origin.
The general expression of $\bar{J}_0(\rho)$ - for an arbitrary a - is given below.

Proposition 6
Let Assumption A hold. Let α and β be as in Proposition 3, and define c as:

$$c = \frac{a^2}{(1-a)^2} \frac{1-a^{2n-2}}{1-a^{2n}}\,.$$

Then the following holds:

$$\lim_{N\to\infty} N\cdot \bar{J}_0(\rho) = \tilde{g}(\rho)^T\, V^T\, \bar{Q}_0\, V\, \tilde{g}(\rho) \tag{10}$$

$$\bar{Q}_0 = \begin{bmatrix} \alpha - c & \beta & 0 & 0 & \cdot \\ \beta & \alpha & \beta & 0 & \cdot \\ 0 & \beta & \alpha & \beta & \cdot \\ 0 & 0 & \beta & \alpha & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

with V defined as in (7). $\square$

Proof: see the Appendix.

For a choice of λ different from 0, one obtains a compromise between (8) and (10). The general expression of $\bar{J}_\lambda(\rho)$, for any λ, is given below.

Proposition 7

Let Assumption A hold. Let c be defined as in Proposition 6 and define c_λ as:

$$c_\lambda = (1-\lambda)\, c\,.$$

Then the following holds:

$$\lim_{N\to\infty} N\cdot \bar{J}_\lambda(\rho) = \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix}^T V^T\, \bar{Q}_\lambda\, V \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix}$$

$$\bar{Q}_\lambda = \left[\begin{array}{ccccc} \lambda \begin{bmatrix} \alpha & \beta & 0 & \cdot & 0 \\ \beta & \alpha & \beta & \cdot & 0 \\ 0 & \beta & \alpha & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \beta \\ 0 & 0 & 0 & \beta & \alpha \end{bmatrix} & \lambda\beta & & & 0 \\ \lambda\beta & \alpha - c_\lambda & \beta & 0 & 0 \quad \cdot \\ & \beta & \alpha & \beta & 0 \quad \cdot \\ 0 & 0 & \beta & \alpha & \beta \quad \cdot \\ & 0 & 0 & \beta & \alpha \quad \cdot \\ & \cdot & \cdot & \cdot & \cdot \quad \cdot \end{array}\right]$$

with V defined as in (7). $\square$

Proof: the proof is easily obtained from Propositions (3) and (6).

4 Simulation example

In this section, we illustrate the tuning of a simple controller - according to the criterion $J_\lambda(\eta,\rho)$ - for a non-minimum phase plant. The plant that we consider has the following

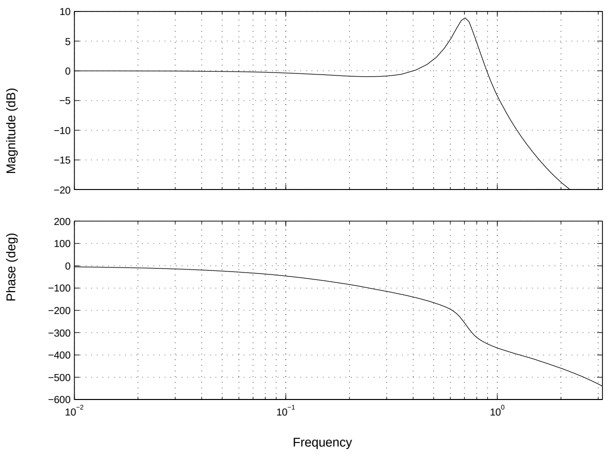

Figure 1: Bode plots of $P(z)$.

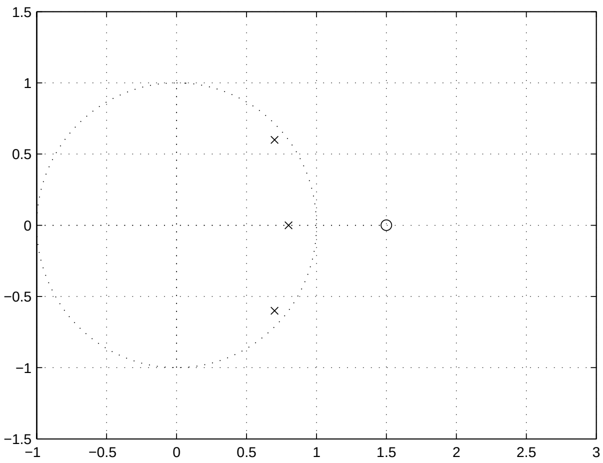

Figure 2: zeros (o) and poles (x) of $P(z)$.

transfer function:

$$P(z) = \frac{0.18 \cdot (-z + 1.5)}{(z - 0.8)(z^2 - 1.4z + 0.85)} .$$

The Bode plots and the pole-zero map of $P(z)$ are displayed in Figures 1 and 2 respectively. The objective is to tune the following simple controller for such plant:

$$C(z, \rho) = \frac{\rho_0 + \rho_1 z^{-1} + \rho_2 z^{-2}}{1 - z^{-1}} .$$

The controller is tuned for a step reference signal using data from noise-free simulations. The reference models are chosen as (we will consider different values of a):

$$\bar{M}(z) = \frac{(1-a)^6 \cdot z^4}{(z-a)^6} \qquad\qquad M(z, \eta) = \sum_{k=1}^{6} \eta_k \, L_k(z, a) .$$

Notice that the desired reference model $\bar{M}(z)$ has the same delay as the plant but does not have the non-minimum phase zero in 1.5. In order to choose the parameter a in the reference models we have performed an iterative procedure. We have started from $a = 0.8$ and tuned the controller for this choice of a. Then, we have progressively reduced the value of a and for each value we have re-tuned the controller starting from the controller tuned for the previous value. At each step, we have also tried different values of λ. For each so-obtained controller, we have registered the settling time of the step response of the corresponding closed-loop transfer function. In the table we report some values of the achieved settling times together with the corresponding values of the design cost function. The minimum settling time (for a given a) was always obtained for $\lambda = 0$.

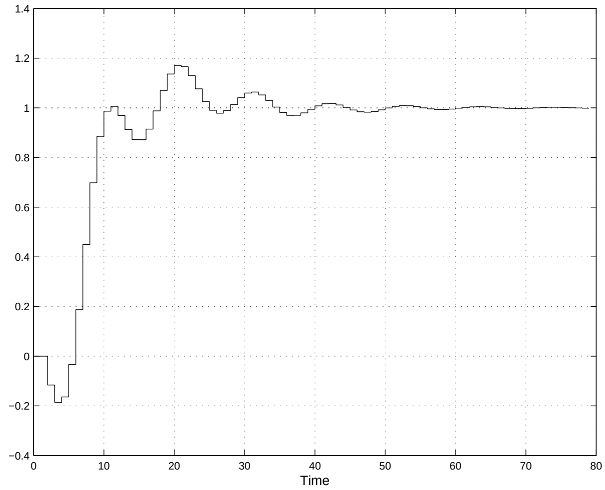

Figure 3: step response of $T(\rho_1^*)$.

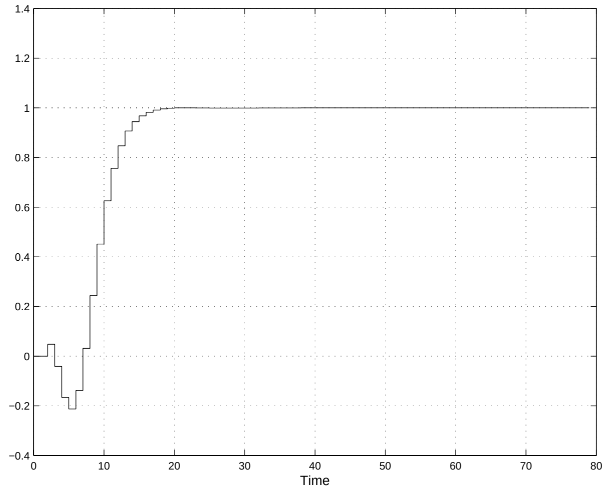

Figure 4: step response of $T(\rho_0^*)$.

	$\lambda = 1$		$\lambda = 0$	
a	Settling time 2%	J_1	Settling time 2%	J_0
0.8	52	$3.6 \cdot 10^{-4}$	52	$4.0 \cdot 10^{-6}$
0.7	49	$5.0 \cdot 10^{-4}$	37	$6.4 \cdot 10^{-5}$
0.6	35	$1.6 \cdot 10^{-3}$	28	$1.2 \cdot 10^{-4}$
0.5	34	$5.0 \cdot 10^{-4}$	15	$9.2 \cdot 10^{-7}$
0.4	38	$1.4 \cdot 10^{-2}$	16	$2.9 \cdot 10^{-7}$
0.3	48	$2.8 \cdot 10^{-2}$	18	$1.0 \cdot 10^{-4}$

In the following we illustrate the case $a = 0.4$.
The controller parameter obtained for $\lambda = 1$ was:

$$\rho_1^* = [\,0.64592 \quad -0.71086 \quad 0.19212\,].$$

The controller and the tunable reference-model parameters obtained for $\lambda = 0$ were:

$$\begin{aligned}
\rho_0^* &= [-0.26580 \quad 0.94611 \quad -0.58753] \\
\eta_0^* &= [-0.00318 \quad -0.07513 \quad -0.02353 \quad 0.518381 \quad 0.448899 \quad 0.134582\,].
\end{aligned}$$

The step response of $T(z, \rho_1^*)$ (i.e. the classical IFT criterion) is displayed in Figure 3. Compare with the step response of $T(z, \rho_0^*)$, obtained from our new adjustable criterion, which is displayed in Figure 4. The pole-zero map of $M(z, \eta_0^*)$ is shown in Figure 5. Note that $M(z, \eta_0^*)$ has the non-minimum phase zero in 1.5 and almost reproduces the delay of $P(z)$ through the zero located very far from the origin. The pole-zero maps of $T(z, \rho_1^*)$ and $T(z, \rho_0^*)$ are displayed in Figures 6 and 7 respectively. Notice how for $\lambda = 0$ the closed-loop poles are actually much closer $a = 0.4$.

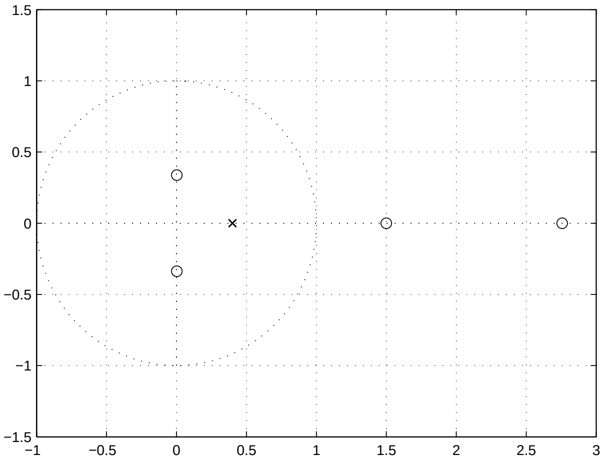

Figure 5: zeros (o) and poles (x) of $M(z, \eta_0^*)$ (+ one zero in -2961.3).

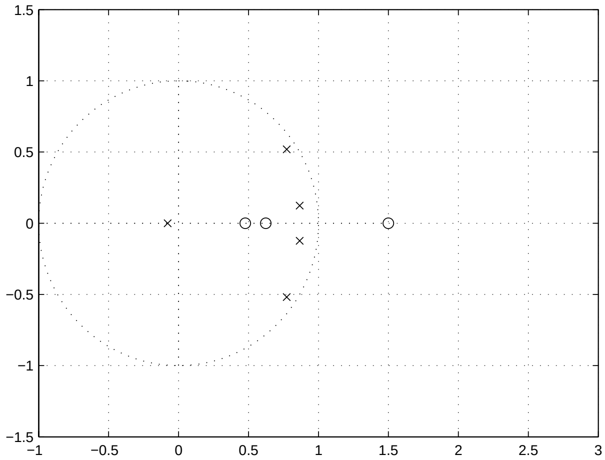

Figure 6: zeros (o) and poles (x) of $T(\rho_1^*)$.

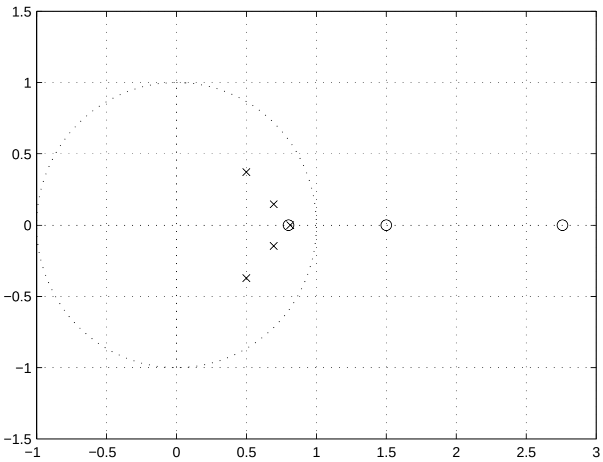

Figure 7: zeros (o) and poles (x) of $T(\rho_0^*)$.

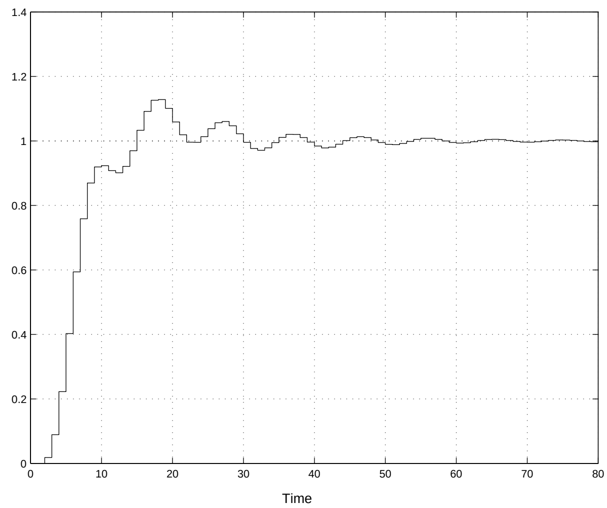

Figure 8: step response of $\bar{T}(\rho_1^*)$.

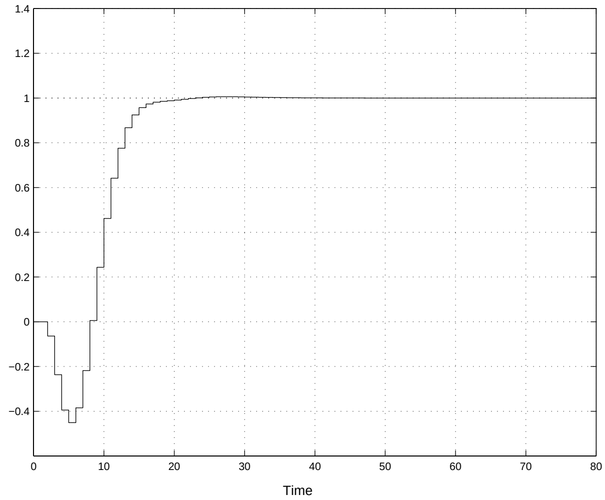

Figure 9: step response of $\bar{T}(\rho_0^*)$.

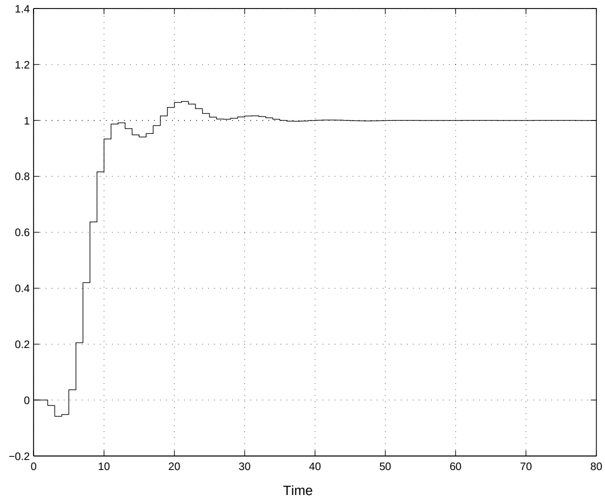

Figure 10: step response of $\bar{T}(\rho_{0.02}^*)$.

Surprisingly enough, the best controller (i.e. $C(z, \rho_0^*)$) comes out to be non-minimum phase.

Let us now consider a different plant with transfer function:

$$\bar{P}(z) = \frac{0.036 \cdot (z + 1.5)}{(z - 0.8)(z^2 - 1.4z + 0.85)} .$$

We consider again the choice $a = 0.4$ in the reference models.
The controller tuned for $\lambda = 1$ (i.e. classical IFT) was given by:

$$\rho_1^* = [\,0.49961 \quad -0.37388 \quad 0.04700\,] .$$

The corresponding closed-loop transfer function presented an oscillatory step response as shown in Figure 8. For the case $\lambda = 0$ we obtained:

$$\begin{aligned} \rho_0^* &= [\,-1.78114 \quad 3.57684 \quad -1.71820\,] \\ \eta_0^* &= [\,-0.08813 \quad -0.27261 \quad 0.02017 \quad 0.56294 \quad 0.51173 \quad 0.26589\,] . \end{aligned}$$

The corresponding step response is displayed in Figure 9. As happened for the first example, also in this case the controller tuned for $\lambda = 0$ turned out to be non-minimum phase and achieved the minimum settling time. On the other hand, in this case the price payed was a very large under-shoot in the initial response. The initial response improved significantly by switching to $\lambda = 0.02$ at the cost of a larger settling time. In this case we obtained:

$$\begin{aligned} \rho_{0.02}^* &= [\,-0.53925 \quad 1.45662 \quad -0.79073\,] \\ \eta_{0.02}^* &= [\,-0.01107 \quad 0.02601 \quad 0.29964 \quad 0.48521 \quad 0.29321 \quad -0.09301\,] . \end{aligned}$$

The corresponding step response is displayed in Figure 10.

5 Conclusions

In this paper we have proposed an adjustable criterion $J_\lambda(\eta, \rho)$ for the tuning of feedback controllers using IFT. Some results on $J_\lambda(\eta, \rho)$ have been given within the framework of the Laguerre expansions. The effectiveness of the criterion has been illustrated also through simulation examples. A deeper analysis of $J_\lambda(\eta, \rho)$ will be the aim of further research. The ability of $M(z, \eta)$ to capture the delay and the non-minimum phase zeros of the plant, the role of the weighting matrix Q_0 in the placement of the closed-loop poles and the sensitivity of the solution to λ require further investigation.

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A Proofs

The following Lemma will be used in the proof of Proposition 1.

Lemma 1

Consider the quadratic function:

$$Y(x) = \frac{1}{2}\, x^T A x - F^T x$$

where $A \in I\!R^{nxn}$ is a symmetric positive definite matrix and $F \in I\!R^n$.
Define $\hat{x}$ as:

$$\hat{x} = \arg\min_x Y(x) \quad \text{subject to} \quad U^T x = d$$

where $U = [1\,1\,\ldots\,1]^T$.
Then, $\hat{x}$ is given by

$$\begin{aligned} \hat{x} &= A^{-1}\left(F + \hat{\lambda}\, U\right) \\ \hat{\lambda} &= \frac{d}{U^T A^{-1} U} - \frac{U^T A^{-1} F}{U^T A^{-1} U}\,, \end{aligned} \tag{11}$$

and the following holds:

$$Y(\hat{x}) = \frac{1}{2}\,\hat{\lambda}^2\, U^T A^{-1} U - \frac{1}{2}\, F^T A^{-1} F\,.$$

□

Proof

The minimizing $\hat{x}$ is obtained, using Lagrange multipliers, by solving the following system of equations with respect to x and λ:

$$\left\{ \begin{array}{l} Ax - F = \lambda U \\ \\ U^T x = d \end{array} \right. .$$

It is easy to show that the solution is given by (11). The expression of $Y(\hat{x})$ is then obtained by simple substitution. □

Proof of Proposition 1
Recall first that:

$$\bar{J}_\lambda(\rho) = \min_\eta J_\lambda(\eta,\rho) \quad \text{subject to} \quad M(1,\eta) = 1$$

where

$$J_0(\eta,\rho) = \frac{1}{N}\sum_{t=1}^{N} \left(T(z,\rho)r(t) - M(z,\eta)r(t)\right)^2 .$$

Define:

$$\begin{aligned} \phi(t) &= [\,L_1(z,a)\,, \ldots\,, L_n(z,a)\,]^T\, r(t) \\ \tilde{\phi}(t) &= [\,L_{n+1}(z,a)\,, \ldots \ldots]^T\, r(t)\,. \end{aligned}$$

Then $\bar{J}_0(\rho)$ can be rewritten as:

$$\bar{J}_0(\rho) = \min_\zeta \frac{1}{N}\sum_{t=1}^{N}\left(\phi(t)^T\,\zeta - \tilde{\phi}(t)^T\tilde{g}(\rho)\right)^2 \quad \text{subject to} \quad U^T\,\zeta = \frac{1-a}{K}\,S^T\tilde{g}(\rho) \qquad (12)$$

where $\zeta = [\,\eta_1 - g_1(\rho), \ldots\, \eta_n - g_n(\rho)\,]^T$, $U = [1\,1\,\ldots\,1]^T$ and $S = [1\,1\,\ldots.]^T$.
The minimizing argument, $\hat{\zeta}(\rho)$, is obtained by Lemma 1 as:

$$\begin{aligned} \hat{\zeta}(\rho) &= A^{-1}\left(B^T\,\tilde{g}(\rho) + \hat{\lambda}(\rho)\,U\right) \\ \hat{\lambda}(\rho) &= \frac{C^T\,\tilde{g}(\rho)}{U^T A^{-1} U} \end{aligned}$$

where

$$\begin{aligned} A &= \frac{1}{N}\sum_{t=1}^{N}\phi(t)\phi(t)^T \\ B &= \frac{1}{N}\sum_{t=1}^{N}\tilde{\phi}(t)\phi(t)^T \\ C &= \frac{1-a}{K}S - BA^{-1}U^T\,. \end{aligned}$$

The $\bar{J}_0(\rho)$ is then obtained by substituting $\hat{\zeta}(\rho)$ to ζ in (12):

$$\begin{aligned} \bar{J}(\rho) &= \tilde{g}(\rho)^T\left[\frac{CC^T}{U^T A^{-1} U} + S\right]\tilde{g}(\rho) \\ S &= \frac{1}{N}\sum_{t=1}^{N}\tilde{\phi}(t)\tilde{\phi}(t)^T - BA^{-1}B^T\,. \end{aligned}$$

□

Proof of Proposition 3

Using Proposition (2) we can write

$$\bar{M}(z)\text{step}(t) - T(z,\rho)\text{step}(t) = [q_1(t)\, q_2(t)\, \ldots]\, V \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix},$$

from which we obtain:

$$J^{\text{MR}}(\rho) = \frac{1}{N} \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix}^T V^T \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \end{bmatrix} [q_1(t)\, q_2(t)\, \ldots]\, V \begin{bmatrix} \bar{m} - \bar{g}(\rho) \\ \tilde{g}(\rho) \end{bmatrix}.$$

The proof is then completed using:

$$\sum_{t=1}^{\infty} q_i(t) q_{i+j}(t) = \begin{cases} \alpha & j = 0 \\ \beta & j = 1 \\ 0 & j > 1 \end{cases},$$

for the proof of the above equations see [6]. □

Proof of Proposition 5

The result of Proposition (4) for the case $a = 0$ holds exactly also for a finite N (see [6]). Hence, we have that - for $a = 0$ - the following holds

$$\hat{\eta}(\rho) - \begin{bmatrix} g_1(\rho) \\ \tilde{g}(\rho) \end{bmatrix} = \left(\sum_{k=n+1}^{\infty} g_k(\rho) \right) \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

By substituting the above expression in $J_0(\eta, \rho)$ we obtain that $\bar{J}_0(\rho) = J_0(\hat{\eta}(\rho), \rho)$ is given by:

$$\bar{J}_0(\rho) = \frac{1}{N} \sum_{t=1}^{N} \left(\left(\sum_{k=n+1}^{\infty} g_k(\rho) \right) z^{-n}\, \text{step}(t) - \sum_{k=n+1}^{\infty} g_k(\rho) z^{-k}\, \text{step}(t) \right)^2.$$

In order to prove the proposition, we have to show that:

$$\left(\sum_{k=n+1}^{\infty} g_k(\rho) \right) z^{-n}\, \text{step}(t) - \sum_{k=n+1}^{\infty} g_k(\rho) z^{-k}\, \text{step}(t) = \begin{cases} 0 & t < n \\ \\ \text{step}(t) - T(z,\rho)\text{step}(t) & t \geq n \end{cases}.$$

For $t < n$ the above equation is obvious. As for $t \geq n$, notice that:

$$\begin{aligned} T(z,\rho)\text{step}(t) &= g_1(\rho)\text{step}(t-1) + \ldots + g_n(\rho)\text{step}(t-n) + \sum_{k=n+1}^{\infty} g_k(\rho) z^{-k}\, \text{step}(t) \\ &= \sum_{k=1}^{n} g_k(\rho) + \sum_{k=n+1}^{\infty} g_k(\rho) z^{-k}\, \text{step}(t) \qquad t \geq n \\ &= 1 - \sum_{k=n+1}^{\infty} g_k(\rho) + \sum_{k=n+1}^{\infty} g_k(\rho) z^{-k}\, \text{step}(t) \qquad t \geq n \end{aligned}$$

and the Proposition is then proved. $\square$

Proof of Proposition 6

Using Proposition (2) we can write:

$$\begin{aligned} M(z,\hat{\eta}(\rho))\mathrm{step}(t) - T(\rho)\mathrm{step}(t) &= \sum_{i=1}^{n-1}\left(\sum_{k=i+1}^{n} -\hat{\eta}_k(\rho) + g_k(\rho)\right) q_i(t) + \left(\sum_{k=n+1}^{\infty} g_k(\rho)\right) q_i(t) \\ &+ \sum_{i=n}^{\infty}\left(\sum_{k=i+1}^{\infty} g_k(\rho)\right) q_i(t)\,. \end{aligned}$$

Since Proposition (4) give us an explicit expression for $g_k(\rho) - \hat{\eta}_k(\rho)$, we have

$$\begin{aligned} M(z,\hat{\eta}(\rho))\mathrm{step}(t) - T(\rho)\mathrm{step}(t) &= \left(\sum_{k=n+1}^{\infty} g_k(\rho)\right) p_n(t) + \sum_{i=n}^{\infty}\left(\sum_{k=i+1}^{\infty} g_k(\rho)\right) q_i(t) \\ &= [q_n(t) + p_n(t) \quad q_{n+1}(t) \ \ \ldots]\ V\,\tilde{g}(\rho) \end{aligned}$$

in which $p_n(t)$ is given by:

$$\begin{aligned} p_n(t) &= \sum_{i=1}^{n-1} c_i q_i(t) \\ c_i &= 1 - \frac{1+a}{1-a^{2n}} \sum_{k=i+1}^{\infty}\left((-a)^{n-i} + (-a)^n(-a)^{k-1}\right) \quad = \frac{(-a)^{n-i} - (-a)^{n+i}}{1-a^{2n}}\,. \end{aligned}$$

Therefore, we obtain that $\bar{J}_0(\rho)$ is given by:

$$\bar{J}_0(\rho) = \frac{1}{N}\,\tilde{g}(\rho)^T V^T \begin{bmatrix} q_n(t) + p_n(t) \\ q_{n+1}(t) \\ \vdots \end{bmatrix} [q_n(t) + p_n(t) \quad q_{n+1}(t) \ \ \ldots]\ V\,\tilde{g}(\rho)\,.$$

The following equations are obtained after some simple but cumbersome calculations:

$$\begin{aligned} \sum_{t=1}^{\infty} p_n(t)^2 &= \frac{a^2 - a^{2n}}{(1-a)^2(1-a^{2n})} \\ \sum_{t=1}^{\infty} \left(q_n(t) + p_n(t)\right)^2 &= \alpha - \frac{a^2}{(1-a)^2}\frac{1-a^{2n-2}}{1-a^{2n}} \\ \sum_{t=1}^{\infty} \left(q_n(t) + p_n(t)\right) q_{n+1}(t) &= \beta\,. \end{aligned}$$

Using such equations the limit expression of $\bar{J}_0(\rho)$ can be finally obtained. $\square$