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ON THE IDENTIFICATION OF FEEDBACK SYSTEMS

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ABSTRACT

The identification of linear systems operating under feedback control is considered. In this paper a number of recent identifiability results are briefly reviewed first; attention is then focused on the "joint process" method, in which a global innovations model is identified for the vector stochastic process made up of all inputs and outputs, from which the open loop and feedback dynamics are subsequently derived. It is shown that without any knowledge about the structure of the "true" system the joint process model can lead to different transfer functions for the open loop model. A set of sufficient conditions are given under which the open loop transfer function and noise dynamics can be uniquely and correctly recovered from the joint process model. It is argued that what might in some cases justify the use of the joint process method rather than direct prediction error methods (treating the input and output data as if the system were operating in open loop) is not its superior identifiability property, but the fact that the model can be identified with covariance factorization methods rather than with maximum likelihood methods that require an a priori parametrization.

I. INTRODUCTION.

The identification of linear systems operating under feedback control is presently the subject of much research. This problem is of great practical interest since many industrial, biological or economic systems operate under some form of feedback. In most practical cases the dynamics of the regulator is unknown (e.g. a manual operator) and the control cannot be disconnected during identification. In addition the system and the regulator are usually perturbed by unknown noise processes.

Fig. 1 represents a closed-loop system with linear open loop dynamics; u is the measurable input signal, s is a measurable externally applied signal, y is the measurable output signal, and w and v are unmeasurable external noise sources that can be assumed white. The signals s and v may or may not be present. In general the regulator can be known or unknown, linear or nonlinear, constant or time-varying. Fig. 2 represents the special case where the regulator is linear and constant; the joint process method, that will be investigated in this paper, is applicable to this particular case.

In many practical situation the identification is performed with the objective of replacing the existing nonoptimal feedback by an optimal regulator. This requires the identification of both the open-loop transfer function $F(z^{-1})$ and the noise dynamics $G(z^{-1})$ (see figures).

In analyzing or designing identification methods for feedback systems the question of identifiability of the open loop transfer function and noise dynamics is of paramount importance. This question has been intensively investigated in recent years. The most comprehensive work on this subject is due to Gustavsson, Ljung and Söderström [1] - [3] ; the survey paper [3] gives a good overview of other results on this subject.

Gustavsson et al. have shown [1] that identifiability depends upon the structure chosen for the model, the identification method, and the experimental conditions. They consider basically three different identification methods :

1°) Indirect identification, whre the open loop process parameters are determined from the estimated closed-loop process parameters using the knowledge of the regulator. The fact that the regulator must be known and noise-free reduces dramatically the applicability of this method.

2°) Direct identification, where a prediction-error method is applied to the input-output data exactly as if the system were operating in open loop. For this method, Gustavsson et al. have shown [1] that the system is uniquely identifiable provided the chosen model structure is able to reproduce the true system transfer function and noise dyna mics for a particular value of the parameter vector θ, and one of the following conditions hold :
   a) the signal s can be chosen to be persistently exciting of high order. Under this condition, Defalque, Gevers and Installé have proposed a particular identification algorithm [4];
   b) the signal v is independent of w and is persistently exciting;
   c) the regulator is nonlinear and nondegenerate;
   d) the regulator is persistently time-varying.

3°) Joint process identification, when the vector stochastic process, made up of the input and output processes u and y, is modelled as the output of a system driven by white noise; the open loop model is subsequently derived from the matrix transfer function of the joint process by matrix manipulations. This last method was first proposed by Phadke and Wu [5] , [6] , and independently derived by Caines and Chan [7] , [8] .

In this paper the joint process method, which has been successfully used by the author for the identification of a glass-furnace process [9] ,will be investigated. In Section II the method will be briefly described. In Section III it will be shown that without some a priori knowledge about the structure of the true system, the joint process method can lead to different (i.e. nonunique) open loop transfer functions and noise models. This is basically due to the fact that several causal and causally invertible spectral factors can be defined fro vector processes. These spectral factors form an equivalence class; unfortunately the open loop transfer functions and noise models that are derived from these equivalent spectral factors are not equivalent; therefore it is in general impossible to recover the correct open loop dynamics from the joint process model. However certain conditions on the delay structure and on the noise structure of the true system will imply that the joint vector process can only be modelled by a unique representative of the class of equivalent spectral factors. In Section IV two such sets of sufficient conditions about the delay structure of the unknown system will be given which, when satisfied, guarantee that the joint process transfer function will uniquely correspond to the correct open loop dynamics. Finally it will be argued in Section V that the use of the joint process method may be justified not so much because it guarantees identiability in cases where the direct method fails, but because in some cases it may be advantageous to use identification algorithms that are applicable to purely nondeterministic processes (such as covariance factorization methods), rather than input-output methods which require an a priori parametrization, i.e. which require more knowledge about the structure of the system.

II. THE JOINT PROCESS METHOD

The joint process method is applicable to the case where the open loop and feedback dynamics are linear (see Fig. 2) and where no external input is available (s = 0). It is desired to identify the open loop dynamics of the process y :

$$y(k) = F(z^{-1})\ u(k) + G(z^{-1})\ w(k) \tag{1a}$$

from measurements of the input and output processes.
The feedback loop is described by :

$$u(k) = H(z^{-1})y(k) + K(z^{-1})v(k) \tag{1b}$$

The vectors y and w are p x 1the vectors u and v are m x 1, w and v are unobservable i. i.d. vector processes with zero mean and covariances Σ_1 and Σ_2, respectively. F, G, H and K are assumed to be regular, stable and inverse stable matrices of appropriate dimensions; in addition it is assumed, without loss of generality, that $G(\infty) = I_p$ and $K(\infty) = I_m$. Assume finally that $E\{\ w(k)v'(j)\} = \delta_{kj}.\ \Sigma_{12}$.

The basic idea of the joint process method is to consider the unmeasurable external white noises w and v as the true inputs to the system. A new observable vector stochastic process z(k), made up of the input and output processes, and a new vector white noise process $\varepsilon(k)$ are defined :

$$z(k) = \begin{bmatrix} y(k) \\ u(k) \end{bmatrix} \qquad \varepsilon(k) = \begin{bmatrix} w(k) \\ v(k) \end{bmatrix}$$

Then we can write z(k) as the output of a white noise driven model

$$z(k) = W(z^{-1})\varepsilon(k) \tag{2}$$

where the transfer function matrix $W(z^{-1})$ can be expressed in terms of the open loop and feedback models as follows:

$$W = \begin{bmatrix} (I - FH)^{-1}G & (I - FH)^{-1}FK \\ (I - HF)^{-1}HG & (I - HF)^{-1}K \end{bmatrix}$$

The covariance matrix of $\varepsilon(k)$ shall be denoted R_ε.

Conversely, the open loop and feedback dynamics F, G, H and K can be recovered from $W(z^{-1})$. Let $W(z^{-1})$ be partitioned into four submatrices of appropriate dimensions

$$W = \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix}$$

Then it is easy to see that

$$F = W_2W_4^{-1} \tag{4a}$$

$$G = W_1 - W_2W_4^{-1}W_3 \tag{4b}$$

$$H = W_3W_1^{-1} \tag{4c}$$

$$K = W_4 - W_3W_1^{-1}W_2 \tag{4d}$$

The idea of the joint process method is to first identify the transfer function matrix $W(z^{-1})$ by modelling the joint input-output process $z(k)$ as the output of a white noise driven model. The open loop transfer function and noise dynamics are subsequently derived from $W(z^{-1})$ through (4).

III. NONUNIQUENESS OF THE OPEN LOOP MODEL

For vector stochastic processes there exist different causal and causally invertible spectral factors $W(z^{-1})$ for the same spectral density matrix function, depending on the structure of the covariance matrix of the vector innovations process. These spectral factors form an equivalence class C_W; they can be obtained from one another by a "similar transformation" [7]: if $W(z^{-1})$ is any particular realization of the process $z(k)$ such that the spectral density $S_z(z)$ can be factored as

$$S_z(z) = W(z^{-1})\, R_\varepsilon\, W'(z),$$

then $W(z^{-1})T$ is also a realization of $z(k)$ if T is any $(p+m)\times(p+m)$ nonsingular constant real matrix. Indeed $z(k)$ can be written as:

$$z(k) = W(z^{-1})T \,.\, T^{-1}\varepsilon(k) = W_1(z^{-1})\varepsilon_1(k).$$

The covariance matrix of $\varepsilon_1(k)$ is R_{ε_1} defined by

$$R_{\varepsilon_1} = T^{-1}\, R_\varepsilon\, (T^{-1})'$$

In order to obtain a unique joint process model Phadke [5] and Chan [7] suggest using a unique "canonical" representation for $W(z^{-1})$; Chan suggests setting $W(\infty) = I$, and denotes the canonical form thus obtained as $W_c(z^{-1})$. Notice that any element $W(z^{-1})$ of the class C_W of equivalent spectral factors can be transformed to the unique representative $W_c(z^{-1})$ through right-multiplication by an appropriate transformation matrix T.

The major problem with the joint process method is that the similar transformations on $W(z^{-1})$ do not preserve the open loop and feedback transfer function matrices F, G, H, K of eqs. (1). This is easy to verify; a simple example will be presented below. In particular it can be shown that, whatever the delay structure of the true system is, the following holds:

Lemma: Let $W(z^{-1})$ be the joint process representation obtained from an arbitrary quadruple $\{F, G, H, K\}$ through (3). If $W_c(z^{-1})$ is the equivalent "canonical" representation, then the corresponding open loop and feedback realization $\{F_c, G_c, H_c, K_c\}$ obtained from $W_c(z^{-1})$ through (4) has a delay in both the open loop and the feedback loop, i.e. $F_c(\infty) = 0$ and $H_c(\infty) = 0$.

Proof: By assumption $W_c(\infty) = I_{p+m}$. In addition (see Section II), $G_c(z^{-1})$ and $K_c(z^{-1})$ are such that $G_c(\infty) = I_p$ and $K_c(\infty) = I_m$. Applying (3) for $W_c(z^{-1})$ with $z = \infty$, and taking into account these various assumptions, it follows immediately that

$$F_c(\infty) = H_c(\infty) = 0.$$

Example: Consider the following feedback system, with instantaneous transmission in the feedforward loop, and a unit delay in the feedback loop:

$$\begin{cases} y(k) = -b\, y(k-1) + a\, u(k) + w(k) \\ u(k) = y(k-1) + v(k) \end{cases}$$

The joint process matrix transfer function is

$$W(z^{-1}) = \left[1 + (b-a)z^{-1}\right]^{-1} \begin{bmatrix} 1 & a \\ z^{-1} & 1+bz^{-1} \end{bmatrix}$$

After transformation to canonical form, we get the following equivalent representation

$$W_c(z^{-1}) = \left[1 + (b-a)z^{-1}\right]^{-1} \begin{bmatrix} 1 & 0 \\ z^{-1} & 1 + (b-a)z^{-1} \end{bmatrix}$$

This leads to the following open loop and feedback models

$$y(k) = (a-b)\; y\,(k-1) + w_c(k) \tag{5a}$$

$$u(k) = y(k-1) + v_c(k) \tag{5b}$$

Obviously the open loop model that would be obtained through the joint process method using the canonical representation is not equivalent with the true open loop model. Notice in particular that the model (5) contains a delay in each loop.

IV. SUFFICIENT CONDITIONS FOR UNIQUENESS

From the lemma and the example it follows that the use of the canonical representation for the joint process matrix transfer function will always lead to a model that has a delay in both loops, even though the true system does not have this property. More generally the fact that the equivalent joint process models lead to nonequivalent open loop and feedback models implies that the joint process identification method is, in general, not applicable. However the method would be applicable if the true system were known to obey certain conditions such that

- either there exists only one element $W°(z^{-1})$ in the class C_W of equivalent spectral factors that satifies these conditions;
- or the class of equivalent spectral factors satisfying these conditions forms a subset $C_W^°$ of C_W such that all elements of $C_W^°$ lead to the same open loop model.

Two such sets of conditions under which the system can be uniquely identified through the joint process method are the following

1°) if the system (1) is known to have a unit delay in both paths, i.e. $F(\infty) = H(\infty) = 0$.

2°) if the noise sources w and v are known to be orthogonal, and if either $F(z^{-1})$ or $H(z^{-1})$ is known to have a unit delay.

To prove the first part, notice that $W(\infty)$, evaluated through (3) with $F(\infty) = H(\infty) = 0$ and $G(\infty) = K(\infty) = I$, gives $W(\infty) = I$. Hence only the canonical form $W_c(z^{-1})$ corresponds to a system with delays in both loops.

To prove the second part, notice that if R_ε is block-diagonal and $F(\infty) = 0$, then $W(\infty)$, evaluated through (3), has the form

$$W(\infty) = \begin{bmatrix} I & 0 \\ H(\infty) & I \end{bmatrix} \tag{6}$$

Let T be an arbitrary real constant nonsingular matrix. Since any equivalent transfer function matrix $W(z^{-1})T$, evaluated at $z = \infty$, must have the form (6), it follows that T must have the form

$$T = \begin{bmatrix} I & 0 \\ T_3 & I \end{bmatrix}$$

with T_3 arbitrary. Since the transformed covariance matrix $T^{-1}R_\varepsilon(T^{-1})'$ must also be block-diagonal, it follows easily that $T = I$. Hence with this second set of assumptions $W(z^{-1})$ is also unique.

V. COMMENTS AND CONCLUSIONS

The two sets of conditions given above may not be the only ones under which the joint process method will lead to a unique (and correct) open loop transfer function and noise model. In general however the joint process method has not proved to have any identifiability property that direct methods do not have, as we have shown above. As a matter of fact, if either one of the two conditions given above holds, a direct prediction error identification method will also yield the correct open loop parameter estimates, as Chan has indicated [7].

This does not mean that the joint process method should be ruled out. Rather we believe that it is useful in all cases where the feedback system to be identified is known to obey one of the two sets of conditions mentioned in Section IV, but where no other knowledge is available about the structure of the system. In these cases it will often be advantageous to use covariance factorization methods on the joint process z(k), at least as a first step. Indeed these methods require no structural knowledge, but they are not applicable to processes with deterministic inputs, whereas maximum likelihood methods require an a priori parametrization of the system and hence some more knowledge on the structure. One such case is the glass-furnace process, where delays are known to exist in each loop, and where the joint process identification method, together with a covariance factorization algorithm, has been successfully applied [9].

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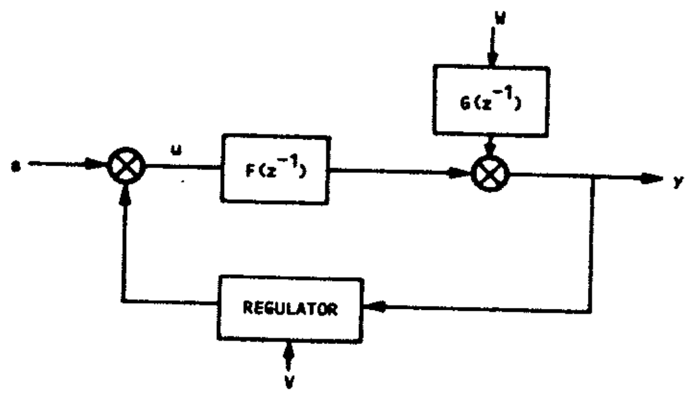

Fig. 1.

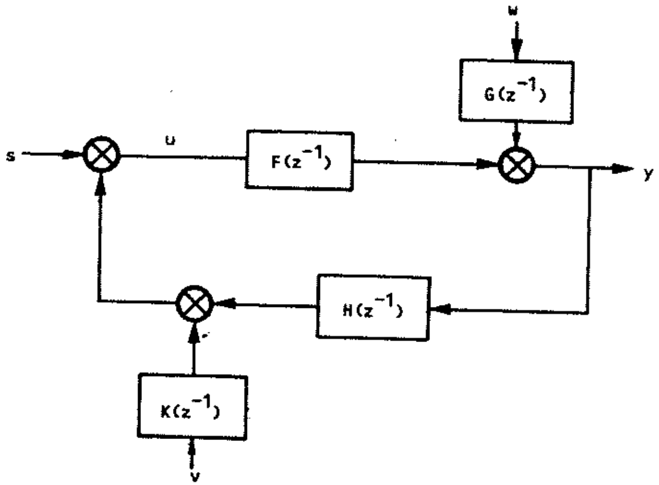

Fig. 2.