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Experimental Restricted Complexity Controller Design

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Abstract: This paper considers a restricted complexity LQG controller design using an existing iterative identification and control design scheme applied to a disturbance rejection problem. The cornerstone of this iterative method is a frequency weighting derived from experimental closed loop input/output signals. This frequency weighting characterises mismatch between two closed loop systems, one in which the controller design is conducted and the other in which the controller is implemented. It is shown that the frequency weighting is mechanism which translates the closed loop mismatch into better control performance when the enhanced LQG controller acts on the true plant. The results obtained provide an intuitive link between the iterative design method and an existing closed loop controller reduction technique.

Keywords:- Iterative Identification/Control Designs, LQG Control, Innovations

1 Introduction

Recently several iterative identification and control design strategies have been proposed that use input-output data collected whilst the plant is operating under closed loop control, see Zang *et al* (1991), Schrama (1992), Bayard (1992), Lee *et al* (1993). Consistent with each of these strategies is the desire to seek an improvement of some performance index with each iteration, and a recognition that identification and control design need to be treated as a joint problem.

This paper considers an iterative identification and control design for disturbance rejection based upon a H_2/LQG iteration as described by Zang *et al* (1992). This scheme shapes the design of an enhanced LQG controller with the objective of ameliorating the global performance of the achieved closed loop system depicted in Fig. 1. Traditional LQG controller design relies on minimising a local performance objective for the designed closed loop system of Fig. 2. The iterative design introduces a closed loop signal based correction in the form of a frequency weighting which modifies the local performance objective such that the controller minimisation incorporates discrepancies between the achieved and the designed closed loop systems. It should be noted that the iterative design uses the frequency transformed designed closed loop system depicted in Fig. 3 in which to conduct the synthesis of the enhanced controller. This allows the controller design to proceed via standard LQG methods.

The frequency weighted H_2/LQG iterative design operates on a class of fixed order controllers. This is a distinct advantage over some other iterative designs (Schrama (1992), Lee *et al* (1993)) which produce increased complexity controllers, for which the order may need to be reduced prior to implementation.

For an optimal closed loop system as depicted in Fig. 4, that is one with full plant and disturbance knowledge, Kwakernaak and Sivan (1972) show that a LQG controller is the optimal solution to an output feedback problem in which the states are unmeasurable and the measurements are noisy. This result uses the separation principle to show that the LQG controller includes an optimal observer to estimate the unmeasurable states. The optimal observer has the property that its innovations process is white, see Kwakernaak and Sivan (1972). The suitability of a restricted complexity LQG controller in the achieved closed loop can be gauged by the whiteness of its state estimator's innovations in that loop, moreover this is a measure which describes the closeness between the closed loop transfer functions of the optimal and achieved closed loop systems. This provides our motivation for investigating, in this paper, the innovations process of an LQG controller produced by the frequency weighted H_2/LQG iterative design. This paper demonstrates that it is the frequency weighting in the iterative design which is the mechanism for maintaining the whiteness of the state estimator innovations process in the achieved closed loop.

The notion of maintaining the whiteness of an innovation process in an LQG controller as a lever to attaining closeness between the achieved and optimal closed loop transfer functions is not new. It has been considered by Liu and Anderson (1986) with respect to controller reduction in the achieved closed loop system given a complete description of the plant and the disturbance. Using stable factorisation and balancing techniques to approximate a right coprime factorisation of the full order LQG controller, Liu and Anderson show the closed loop controller reduction results in a reduced order LQG controller in which the state estimator's innovations process is white. This reduction technique does not require a frequency weighting

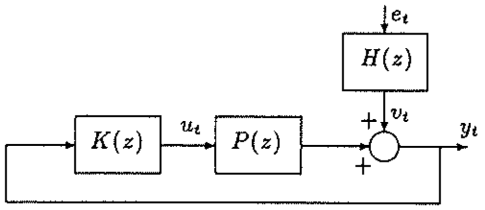

Fig. 1. The achieved closed loop system consists of a controller $K(z)$ acting upon the true plant $P(z)$. v_t is the additive output disturbance.

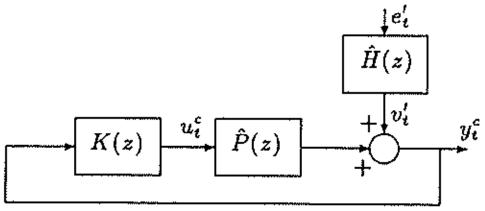

Fig. 2. The designed closed loop system consists of a controller $K(z)$ acting upon the plant model $\hat{P}(z)$. v'_t is a model of the additive output disturbance.

in order to maintain the whiteness of the innovations process. Furthermore, Liu and Anderson have derived an upper bound on the L_∞ norm for the closed loop mismatch between the optimal and achieved closed loops given an exact plant and disturbance representation.

The main contribution of this paper is the connection between frequency weighted iterative design using inexact plant and disturbance description and closed loop controller reduction based on full plant and disturbance knowledge. Both methods deliver LQG controllers which when implemented in the achieved closed loop system attempt to maintain the whiteness of the state estimator innovations process, thereby reducing the difference between the achieved and optimal closed loop systems. Hence, these two design procedures reach the same endpoint of having restricted complexity controllers in the achieved closed loop. In both cases, the achieved closed loop is in some sense close to the optimal closed loop. The advantage of the iterative design is that full plant and disturbance knowledge are not necessary.

This paper is organised as follows. Section 2 details preliminary and motivational aspects behind the iterative identification/control design of Zang *et al* (1992). Section 3 presents the theory which describes the mechanism for maintaining the whiteness of the Kalman Predictor innovations in the achieved closed loop with LQG controllers produced by the iterative design. An example promoting this theory is given in Section 4. Section 5 concludes.

2 Frequency Weighted H_2/LQG Iterative Control Design

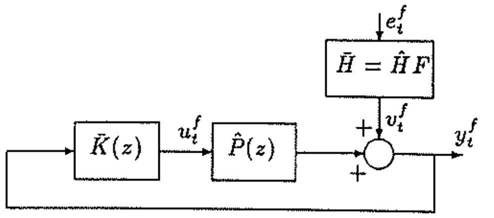

Fig. 3. The transformed designed closed loop system consists of a controller $\bar{K}(z)$ acting upon the plant model $\hat{P}(z)$. $\hat{H}F$ is the frequency transformed disturbance model and controller. $\bar{K}$ is the frequency transformed controller.

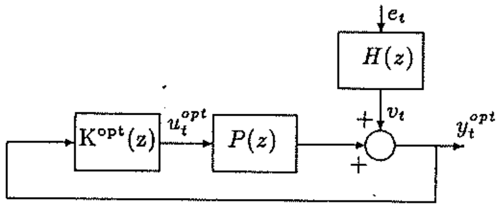

Fig. 4. The optimal closed loop system consists of the truly optimal controller $K^{opt}(z)$ designed using full knowledge of both the true plant $P(z)$ and the true output disturbance $H(z)$.

The control problem considered in this paper involves the rejection of an additive disturbance v_t which acts upon a plant output y_t. Without any feedback control the plant output y_t is given by

$$y_t = P(z)u_t + v_t, \tag{1}$$

where u_t is the plant input and $P(z)$ is a strictly proper rational stable transfer function. Furthermore, v_t is assumed to be a quasi-stationary zero mean stochastic process represented by

$$v_t = H(z)e_t, \tag{2}$$

where e_t is a white noise process with zero mean and variance σ^2. $H(z)$ is a proper stable rational transfer function.

In many cases, the plant and disturbance transfer functions are not known exactly. Associated with the plant and disturbance are a set of parameterised models,

$$\left\{\hat{P}(z,\theta), \hat{H}(z,\theta),\ \theta \in D_\theta \subset R^d\right\},$$

where D_θ in R^d is a subset of admissible values. This set is not presumed to contain the true plant and disturbance $\{P(z), H(z)\}$.

A particular model in that model set, driven by an input u^c_t, will produce an output described by

$$y^c_t = \hat{P}(z,\theta)u^c_t + \hat{H}(z,\theta)e'_t, \tag{3}$$

for a specific value of θ, where e'_t is a white noise process and $\hat{P}(z,\theta)$ and $\hat{H}(z,\theta)$ are strictly proper rational transfer functions.

A controller K can be implemented in closed loop with the true plant and disturbance as depicted by the achieved closed loop system of Fig. 1. The global performance of the controller in the achieved closed loop is given by an LQ cost criterion of the form,

$$J^* = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N} (y_t)^2 + \lambda(u_t)^2\right\}, \quad (4)$$

where λ is a penalty imposed upon the control action.

The iterative scheme proposed by Zang *et al* (1991) uses two frequency weightings F_1 and F_2 to quantify a signal-based model mismatch. That is,

$$F_1 = \{\frac{\Phi_y}{\Phi_{y^c}}\}^{1/2} \quad ; \quad F_2 = \{\frac{\Phi_u}{\Phi_{u^c}}\}^{1/2}, \quad (5)$$

where $\Phi_y, \Phi_{y^c}, \Phi_u, \Phi_{u^c}$ are the spectra of the corresponding closed loop signals. These signals are all readily available for measurement.

Transfer function expressions for the above ratios between the corresponding achieved and designed closed loop signals are given by,

$$\frac{y_t}{y_t^c} = \frac{u_t}{u_t^c} = \frac{H[1+\hat{P}K]}{\hat{H}[1+PK]}, \quad (6)$$

where K is a frequency weighted certainty equivalence feedback controller.

For the disturbance rejection case, the only case consider in this paper, it is apparent that the frequency weightings F_1 and F_2 are equivalent. Define a frequency weighting F as,

$$F \triangleq F_1 = F_2. \quad (7)$$

The frequency weighted local performance LQ criterion associated with the designed closed loop system is given by,

$$J^c = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N} [F(y_t^c)]^2 + \lambda[F(u_t^c)]^2\right\}, \quad (8)$$

$$= \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N} [1+\lambda|K|^2]\frac{|\hat{H}|^2}{|1+\hat{P}K|^2}\right\}. \quad (9)$$

Here, the frequency weightings serve as a mechanism by which to manipulate the iterative design procedure in order to deliver controllers which accomplish high performance in the achieved closed loop system. This manipulation recognises that controller performance is always measured in the achieved closed loop system.

The standard non-frequency weighted version of the above local performance LQ criterion (8,9) is given by,

$$\hat{J} = \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N} (y_t^c)^2 + \lambda(u_t^c)^2\right\}, \quad (10)$$

$$= \lim_{N\to\infty} \frac{1}{N} E\left\{\sum_{t=1}^{N} [1+\lambda|K|^2]\frac{F|\hat{H}|^2}{|1+\hat{P}K|^2}\right\}. \quad (11)$$

Consideration of minimisation criteria for J^c (9) and $\hat{J}$ (11) reveals that the only frequency transformation required to mould the frequency weighted LQG control problem into a standard non-frequency weighted LQG problem is on the disturbance model $\hat{H}$. That is,

$$\bar{H} = \hat{H}F. \quad (12)$$

The standard non-frequency weighted LQG design delivers a standard LQG controller $\bar{K}$ for the transformed designed closed loop system as depicted in Fig. 3. Again, perusal of the minimisation criteria for J^c (9) and $\hat{J}$ (11) discloses the equivalence of the frequency weighted LQG controller K in the designed closed loop with the standard LQG controller $\bar{K}$ in the transformed designed closed loop. That is,

$$K = \bar{K}. \quad (13)$$

The certainty equivalence principle is evoked when the frequency weighted LQG controller K is implemented in the achieved closed loop.

Having sketched the canvas outlining the controller enhancement of the iterative design, the painting of a picture which captures the role of the frequency weighting with respect to the state estimator innovations of the enhanced controller can now begin.

3 State Estimator Innovations within the Enhanced LQG Controller

Consider the design of the certainty equivalence controller $\bar{K}$ for the designed closed loop system of Fig. 3. Suppose that the plant model $\hat{P}$ can be written in state space form, that is

$$x_{t+1} = Ax_t + Bu_t^f, \quad (14)$$
$$y_t^f = Cx_t + v_t^f, \quad (15)$$

where A, B, C are the system matrices, v_t^f is the additive disturbance acting on the plant model output y_t^f. A similar state space representation can be derived for the disturbance model $\bar{H}$. That is,

$$x_{t+1}^d = A^d x_t^d + B^d p_t^f, \quad (16)$$
$$v_t^f = C^d x_t^d + q_t^f, \quad (17)$$

where A^d, B^d, C^d are the system matrices, p_t^f and q_t^f are respectively, the process and measurement noises associated with the disturbance model.

As described by Bitmead *et al* (1990), state space realisation of the plant and disturbance models can be combined into a composite state space representation given by,

$$x_{t+1}^f = \begin{pmatrix} A & 0 \\ 0 & A^d \end{pmatrix} x_t^f + \begin{pmatrix} B \\ 0 \end{pmatrix} u_t^f + \begin{pmatrix} 0 \\ B^d \end{pmatrix} p_t^f, \quad (18)$$

$$\triangleq A^m x_t^f + B^m u_t^f + W^m p_t^f, \quad (19)$$

$$y_t^f = (C \quad C^d)\, x_t^f + q_t^f, \quad (20)$$
$$\triangleq C^m x_t^m + q_t^f. \quad (21)$$

The Kalman predictor state estimator for the above composite state vector x_{t+1}^f may now be constructed as,

$$\hat{x}_{t+1|t}^f = A^m \hat{x}_{t|t-1}^f + B^m u_t^f + M^p(y_t^f - C^m \hat{x}_{t|t-1}^f), \quad (22)$$

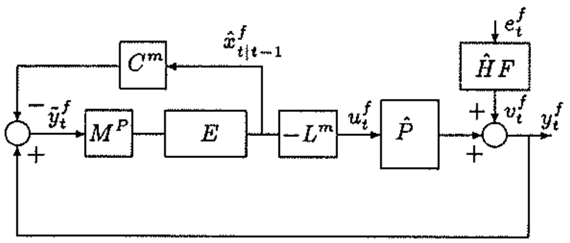

Fig. 5. The transformed designed closed loop system showing the Kalman Predictor Innovations $\tilde{y}^f_t$. Note $E = (zI - A^m + B^m L^m)^{-1}$.

$$= (A^m - M^P C^m)\hat{x}^f_{t|t-1} + B^m u^f_t + M^P y^f_t, \quad (23)$$

where M^P is Kalman predictor gain.

The frequency weighted LQ criterion (8) applied with state estimation gives an LQG control law of the form,

$$u^f_t = -(\,L^p \quad L^d\,)\,\hat{x}^f_{t|t-1}, \quad (24)$$
$$\triangleq -L^m \hat{x}^f_{t|t-1}, \quad (25)$$

where L^p, L^d are LQ gains associated with the plant and disturbance states, and $\hat{x}^f_{t|t-1}$ is the state estimate given by the Kalman predictor (23).

Combining the Kalman predictor (23) with the LQ feedback control law (25) yields

$$u^f_t = -L^m(zI - A^m + B^m L^m)^{-1} \times M^p[y^f_t - C^m \hat{x}^f_{t|t-1}], \quad (26)$$
$$= -L^m(zI - A^m + B^m L^m)^{-1} M^p \tilde{y}^f_t, \quad (27)$$

where $\tilde{y}^f_t$ is the Kalman predictor innovations process in the transformed designed closed loop. By definition, the innovations process is the estimation error between the predicted output, $C^m \hat{x}^f_{t|t-1}$, and the measured output, y^f_t, that is,

$$\tilde{y}^f_t \triangleq -C^m \hat{x}^f_{t|t-1} + y^f_t. \quad (28)$$

Fig. 5 explicitly shows this innovations process in the transformed designed closed loop.

Using (23) and (25) to substitute for the estimated state vector $\hat{x}^f_t$ in terms of the plant output y^f_t in the above equation (28), the transformed designed closed loop innovations process $\tilde{y}_t$ becomes

$$\tilde{y}^f_t = [I + C^m(zI - A^m + B^m L^m)^{-1} M^p]^{-1} y^f_t. \quad (29)$$

This innovations process can be expressed in terms of the white noise sequence e^f_t which drives the frequency transformed disturbance model $\bar{H}$, that is

$$\tilde{y}^f_t = [I + C^m(zI - A^m + B^m L^m)^{-1} M^P]^{-1} \times [1 + \hat{P}\tilde{K}]^{-1} \hat{H} F e^f_t, \quad (30)$$
$$\triangleq T(z) e^f_t. \quad (31)$$

As the innovations in the transformed designed closed loop system are white, $T(z)$ must be all-pass.

Now consider the implementation of the above non-frequency weighted certainty equivalence LQG controller $\tilde{K}$ in the achieved closed loop. Recall that equation (13) implies that this controller $\tilde{K}$ from the transformed designed closed loop is the same as a frequency weighted LQG controller K implemented in the achieved closed loop. Using the innovations definition given in (28), the Kalman predictor innovations process $\tilde{y}_t$ for the achieved closed loop system is given by,

$$\tilde{y}_t = -C^m \hat{x}^f_{t|t-1} + y_t, \quad (32)$$
$$= [I + C^m(zI - A^m + B^m L^m)^{-1} M^P]^{-1} \times [1 + PK]^{-1} H e_t, \quad (33)$$
$$= T(z) \frac{[1 + \hat{P}\tilde{K}]H}{[1 + PK]\hat{H}F} e_t. \quad (34)$$

The full order frequency weighting F from (5) can be expressed in terms of spectral factors given by the achieved and designed closed loop sensitivity functions. That is, $F(z)$ equals

$$\left\{ \frac{H(z)[1 + \hat{P}(z)K(z)]\, H(z^{-1})[1 + \hat{P}(z^{-1})K(z^{-1})]}{\hat{H}(z)[1 + P(z)K(z)]\, \hat{H}(z^{-1})[1 + P(z)^{-1}K(z^{-1})]} \right\}^{1/2}. \quad (35)$$

The minimum phase spectral factor, $F(e^{j\omega})$, satisfies

$$|F(e^{j\omega})| = \left| \frac{H(e^{j\omega})[1 + \hat{P}(e^{j\omega})K(e^{j\omega})]}{\hat{H}(e^{j\omega})[1 + P(e^{j\omega})K(e^{j\omega})]} \right|. \quad (36)$$

Substituting the minimum spectral factor of F (36), and recalling the relationship (13), causes (34) to yield,

$$\tilde{y}_t = T(z) e_t. \quad (37)$$

Hence the effect of the frequency weighting F is to maintain the whiteness of the innovations process in the achieved closed loop. It should also be noted that the innovations process in the achieved closed loop will not be white when the controller is delivered based upon a non-frequency weighted controller synthesis in the designed closed loop system. This result reinforces the focus of the frequency weighted iterative design toward the achieved closed loop system and cements its connection, as alluded to in the introduction, with closed loop controller reduction given full plant and disturbance knowledge.

Zang et al (1992) have shown for an exact plant description, $\hat{P} = P$, and an inexact disturbance description, $\hat{H} \neq H$, the iterative design will deliver the truly optimal controller in one iteration. With an inexact plant description, $\hat{P} \neq P$, and an exact disturbance description, $\hat{H} = H$, the iterative design will achieve approximately the same performance for the achieved and frequency transformed designed closed loop systems. This paper generalises these findings for the case of inexact plant and disturbance knowledge, i.e $\hat{P} \neq P, \hat{H} \neq H$.

The iterative design algorithm uses a fixed order auto-regressive (AR) models to approximate signals $\Phi_y^{1/2}, \Phi_{y^c}^{1/2}$, from finite length data sequences y_t, y^c_t, before computing the frequency weighting. AR models were selected for use in the iterative algorithm as :-

- they produce stable frequency weightings.
- many commercial software packages include AR identification algorithms.

Third order AR models were perceived to be sufficiently accurate to give a good approximation of the frequency weighting in many examples.

The degree of approximation in obtaining the frequency weighting determines the ability of the frequency weighting F to cancel sensitivity terms in equation (34) which represents the Kalman predictor innovations $\tilde{y}_t$ in the achieved closed loop. Therefore, in practice the role of the frequency weighting in maintaining the whiteness of the state estimator innovations in the achieved closed loop is an approximate one.

An example demonstrating the influence of the frequency weighting in the iterative design is given in the next section.

4 Example

The example considered below demonstrates that the Kalman predictor innovations in the achieved closed loop are initially whitened and subsequently maintained white by the frequency weighted iterative design.

The real plant under investigation is *7th* order of the following form,

$$y_t = P(z)u_t + v_t = \frac{B(z)}{A(z)} + H(z)e_t \qquad (38)$$

where the coefficients of the polynomials $A(z), B(z)$, represented in vector form as a and b, are respectively

$$\begin{aligned} a &= [1\ 0.0049\ \ -0.0848\ \ -0.1953\ 0.1450 \\ &\qquad -0.0159\ \ -0.0505\ \ 0.0145] \\ b &= [0\ \ 0.5000\ \ 1.5059\ \ 0.8575\ \ 0.0897 \\ &\qquad 0.5463\ \ 0.0738\ \ 0.0002] \end{aligned}$$

This is a stable non-minimum phase plant with unit delay. The disturbance filter is high pass with a transfer function

$$H(z) = \frac{1 - 0.975z}{1 + 0.1z}$$

Fig. 6 gives the frequency response for the true plant and the true disturbance.

The iterative identification and control design performed on this plant uses a composite iterative scheme based on refinements to the original iterative design methodology as proposed by Partanen and Bitmead (1993). This composite iterative design performs a model adjustment and controller enhancement during the first iteration, subsequent iterations directly refine the controller without a model adjustment phase. The advantage of this scheme is that only one closed loop identification experiment with a suitably selected excitation need be performed. With this iterative design 3rd order plant models and 1st order disturbance models were identified using a Box-Jenkins algorithm to minimise sum-square filtered prediction errors. Identification of AR models for computation of the frequency weighting was conducted using 4096 sample length closed loop input/output data sequences. The frequency weighted local performance criterion includes a low penalty cost on control action, that is $\lambda = 0.01$. Eight iterations were performed.

Fig. 7 plots the Kalman predictor innovations in the achieved closed loop system for the first three iterations of the design. The solid line in Fig. 7 gives Kalman predictor innovations of the full order optimal LQG controller (designed using the true plant and the true disturbance), predictably this innovations process is white. The initial iteration is based upon an achieved closed system with an initial non-frequency weighted LQG controller K_0 derived from an initial plant model $\hat{P}_0$ which captures only the low pass nature of the true plant, hence the Kalman predictor innovations (dashed line) in the initial achieved closed loop are not white. After two iterations Fig. 7 shows the iterative scheme has delivered an LQG controller with white Kalman predictor innovations in the achieved closed loop. Numerical approximations have given an imperfect pole/zero cancellation for the Kalman predictor innovations near dc after the second iteration. This is not significant due to the near zero disturbance energy around dc. Further iterations did not alter the whiteness of the Kalman predictor innovations in the achieved closed loop.

5 Conclusions

This paper has considered an existing iterative identification and control design for disturbance rejection. It has been shown that for this iterative design, a frequency weighting is responsible for maintaining the whiteness of the Kalman predictor innovations in the achieved closed loop system. This result links the iterative design with an existing closed loop controller reduction technique, with respect to the difference between the achieved and optimal closed loop transfer functions for which an upper bound already exists.

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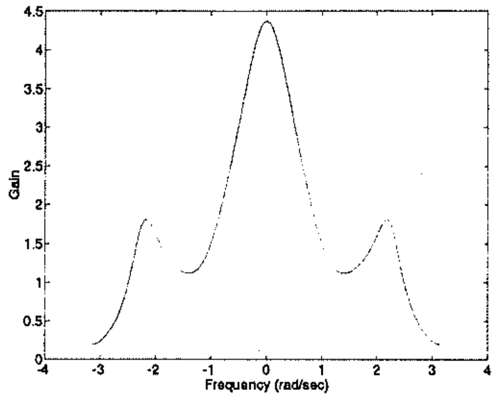

Fig. 6. Bode plot of the True Plant P (solid line) and the True Disturbance H (dotted line).

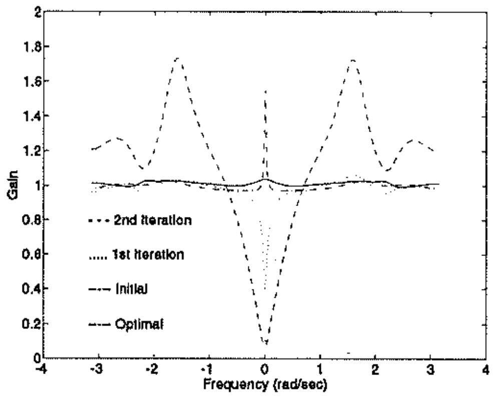

Fig. 7. Bode plot of the Kalman Predictor Innovations $\tilde{y}_t$ in the Achieved Closed Loop System.

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