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SUFFICIENT EXPERIMENTAL CONDITIONS FOR THE CONVERGENCE OF AN ADAPTIVE OBSERVER FOR NONLINEAR BIOCHEMICAL PROCESSES

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Abstract. An adaptive observer/identifier is presented for the on-line estimation of states and parameters of fermentation processes in two typical applications. A proof of convergence is given under realistic experimental conditions. Simulation results illustrate the behaviour of our observer.

Keywords. Adaptive observers; fermentation processes; nonlinear systems.

INTRODUCTION

A biochemical (or fermentation) process is a process of growth of microorganisms (called the biomass) by the consumption of an appropriate substrate under suitable environmental conditions. A critical issue in controlling fermentation processes is that reliable and cheap sensors for on line measurement of the main biological variables (i.e. biomass, substrate or byproducts concentrations) are most often not available (see e.g. Dochain and Bastin, 1987). The use of adaptive observers as "software sensors" for some of these variables can therefore constitute a valuable alternative.

The purpose of this paper is to describe sufficient (and realistic !) experimental conditions which guarantee the convergence of two such adaptive observers for nonlinear biochemical processes.

THE BASIC NON LINEAR STATE SPACE MODEL

We consider the case of an anaerobic fermentation, characterized by a Blackman's kinetics, with a growth-associated byproduct in liquid phase. The dynamical behaviour of this process, in a stirred tank reactor, is described by the following multilinear state-space model :

$$\dot{z}_1 = \alpha z_1 z_2 - u_2 z_1 \quad (1.a)$$

$$\dot{z}_2 = -\beta z_1 z_2 + u_1 u_2 - u_2 z_2 \quad (1.b)$$

$$\dot{z}_3 = \gamma z_1 z_2 - u_2 z_3 \quad (1.c)$$

with the states: z_1 = biomass concentration
z_2 = substrate concentration
z_3 = product concentration
the inputs: u_1 = influent substrate concentration
u_2 = dilution rate
the parameters : α, β, γ constant and strictly positive

We assume that the inputs u_1 and u_2 are known on line. We shall describe adaptive observers for the system (1.a-c) in the two following cases :

I. - the output $y(t) = z_1(t)$ is measured on line
- the adaptive observer allows the on line estimation of the state z_2 and the parameters α and β.

II. - the output $y(t) = z_3(t)$ is measured on line
- the adaptive observer allows the on line estimation of the state z_2 and the parameters α and $\delta = \beta/\gamma$

EXPERIMENTAL CONDITIONS

c1. The initial biomass and substrate concentrations are strictly positive :

$$z_1(0) > 0 \qquad z_2(0) > 0$$

C2. The dilution rate $u_2(t)$ is strictly positive and constant.

c3. The influent substrate concentration $u_1(t)$ is
. persistently exciting of order 2
. piecewise continuous
. bounded : $0 < 2u_2 \frac{\beta max}{\alpha min} < u_1(t) \leq \bar{u}_1$

where αmin and βmax are respectively known lower and upper bounds on α and β, such that $2\beta max \geq \alpha min$.

APPLICATION I : *Estimation of $z_2(t)$ from measurements of $z_1(t)$*

We first introduce the following state transformation :

$$x_1 = z_1$$

$$x_2 = \frac{\beta}{\alpha} z_1 + z_2$$

Then, the state-space model (1.a-b) is equivalent to:

$$\dot{x}_1 = \alpha x_1 x_2 - \beta x_1^2 - u_2 x_1 \quad (2.a)$$

$$\dot{x}_2 = (u_1 - x_2) u_2 \quad (2.b)$$

$$y = x_1 \quad (2.c)$$

For this system, we consider the following adaptive observer/identifier :

$$\dot{\hat{x}}_2 = (u_1 - \hat{x}_2)\, u_2 \quad (3)$$

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \frac{1}{s+c} \begin{bmatrix} y \\ y\hat{x}_2 \\ -y^2 \\ -u_2 y \end{bmatrix} \quad (4)$$

$$\hat{y} = cv_0 + \hat{\alpha}v_1 + \hat{\beta}v_2 + v_3 \quad (5)$$

$$\begin{bmatrix}\dot{\hat{\alpha}}\\ \dot{\hat{\beta}}\end{bmatrix} = \Gamma\varphi\ (y-\hat{y}) \quad (6)$$

$$\dot{\Gamma} = -\Gamma\varphi\varphi^T\Gamma \qquad \Gamma(0) = k_0 I > 0 \quad (7)$$

$$\varphi = \begin{bmatrix}v_1\\ v_2\end{bmatrix} \quad (8)$$

$$\dot{\hat{z}}_2 = -\hat{\beta}z_1\hat{z}_2 + u_1u_2 - u_2\hat{z}_2 \quad (9)$$

. c and k_0 are arbitrary positive constant design parameters

. the initial conditions $v_i(0)$ $(i=0,\dots,3)$ are arbitrary

. the initial estimates $\alpha(0)$, $\beta(0)$, $x_2(0)$ are arbitrary but non negative

. the implementation (6)-(7)-(8) of the adaptive parameter estimator is a continuous time least-squares implementation with $\varphi(t)$ the regression vector.

APPLICATION II : ***Estimation of $z_2(t)$ from measurements of $z_3(t)$***

We now introduce the following state transformation:

$$x_1 = z_3$$
$$x_2 = z_2 + \frac{\beta}{\gamma} z_3$$
$$x_3 = z_1 - \frac{\alpha}{\gamma} z_3$$

Then the state-space model (1.a-b-c) is equivalent to :

$$\dot{x}_1 = \gamma x_2x_3 - \beta x_1x_3 + \alpha x_1x_2 - \frac{\alpha\beta}{\gamma} x_1^2 - u_2x_1 \quad (10.a)$$
$$\dot{x}_2 = (u_1-x_2)\ u_2 \quad (10.b)$$
$$\dot{x}_3 = -u_2x_3 \quad (10.c)$$
$$y = x_1$$

It clearly follows from (10.c) and condition c3 that

$$\lim_{t\to\infty} x_3 = 0$$

On the other hand, it is easily shown from conditions c1, c2, c3, that x_1 and x_2 are bounded (see e.g. Dochain and Bastin, 1987).

Hence, the first two terms in (10.a) tend to zero; and, for sufficiently large t, the model (10.a-b) reduces to :

$$\dot{x}_1 = \alpha x_1x_2 - \delta x_1^2 - u_2x_1 \quad (11.a)$$
$$\dot{x}_2 = (u_1-x_2)\ u_2 \quad (11.b)$$
$$y = x_1 \quad (11.c)$$

This model is completely equivalent to the model (2.a-c) and a similar identifier/observer can be formulated for the on line estimation of α, δ, x_2 and z_2.

CONVERGENCE ANALYSIS

We define the estimation errors :

$$\tilde{y} = y-\hat{y} \qquad \tilde{x}_2 = x_2-\hat{x}_2 \qquad \tilde{z}_2 = z_2-\hat{z}_2$$
$$\tilde{\alpha} = \alpha-\hat{\alpha} \qquad \tilde{\beta} = \beta-\hat{\beta}$$

and $\tilde{\theta} = \begin{bmatrix}\tilde{\alpha}\\ \tilde{\beta}\end{bmatrix}$

From (2) to (9), the following 'error system' can be derived :

$$\dot{\tilde{x}}_2 = -u_2\tilde{x}_2 \quad (12.a)$$
$$\dot{\tilde{\theta}} = -\Gamma\varphi\ (\varphi^T\tilde{\theta} + \epsilon) \quad (12.b)$$

where ϵ is bounded by an exponentially decaying function.

The global convergence of the adaptive observer is then established by :

Theorem : Under conditions c1 to c3

i) $\lim_{t\to\infty} \tilde{x}_2 = 0$

ii) $\lim_{t\to\infty} \tilde{\theta} = 0$

iii) $\lim_{t\to\infty} \tilde{z}_2 = 0$

Proof :

i) follows from (12.a) and c3

ii) a) using c1-c3 it is easily shown that $\varphi(t)$ is bounded

b) $\varphi(t)$ is p.e. by the lemma hereafter and by a result of Boyd and Sastry (1983)

then $\lim_{t\to\infty} \tilde{\theta}(t) = 0$

by a result of Kreisselmeier (1977)

iii) follows immediately from (9), i) and ii).

Lemma :

Under conditions c1 to c3
if $u_2 \leq x_1(o) \leq x_1$ and $2u_2\beta max/\alpha min \leq x_2(o) \leq x_1$, then the vector

$\xi(t) = (yx_2, -y^2)$ is a p.e. signal.

Proof :

a) Under the above assumptions, it is easily shown that the state variables x_1, and x_2 are bounded as follows, for all t :

$$0 < u_2 \leq x_1 \leq \bar{u}_1$$

$$0 < 2u_2 \frac{\beta max}{\alpha min} \leq x_2 \leq \bar{u}_1$$

b) From (2.b) and condition C3, we conclude that x_2 is p.e. of order 2 (see Mareels and Gevers, 1988).

c) We define the following time transformation

$$s(t) = \int_0^t x_2(\tau)d\tau$$

d) We define the functions $y_1(s)$ and $y_2(s)$ by :

$x_1(t) = (y_1 \circ s)(t) \qquad x_2(t) = (y_2 \circ s)(t)$

where o stands for composition of functions.

e) It follows from a), b), c) and from the definition of y_2 that :

x_2 p.e. of order 2 => (y_2 $\dot{y}_2$) p.e

=> ($\alpha y_2 - u_2$ $\alpha\dot{y}_2$) p.e

f) Now equation (2,a) can be rewritten as follows

$$\frac{dy_1}{ds} = -\beta y_1 + w$$

with $w = \alpha y_2 - u_2$

Then we have the following succession of implications (see Mareels and Gevers, 1988) :

w p.e of order 2 => (y_1 w) p.e

=> (y_1 $\dot{y}_2$) p.e

=> (x_1 x_2) p.e

=> ($x_1 x_2$ $-x_2^2$) p.e

which completes the lemma.

It must be emphasized that the bounds on x_1 and x_2 (point a)) are critical in deriving all the implications of this theorem.

COMMENT

The global stability of the adaptive observer in the more realistic case of time varying (state dependent) parameters α and β can also be established (see Mareels et al. (1987) and Dochain and Bastin (1988) for further details).

SIMULATION RESULTS

Simulation experiments of the above estimation algorithm (3)(9) have been carried out . These are illustrated in fig.1.

The system (1) has been simulated under the following conditions :

$\alpha = 0.4$ $\qquad \beta = 0.1$

$x_1(0) = 0.7$ $\qquad x_2(0) = 0.25$

$u_1(0) = 2$ $\qquad u_2(0) = 0.1$ (day^{-1})

Random amplitude step functions of the inputs signals u_1 and u_2 have been applied to the system

A least-square discretized version of algorithm (4)-(8) has been implemented.

The following initial values have been used :

$\hat{\alpha}(0) = \hat{\beta}(0) = 0$

$\hat{z}_2(0) = 0.25 \qquad \hat{x}_2(0) = 2$

Fig. 1c and 1d show the convergence of the estimates α and β to their true constant values, while the evolution of the substrate concentration $z_2(t)$ and of its estimate $z_2(t)$ is shown in fig.1e.

The actual biological parameters, i.e. the specific growth rate $\mu \triangleq \alpha z_2$ and the yield coefficient $k_1 \triangleq \beta/\alpha$, can be estimated from the above estimates as follows :

$$\hat{\mu} = \hat{\alpha}\hat{z}_2$$

$$\hat{k}_1 = \frac{\hat{\beta}}{\hat{\alpha}}$$

The estimation results are presented in fig.1f and 1g. Note the very good convergence of the estimate k_1 to its true value, and the ability of the estimation algorithm to follow (after an initial transient) the variations of $z_2(t)$ and $\mu(t)$.
It is also worth noting that in fig.1, condition c2 has not been respected (since a non constant input u_2 has been applied to the system). In fact, it has been observed that in practice, such a time-varying input u_2 induces more excitation to the estimator.

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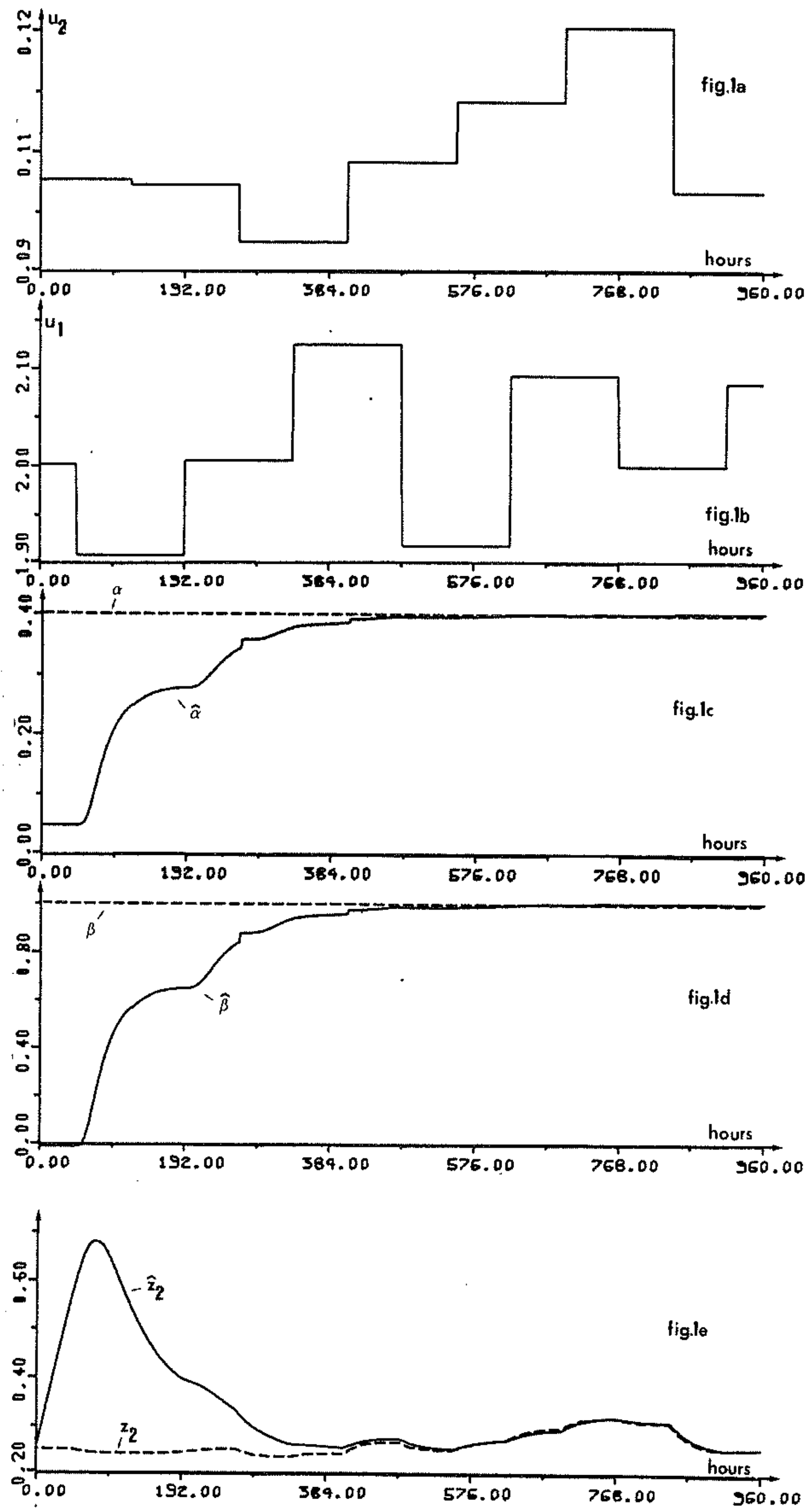

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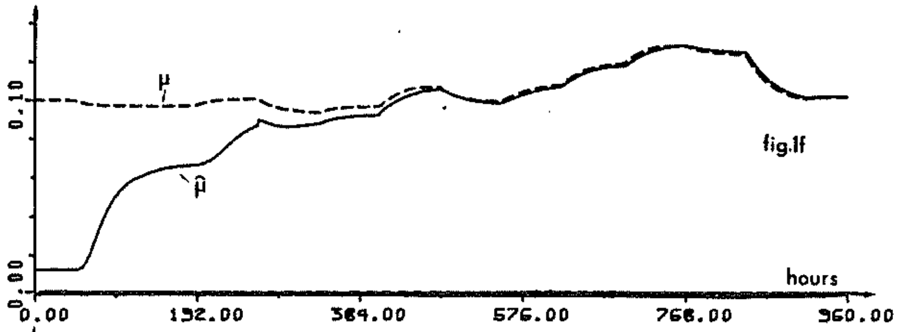
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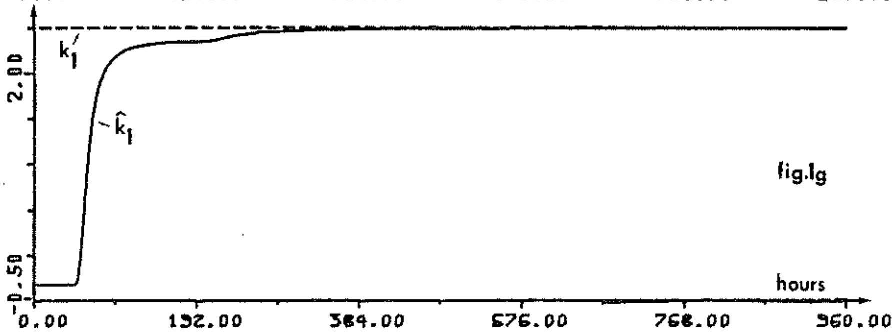
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