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A NEW AND WIDER CLASS OF OVERLAPPING FORMS FOR THE PRESENTATION OF MULTIVARIABLE SYSTEMS

Michel R. Gevers

Department of Systems Engineering, Research School of Physical Sciences
Australian National University, GPO Box 4, Canberra, ACT 2601, Australia

Ah Chung Tsoi

Department of Electrical and Electronic Engineering
University of Auckland, Auckland, New Zealand

Abstract One approach to the identification of multivariable linear systems is to use one of several "overlapping" or "pseudocanonical" forms. They are uniquely identifiable. The structure of each pseudocanonical form is determined by a set of structure indices, that indicate which particular rows of the Hankel matrix of Markov parameters have been selected to form a basis. In this paper, we relax the traditional constraints on the selection of these rows. This allows for more flexibility and enhances the chances of obtaining a numerically well-conditioned basis.
Keywords Identification, multivariable systems, state-space parametrization.

INTRODUCTION

A problem which has been the subject of many studies in linear multivariable systems theory is the determination of uniquely identifiable parametrizations for state-space or autoregressive moving average (ARMA) models. One approach that has received increasing attention is the use of overlapping parametrizations (also called pseudo-canonical forms). This concept was first suggested by Glover and Willems (1974), and studied by Ljung and Rissanen (1976), Van Overbeek and Ljung (1982), Picci (1980), Rissanen (1981), Deistler and Hannan (1981), Wertz, Gevers, and Hannan (1982), Gevers and Wertz (1981), Guidorzi and Beghelli (1982), and Correa and Glover (1982). It has been shown that the set of all finite dimensional linear systems can be represented by a finite number of uniquely identifiable parametrizations. Each parametrization is characterised by a set of integers known as structure indices. Each system can be represented almost surely in any one of those pseudocanonical forms, and any two parametrizations describing the same system are related by a similarity transformation.

The structure indices are determined by the particular way in which a basis is selected for the rows of the Hankel matrix of impulse responses (or Markov parameters). In order to obtain a representation that contains a small number of uniquely identifiable parameters, certain rules must be imposed for the selection of this basis. All overlapping forms described so far have involved the following two selection rules (see Guidorzi and Beghelli (1982), Correa and Glover (1982), Wertz, Gevers and Hannan (1982), Van Overbeek and Ljung (1982), Gevers and Wertz (1982)):

(i) a block selection rule: if the dimension output vector is p, then an entire block of p rows is chosen;

(ii) a Hankel chain selection rule: if a vector is in the basis, then its corresponding predecessor vector is also in the basis (the predecessor of a vector in the Hankel matrix is the one that is located p rows above that vector).

Most authors further refined rule (i) by imposing that the first block row had to be selected. Gevers and Wertz (1984) showed that this may lead to a singular leading coefficient matrix for the corresponding matrix fraction description (MFD) form. They proposed an alternative selection procedure that produces an identity leading coefficient matrix whill still retaining the block selection rule.

In this paper, we show that one can actually relax the block selection rule (i), thereby increasing the flexibility in the choice of a basis. Once a basis has been selected with our new rules, there are a number of ways to construct a state-space representation; the most obvious way is to include in the state the components that correspond to the selected basis rows of the Hankel matrix. This leads to a minimal representation. Alternatively one can include additional components in the state-vector, leading to a non-minimal representation; the most obvious way is to include in the state the components that correspond to the selected basis rows of the Hankel matrix. This leads to a minimal representation. Alternatively one can include additional components in the state-vector, leading to a non-minimal representation. The reason for doing so is that the parameters of this non-minimal representation can be easily related to MFD forms constructed with these new selection rules. These relationships have been established in Gevers and Tsoi (1984), where an alternative non-minimal state-space representation was used. Here we shall limit ourselves to state-space forms and establish the connections between the minimal and non-minimal state-space.

SELECTING A BASIS OF THE HANKEL MATRIX: A NEW SET OF RULES

We consider a p-dimensional stationary full rank zero-mean stochastic process $\{y_t\}$ with rational spectrum. The linear least-squares predictor of $\{y_t\}$ given the past history of the process is of full rank. Then $\{y_t\}$ can be described up to second order statistics by a state-space representation

$$x_{t+1} = Fx_t + Ke_t$$
$$y_t = Hx_t + e_t \qquad (2.1)$$

where x_t is a n-dimensional state vector, y_t is a p-dimensional white noise sequence with covariance matrix Q, and F, K, H are constant matrices of appropriate dimensions. F is assumed to be stable.

In this paper, we shall not assume n in (2.1) to be minimum. From (2.1) it is simple to show that

$$y_t = \sum_{i=0}^{\infty} R_i \, e_{t-i} \qquad (2.2)$$

where the R_i are p x p matrices, known as the Markov parameters or the impulse response matrices. Furthermore,

$$R_i = HF^{i-1}K, \quad i = 1,2,\ldots : \quad R_0 = I \qquad (2.3)$$

By demanding that the casual inverse of y_t exists, ie

$$e_t = \sum_{i=0}^{\infty} N_i \, y_{t-i} \qquad (2.4)$$

where $N_0 = I_p$, and $N(z) = \sum_{i=0}^{\infty} N_i z^{-i}$ has no poles outside the unit circle, it is possible to define

$$e_t \triangleq y_t - \hat{y}_{t/t-1}$$

where $\hat{y}_{t/t-k}$ is the linear least squares k-step ahead predictor of y_t.

Note that

$$\hat{y}_{t+j/t-1} = \sum_{i=j+1}^{\infty} R_i \, e_{t+j-i}, \quad j = 0,1,2,\ldots$$

Similarly,

$$\hat{y}_{t+j+1/t} = \sum_{i=j+1}^{\infty} R_i \, e_{t+j+1-i}, \quad j = 0,1,2,\ldots$$

Let

$$\hat{Y}_N(t) \triangleq \begin{bmatrix} \hat{y}_{t/t-1} \\ \hat{y}_{t+1/t-1} \\ \vdots \\ \hat{y}_{t+N-1/t-1} \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & R_3 \ldots \\ R_2 & R_3 & R_4 \ldots \\ \vdots & \vdots & \vdots \\ R_N & R_{N+1} & \ldots \end{bmatrix} \begin{bmatrix} e_{t-1} \\ e_{t-2} \\ \vdots \\ \end{bmatrix} \qquad (2.5)$$

$$= H_{N,\infty} \begin{bmatrix} e_{t-1} \\ e_{t-2} \\ \vdots \end{bmatrix}$$

Hence

$$\hat{Y}_N(t+1) = \begin{bmatrix} y_{t+1/t} \\ y_{t+2/t} \\ \cdot \\ \cdot \\ \cdot \\ y_{t+N/t} \end{bmatrix} = \begin{bmatrix} R_1 & R_2 & R_3 \cdots \\ R_2 & R_3 & R_4 \cdots \\ R_3 & R_4 & R_5 \cdots \\ \cdot & & \\ \cdot & & \\ R_N & R_{N+1} \ldots\ldots & \end{bmatrix} \begin{bmatrix} e_t \\ e_{t-1} \\ e_{t-2} \\ \\ \\ \end{bmatrix}$$

$$= \begin{bmatrix} R_2 & R_3 & R_4 \cdots \\ R_3 & R_4 & R_5 \cdots \\ R_4 & R_5 & R_6 \cdots \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ R_{N+1} & R_{N+2} & \cdots \end{bmatrix} \begin{bmatrix} e_{t-1} \\ e_{t-2} \\ e_{t-3} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} + \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ \cdot \\ \cdot \\ R_N \end{bmatrix} e_t \qquad (2.6)$$

Comparing (2.6) with (2.1) and (2.5), it is obvious that the state vector x_t forms a basis of $\hat{Y}_N(t)$

Let the set of selected basis components be denoted by $I = \{i_1, i_2, \ldots i_n\}$. Also, let the [j+ k-1 p]th component of $\hat{Y}_N(t)$ be denoted by $\hat{y}_{jk}(t-1)$. Let R^j denote the jth block row of $H_{N,\infty}$, and r_{ij} the ith row of R^j, ie

$$R^j = [R_j \; R_{j+1} \ldots] = \begin{bmatrix} r_{1j} \\ r_{2j} \\ \cdot \\ \cdot \\ \cdot \\ r_{pj} \end{bmatrix}$$

Then, r_{ij} = [i+(j-1)p]th row of $H_{N,\infty}$. Let $r_{ij}(k)$ be a row p-vector made up of the kth set of p elements of row r_{ij}

$$\text{Thus, } H_{N,\infty} = \begin{bmatrix} r_{11} \\ r_{21} \\ \vdots \\ r_{p1} \\ \cdots \\ r_{12} \\ \vdots \\ r_{p2} \\ \cdots \\ \vdots \\ \end{bmatrix}$$

Let x_t be the basis. Then

<u>Definition 2.1</u> For given i, i=1,2,..p, denote

(a) $S_i \triangleq (\hat{y}_{ik}(t-1) \,|\, \hat{y}_{ik}(t-1) \in x_t$ for some $k \geq 1)$

(b) Let n_i = number of elements in S_i. Then n_1, $n_2 \ldots\ldots n_p$ will be called the <u>structure indices.</u>
In addition, $n = \sum_{i=1}^{p} n_i$

(c) $m_i = \min_{m=1,2\ldots} \{m \,|\, \hat{y}_{im}(t-1) \in x_t\}$, $i = 1,2,\ldots,p$

(d) $s_i = m_i + n_i - 1$ and $s \triangleq \max_{i=1,\ldots,p} \{s_i\}$

<u>Example</u>

Suppose p = 2, n = 5, I = {1, 4, 5,6, 8}, then

$$S_1 = \{\hat{y}_{11}(t-1), \; \hat{y}_{13}(t-1)\},$$

$$S_2 = \{\hat{y}_{22}(t-1), \; \hat{y}_{23}(t-1), \; \hat{y}_{24}(t-1)\}$$

and $n_1 = 2$, $n_2 = 3$; $m_1 = 1$; $m_2 = 2$; $s_1 = 2$, $s_2 = 4$
We wish to impose the following <u>selection rules</u>
<u>Rule 1:</u> $S_i \neq 0$, or equivalently, $n_i \geq 1$, i=1,2,..p
ie every one of the p components of y(t) appears at least once.
<u>Rule 2:</u> for i = 1, 2,...,p
$\{\hat{y}_{i,m_i}(t-1), \; y_{i,m_i+1}(t-1)m, \ldots y_{i,s_i}(t-1)\} \in x_t$, ie, the n_i components of x_t whose first index is i appear in n_i successive blocks. This is the Hankel chain rule.
<u>Rule 3:</u> $m_i=1$ for at least one i in (1, 2,...,p)

Comment:

Rules 1 to 3 are a relaxation of the selection rule imposed by Van Overbeek and Ljung (1982), Wertz, Gevers and Hannan (1982), Gevers and Wertz (1984) in that we do not require to have one full block $\hat{y}_{jk}(t-1)$, $j = 1, 2, \ldots, p$ in the basis x_t. Rules 1 to 3 cannot be relaxed further. If they were not imposed, then the resultant parametrization would be over-parametrized. Equivalently, the F matrix might become full.

Example

With $p = 2$, $n = 5$, the following are a few candidate selections:

{1,2,3,4,5}, {1,3,4,6,8}, {1,3,6,8,10}
{1,4,6,8,10}, {1,6,8,10,12} {11,13,2,4,6}

STATE-SPACE REPRESENTATION

We will now define two state vectors which will lead to two different state-space representations, the first one minimal, the second one not.

Representation 1

$$x_t = [\hat{y}_{1,m_1}(t-1)\ \hat{y}_{1,m_1+1}(t-1) \ldots \hat{y}_{1,s_1}(t-1) \,\vdots\, \ldots \,\vdots\, \hat{y}_{p,m_p}(t-1), \ldots \hat{y}_{p,s_p}(t-1)]^T$$

The dimension of x_t is $n = \sum_{i=1}^{p} n_i$. The state-space representation is characterized by the matrices H, F, K as shown in (2.1). They are obtained by expressing $\hat{y}_{1,m_1+n_1}(t-1), \ldots \hat{y}_{2,m_2+n_2}(t-1), \ldots \hat{y}_{p,m_p+n_p}(t-1)$ as a function or x_t, or equivalently, by expressing $r_{1,m_1+n_1}, r_{2,m_2+n_2}, \ldots r_{p,m_p+n_p}$ as a function of the corresponding basis rows of $H_{N,\infty}$. The corresponding rows of $H_{N,\infty}$ may be represented as:-

$$R = \begin{bmatrix} r_{1,m_1} \\ \vdots \\ r_{1,s_1} \\ \vdots \\ r_{p,m_p} \\ \vdots \\ r_{p,s_p} \end{bmatrix}$$

Thus, for $i = 1,2,\ldots p$, there exists unique $\alpha_{ik\ell}$ such that

$$r_{i,m_i+n_i} = \sum_{k=1}^{p} \sum_{\ell=m_k}^{s_k} \alpha_{ik\ell}\, r_{k\ell} \tag{3.1}$$

By the Hankel structure, it follows that

$$r_{i,m_i+n_i+q} = \sum_{k=1}^{p} \sum_{\ell=m_k}^{s_k} \alpha_{ik\ell}\, r_{k,\ell+q}, \quad q \geq 0 \tag{3.2}$$

In addition

$$\hat{y}_{i,m_i+n_i}(t-1) = \sum_{k=1}^{p} \sum_{\ell=m_k}^{s_k} \alpha_{ik\ell}\, \hat{y}_{k\ell}(t-1) \tag{3.3}$$

Hence, we have

$$x_{t+1} = Fx_t + Ke_t$$
$$y_t = Hx_t + e_t \tag{3.4}$$

where

$$F = \left[\begin{array}{ccc|c|ccc} 0 & I_{n_1-1} & & & & \bigcirc & \\ \vdots & & & & & & \\ 0 & & & \cdots & & & \\ \alpha_{11m_1} & \cdots & \alpha_{11s_1} & \cdots & \alpha_{1pm_p} & \cdots & \alpha_{1ps_p} \\ \hline & \vdots & & \ddots & & & \\ \hline & & & & 0 & & \\ & \bigcirc & & & \vdots & I_{n_p-1} & \\ & & & \cdots & 0 & & \\ \alpha_{p1m_1} & \cdots & \alpha_{p1s_1} & \cdots & \alpha_{ppm_p} & \cdots & \alpha_{pps_p} \end{array}\right]$$

n x n matrix

$$H = \begin{bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_p \end{bmatrix}, \text{ a } p \times n \text{ matrix}$$

where

$$h_i = [\underbrace{0 \ldots 0}_{n_1} \,\vdots\, \ldots \,\vdots\, \underbrace{1\ 0 \ldots 0}_{n_i} \,\vdots\, \ldots \,\vdots\, \underbrace{0 \ldots 0}_{n_p}] \quad \text{if } m_i = 1$$

and

$$h_i = [\beta_{i11m_1} \ldots \beta_{i11s_1} \,\vdots\, \beta_{i12m_2} \ldots \beta_{i12s_2} \,\vdots\, \ldots \,\vdots\, \beta_{i1pm_p} \ldots \beta_{i1ps_p}] \quad \text{if } m_i > 1$$

The coefficients $\beta_{ijk\ell}$ are obtained from the following $p\,(\sum_{i=1}^{p} m_i) - p$ relationships for

$i = 1,2,\ldots,p$; $j=1,\ldots,m_i-1$

$$r_{ij} = \sum_{k=1}^{p} \sum_{\ell=m_k}^{s_k} \beta_{ijk\ell}\, r_{k\ell} \tag{3.5}$$

$$K = \begin{bmatrix} r_{1m_1}(1) \\ \vdots \\ r_{1m_1+n_1-1}(1) \\ \vdots \\ r_{pm_p}(1) \\ \vdots \\ r_{pm_p+n_p-1}(1) \end{bmatrix}, \text{a } m \times p \text{ matrix}$$

Comment

The total number of "free" parameters for this representation is:-

in F : pn
in K : pn
in H : vn, where v = number of m_i that are > 1 $(0 \leq v \leq p-1)$

Hence, the number of parameters involved lies between 2np and 3np-n

Representation 2

The state vector is defined as follows:

$$\bar{x}_t = [\hat{y}_{11}(t-1), \hat{y}_{12}(t-1), \ldots \hat{y}_{1s_1}(t-1) \,\vdots\, \ldots \,\vdots\, \hat{y}_{p1}(t-1), \hat{y}_{p2}(t-1), \ldots \hat{y}_{ps_p}(t-1)]^T$$

The corresponding state-space representation is characterized by the matrices $\bar{H}$, $\bar{F}$, $\bar{K}$. The coefficients of $\bar{F}$ are obtained by the same linear combinations as before, see(3.3). However here some zeros must be added in the last row of each block of $\bar{F}$, since the basis elements.

$$[y_{1m_1}(t-1) \ldots \hat{y}_{1s_1}(t-1) \,\vdots\, \ldots \,\vdots\, \hat{y}_{pm_p}(t-1) \ldots \hat{y}_{ps_p}(t-1)]$$

now constitute a subset of the components of x_t. The rows of $H_{N,\infty}$ corresponding to x_t are given by

$$\bar{R} = \begin{bmatrix} r_{11} \\ \vdots \\ r_{1s_1} \\ \hline \vdots \\ \hline r_{p_1} \\ \vdots \\ r_{ps_p} \end{bmatrix}$$

We therefore have

$$\begin{cases} \bar{x}_{t+1} = \bar{F}\bar{x}_t + \bar{K}e_t \\ y_t = \bar{H}\bar{x}_t + e_t \end{cases} \qquad (3.6)$$

where

$$F = \left[\begin{array}{cc|c|cc} \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & I_s -1 & \cdots & & \bigcirc \\ 0\ldots0 & \alpha_{11m_1} \cdots \alpha_{11s_1} & & 0\ldots0 & \alpha_{1pm_p} \cdots \alpha_{1ps_p} \\ \underbrace{}_{m_1-1} \vdots & & & & \\ \hline & \bigcirc & \cdots & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & I_{s_p-1} \\ 0\ldots0 & \alpha_{p1m_1} \cdots \alpha_{p1s_1} & & \underbrace{0\ldots0}_{m_p-1} & \alpha_{ppm_p} \cdots \alpha_{pps_p} \end{array}\right]$$

$$\bar{H} = \left[\begin{array}{c|c|c|c} 1\ 0 \ldots 0 & & & \\ & 1\ 0 \ldots 0 & \cdots & \\ & & & 1\ 0 \ldots 0 \end{array}\right]$$

$$\bar{K} = \begin{bmatrix} r_{11}(1) \\ \vdots \\ r_{1s_1}(1) \\ \hline \vdots \\ \hline r_{p1}(1) \\ \vdots \\ r_{ps_p}(1) \end{bmatrix}$$

The following relationship exists between the representations 1 and 2.

Lemma 3.1

n of the rows of $\bar{K}$ appear directly in K. The other rows of $\bar{K}$ are related to those of K via the $\beta_{ijk\ell}$:

$$r_{ij}(1) = \sum_{k=1}^{p} \sum_{\ell=m_k}^{s_k} \beta_{ijk\ell}\ r_{k\ell}(1) \qquad \begin{matrix} i = 1,\ldots,p \\ j = 1,\ldots,m_i-1 \end{matrix} \qquad (3.7)$$

The coefficients of H in representation 1 are obtained from the relations (3.7) for j = 1.

Proof: The proof follows the definition (3.5) of the $\beta_{ijk\ell}$

Comment

To construct $\bar{K}$, we will usually need some $\beta_{ijk\ell}$ which do not appear in F or H. In fact, if $m_i > 2$, we need $\beta_{ijk\ell}$ for $j = 2, \ldots, m_i-1$

Example

Let p = 2, n = 4, and I = {1, 3, 6, 8}. Thus $n_1 = 1$, $m_1=1$; $n_2 = 2$, $m_2 = 3$; and $s_1 = 2$, $s_2 = 4$

Representation 1

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha_{111} & \alpha_{112} & \alpha_{123} & \alpha_{124} \\ 0 & 0 & 0 & 1 \\ \alpha_{211} & \alpha_{212} & \alpha_{223} & \alpha_{224} \end{bmatrix}, \qquad K = \begin{bmatrix} r_{11}(1) \\ r_{12}(1) \\ r_{23}(1) \\ r_{24}(1) \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \beta_{2111} & \beta_{2112} & \beta_{2123} & \beta_{2124} \end{bmatrix}$$

and x_t contains the components indexed (1,3,6,8) of $\hat{Y}_N(t)$

Representation 2

$$\bar{F} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \alpha_{111} & \alpha_{122} & 0 & 0 & \alpha_{123} & \alpha_{124} \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \alpha_{211} & \alpha_{212} & 0 & 0 & \alpha_{223} & \alpha_{224} \end{bmatrix} \qquad \bar{K} = \begin{bmatrix} r_{11}(1) \\ r_{12}(1) \\ \hline r_{21}(1) \\ r_{22}(1) \\ r_{23}(1) \\ r_{24}(1) \end{bmatrix}$$

and $\bar{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

and $\bar{x}_t$ contains the components indexed (1,3,2,4,6,8) of $\hat{Y}_N(t)$

Finally we have the following result concerning Representation 2.

Lemma 3.2:
Representation 2 is completely observable, but not completely controllable.

Proof
From the form of $\bar{F}$ and $\bar{K}$ in (3.6), we have

$$\bar{F}\bar{K} = \left[\begin{array}{l} r_{12}(1) \\ r_{13}(1) \\ \vdots \\ r_{1,s_1}(1) \\ r_{1,s_1+1}(1) \quad \leftarrow \\ \hline r_{22}(1) \\ \vdots \\ r_{2,s_2+1}(1) \quad \leftarrow \\ \hline \vdots \\ \hline r_{p2}(1) \\ \vdots \\ r_{p,s_p+1}(1) \quad \leftarrow \end{array}\right] \quad \text{and} \quad \bar{F}^2\bar{K} = \left[\begin{array}{l} r_{13}(1) \\ \vdots \\ r_{1,s_1+1}(1) \\ r_{1,s_1+2}(1) \quad \leftarrow \\ \hline r_{23}(1) \\ \vdots \\ r_{2,s_2+2}(1) \quad \leftarrow \\ \hline \vdots \\ \hline r_{p3}(1) \\ \vdots \\ r_{p,s_p+2}(1) \quad \leftarrow \end{array}\right]$$

where the rows indicated by the arrows are obtained by applying (3.2) with $q = 1$ and 2, respectively. Hence $[\bar{K}, \bar{F}\bar{K}, \bar{F}^2\bar{K}, \ldots] = R$. Thus rank $R = n$. Complete observability follows from the fact that the observability matrix is a matrix obtained by a permutation of the rows of the identity matrix.

CONCLUSIONS

We have relaxed the usual selection rules for the selection of a basis for the rows of the Hankel matrix of Markov parameters. Once a set of basis vectors has been selected, with these more flexible rules, there are different ways of defining a state vector. We have exhibited two such state-space representations, a minimal one and a non-minimal one, and we have shown the relationships between the two.

In Ljung and Rissanen (1976), it has been argued that, for numerical reasons, one should select a state with least complexity. This idea has given rise to a number of methods for the selection of a "well-conditioned" state, using either complexity measures or orthogonality ideas (see eg, Van Overbeek and Ljung (1982), Wertz, Gevers and Hannan (1982)). Even though the parameterizations we have presented here are more complex and contain more parameters than the usual ones, by allowing more flexibility in the choice of the state we have increased the chances of obtaining a "well conditioned" state.

In Gevers and Tsoi (1984) we have also described MFD forms using these new selection rules, and we have argued that they might be useful for the description of time series processes with very different time scales.

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