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Adaptive optimal control and GPC : robustness analysis *

Robert R. Bitmead*, Michel Gevers, Vincent Wertz

*Dept Systems Engineering, Australian National University, and
Laboratoire d'Automatique, Université Catholique de Louvain,
B-1348 Louvain-la-Neuve, Belgium

Abstract. -The explicit connection is made between the (nonadaptive) control law design stage of Predictive Adaptive Control and recent techniques of Linear Quadratic Gaussian control with Loop Transfer Recovery for robustness enhancement. The inherent controller design robustness of these methods is examined in terms of the nonadaptive closed loop properties. With this latter information, we then pose the question of to which plant model does the Recursive Least Squares identification stage converge in closed loop. We can demonstrate the robust interplay of the identifier and of the controller in this class of adaptive control methods. Design guidelines for a Candidate Robust Adaptive Predictive Controller follow.

1 Introduction

Predictive adaptive control laws have achieved a significant level of acceptability and practical success in industrial process control applications. Their *raison d'être* is typically advanced as an heuristic generalization of minimum variance adaptive control, where the control, u_k, is chosen to minimize the plant output, y_{k+1}, variance from a reference, w_{k+1}, i.e. $E(y_{k+1} - w_{k+1})^2$, at each time k [1]. Several generalizations followed and culminated into a multistep predictive controller called GPC [2]. There, u_k to u_{k+N_u-1} are chosen to minimize,

$$J(N_1, N_2, N_u) = E\{\sum_{j=N_1}^{N_2} [y_{k+j} - w_{k+j}]^2 + \sum_{j=1}^{N_u} \lambda_j [u_{k+j-1}]^2\} \quad (1)$$

subject to $u_{k+i} = 0, \quad i = N_u, \ldots, N_2.$

The identification component of these adaptive controllers is usually chosen to be a variant of Recursive Least Squares (RLS).

The criterion (1) may be interpreted as a receding horizon Linear Quadratic (LQ) control objective, i.e. at each time k the criterion may be viewed as a finite horizon LQ problem with this horizon receding into the future as the time advances. Thus asymptotically the control law solving this problem will yield a fixed state variable feeback controller, see [2,3,4].

In order to solve for the control value, u_k, for this problem it is necessary to generate predictions of the future values of the plant output , y_{k+i}. The mechanism of generating these predictions does perform the same rôle as does a state estimator or observer, [5,6] and, as stated in [8],the selection of the predictor/observer polynomial, $C(z)$, in practice has a dramatic influence upon the success of the adaptive controller. Thus the predictive control strategy is composed of the same elements as a linear state variable feedback strategy with incomplete measurements and a quadratic cost, which we denominate (somewhat too generally) as Linear Quadratic Gaussian (LQG) control.

Since the predictive control criterion has a direct interpretation as an equivalent LQG problem, one is led to ask whether this inherent control strategy might be robust as a nonadaptive control law. LQG has been the subject of considerable recent robustness analysis. Specifically, LQG Loop Transfer Recovery (LTR) [7] theory has been developed to allow the recovery in LQG designs of the robustness known to be present in full state feedback LQ control. We shall interpret the predictive controller design in terms of the LQG/LTR theory and show how the rôle of particular elements of the design, such as λ in (1), is the same as that of certain other variables in LTR methods.

The robustness of LQG/LTR control systems is measured by the ability to maintain closed loop stability in the face of multiplicative perturbations to the open loop nominal plant. We show that the multiplicative error between the plant and its nominal model is related to the relative error of the frequency response of the open loop system.

In adaptive control systems, one is concerned not just with controller design and its robustness to externally imposed modelling errors, but also with on-line plant identification where two facets of the same issue arise: 'What is the effect of the control law upon the model identified in closed loop?' and/or 'To what extent is the identifier capable of providing a model compatible with the controller robustnesss?'. The Return Difference Equalities (EDR) of optimal filtering and control, i.e. LQG, allows us to conclude that the model fit achieved via the use of this control law dovetails precisely acccording to a relative error criterion. In this way, the adaptation helps to tune the model to

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fit the controller robustness, thereby improving closed loop robustness overall.

Design guidelines for a Candidate Robust Adaptive Controller follow from both the LQG/LTR analysis and study of the interplay between identification and control.

2 Predictive Control and LQG

Let the open loop plant have a state variable description as follows,

$$x_{k+1} = Fx_k + Gu_k \quad (2)$$
$$y_k = Hx_k. \quad (3)$$

2.1 The Control Law

We make the following observations concerning the equivalence between GPC and LQG;

- the criterion (1) is quadratic in the plant state, x_k, and the input, u_k.
- the weighting matrices associated with the state and with the control are, respectively, $Q = H^T H$ and $R = \lambda I$, ignoring the differences in horizons N_2 and N_u. Incorporating these horizons and N_1 requires the introduction of some zero Q values and some infinite R's for certain time indices [4].
- the solution of this standard finite horizon LQ problem may be stated in terms of a time-varying linear state variable feedback law,

$$u_{k+N-j} = -(G^T P_{j-1} G + R_{j-1})^{-1} G^T P_{j-1} F \hat{x}_{k+N-j}. \quad (4)$$

  Here $\hat{x}_k$ is a state estimate produced by an observer, P_j is the solution of a Riccati difference equation:

$$P_{j+1} = F^T P_j F - F^T P_j G (G^T P_j G + R_j)^{-1} G^T P_j F + Q_j \quad (5)$$

- the control law (??), in the predictive control situation, only results in u_k being applied from this entire finite time solution. A new finite horizon problem is solved for u_{k+1} with the same horizon. Thus a stationary control law arises with a gain only dependent upon P_{N-1}. This is the principle behind receding horizon LQ control.
- the reference signal, w_k, may be introduced into the picture to yield a tracking problem [9]. This solution will have the same stability properties as the regulation problem but will include a reference signal precompensator.
- the asymptotic stability of the closed loop with a receding horizon LQ control law is not guaranteed, since the finite horizon subproblems do not have any connection to infinite horizon properties such as stability. The asymptotic stability is assured for the closed loop of an infinite horizon LQ problem with $[F, Q^{1/2}]$ stabilizable.

2.2 The State Estimator

The linear state variable control laws above are all implemented using state estimates, $\hat{x}_k$, since full state information is not available. We make the following remarks concerning the generation of these state estimates;

- because the plant is assumed strictly proper it is possible to produce state estimates using an observer with a direct feedthrough term without encountering algebraic loop problems. Such an observer has the form,

$$\hat{x}_{k+1} = (F - MHF)\hat{x}_k + (G - MHG)u_k + My_{k+1} \quad (6)$$

  where the eigenvalues of $F - MHF$ may be arbitrarily placed by choice of M provided $[F, HF]$ is an observable pair.
- if we presume that output measurements are corrupted by noise, as is the state, then an alternative derivation of an observer may be carried out based upon optimal state estimation, Kalman Filtering (KF) [10]. Then the filter gain, M, is designed via the solution of the filtering algebraic Riccati equation as follows, with measurement noise covariance Q_o and independent process noise covariance R_o,

$$M = \Sigma H^T (H\Sigma H^T + R_o)^{-1} \quad (7)$$
$$\Sigma = F\Sigma F^T - F\Sigma H^T (H\Sigma H^T + R_o)^{-1} H\Sigma F^T + Q_o. \quad (8)$$

- note the structural similarity (duality actually) between the LQ control solution and the KF solution. At least this is true inasmuch as Riccati difference equations of similar form are used. The actual duality exists between the LQ control of a plant with delay and the Kalman one-step-ahead predictor for the same plant, but the observer/KF with direct feedthrough is not dual to the LQ controller for a plant with delay.
- by careful choice of the state coordinate basis, it is possible to formulate state estimates directly as future plant output predictions [6]. In this formulation, the observer characteristic polynomial is identified with the predictor polynomial.

Hence, the nonadaptive predictive control law (perhaps with some slight modification for guaranteed stability) falls under the ambit of stationary infinite horizon LQ control implemented with the use of an observer. With only a slight abuse of notation we shall denominate this Linear Quadratic Gaussian (LQG) control in the sequel.

3 LQG Robustness and LTR

There has been considerable activity for an extended period on issues associated with the robustness of feedback control systems. Here we shall state a subset of results which pertain to the robustness of LQG control. The path followed here is close to a single-input/single-output, discrete-time version of

Lehtomaki *et al.* [11], Doyle [12], Stein and Athans [7], and Kwakernaak [13] and shall culminate with the work of Maciejowski [14]. Extensions to multivariable systems are direct from the referred works but would hinder clarity here.

3.1 Closed Loop Robustness

Here we shall consider criteria for the preservation of stability when a given feedback controller, $C(z)$, designed on the basis of a nominal plant, $P(z)$, is connected in closed loop with an actual plant, $\tilde{P}(z)$, which may differ from $P(z)$.

Denote the actual plant, $\tilde{P}(z)$, as being a multiple of the nominal $P(z)$,

$$\tilde{P}(z) = L(z)P(z), \qquad (9)$$

where the multiplicative perturbation is $L(z)$ and pole-zero cancellations may occur in (??). Denote the nominal and actual controller/plant cascades by $P(z)C(z) = G(z)$ and $\tilde{P}(z)C(z) = \tilde{G}(z)$.Then, an important condition for the preservation of stability under multiplicative perturbation is the following,

$$|L^{-1}(z)-1| < \min\left[1, |1 + G(z)|\right] \text{ at each } z \in \Omega, \quad (10)$$

where Ω is a contour consisting of the unit circle indented around unit circle zeros of the open loop characteristic polynomial of $G(z)$.

This result links the closed loop robustness to multiplicative perturbation $L(z)$ with the value of the frequency response of the return difference, $[1 + G(z)]$, of the nominal controlled system. To gain a further appreciation of the statement, consider the following expression of the final inequality,

$$\begin{aligned} L^{-1}(z) - I &= [G(z)L(z)]^{-1}[G(z) - G(z)L(z)] \\ &= \tilde{P}^{-1}(z)[P(z) - \tilde{P}(z)]. \qquad (11) \end{aligned}$$

That is, the relative error between the nominal and actual plant has to be bounded by the magnitude of the frequency response of the return difference of the nominal controller, in order to guarantee closed loop stability.

3.2 LQ and KF Robustness

If we denote the feedback gain in the LQ solution (??) by K and regard the signal $z_k = -K\hat{x}_k$ as a fictitious output in a unity feedback representation of the LQ plant then one has for the cascaded plant/controller transfer function, $G(z)$ of the previous section,

$$G(z) = -K(zI - F)^{-1}G. \qquad (12)$$

We may now state the discrete Return Difference Equality of optimal control. This is a relationship satisfied by the solution of an infinite horizon time-invariant LQ problem. It follows simply from the algebraic Riccati equation.

$$\begin{aligned} &R + G^T(z^{-1}I - F)^{-T}Q(zI - F)^{-1}G = \\ &\quad [I - K(z^{-1}I - F)^{-1}G]^T(G^TPG + R) \\ &\quad [I - K(zI - F)^{-1}G] \qquad (13) \end{aligned}$$

We consider this equality for z on the unit circle and note that the left hand side is a spectrum which consists of a strictly positive constant part, R, and a strictly proper nonnegative part. Thus from (??) we have directly that

$$|I - K(zI-F)^{-1}G| \geq \frac{R}{G^TPG + R}, \qquad \text{for all } z \in \Omega. \quad (14)$$

Referring now to (??) and (??), we see that LQ full state feedback possesses a natural robustness to multiplicative perturbation of the plant system, because $I + G(z) = I - K(zI - F)^{-1}G$.

For the Kalman Filter, there exists the dual Return Difference Equality, and, by direct analogy to the LQ case, we have that

$$|I - H(zI-F)^{-1}M| \geq \frac{R_o}{H\Sigma H^T + R_o}, \qquad \text{for all } z \in \Omega \quad (15)$$

where R_o is the measurement noise covariance matrix and Q_o is the process noise covariance matrix. Thus the Kalman Filter also possesses an inherent degree of robustness to multiplicative mismodelling of the plant.

3.3 Guaranteed LQ Margins with Observers

"There is none!" [12].

That is, it is possible to find examples of the complete loss of the above robustness to multiplicative perturbation when either the LQ controller is implemented using an observer, LQG, or the Kalman Filter is used when the plant is operating under state-estimate feedback.

The issue is that the plant/controller cascade, $G(z)$ above, is replaced as follows for the LQ (so-called input preserving) problem with an observer possessing a direct feedthrough term,

$$\begin{aligned} G(z) &= -K \times \{I + (I - MH)(F + GK) \\ &\quad [zI - (I - MH)(F + GK)]^{-1}\} \\ &\quad \times MH(zI - F)^{-1}G. \qquad (16) \end{aligned}$$

For the KF, or output preserving problem with an observer possessing a direct feedthrough term,

$$\begin{aligned} G(z) = &-H(zI - F)^{-1}GK \\ &\times\{I + (I - MH)(F + GK) \\ &[zI - (I - MH)(F + GK)]^{-1}\} \times M. \quad (17) \end{aligned}$$

This alteration from the ideal $G(z)$ has the potential effect that all robustness of the closed loop may be lost, since we no longer have an automatic lower bound upon $|I + G(e^{j\omega})|$ given by the return difference equality. It may well be the case that the new closed loop is actually more robust than the ideal (see the examples in [16]). What is at issue is that the 'guaranteed' margin implied by the return difference equalities is lost.

3.4 Loop Transfer Recovery

Loop Transfer Recovery (LTR) refers to a design methodology whereby the robustness guarantee of LQ and KF can be recovered for certain plants operating under LQG. The better known theory for LTR is in continuous time, see [7,15], where the KF and LQ are dual. The design methodology runs as follows;

1. perform an LQ design according to normal rules.
2. design an observer by using the Kalman Filter with covariance matrices Q_o and $R_o = \rho I$, with the positive real parameter $\rho \to 0$.
3. if the open loop plant, $P(s) = H(sI - F)^{-1}G$, is minimum phase, i.e. it possesses no zeros which are in the right half complex plane, then $G_{LQG}(s) \to G_{LQ}(s)$ as $\rho \to 0$ for all s. If $P(s)$ is nonminimum phase then this convergence does not occur.

An alternative design procedure is the dual of the above;

1. perform a KF design according to normal rules.
2. design a state variable feedback via LQ with weighting matrices Q and $R = \lambda I \to 0$.
3. if $P(s)$ is minimum phase then $G_{LQG}(s) \to G_{KF}(s)$ as $\lambda \to 0$ for all s.

Several points are in order here;

- The principle of these LTR methods is that by performing a singular optimal filtering (control) design in the LQG controller, the extra dynamics appearing in $G_{LQG}(s)$ are forced to cancel with the (stable) open loop plant zeros, thereby yielding the required LQ or KF $G(s)$, which possesses an inherent degree of robustness.
- While the method guarantees nothing for nonminimum phase plants, there abound claims that the methodology performs well with many such systems. The question is then what value of ρ or λ is a suitable stopping point.
- The robustness is achieved at the expense of performance, since the nominal behaviour is detuned from the natural LQG settings, Q, R, Q_o, R_o.
- It is generally agreed that the latter (KF followed by singular optimal control) method is preferable to the LQ-first approach.

In discrete time, the situation is similar but deceptively different, as has been investigated by Maciejowski [18] with the following remarks being pertinent.

Remark 1 *In discrete time there is a distinction between the Kalman Filter and the Kalman one-step-ahead predictor. This distinction does not exist in continuous-time.*

Remark 2 *LTR is only possible in discrete time using the true Kalman Filter followed by singular optimal control design. The nonduality of discrete time LQ and KF for a strictly proper plant causes the discrepancy between design approaches.*

Remark 3 *The LTR return difference converges to the KF return difference as the LQ control weighting $\lambda \to 0$ if and only if the open loop plant $P(z)$ is minimum phase and minimum delay, i.e. no zeros outside the unit circle and* $\det HG \neq 0$, *i.e. a unit delay.*

To summarize, the discrete time LQG/LTR design follows;

1. perform a Kalman Filter (with direct feedthrough) design for the open loop plant.
2. conduct a singular optimal control design for the LQ feedback with weighting matrices Q and $R = \lambda I$.
3. if the open loop plant is minimum phase and minimum delay, then $G_{LQG}(z) \to G_{KF}(z)$ for all z as $\lambda \to 0$. Otherwise one must cease the design at a nonzero value of λ — the empirical claim being that this works well for many systems.

We now claim that Predictive Control, (PC), as a nonadaptive control law, implements *ipso facto* an LQG/LTR design. Specifically, we have shown the control objective of Generalized Predictive Control (1) to be explicitly a receding horizon LQ criterion which, if asymptotic stability modifications are made, is identical to an infinite horizon LQ criterion with weighting matrices $Q \geq H^T H$ and $R = \lambda I$.

The positive value λ is selected as a somewhat arbitrary control input weighting. Our thesis here is that λ is a small correction to our desired controller criterion selected to achieve closed loop robustness to our design. That is the λ's in Predictive Control and in LQG/LTR are the same.

To complete our presciption of Predictive Control as LQG/LTR, we note that the 'missing link' in the PC design is the specification of the predictor polynomials, $C(z)$. A major feature of PC is that the choice of $C(z)$ is recognized as crucial for success and also is presumed to have been fixed before the control design stage and tuning is begun. Given the identity between these predictors and observers, we are now able to associate with each fraction of the PC design an equivalent from discrete time LQG/LTR. Mohtadi [5] advocates the use of $C(z)$ which reflect the plant and measurement noise models and which implement delay-free predictions of the plant output. The connection to Kalman Filtering theory is clear and indeed suggests how this component of PC should be designed. By careful choice of KF weighting matrices it is possible to maintain the coordinate-free state variable controller design [16].

4 Adaptation and Robustness

The major application of Predictive Control laws is in the area of adaptive process control. That is the nonadaptive control law design is coupled with an on-line Recursive Least Squares (RLS) parameter estimator. The questions then concern the interplay between the adaptation and the controller robustness achieved via the LQG/LTR connection. We first consider how the control signal, u_k, affects the selection of the nominal plant transfer function, $P(z)$, and then move on to consider the features of the control in the Predictive Controller or LQG/LTR.

According to the recent theory of Ljung [8], the minimization of a Least Squares prediction error criterion between asymptotically large, stationary data sets of plant input and output can, via Parseval's identity, be interpreted in terms of a frequency response deviation minimization. Specifically, we presume a plant structure

$$y_k = \tilde{P}(z)u_k + v_k, \qquad (18)$$

where v_k is the measurement noise, and we presume a model structure

$$\hat{y}_k = P(z,\theta)u_k + H(z,\theta)\xi_k, \qquad (19)$$

where ξ_k is the prediction error of the model (??) and $P(z,\theta)$ and $H(z,\theta)$ are parametrized transfer functions. The Least Squares solution is sought as follows

$$\min_{\theta\in\Theta} \mathrm{E}\left[H^{-1}(z,\theta)(y_k - P(z,\theta)u_k)\right]^2. \qquad (20)$$

This may equally well be regarded as a frequency domain minimization

$$\min_{\theta\in\Theta} \int_{-\pi}^{\pi} [|\tilde{P}(e^{j\omega}) - P(e^{j\omega},\theta)|^2\Phi_{uu}(\omega) + \Phi_{vv}(\omega)]\frac{d\omega}{|H(e^{j\omega},\theta)|^2}. \qquad (21)$$

From (??) we see the rôle played in the selection of plant model, $P(z,\theta)$, by the input spectrum as well as the parts played by the noise spectrum and the class of noise models. For our (illustrative) purposes here we shall assume in what follows that we are operating with small noise, $\Phi_{vv}(\omega) \ll \Phi_{uu}(\omega)$, and that we have chosen no noise model, $H(z,\theta) = 1$. Further, we shall assume that the on-line use of RLS with prediction error updates yields the true Least Squares prediction error minimizing value of θ. Then the simplified identification criterion associated with the adaptation of the controller is

$$\min_{\theta\in\Theta} \int_{-\pi}^{\pi} \left|\tilde{P}(e^{j\omega}) - P(e^{j\omega},\theta)\right|^2 \Phi_{uu}(\omega)\, d\omega. \qquad (22)$$

We next pose the question of what is the nature of $\Phi_{uu}(\omega)$ for PC or LQG/LTR. Recall that our control law is given by

$$u_k = -K\hat{x}_k + w_k, \qquad (23)$$

where w_k is the reference signal and the state variable feedback gain, K, is given by the solution of a singular optimal predictive control problem, i.e. $R = \lambda \to 0$ and $Q = H^T H$ fixed. The closed loop transfer function between the external reference signal, w_k, and the control signal, u_k, is precisely the inverse of the (input preserving or LQ) return difference. For the nominal plant, the observer dynamics cancel from this transfer function to yield the closed loop relation

$$u_k = [I - K(zI - F)^{-1}G]^{-1}w_k. \qquad (24)$$

To ascertain the spectrum of u_k we need to analyse this return difference with $Q = H^T H$ and $R = \lambda \to 0$. The following relation has been shown (see [16]);

$$\Phi_{uu}(\omega) \approx P^{-1}(e^{j\omega})\Phi_{ww}(\omega)P^{-1}(e^{-j\omega}). \qquad (25)$$

This expression for the nominal closed loop control spectrum stems directly from the singular optimal control element of its genesis.

The control spectrum from (??) may now be substituted into the identification criterion (??) to yield the equivalent adaptive predictive control identification criterion;

$$\min_{\theta\in\Theta} \int_{-\pi}^{\pi} \left|\frac{\tilde{P}(e^{j\omega}) - P(e^{j\omega},\theta)}{\tilde{P}(e^{j\omega})}\right|^2 \Phi_{ww}(\omega)\, d\omega. \qquad (26)$$

That is to say, using (??), the adaptation criterion is identical to the modelling criterion

$$\min_{\theta\in\Theta} \int_{-\pi}^{\pi} \left|L(e^{j\omega}) - 1\right|^2 \Phi_{ww}(\omega)\, d\omega. \qquad (27)$$

To draw these many threads together at this stage, we have shown the following;

- The LQG/LTR control law is implicit in the statement of Predictive Control.
- This control law is naturally robust to unmodelled multiplicative plant perturbations, $L(z)$ provided $|L^{-1}(z) - I| < |I + G(z)|$ for all $z \in \Omega$.
- The LQG/LTR control law engenders a closed loop control spectrum which causes the adaptation component to minimize a weighted integral of $\left|L(e^{j\omega}) - 1\right|^2 \Phi_{ww}(\omega)$.
- The adaptation and control stages may thus be seen to be mutually supporting in terms of their effects vis-à-vis the closed loop robustness requirements.
- One may use the above results to indicate how the reference signal should be chosen to achieve maximal robustness and also how one might choose identification prefilters better to enhance robustness, if extra information is available. By careful choice of w_k and prefilters, one may encourage the modelling to fit best precisely at those frequencies where $[I + G(z)]$ is small. In this way, designs exceeding straight H_∞ (worst case) robustness design can be achieved.

The analysis so far has shown that, when identification is performed in closed loop in an off-line manner, the regulator influences the frequency spectrum of the plant input and therefore affects the bias distribution of the estimated model. In an adaptive control situation, the regulator parameters will be continuously adjusted as a function of the on-line estimates of the plant parameters. As a consequence, there is no guarantee, in an adaptive closed loop, that the solution of the recursive parameter will converge, and, even if it does so, it is not clear what the meaning of the convergence point might be. This situation has been studied in [16], where it is shown that under appropriate assumptions, including persistence of excitation of the regressors and slow adaptation, the solution of the parameter update equation will converge to a point that is close to the asymptotic solution of the off-line parameter estimation criterion.

5 A Candidate Robust Adaptive Predictive Controller

Following the analysis which has been developped in the previous sections, we may now start the synthesis part by proposing an adaptive controller coupling the robustness theory of LQG/LTR linear control law design with Least Squares identification modelling ideas. Actually, what follows here is not the precise description of an adaptive controller ready for industrial use. Instead, we give guidelines which, if used, can lead to several adaptive predictive controllers which will be robust (at least this is our claim supported by the previous analysis) to perturbations and unmodelled dynamics. The candidate robust adaptive controller should consist of the interconnection of the following components (see [16] for details):

- An RLS parameter identifier with slow adaptation, fixed noise model based on prior knowledge, signal filters rolling off outside the reference bandwidth and around the desired closed loop bandwidth, and a reference signal consisting of the desired trajectory plus (if necessary) a perturbation which contains sufficient spectral support to dominate the output disturbance over the closed loop bandwidth. Further, additional features such as normalization, deadzones and leakage should be considered.

- An LQG controller with a Kalman filter design based on a coupled plant and noise model, with measurement noise power determined by a scalar ρ and plant process noise covariance GG^T, and an LQ tracking control law design based on coupled plant, noise and reference models with state weighting being H^TH and control weighting λ.

The robustness of the control law stage is achieved by two features. Firstly, the Kalman filter is designed to have a degree of robustness, and secondly, the LQG/LTR methodology is used to strive to achieve a closed loop having the equivalent KP robustness. The factor influencing the robustness of the KF is the value of ρ. Clearly, a more robust KP is obtained by choosing a bigger ρ, but this causes a more cautious predictor having slower response. One is sacrificing the closed loop performance for the sake of ameliorating the robustness.

The design variable complementary to ρ is the LQ weighting λ. The LQG/LTR theory dictates that the selection of λ should be made after the selection of ρ and should, indeed, be to take λ as small as is feasible before the control signal in the closed loop begins to fail to comply with its design limits. Thus ρ is chosen first and then a suitably small value of λ is sought.

Examples of this prototype controller can be found in [16].

6 Conclusion

We have analyzed the connections between an existing popular adaptive control scheme and the recent theory of robustness of LQG systems to show that this control law embodies the elements of LQG/LTR design together with an interrelating adaptation phase whose objective is interpretable in terms of best fitting the control robustness criterion. In this fashion we have shown how adaptation and robustness may be mutually supportive and, further, have demonstrated both why this adaptive control method has been found to work well in many circumstances and how it might be modified better to take into account further plant knowledge and more sophisticated design.

The pleasing feature of this work is that full contact is established between practical adaptive control and sophisticated control design theories. This should reinforce both camps.

7 References

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