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IDENTIFIABILITY OF CLOSED LOOP SYSTEMS USING THE JOINT INPUT-OUTPUT IDENTIFICATION METHOD.

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Abstract. The identifiability of multiple input multiple output linear dynamic systems operating in closed loop is considered for the case where the plant and the regulator dynamics are both linear and time-invariant. Two basic identification methods have been proposed for such systems : the joint input-output method, in which the input and output processes are modeled jointly as the output of a white noise driven system ; and the direct method, in which a prediction error method is used on the input-output data just as if the system were in open loop. Previously obtained identifiability results for the joint input-output method are extended to a variety of new situations, including a possible one-sided correlation between the regulator noise and the process noise. These new identifiability conditions are now very close to those obtained for the direct method.

Keywords. Identification ; multivariable systems ; closed loop systems ; spectral factorization ; stochastic systems.

I. INTRODUCTION

The identifiability of multiple input multiple output (MIMO) linear dynamic systems operating in closed loop has been the subject of much research in recent years. See [1] for an excellent survey on this subject. The question at hand is whether the forward path dynamics (i.e. the plant or process dynamics) can be identified from input and output measurements despite the presence of feedback whilst the measurements are taken. We shall restrict our attention here to the case where the feedback dynamics are unknown (e.g. the feedback is a manual operator), and where no external input perturbation signal can be applied for identification purposes.

For such a case two major identification methods have been proposed, which we shall briefly recall :

1) the direct identification method I_1 : an open loop model is identified using a prediction error method on the plant input-output data just as if the system were in open loop. This method has been proposed and extensively studied by L. Ljung, I. Gustavsson and T. Söderström [2].

2) the joint input-output identification method I_2 : the input and output processes are first modeled jointly as the output of a system driven by white noise. The plant dynamics are subsequently derived from the joint model by matrix operations. This method has been proposed by P.E. Caines and C.W. Chan [3], and independently by M.S. Phadke [4] - [5].

The method I_1 has the major advantage over I_2 that it allows for a wide variety of possible structures for the unknown regulator (namely the regulator can be nonlinear or time varying), whereas the application of I_2 is limited to systems with a linear and time-invariant regulator. Actually, the feedback dynamics are not even identified with I_1, whereas I_2 identifies both the plant and the feedback dynamics, even if this is in most practical cases unnecessary. On the other hand, when no a priori knowledge is available about the structure of the system, I_2 has the advantage that it allows the use of nonparametric methods, such as spectral or covariance factorization methods, whereas I_1 requires that a parametric structure be chozen a priori.

The identifiability of MIMO linear dynamic systems operating in closed-loop has been discussed in a number of papers under different sets of assumptions on the structure of the system, the regulator and the noise sources. The most general sets of identifiability conditions have been obtained for the direct method by T. Söderström, L. Ljung and I. Gustavsson [6], and for the joint input-output method by T. Ng, G. Goodwin and B. Anderson [7]. It turns out that, in the case of a time-invariant linear system with a time-invariant linear feedback (i.e. the only case where a comparison between I_1 and I_2 can be made), the two sets of sufficient conditions for identifiability derived in [6] and [7] are not identical. In particular for I_2 the conditions derived in [7] require that there be no correlation at all between the process noise m_i and the regulator noise n_i (see Fig.1) whereas for I_1 a one-sided correlation is allowed [6], namely a model of the form $n_1 = L(z)w_1 + \tilde{w}_1$, where $L(z)$ is a causal stable filter, and $\tilde{w}_1$ is orthogonal to w_1 ; moreover, $G(z)$ is restricted to being minimum phase, a nontrivial restriction when $L(z)$ is nonzero.

The reason for this discrepancy is that the identifiability conditions for I_2 have been derived under the assumption that both the plant and the feedback dynamics must be identifiable. In fact only a model of the plant is required, and it turns out that there is an equivalence class of spectral factors of the joint input-output process that is uniquely related to the same forward path dynamics (i.e. $F(z)$ and $G(z)$ in Fig.1), but that produces different feedback dynamics. We show in this paper that it is possible to identify $F(z)$ and $G(z)$ (but not $H(z)$ and $K(z)$) using the joint method I_2 when there is a one-sided correlation between the process noise and the regulator noise, namely a model of the form $n_1 = L(z)m_1 + \tilde{m}_1$, where $L(z)$ is a strictly causal stable filter and $\tilde{m}_1$ is orthogonal to m_1. The new set of identifiability conditions for I_2 is therefore weaker thant that derived in [7] and is now very similar to that obtained for I_1, in [6].

The paper is organized as follows. In section II the notations are introduced, the assumptionson the structure of the system and the noise sources are stated, and the identifiability results obtained in [6] and [7], for I_1 and I_2 respectively, are summarized. Section III gives some basic results in spectral factorization of multivariable processes, and establishes some relations between the matrix transfer functions of the input-output model and the spectral factors of the spectral density of the joint input-output process. Section IV gives a new set of sufficient conditions for the identifiability of the global model using I_2, i.e. for the case where it is desired to recover both the open loop and the feedback dynamics. In section V identifiability conditions are derived for I_2 in the case where only the process dynamics need be recovered, as is usually the case in practice. Due to space limitations all the lengthy proofs are omitted ; the reader is referred to the full version of the paper for these proofs.

II. STATEMENT OF THE PROBLEM.

In this paper we examine the identifiability of closed loop systems in the case where the process and the regulator dynamics are linear and time-invariant, and where the disturbances in the feedback path are a full rank stochastic process. The class of systems considered here is depicted in Fig.1, where $u_1 \in R^m$ is the process input, $y_1 \in R^p$ is the process output, $m_1 \in R^p$ is the noise in the forward path, $n_1 \in R^m$ is the noise in the feedback path, and $F(z)$, $G(z)$, $H(z)$ are causal rational transfer function matrices.

We further assume that m_1 is the output of a filter driven by white noise w_1 of covariance Q_{11} and with matrix transfer function $G(z)$. Assumptions on n_1 will be made later on.

The set-up of Fig.1 models the interconnection structure of a physical closed-loop system, except in one respect. In the physical system, noise signals may be introduced at some part internal to the plant and/or feedback controller. These noises have been referred to the outputs of the plant and controller in the model.

The following three assumptions on the closed loop system will be made throughout this paper :

2.1) there exists a delay somewhere in the closed loop system, i.e. $F(\infty)H(\infty) = 0$, where $F(\infty) = \lim_{z\to\infty} F(z)$

2.2) The joint process $[y_1^T, u_1^T]^T$ is stationary.

2.3) The joint process $[y_1^T, u_1^T]^T$ is a full rank bounded stochastic process.

Comments. (a) Assumption 2.2) clearly requires the closed-loop system to be asymptotically stable. It also apparently requires $G(z)$ to be asymptotically stable. This is however not quite so. Suppose the actual physical system has stationary noise entering at some internal part of the plant $F(z)$ in front of some instability. Then in referring that noise to the output, $G(z)$ will acquire one or more unstable poles. Technically, it is necessary and sufficient to have

McMillan degree of unstable part of $[F(z) \vdots G(z)]$ = McMillan degree of unstable part of $F(z)$

as well as closed loop asymptotic stability in order that Fig.1 correspond to a physical set-up with stationary $[y_1^T \; u_1^T]^T$

(b) Assumption 2.3) imposes that $[m_1^T \; n_1^T]^T$ be a full rank process.

The identifiability question can now be stated as follows : assuming that the process has the structure depicted in Fig.1 with the assumptions described above, is it possible to obtain consistent estimates of the transfer function matrices $F(z)$ and $G(z)$ from measurements of y_1 and u_1 ? The answer to this question will depend on the model structure chozen, the experimental conditions (i.e. the feedback structure) and the identification method.

a) The model structure : in case a parametric identification method is used, we shall assume that the class of models considered, $\mathcal{M}(\theta)$, contains the true system for a particular value of the parameter vector.

b) The experimental conditions : these have been partly specified already with the assumption that only linear time-invariant regulators are considered with a persistently exciting regulator noise. Different sets of assumptions will be made on the correlation between n_1 and m_1.

c) The identification method : two methods will be considered, the direct identification method I_1 and the joint input-output method I_2.

For I_1, a prediction error method is used on the data $\{u_i, y_i\}$ just as if they had been obtained in open loop. The estimate $\hat{\theta}_N$ is the minimizing element of a scalar function of $\frac{1}{N}\sum_1^N \varepsilon_i \varepsilon_i^T$, where ε_i is the prediction error on the linear least squares estimate $\hat{y}_{i/i-1}(\hat{\theta})$ of y_i given $\{U^{i-1}, Y^{i-1}\}$, the data up to time $i-1$.

With I_2 a white noise driven model is first obtained for the joint process $\begin{bmatrix} y_i \\ u_i \end{bmatrix}$.

The process model and also the regulator model are subsequently derived from the transfer function model of the joint process as will be shown later.

With the experimental conditions we have described so far and with the assumptions 2.1) and 2.3), the following identifiability results have been obtained.

Identification method I_1 (see [6]) :

Let $F_s(z)$, $G_s(z)$ denote the true system quantities and $F^\theta(z)$, $G(z)$ denote the transfer function matrices associated with the parameter value θ.

Let $D_s = \{\theta | F_s(z) = F_\theta(z),\ G_s(z) = G_\theta(z)\}$.

Then $F_s(z), G_s(z)$ is identifiable, i.e. $\hat{\theta}_N \to D_s$ as $N \to \infty$, whenever D_s is nonempty and the following conditions hold :

2.4) $n_i = K(z)v_i + L(z)w_i$

where $K(z)$ is of full normal row rank, and both $K(z)$ and $L(z)$ are causal* and asymptotically stable, $\{v_i\}$ is persistently exciting of any finite order and independent of the noise $\{w_i\}$

2.5) $F_s(\infty)L(\infty) = 0$

2.6) $G_s(z)$ has all its poles and zeros strictly inside the unit circle and $G_s(\infty) = I$.

The proof is given in [6].

* This means that $K(\infty)$ and $L(\infty)$ are finite, but not necessarily zero.

Comment: Assumptions 2.4) and 2.5) mean that a "one-sided correlation" is admissible between the regulator noise n_i and the process noise w_i, but with at least one delay from w_i to the "deterministic" output $F(z)u_i$. Assumption 2.6), though apparently not very restrictive (cfr. the spectral factorization theorem) does in fact constitute a nontrivial restriction, and it will play a crucial part in explaining the different identifiability conditions obtained for I_1 and I_2.

Identification method I_2 (see [7])

The system $\{F,G,H,K\}$ is identifiable in the sense that F and H may be found, and G and K found up to para-unitary equivalence (see below), if

2.7) $n_i = K(z)v_i$ (2.1)

(see Fig.2), where $K(z)$ is of full normal row rank, and where $\{v_i\}$ is a white noise sequence.

2.8) $\{v_i\}$ and $\{w_i\}$ are uncorrelated, i.e.

$$E\{\begin{bmatrix} w_i \\ v_i \end{bmatrix} (w_j^T v_j^T)\} = Q\delta_{ij} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix}\delta_{ij} \quad (2.2)$$

with Q a block-diagonal positive definite matrix.

2.9) $F(\infty) = H(\infty) = 0$

The proof is given in [7].

Comment: Assumptions 2.3), 2.7) and 2.8) force $\{m_i\}$ to be a full rank process independent of $\{n_i\}$, and accordingly $G(z)$ must have full normal row rank. Assumptions 2.7) - 2.8) show that the sufficient conditions derived for I_2 allow no correlation at all between n_i and w_i, whereas the conditions derived for I_1 allow one-sided correlation. Assumption 2.9) shows that a delay is required in each path, whereas only one delay in the closed loop is required for I_1. On the other hand, no assumption is made about the zeros of $G(z)$. Notice also that the conditions for I_2 are on the identifiability of the quadruple $\{F,G,H,K\}$ (see Fig.2), whereas in fact only $\{F,G\}$ need to be identified. This is because method I_2 is based on the relationship that exists between $\{F,G,H,K\}$ and the spectral density $\phi_{yu}(z)$ of the joint process $\begin{bmatrix} y_i \\ u_i \end{bmatrix}$

$$\phi_{yu}(z) = W(z)QW^*(z) \quad (2.3)$$

where $W^*(z)$ denotes $W^T(z^{-1})$, Q is defined by (2.2), and $W(z)$ is given by†

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} = \begin{bmatrix} (I-FH)^{-1}G & (I-FH)^{-1}FK \\ (I-HF)^{-1}HG & (I-HF)^{-1}K \end{bmatrix} \quad (2.4)$$

[The inverses exist by assumption (2.1)].

There exists a one-to-one correspondence between W and the quadruple $\{F,G,H,K\}$ [7]. The latter can be uniquely recovered from W as follows :

$$F = W_{12}W_{22}^{-1} \qquad H = W_{21}W_{11}^{-1}$$
$$G = W_{11} - W_{12}W_{22}^{-1}W_{21} \qquad K = W_{22} - W_{21}W_{11}^{-1}W_{12} \quad (2.5)$$

Method I_2 therefore consists in first estimating the transfer function W for the joint process, and then computing $\{F,G,H,K\}$ from (2.5). However the factorization (2.3) of $\phi_{yu}(z)$ is nonunique, and different spectral factors W will normally lead to different sets of quadruples $\{F,G,H,K\}$. The identifiability conditions 2.1) to 2.3) and 2.7) to 2.9) insure that if $\{\bar{F},\bar{G},\bar{H},\bar{K}\}$ is the quadruple obtained from the

† For the sake of simplicity we omit the argument z whenever possible.

unique minimum phase spectral factor $W(z)$ for which $W(\infty) = I$, while $\{F,G,H,K\}$ are the true or physical transfer functions, then

$$F = \bar{F},\ G = \bar{G}V_{11},\ H = \bar{H},\ K = \bar{K}V_{22} \qquad (2.6)$$

where $V_{11}(z)$ and $V_{22}(z)$ are rational matrices such that

$$V_{11}(z)Q_{11}V_{11}^{*}(z) = \bar{Q}_{11},\ V_{22}(z)Q_{22}V_{22}^{*}(z)=\bar{Q}_{22} \quad (2.7)$$

with Q_{11} and Q_{22} real symmetric positive definite. In other words, if the identifiability conditions for I_2 are satisfied, the matrix transfer functions $\bar{F},\bar{H}$ obtained from the minimum phase spectral factor are identical to the open loop and feedback transfer functions, while the noise models obtained from the minimum phase factorization are identical up to right multiplication by scaled para-unitary matrices $V_{11}(z)$ and $V_{22}(z)$ which do not alter the spectral density of n_1 and m_1 (cfr the spectral factorization theorem).[1]

From the preceding discussion on the nonuniqueness of the spectral factorization of $\phi_{yu}(z)$ it is clear that to guarantee identifiability with I_2, one must find a set of conditions on the structure of the process and the regulator such that

- either there exists only one spectral factor W that satisfies these conditions, in which case the true $\{F,G,H,K\}$ are related to this factor W by (2.5) ;
- or there exists a class of spectral factors satisfying these conditions, and the quadruples $\{\bar{F},\bar{G},\bar{H},\bar{K}\}$ corresponding to all the members of this class are related to the true quadruple by the relations (2.6).

On the other hand, we have already pointed out that in most practical cases, only F and G need to be recovered. Therefore the question of identifiability using method I_2 should be rephrased as follows : What are the conditions on the structure of the process and the regulator such that all possible spectral factors satisfying these conditions will lead to

$$F = \bar{F} \quad ,\ G = \bar{G}V_{11} \qquad (2.8)$$

with $V_{11}(z)$ as above. Here F,G are the transfer function matrices corresponding (through (2.5)) to an arbitrary member of the class of admissible spectral factors and $\bar{F},\bar{G}$ are the matrices corresponding to a particular member of that class (e.g. the unique minimum phase spectral factor $\bar{W}(z)$ for which $\bar{W}(\infty) = I$).

In order to investigate these questions, we first briefly review some results on spectral factorization, and on the relations between the spectral factors $W(z)$ of $\phi_{yu}(z)$ and the matrix transfer functions F,G,H,K.

III. RELATIONS BETWEEN SPECTRAL FACTORS OF $\phi_{yu}(z)$ AND THE CLOSED LOOP MODEL.

We first recall some basic results about spectral factorization theory. These results are discrete-time extensions of continuous-time results of D.C. Youla [8]. The proofs of [8] can easily be extended and will therefore be omitted.

Spectral factorization theorem :

Let $\phi(z)$ be the nxn rational spectral density matrix, of full normal rank, of a real stationary stochastic process.

a) There exists a real nxn rational matrix $H(z)$ such that $\phi(z) = H(z)H^{*}(z)$, where $H(\infty)$ is finite and nonsingular, $H(z)$ is stable (i.e. $H(z)$ is analytic in $|z|\geq 1$) and minimum phase (i.e. $H^{-1}(z)$ exists and is analytic in $|z|>1$). If $\phi(z)$ is positive definite on $|z|=1$, $H^{-1}(z)$ is analytic in $|z|\geq 1$.

b) Any other factorization $\phi(z)=K(z)K^{*}(z)$, in which $K(z)$ is real rational, is such that $K(z)=H(z)U(z)$ where $U(z)$ is a real rational, para-unitary matrix (i.e. $U(z)U^{*}(z)=I$). Moreover $U(z)$ is stable if and only if $K(z)$ is stable.

c) Any other factorization $\phi(z)=K(z)K^{*}(z)$, in which $K(\infty)$ is finite and nonsingular, $K(z)$ is real rational, stable and minimum phase, is such that $K(z)=H(z)T$ where T is a real orthogonal matrix.

d) There exists a unique factorization of the form $\phi(z)=\bar{W}(z)\bar{Q}\bar{W}^{*}(z)$, in which $\bar{W}(z)$ is nxn real rational, stable, minimum phase and such that $\bar{W}(\infty)=I$, with $\bar{Q}$ positive definite symmetric.

e) Any other factorization of the form $\phi(z)=W(z)QW^{*}(z)$ in which $W(z)$ is real rational, and Q is positive definite symmetric, is such that $W(z) = \bar{W}(z)P(z)$, where $P(z)$ is a real rational scaled para-unitary matrix, i.e. $P(z)QP^{*}(z)=\bar{Q}$. Moreover $P(z)$ is stable if and only if $W(z)$ is stable.

f) Any other factorization of the form $\phi(z)=W(z)QW^{*}(z)$ in which $W(\infty)$ is finite and nonsingular, $W(z)$ is nxn real rational, stable and minimum phase, and Q is positive definite symmetric is such that $W(z)=\bar{W}(z)P$, where P is a real nonsingular constant matrix, with $PQP'=\bar{Q}$.

Definition 1 :

We shall call $\bar{W}(z)$, defined in part d) of the Spectral Factorization Theorem, the normalized minimum-phase stable factor (NMSF) of $\phi(z)$.

Going back to the closed loop models described in Section II (Figs 1 and 2), we shall also introduce the following definition.

Definition 2 :

Given a stationary process $\begin{bmatrix} y_1 \\ u_1 \end{bmatrix}$, generated by a closed loop system of the form of Fig.1 and obeying the assumptions 2.1) to 2.3), we shall call $\{\bar{F},\bar{G},\bar{H},\bar{K}\}$ the normalized minimum-phase realization (NMR) of the joint process $\begin{bmatrix} y \\ u \end{bmatrix}$, where $\{\bar{F},\bar{G},\bar{H},\bar{K}\}$ is derived via (2.5) from the NMSF $W(z)$ of $\phi_{yu}(z)$.

Notice also that to the unique NMSF $\bar{W}(z)$ of the joint spectral density $\phi_{yu}(z)$, there corresponds a unique (innovations) covariance matrix $\bar{Q}$, which can be suitably partitioned according to the dimensions of y and u :

$$\bar{Q} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} \\ \bar{Q}_{21} & \bar{Q}_{22} \end{bmatrix}$$

We can now establish some relations between the structure or the properties of the closed loop representation $\{F,G,H,K\}$ (Fig.2) and the properties of the corresponding joint process matrix transfer function $W(z)$.

Lemma 1.

Consider the closed loop system described in Fig.2, and let $W(z)$ be defined by (2.4). There exists a delay in both the feedforward path and the feedback path if and only if $W(\infty)$ is block diagonal.

Proof : Follows immediately from (2.4)-(2.5) established at $z=\infty$.

Corollary 1 : The normalized minimum-phase realization (NMR) of the joint process $\begin{bmatrix} y \\ u \end{bmatrix}$ with spectral density $\phi_{yu}(z)$ has a delay in both the feedforward and the feedback path.

Proof: Trivial, since $\bar{W}(\infty)$ is diagonal.

Lemma 2.

Consider the joint process $\begin{bmatrix} y \\ u \end{bmatrix}$ described by Fig.2 and obeying assumptions 2.1) to 2.3) Let $W(z)$ be a matrix transfer function model for this process defined by (2.4). Suppose that $F(z)$ and $H(z)$ are stable. Then $W(z)$ is minimum phase if and only if $G(z)$ and $K(z)$ are minimum phase.

It is crucial that the assumption of stable $F(z)$ and $K(z)$ be made, since otherwise minimum phase $G(z),K(z)$ can yield nonminimum phase $W(z)$. Consider, for example, the quadruple

$$F(z)=\frac{1}{z-2} \quad G(z)=1 \quad H(z)=-1.5 \quad K(z)=1$$

which yields the nonminimum phase

$$W(z) = \begin{bmatrix} z-2 & -1 \\ 1.5 & 1 \end{bmatrix}^{-1} \begin{bmatrix} z-2 & 0 \\ 0 & 1 \end{bmatrix}$$

We are now in a position to state precisely what is meant by the identifiability of a closed loop system using method I_2. Clearly this notion is related to the question whether the particular spectral factor $W(z)$ that is computed from $\phi_{yu}(z)$ will lead to the exact matrix transfer functions via (2.5). But it turns out that all spectral factorizations may not be compatible with the a priori knowledge about the structure of the system. Or, more precisely, all spectral factorizations may not lead to a $\{F,G,H,K\}$ quadruple and to a Q matrix that are compatible with this a priori knowledge. For example, by Corollary 1, the NMSF leads to a NMR which has a delay in both $F(z)$ and $H(z)$. If it is known that the true system has a delay in $F(z)$ but not in $H(z)$, then this realization is not compatible with the a priori knowledge. Therefore we introduce the notion of admissible spectral factorization.

Definition 3 :

Given a stationary process $\begin{bmatrix} y \\ u \end{bmatrix}$, generated by a closed loop system of the form of Fig.1 and obeying the assumptions 2.1) to 2.3), we shall say that the spectral factorization $\phi_{yu}(z)=W(z)QW^*(z)$ is admissible if the corresponding realization $\{F,G,H,K\}$ and the covariance matrix Q are compatible with the a priori knowledge of the closed loop system.

To any set of a priori conditions on the closed loop system there shall correspond a class of admissible spectral factorizations $\{W(z),Q\}$. With the identification method I_2, a particular (canonical) element of this class will be selected. This element will be minimum phase and nonsingular at $z=\infty$. (For example : if it is known that $F(\infty) = H(\infty) = 0$, then in the class of all spectral factors $W(z)$ that have $W(\infty)$ block diagonal - cfr. Lemma 1 - the NMSF will be chozen). For a description of various canonical factors, see e.g. [4]. The question now is whether this canonical element $\{\bar{W}(z),\bar{Q}\}$ will lead to the correct transfer function matrices $\{F,G,H,K\}$ and the correct noise covariances.

Definition 4 :

a) Given a set of a priori assumptions on the structure of a closed loop system of the form of Fig.1, we shall say that the open loop transfer functions $\{F,G\}$ are recoverable from $\phi_{yu}(z)$ if

$$F = \bar{F}, \quad G = \bar{G}V_1 \qquad (3.1a)$$

where $\{\bar{F},\bar{G}\}$ are obtained from the admissible canonical factorization $\{\bar{W},\bar{Q}\}$ via (2.5), and $V_1(z)$ is a scaled para-unitary matrix such that

$$V_1(z)Q_{11}V_1^*(z) = \bar{Q}_{11} \qquad (3.1b)$$

b) Given a set of a priori assumptions on the structure of a closed loop system of the form of Fig.2, we shall say that the transfer functions $\{F,G,H,K\}$ are recoverable from $\phi_{yu}(z)$ if

$$F = \bar{F}, \ G = \bar{G}V_1, \quad H = \bar{H}, \ K = \bar{K}V_2 \qquad (3.2a)$$

where $\{\bar{F},\bar{G},\bar{H},\bar{K}\}$ are obtained from the admissible canonical factorization $\{\bar{W},\bar{Q}\}$ via (2.5), $V_1(z)$ obeys (3.1b) and $V_2(z)$ obeys

$$V_2(z)Q_{22}V_2^*(z) = \bar{Q}_{22} \qquad (3.2b)$$

Definition 4 is a precise statement for the identifiability of a closed loop system (the forward path, or the global model) using the joint input-output identification method I_2. In this definition we have considered that the matrices G and K may differ from one another (and hence from the true values), by right multiplication by a scaled para-unitary matrix. This ambiguity occurs even in the open loop case (cfr. the spectral factorization theorem) and does not influence the input-output characteristics of the forward or the feedback path.

Comments :

1) It might happen that the class of admissible spectral factors contains only one element, in which case the system is always identifiable. It might also happen that the quadruples {F,G,H,K} obtained from all admissible spectral factors are all related by (3.2a); in the latter case any arbitrary admissible factorization (and not just the canonical one) will lead to the correct model.

2) Definition 4 is an extension of the definition given by Ng et al. [7] for the identifiability of {F,G,H,K} . In [7] the system transfer functions {F,G,H,K} are called recoverable from $\phi_{yu}(z)$ if (3.2) holds, with $\{\bar{F},\bar{G},\bar{H},\bar{K}\}$ being the NMR of $\phi_{yu}(z)$. However, as we have shown, the NMR may not be an admissible realization. In such a case {F,G,H,K} might still be recoverable from another admissible realization.

IV. IDENTIFIABILITY CONDITIONS FOR THE TRANSFER FUNCTIONS {F,G,H,K} USING I_2.

In this section we consider the case where the closed loop process is assumed to have the form depicted in Fig.2, and where it is desired to recover the whole quadruple {F,G,H,K} from $\phi_{yu}(z)$ using method I_2. The first lemma, which is a slight generalization of a result of Ng et al. [7], is a necessary and sufficient condition for identifiability.

Lemma 3.
Consider the closed loop process depicted in Fig.2 and obeying the assumptions 2.1) to 2.3). Let $\bar{W}(z)$ be the canonical spectral factor chozen in the class of admissible spectral factors of $\phi_{yu}(z)$ and let W(z) be obtained from the true {F,G,H,K} through (2.4). Then {F,G,H,K} are recoverable from $\phi_{yu}(z)$ if and only if the transformation from W(z) to $\bar{W}(z)$ is block-diagonal, i.e.

$$W(z) = \bar{W}(z) \begin{bmatrix} V_1(z) & 0 \\ 0 & V_2(z) \end{bmatrix} \qquad (4.1)$$

Proof :
Using (2.4)-(2.5) shows that (3.2a) is equivalent with (4.1).

The following theorem gives alternative sets of sufficient conditions for the identifiability of {F,G,H,K} under two different sets of assumptions on the structure of the closed loop system.

Theorem 1.
Consider the closed loop process depicted in Fig.2 and obeying the assumptions 2.1) to 2.3) and 2.7). Then {F,G,H,K} is recoverable from $\phi_{yu}(z)$ if it is known a priori that either one of the following conditions holds :
i) there exists at least a unit delay in one of F and H ; m and n are uncorrelated ;
ii) there exists at least a unit delay in both F and H ; F and H are stable, while G and K are minimum phase with $G(\infty)$ and $K(\infty)$ nonsingular.

Comments : The first set of assumptions (i.e. part i) represents a strict improvement over the results of [7].
Part ii) is a new result. The proof of theorem 1 is given in the full version of this paper.

V. IDENTIFIABILITY CONDITIONS FOR THE FORWARD PATH {F,G} USING I_2.

In this section we consider closed loop systems in which a one-sided correlation is allowed between the regulator noise n and the process driving noise w. Such systems can be described by the following structure (see Fig. 3)

$$y_i = F(z)u_i + G(z)w_i \qquad (5.1a)$$
$$u_i = H(z)y_i + n_i \qquad (5.1b)$$
$$n_i = K(z)v_i + L(z)w_i \qquad (5.1c)$$

with the assumptions 2.1) to 2.4). Various possibilities will be considered for L(z). Notice that the system (5.1) comprises the closed loop process examined in section IV as a special case, namely for L(z)=0. The noise $\{v_i\}$ is considered to be unmeasurable and, by assumption 2.4) we can write, without loss of generality, that

$$E\left\{\begin{bmatrix} w_i \\ v_i \end{bmatrix} \begin{bmatrix} w_j^T & v_j^T \end{bmatrix}\right\} = Q\delta_{ij} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix}\delta_{ij} \qquad (5.2)$$

The spectral density of the joint process $\begin{bmatrix} y \\ u \end{bmatrix}$ is $\phi_{yu}(z)=W(z)QW^*(z)$, where W(z) is uniquely expressed in terms of {F,G,H,K,L} as follows

$$W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} = \begin{bmatrix} (I-FH)^{-1}(G+FL) & (I-FH)^{-1}FK \\ (I-HF)^{-1}(HG+L) & (I-HF)^{-1}K \end{bmatrix} \qquad (5.3)$$

{F,G,H,K,L} can of course not be recovered uniquely from W(z). However we have the following result.

Lemma 4.
Let (F,G,H,K,L) be the transfer function matrices defined by (5.1) and let W be defined by (5.3). Then, provided that W_{22} has full normal row rank, so that a right inverse W_{22}^{-1} exists, F and G are uniquely expressible in terms of W as follows :

$$F = W_{12}W_{22}^{-1} \qquad G = W_{11}-W_{12}W_{22}^{-1}W_{21} \qquad (5.4)$$

Proof : Using the matrix identity $(I-FH)^{-1}F = F(I-HF)^{-1}$, it follows from (5.3) that $W_{12}W_{22}^{-1} = F$, and that
$W_{11}-W_{12}W_{22}^{-1}W_{21} = (I-FH)^{-1}(G+FL)-F(I-HF)^{-1}(HG+L) = G.$

The question naturally arises as to when W_{22} has full normal row rank.

Lemma 5.
Let W(z) be constructed as above with the assumptions 2.1) to 2.4). Sufficient conditions for $W_{22}(z)$ to have full normal row rank are the following :
a) W(z) is square, or, equivalently, G(z) and

K(z) are square

b) $L(z)=L^{\circ}(z)G(z)$ for some $L^{\circ}(z)$, not necessarily proper.

Given a closed loop process of the form depicted in Fig.3, there corresponds under assumptions 2.1) to 2.4) a unique W(z) and Q to it via (5.3) and (5.2) and therefore a unique $\phi_{yu}(z)=W(z)QW^{*}(z)$. The question we examine in this section is under what conditions can we recover F and G from $\phi_{yu}(z)$. Given $\phi_{yu}(z)$ and given some a priori knowledge on the structure of the closed loop system (5.1)(e.g. on the delays in F,H,L), we postulate one can always obtain an admissible canonical factorization $\phi_{yu}(z)=\bar{W}(z)\bar{Q}\bar{W}^{*}(z)$ of $\phi_{yu}(z)$. From $\bar{W}$ and $\bar{Q}$ one can obtain a unique representation for $\begin{bmatrix} y \\ u \end{bmatrix}$ in the form of Fig.2, where $\{\tilde{F},\tilde{G},\tilde{H},\tilde{K}\}$ are derived from $\bar{W}$ via the expression (2.5). Now the question of identifiability is : under what a priori conditions do we have $F=\tilde{F}$, $G=\tilde{G}V_1$, where V_1 is a scaled para-unitary matrix satisfying (3.1b). Those identifiability conditions will now be derived.

Comment : We recall that we are only concerned with recoverability of F and G. It is clear that H,K and L can not be recovered from $\phi_{yu}(z)$. Actually given the closed loop system described by eqs.(5.1) and Fig.3, a closed loop model can be obtained with the same spectrum and having the form of Fig.2

$$y_i = \tilde{F}u_i + \tilde{G}w_i \qquad (5.5a)$$
$$u_i = \tilde{H}y_i + \tilde{K}v_i \qquad (5.5b)$$

where $\tilde{F},\tilde{G},\tilde{H},\tilde{K}$ are derived from W defined in (5.3) by the equations (2.5). This leads to :

$$\tilde{F} = F\ ,\quad \tilde{G} = G \qquad \text{(see lemma 4)} \qquad (5.6a)$$
$$\tilde{H} = (I+LG^{-1}F)^{-1}(H+LG^{-1}),\tilde{K} = (I+LG^{-1}F)^{-1}K \quad (5.6b)$$

The following lemma gives a necessary and sufficient condition for identifiability of {F,G}

Lemma 6.

Consider the closed loop process depicted in Fig.3 and obeying assumptions 2.1) to 2.4). Suppose there exists a canonical minimum phase spectral factor $\bar{W}(z)$ in the class of spectral factors of $\phi_{yu}(z)$ [i.e. $\phi_{yu}(z)=\bar{W}(z)\bar{Q}\bar{W}^{*}(z)$ for a block diagonal $\bar{Q}$, $\bar{W}(\infty)$ is nonsingular and consistent with knowledge of delays of the blocks of the overall system, and $\bar{W}(z)$ is a minimum phase spectral factor computable uniquely from $\phi_{yu}(z)$] . Let W(z) be obtained from the true $\{F,G,H,K,L\}$ through (5.3). Then the pair {F,G} is recoverable from $\phi_{yu}(z)$ if and only if the scaled para-unitary transformation from W(z) to $\bar{W}(z)$ (whose existence is guaranteed by the spectral factorization theorem) is lower block triangular, i.e.

$$W(z) = \bar{W}(z)\begin{bmatrix} V_{11}(z) & 0 \\ V_{21}(z) & V_{22}(z)\end{bmatrix} \quad (5.7a)$$

with V_{22} possessing a right inverse, sufficient conditions for this being conditions a) or b) of lemma 5. Moreover, if W(z) is square, the transformation from $\bar{W}$ to W is block diagonal, i.e.

$$W(z) = \bar{W}(z)\begin{bmatrix} V_{11}(z) & 0 \\ 0 & V_{22}(z)\end{bmatrix} \qquad (5.7b)$$

Theorem 2.

Consider the closed loop process depicted in Fig.3 and obeying assumptions 2.1) to 2.3) and the following variation of 2.4): The regulator noise $\{n_i\}$ is related to the process noise $\{m_i\}$ by

$$n_i = L^{\circ}(z)m_i + K(z)v_i \qquad (5.8)$$

where $L^{\circ}(z)$ is causal , and where $\{v_i\}$ is white noise, independent of $\{w_i\}$, so that (5.2) holds. Suppose moreover that F(z) is stable (and so G(z) is also stable), and that any instability in $L^{\circ}(z)$ can only come from referring the feedback controller noise from an internal point to the output of H(z), so that the McMillan degree of the unstable part of $[K(z) \vdots L^{\circ}(z)]$ = McMillan degree of the unstable part of $[H(z) \vdots K(z) \vdots L^{\circ}(z)]$. Finally suppose that either $F(\infty)$ is zero or $H(\infty)$ and $L(\infty)$ are zero, while all these matrices are finite. Then {F,G} is recoverable from $\phi_{yu}(z)$.

Comment.

The relation between n and m is a special case of the model (5.1) obtained for $L(z) = L^{\circ}(z)G(z)$, so that (5.8) can also be expressed as

$$n_i=L^{\circ}(z)G(z)w_i+K(z)v_i \qquad (5.9)$$

(This form for L(z) was also encountered in Lemma 5). Notice also that the assumptions of the theorem imply that the correlation between the regulator noise $\{n_i\}$ and process noise $\{w_i\}$ is one-sided, i.e.

$$E\left[n_i w_j'\right] = 0 \quad i<j \ (i\leq j \text{ if } L(\infty)=0) \ (5.10)$$

It should be noticed that in Theorem 2 we have made the rather strong assumption (5.8),whereas for the identification method I_1 the more general model (5.1c) was allowed for the one-sided correlation between n and w (see assumption 2.4). However with I_1 the minimum phase assumption on G(z) was included in the identifiability conditions (see assumption 2.6), which is not the case in Theorem 2. Actually, this assumption is more restrictive, since the I_1 problem can be replaced by an I_2 problem in which $L^{\circ}(z)=L(z)G^{-1}(z)$, with $L^{\circ}(z)$ causal and stable by assumptions 2.4) and 2.6).

The assumptions on $F(\infty)$ and $K(\infty)$ of theorem 2 are consistent with assumptions 2.1) and 2.5) in method I_1, while the requirement in Theorem 2 that F(z) be stable is apparently not required in I_1. Actually, the proof in [6] of I_1 is somewhat abbreviated, and it is not clear to us that one can dispense with stability of F(z) in method I_1. In the full version of this paper we show by a counterexample that the stability of F(z) is indeed necessary for I_2. We show now by another counterexample that when G(z) is not minimum phase, a model of the form (5.1c) for the regulator noise will in general not allow recoverability of {F,G}.

Consider the case when, see Fig.3,

$$F=z^{-1} \quad G=\frac{1-2z^{-1}}{1-0.5z^{-1}} \quad H=0.5z^{-1} \quad K=1 \quad L=z^{-1}$$

with $\{w_i\}$, $\{v_i\}$ independent, zero mean, white noise sequences. Then

$$W(z)=\begin{bmatrix}\dfrac{1-2z^{-1}+z^{-2}-0.5z^{-3}}{1-0.5z^{-1}-0.5z^{-2}+0.25z^{-3}} & \dfrac{z^{-1}}{1-0.5z^{-2}}\\[2ex] \dfrac{1.5z^{-1}-1.5z^{-2}}{1-0.5z^{-1}-0.5z^{-2}+0.25z^{-3}} & \dfrac{1}{1-0.5z^{-2}}\end{bmatrix}$$

A factorization $\Phi_{yy}=W(z)W^{*}(z)=\tilde{W}(z)\tilde{Q}\tilde{W}^{*}(z)$ with $\tilde{W}(z)$ the NMSF yields

$$\tilde{W}(z)=\begin{bmatrix}\dfrac{1-0.8z^{-1}+0.4z^{-2}-0.2z^{-3}}{1-0.5z^{-1}-0.5z^{-2}+0.25z^{-3}} & \dfrac{0.4z^{-1}-0.2z^{-2}-0.15z^{-3}}{1-0.5z^{-1}-0.5z^{-2}+0.25z^{-3}}\\[2ex] \dfrac{0.9z^{-1}-0.6z^{-2}}{1-0.5z^{-1}-0.5z^{-2}+0.25z^{-3}} & \dfrac{1-0.2z^{-1}-0.45z^{-2}}{1-0.5z^{-1}-0.5z^{-2}+0.25z^{-3}}\end{bmatrix}$$

and $\tilde{Q}=\begin{bmatrix}3.4 & -1.2\\ -1.2 & 1.6\end{bmatrix}$

It is easy to see that $\tilde{W}_{12}\tilde{W}_{22}^{-1}\neq F$ as it should.

In the next theorem we impose further constraints on F,G,H and K than earlier, but drop the requirement that $\{v_i\}$, $\{w_i\}$ be uncorrelated.

Theorem 3.

Consider the closed-loop process depicted in Fig.3, obeying assumptions 2.1) to 2.3) and assumption 2.4) save that $\{v_i\}$ and $\{w_i\}$ are white but correlated with one another :

$$E\left(\begin{bmatrix}w_i\\ v_i\end{bmatrix}\begin{bmatrix}w_j^T & v_j^T\end{bmatrix}\right)=\begin{bmatrix}Q_{11} & Q_{12}\\ Q_{21} & Q_{22}\end{bmatrix}\delta_{ij}=Q\delta_{ij}$$

Suppose further that F(z), H(z) and L(z) are stable, that G(z) and K(z) are stable and minimum phase, that $F(\infty)$ is zero, and that $G(\infty)$ and $K(\infty)$ are nonsingular. Then the pair {F,G} is recoverable.

Conclusions.

We have shown that the identifiability conditions for I_2 could be modified if the point of view is taken that only the forward path model {F,G} is to be identified, which is the case in practice. With this approach in mind, the method I_2 is now applicable to a variety of new situations and the identifiability conditions for I_2 are now very similar to those for I_1. The proofs and more additional details and examples will appear in the full version of this paper.

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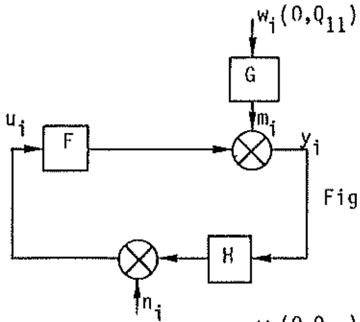

Fig.1.

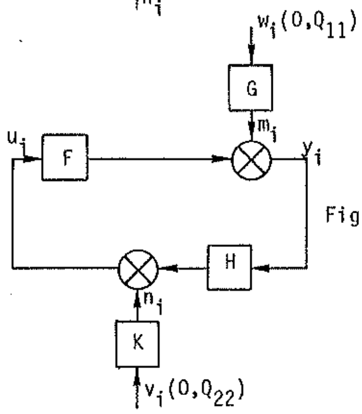

Fig.2

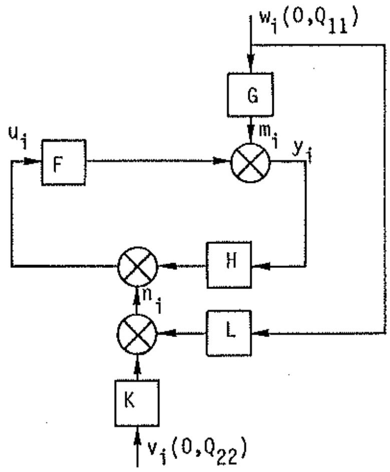

Fig.3