C40 CDC 1988,

A PARAMETER ESTIMATION ALGORITHM FOR CONTINUOUS TIME STOCHASTIC ADAPTIVE CONTROL

M. Gevers and V. Wertz

Louvain University, Batiment Maxwell, B-1346 Louvain-La-Neuve, Belgium

G.C. Goodwin

Department of Electrical & Computer Engineering University of Newcastle, N.S.W., Australia 2308

ABSTRACT

We present an adaptive parameter estimator for continuous time stochastic systems which results in globally stable adaptive control algorithms. Our analysis method is inspired by Goodwin and Mayne [1] in that properties of the parameter estimator are established that hold irrespective of the control law. Compared with comparable discrete-time schemes, the success of our algorithm hinges crucially on a modification of the parameter update law which ensures that the normalised regression vector remains uniformly bounded.

I. INTRODUCTION

A number of global stability results have been established for discrete stochastic adaptive control algorithms in the case of known model structure [2], [3]. However, a realistic continuous-time result has been elusive because of the difficulty of handling continuous-time white noise properly. Preliminary results have had to rely upon data dependent assumptions on the normalised regression vectors, that were unlikely to be true [4].

Here we present a globally stable continuous time stochastic adaptive control algorithm that does not require such assumptions. The main feature of our algorithm is a new normalization technique which guarantees that the normalised regression vector $\frac{\varphi^T\varphi}{r}$ is uniformly bounded. This allows us to establish properties of the parameter estimator which hold irrespective of the control law, in particular, the absence of finite escapes. The parameter estimator can be combined with a wide class of certainty equivalence control laws to establish almost sure global stability of the resultant algorithm.

Due to lack of space, we present the main ideas and results without proofs; the reader is referred to the full paper [5] for details and proofs.

II. THE MODEL

Consider the following single input single output state space innovations representation

$$\begin{cases} dx_t = Ax_t dt + Bu_t dt + Kd\omega_t \\ dy_t = Cx_t dt + d\omega_t \end{cases} \tag{2.1}$$

where ω_t is a Wiener process with incremental covariance $\sigma^2 dt$. Notice that y_t is the integral of the output. We denote by $\mathcal{F}_t$ the increasing σ-fields generated by ω_t and the unknown initial condition x_0, and we assume that $\|x_0\|^2$ bounded. Without loss of generality, we assume that the model (2.1) is in observer form:

$$A = \begin{bmatrix} -a_{n-1} & & & \\ -a_{n-2} & & I_{n-1} & \\ \vdots & & & \\ -a_0 & 0 & \cdots & 0 \end{bmatrix} \quad B = \begin{bmatrix} b_{n-1} \\ \vdots \\ \vdots \\ b_0 \end{bmatrix} \quad K = \begin{bmatrix} k_{n-1} \\ \vdots \\ \vdots \\ k_0 \end{bmatrix} \tag{2.2}$$

$$C = [\,1 \quad 0 \quad \cdots \quad 0\,]$$

The parameters a_i, b_i and k_i are assumed unknown but constant.

With proper care taken for the initial conditions, the model (2.1)-(2.2) can be rewritten in linear regression form as follows (see [5] for details):

$$dy_t = \phi_t^T \theta dt \tag{2.3}$$

Here

$$\theta = \begin{bmatrix} G \\ B \\ K-G \end{bmatrix} \quad \text{with } G = \begin{bmatrix} g_{n-1} \\ \vdots \\ g_0 \end{bmatrix} \tag{2.4}$$

where $g_i \triangleq e_i - a_i$, $i=0, \ldots, n-1$, and the e_i are arbitrary subject to $E(\rho) \triangleq \rho^n + e_{n-1}\rho^{n-1} + \cdots + e_0$ having all its zeros in the open left half plane.

Using the operator notation $\rho \triangleq \frac{d}{dt}$, the regressor ϕ_t is obtained as the solution of the following equations

$$\phi_t^T = \frac{1}{E(\rho)} [\rho^{n-1}dy, \cdots, dy \,;\, \rho^{n-1}u, \cdots, u \,;\, \rho^{n-1}d\omega, \cdots, d\omega]_t \tag{2.5}$$
$$= [(\phi_t^y)^T, (\phi_t^u)^T, (\phi_t^\omega)^T]$$

The equations (2.5) are, in fact, to be regarded as integral equations with appropriate choice of initial conditions: see [5].

The model (2.3)-(2.5) is not in a form suited for parameter estimation, because the last n terms of ϕ_t depend on the unmeasured noise ω_t. Thus, assuming that $\hat{\theta}_t$ is an $\mathcal{F}_t$-measurable estimate of θ we define the predicted output $d\hat{y}_t$ via a pseudo-regressor ψ_t as follows

$$d\hat{y}_t = \psi_t^T \hat{\theta}_t dt = [(\psi_t^y)^T, (\psi_t^u)^T, (\psi_t^e)^T]\, \hat{\theta}_t dt \tag{2.6}$$

where $\psi_t^y = \phi_t^y$, $\psi_t^u = \phi_t^u$ and ψ_t^e is the solution of

$$\psi_t^e = \frac{1}{E(\rho)} [\rho^{n-1}de, \cdots, de]_t^T \quad , \quad \psi_0^e = 0 \tag{2.7}$$

with

$$de_t \triangleq dy_t - d\hat{y}_t \tag{2.8}$$

We now denote

$$\eta_t \triangleq \phi_t^T \theta - \psi_t^T \hat{\theta}_t \tag{2.9}$$

and notice that η_t is the "deterministic part" of the prediction error:

$$de_t - d\omega_t = \eta_t dt \tag{2.10}$$

It can then be shown [5] that

$$D(\rho)\eta_t = E(\rho)[-\psi_t^T \tilde{\theta}_t] \tag{2.11}$$

where $D(\rho) \triangleq \det(\rho I - A + KC)$ and $\tilde{\theta}_t \triangleq \hat{\theta}_t - \theta$. The scenario has now been set up to introduce a particular parameter estimator to construct $\hat{\theta}_t$.

III. THE ESTIMATION ALGORITHM

We introduce the following estimation algorithm

$$d\hat{\theta}_t = \frac{\psi_t}{r_t}(dy_t - \psi_t^T \hat{\theta}_t\, dt) \tag{3.1}$$

where

$$r_t = \sup_{0\le\tau\le t} \|\psi_\tau\|^2 + \int_0^t \|\psi_\tau\|^2 d\tau + c_0,\ \neq c_0 > 0 \tag{3.2}$$

and c_0 is any positive deterministic number. Note that $\hat{\theta}_t$ is obtained by an Ito integral which makes sense locally (and even globally as is established below) since by construction $\frac{\psi_t}{r_t}$ is $\mathcal{F}_t$-measurable and continuous. Using (3.1), (2.6), (2.8) and (2.10), we see that the parameter error, $\tilde{\theta}_t$, is the solution of the following stochastic differential equation

$$d\tilde{\theta}_t = \frac{\psi_t}{r_t}\eta_t\, dt + \frac{\psi_t}{r_t} d\omega_t \tag{3.3}$$

The analysis of our estimation algorithm now revolves around establishing properties of the solutions of (3.3). Recall that η_t and $\psi_t^T\tilde{\theta}_t$ are related by (2.11). The first result implies that the solutions $\tilde{\theta}_t$ almost surely have no finite escape time.

Lemma 3.1

The solutions of (3.3) satisfy

$$E\{\sup_{0\le t\le T} |\tilde{\theta}_t|^{2p}\} < (K + 3E\{|\tilde{\theta}_0|^{2p}\})e^{KT} \tag{3.4}$$

for any $p > 0$, where K depends only on p, T, c_0 and the bound on the initial condition $\|x_0\|$.

We now present the main properties of our parameter estimator.

Theorem 3.1

Consider the model (2.1) and the estimator (3.1)-(3.2) with ψ_t defined by (2.5)-(2.7). Assume that the zeros of $D(\rho)$ and $E(\rho)$ are strictly in the left half plane, and that $\frac{D}{E}$ is input strictly passive.

Then:

1) $$\limsup_{t\to\infty} \|\tilde{\theta}_t\| < \infty \quad \text{a.s.} \tag{3.5}$$

2) $$\limsup_{t\to\infty} \int_0^t \frac{\eta_\tau^2}{r_\tau} dt < \infty \quad \text{a.s.} \tag{3.6}$$

3) For all finite Δ,
$$\lim_{t\to\infty} \sup_{0\le T\le\Delta} \|\hat{\theta}_{t+T} - \hat{\theta}_t\| = 0 \quad \text{a.s.} \tag{3.7}$$

A key feature of our estimator is our choice of r_t, which differs from the choice usually made in the discrete time case. Our choice guarantees that $\frac{\|\psi_t\|^2}{r_t} \le 1 \quad \forall\, t$ and this is crucial in establishing (3.6). In [4] r_t is defined as $\int_0^t \|\psi_\tau\|^2 d\tau$ and, as a consequence, all the convergence proofs require the assumptions that $\frac{\|\psi_t\|^2}{r_t}$ is almost surely bounded. This is rather unrealistic given that ψ_t contains signals driven by a Wiener process.

Given an adaptive control law that guarantees that the signals in the closed loop satisfy a certain growth condition, a further boundedness property can be established.

Lemma 3.2

Consider the model (2.1) or (2.3), and assume that the parameter estimator is such that (3.6) holds, and that the controller is such that the following growth condition holds

$$\frac{r_t}{t} \le C + \frac{K_1}{t}\int_0^t \eta_\tau^2 d\tau \quad \text{a.s.}$$

where C and K are finite positive constants. Then

1) $$\limsup_{t\to\infty} \frac{r_t}{t} \le K_2 < \infty \quad \text{a.s.}$$

2) $$\lim_{t\to\infty} \frac{1}{t}\int_0^t \eta_\tau^2\, d\tau = 0 \quad \text{a.s.}$$

Recall that η_t can be thought of as the "deterministic part" of the prediction error. Comparing with the continuous-time deterministic algorithms of [1], we have not proved the uniform boundedness of $\frac{\eta_t}{r_t^{1/2}}$ or of $\dot{\hat{\theta}}$ (if that quantity existed). However, the parameter estimator properties that have just been described are all that is needed to establish global boundedness of a class of indirect certainty equivalence adaptive control algorithms, such as pole-placement algorithms. We refer the reader to [5] for a presentation and proof of these results.

CONCLUSIONS

An adaptive parameter estimator for continuous time stochastic systems has been described. A key feature of this estimator is the normalization which ensures that no finite escapes occur. We also establish properties of the estimator which hold for very general classes of input signals. These properties can be used to establish global stability of continuous time stochastic adaptive control algorithms.

References

[1] Goodwin, G.C. and D.Q. Mayne, 'A parameter estimation perspective of continuous time model reference adaptive control', *Automatica*, Vol.23, No.1, January 1987pp.57-70.

[2] Goodwin, G.C., P.J. Ramadge and P.E. Caines, 'Discrete time stochastic adaptive control', SIAM J. Control and Optimization, Vo.19, No.6, November 1981, pp.829-853.

[3] Kumar, P.R., 'A survey of some results in stochastic adaptive control', SIAM J. Control and Optimization, Vo.23, No.3, 1985, pp.329-380.

[4] Chen, H.F., Recursive Estimation and Control for Stochastic Systems, Wiley 1985, New York.

[5] Gevers, M., G.C. Goodwin and V. Wertz, 'Continuous time stochastic adaptive control', Tech. Report EE8743, University of Newcastle, submitted to SIAM J. Control and Optimization.