INTRODUCING CAUTION IN ITERATIVE CONTROLLER DESIGN

Robert R. Bitmead* Michel Gevers** Ari G. Partanen***

* Cooperative Research Centre for Robust & Adaptive Systems, Department of Systems Engineering, RSISE, Australian National University, Canberra ACT 0200, AUSTRALIA. Phone: +61 6 279 8612, fax +61 6 279 8615, email bob@syseng.anu.edu.au
** Centre for Systems Engineering and Applied Mechanics (CESAME), Batiment Euler, University of Louvain, B-1348 Louvain-la-Neuve BELGIUM. Phone: +32 10 47 2590, fax +32 10 47 2180, email gevers@auto.ucl.ac.be
*** CRA Advanced Technical Development, Locked Bag 347, Bentley DC, WA, 6983 AUSTRALIA. Phone: +61 9 470 7721, fax +61 9 470 5379, email ari.Partanen@restech.cra.com.au

Abstract: Iterative identification and control design extend classical design methods by using on-line experimental performance data to adjust the feedback regulator. The inclusion of this new information concerning achieved controller properties permits a focus on actual performance, as opposed to designed performance. In the context of Robust Control, this is a fundamentally important piece of information. However, controller adjustment for performance improvement needs to be balanced against closed-loop stability requirements. To achieve this balance it is critical not to modify the controller too outlandishly in any one step but, rather, to apply some cautious updating procedure to limit the change to the controller. Likewise, alterations to the desired controlled performance need to be made gradually rather than dramatically. Our approach in this paper is to explore the origin of this caution and to present some methods derived and applied in a practical problem of sugar-cane crushing mill control.

Keywords: Iterative Design, System Identification, Control Design, Simultaneous Stabilization

1. INTRODUCTION

Iterative Control Design is an approach to combined identification and model-based controller design which makes explicit the interactions between modelling (using data from the closed loop with the current controller) and subsequent updating and implementation of that controller based on the latest model. The heredity of these methods is found in both Adaptive Control and Robust Control. Robust Control accommodates modelling error in developing a control design. Cognisance is not taken of subsequent implemented closed-loop performance, however. Adaptive Control, on the other hand, considers simultaneous model and controller adjustment using closed-loop data. The adjustment rate of Adaptive Control, however, is relatively fast compared with the Iterative Control Design paradigm and leads to nonlinear interactions which are difficult to analyse. Iterative methods might well be con-
considered as very slowly adapting block adaptive control.

Several research groups have developed specific approaches to iterative design [1–8]. Central to all methods is the focus on the manipulation of the identification criterion to reflect the control objective and, conversely, the use of robust control design to accommodate the identified approximate models. The scheme upon which we concentrate here is affectionately known as the Zangscheme [1]. This approach demonstrates many features in common with those of others, with some variations due to Partanen et al in which model updates may be by-passed to yield a direct method [8].

Figure 1 depicts the two central control loops:

- \( \mathcal{L}(P, C) \) — the achieved loop containing the real plant, \( P \), and real disturbance, \( \nu_t = H e_t \), operating under feedback control with controller, \( C \), and external reference, \( \nu_t \);
- \( \mathcal{L}(\hat{P}, C) \) — the designed loop containing the plant model, \( \hat{P} \), and modelled disturbance, \( \nu_t = H e_t \), operating (i.e. simulating in the design computer) under feedback control with the same controller, \( C \), and external reference, \( \nu_t \).

The central idea of iterative control is to recognise that, while design takes place on the designed loop \( \mathcal{L}(\hat{P}, C) \), actual performance specifications need to be met on the achieved loop \( \mathcal{L}(P, C) \) exclusively. Closed-loop modelling serves to bring the models \( \hat{P} \) and \( H \) close to their real counterparts \( P \) and \( H \) in the sense that a modelling criterion is minimized. Control design necessarily takes place on the \( \mathcal{L}(P, C) \) design loop since the actual \( P \) and \( H \) are unknown.

For the Zangscheme, iterative modelling and design operate in a least-squares/LQ framework. Closed-loop modelling minimizes the criterion

\[
J^{id} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left[ (y_i - y_i^*)^2 + \lambda (u_i - u_i^*)^2 \right].
\]

with controller \( C \) fixed in both \( \mathcal{L}(P, C) \) and \( \mathcal{L}(\hat{P}, C) \) loops. Control design minimizes

\[
J^c = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left[ (F_1 y_i^2) + \lambda (F_2 y_i^2) \right],
\]

where \( F_1(z) \) and \( F_2(z) \) are frequency weighting filters. The actual focus of attention is not upon \( J^c \) but upon the achieved performance in the achieved loop:

\[
J^{ach} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \left[ (y_i)^2 + \lambda (u_i)^2 \right].
\]

The outcome of iterative control is to define a sequence of models \( \{\hat{P}_i\} \) (denoting combined plant and disturbance models) and a sequence of controllers \( \{C_i\} \), which are connected via the successive solution of control-design and closed-loop identification minimizations. The interdependence of these sequences is that \( C_i = C_i(\hat{P}_i) \) and \( \hat{P}_i = \hat{P}_i(P, C_{i-1}) \).

Performance is the focus of the above iterative scheme with the concentration being on minimizing \( J^{ach} \) via appropriate \( J^{id} \) and \( J^c \) minimization stages. Robust stability is a feature which needs separate inclusion or treatment [1]. Detuning of the performance objective is a frequent approach to introducing stability robustness. Here we shall demonstrate that cautious adjustment of controllers accommodates robust stability to some extent.

2. STABILIZABILITY CONSTRAINTS AND CAUTION

Consider the closed-loop transfer function of the achieved loop \( \mathcal{L}(P, C) \)

\[
\mathcal{T}(P, C) = \frac{1}{1 + PC} + \frac{C}{1 + PC}. \tag{2}
\]

If \( \mathcal{L}(P, C) \) is stable, then we may follow Vinnicombe [17] in defining the stability margin of this loop as

\[
b_{P,C} \triangleq \| \mathcal{T}(P, C) \|_{\infty}^{-1}. \tag{3}
\]

Further, we may define the distance between two plants, \( P_0 \) and \( P_1 \), as

\[
\delta_0 = \left\| (1 + P_1 P_1^*)^{-\frac{1}{2}} (P_1 - P_0)(1 + P_0^* P_0)^{-\frac{1}{2}} \right\|_{\infty} \leq 1,
\]

and the winding number condition on the pair \( (P_0, P_1) \),

\[
\left\| [1 + P_1^* P_0^*] \neq 0 \quad \forall \omega, \quad \text{and} \quad \text{wnc}(1 + P_1^* P_0^*) + \eta(P_0) - \eta(P_1) = 0 \right\}, \tag{4}
\]

where \( \text{wnc}(\cdot) \) is the winding number or number of counter-clockwise encirclements of the origin of the Nyquist plot and \( \eta(\cdot) \) is the number of right half-plane poles. We may then appeal to the following theorem of Vinnicombe [17].

**Theorem 1. (Vinnicombe).** Given a plant \( P_0 \) and stabilizing compensator \( C_0 \), the closed loop formed by \( C_0 \) and another plant \( P_1 \), with \( (P_0, P_1) \) satisfying (4), will be stable for all \( P_1 \) satisfying

\[
\delta_0(P_0, P_1) \leq \beta \text{ if and only if } b_{P_0, C_0} > \beta.
\]

This provides a connection between simultaneous stabilization of two plants, design properties and distance between the plants as measured by the \( \delta_0 \).
metric. It is our intention to apply this to consider the successive elements of iterative control.

Consider the $i$-th stage of an iterative modelling and control design;

- with controller $C_{i-1}$ operating on $P$, fit a new model $\hat{P}_i$ based on closed-loop data from $L(P, C_{i-1})$,
- with model $\hat{P}_i$ design a new controller $C_i$.

The nature of these design steps is to include implicit simultaneous stabilizability assumptions as follows.

**Stability Assumption 1.** $\hat{P}_i$ should be stabilized by $C_{i-1}$.

**Stability Assumption 2.** $C_i$ should stabilize $\hat{P}_i$ and $P$.

Drawing on the connections between stabilizability and metric distance between plants, an immediate translation of these Stability Assumptions is possible.

**Theorem 2.** Subject to winding number conditions holding on $(\hat{P}_i, \hat{P}_{i-1})$, $(\hat{P}_i, P)$ and $(C_i, C_{i-1})$, the Stability Assumptions above require that

\[
\delta_\nu(\hat{P}_i, P) \leq b_{P, C_{i-1}}, \quad (5)
\]

\[
\delta_\nu(\hat{P}_i, \hat{P}_{i-1}) \leq b_{\hat{P}_{i-1}, C_{i-1}}, \quad (6)
\]

\[
\delta_\nu(C_i, C_{i-1}) \leq \min \left[ b_{\hat{P}_i, C_{i-1}}, b_{P, C_{i-1}} \right]. \quad (7)
\]

Direct interpretation of the above metric conditions is possible and will be amplified in the next section.

- (5) dictates that the new model $\hat{P}_i$ must be an adequate model of the true plant $P$ as specified by current closed-loop achieved performance;
- (6) is straightforward, if $\hat{P}_i$ is artificially restricted to the class of systems stabilized by $C_{i-1}$, subject to the continued satisfaction of the winding number condition,

- (7) provides an upper bound on the permissible variation of subsequent controllers $C_i$ and $C_{i-1}$ in terms of the worse of achieved and designed performance measures with the existing controller $C_{i-1}$.

These constraints on successive models and controllers in iterative control follow immediately from the stabilization conditions. As we shall see, the identification constraint (5) represents a condition whose satisfaction is mostly connected to the adequacy of the model class. Inequality (7) limits the available magnitude of controller adjustment, we call this **caution** in iterative control design.

We make several observations concerning caution. Note firstly that the stability margin $b_{P, C}$ is the inverse of the infinity-norm of $T(P, C)$. That is, the larger is $\|T(P, C)\|_\infty$, the smaller is the margin $b_{P, C}$.

1. Condition (5) implies that poor achieved performance imposes tighter constraints on open-loop plant fit.
2. Condition (6) implies that poor designed performance imposes tighter constraints on permissible model variation, at least modulo the satisfaction of the winding number condition.
3. Condition (7) implies that poor achieved performance from the previous controller or poor 'designed' performance of the new model with the previous controller both limit allowable controller modifications.

### 3. METRIC DISTANCE, PERFORMANCE AND CAUTION

**Metric Distance**

The metric distance $\delta_\nu(\cdot, \cdot)$ above provides the natural measure of plant/controller deviation. While it is a measure computable directly from the frequency responses of the two systems involved, it also has a simple interpretation via normalized coprime factors. In this regard, $\delta_\nu$ is viewed as a natural replacement for the gap metric [10].
Suppose that \( P_0 \) and \( P_1 \) are two systems stabilized by a common controller \( C_0 \). Further suppose that \( P_0 \) and \( C_0 \) have coprime factors \([N_0, D_0]\) and \([X_0, Y_0]\) over \( S \), the ring of stable, rational transfer functions with \([N_0, D_0]\) being normalized, i.e. \( N_0^* N_0 + D_0^* D_0 = 1 \). (We are working here over the class of single-input single-output systems, only for notational convenience.) Then, since \( P_1 \) is also stabilized by \( C_0 \), it has a (not necessarily normalized) coprime factorization

\[
P_1 = D_1^{-1} N_1 = [D_0 + Q_1 X_0]^{-1} [N_0 - Q_1 Y_0].
\]

Using these coprime factorizations and the stabilization property that \( D_0 Y_0 + N_0 X_0 = 1 \), we may compute

\[
\delta_\nu(P_0, P_1) = \left\| (1 + P_1 P_1^*)^{-\frac{1}{2}} (P_1 - P_0) (1 + P_0^* P_0)^{-\frac{1}{2}} \right\|_{\infty}.
\]

\[
= \left\| (D_1 D_1^* + N_1 N_1^*)^{-\frac{1}{2}} (D_0 N_1 - D_1 N_0) \times (D_0^* D_0 + N_0^* N_0)^{-\frac{1}{2}} \right\|_{\infty}
\]

\[
= \left\| (D_1 D_1^* + N_1 N_1^*)^{-\frac{1}{2}} (-Q_1 [D_0 Y_0 + N_0 X_0]) \times (D_0^* D_0 + N_0^* N_0)^{-\frac{1}{2}} \right\|_{\infty}
\]

\[
= \left\| (D_1 D_1^* + N_1 N_1^*)^{-\frac{1}{2}} Q_1 \right\|_{\infty}.
\]

That is, the metric distance between simultaneously stabilized plants \( P_0 \) and \( P_1 \) is strongly tied to the infinity norm of the difference between their normalized coprime factors.

**Performance**

Using non-normalized coprime factors one may also connect the respective closed-loop performance of the two loops. Note that the stability condition of \( \mathcal{L}(P_0, C_0) \) implies that, without loss of generality, we may take

\[
D_0 Y_0 + N_0 X_0 = 1.
\]

This, in turn, implies that we may rewrite the closed-loop transfer functions as

\[
\mathcal{T}(P_0, C_0) = \begin{pmatrix} X_0 & D_0 \\ -Y_0 & N_0 \end{pmatrix} \begin{pmatrix} Q_1 \\ 1 \end{pmatrix}
\]

\[
\times \begin{pmatrix} R_1 & 1 - R_1 Q_1 \end{pmatrix} \begin{pmatrix} N_0 & -D_0 \\ Y_0 & X_0 \end{pmatrix}.
\]

Since \( P_1 \) is also stabilized by \( C_0 \), we have

\[
\mathcal{T}(P_1, C_0) = \begin{pmatrix} D_1 \\ N_1 \end{pmatrix} \begin{pmatrix} Y_0 & X_0 \end{pmatrix}.
\]

Introducing controller \( C_1 \) which stabilizes \( P_1 \), we recognize that, since \( P_1 \) is stabilized by both \( C_0 \) and \( C_1 \), \( C_1 \) may be written

\[
C_1 = [Y_0 + R_1 N_1]^{-1} [X_0 - R_1 D_1],
\]

where \( R_1 \in S \). Clearly,

\[
\mathcal{T}(P_1, C_1) = \begin{pmatrix} D_1 \\ N_1 \end{pmatrix} \begin{pmatrix} Y_0 + R_1 N_1 & X_0 - R_1 D_1 \end{pmatrix}.
\]

**Theorem 3.** Consider the two plants \( P_0 \) and \( P_1 \) and two controllers \( C_0 \) and \( C_1 \) with the properties

- \( C_0 \) stabilizes \( P_0 \) and \( P_1 \),
- \( C_1 \) stabilizes \( P_1 \).

Let \( P_0 \) and \( C_0 \) possess coprime factorizations \([N_0, D_0]\) and \([X_0, Y_0]\) respectively.

Then for some \( Q_1 \in S \), \( P_1 \) possesses a coprime factorization

\[
[N_1, D_1] = [N_0 - Q_1 Y_0, D_0 + Q_1 X_0],
\]

and for some \( R_1 \in S \), \( C_1 \) possesses a coprime factorization

\[
[X_1, Y_1] = [X_0 - R_1 D_1, Y_0 + R_1 N_1].
\]

Further, the closed-loop performance transfer functions of \( \mathcal{L}(P_1, C_1) \) may be written with the following decomposition over \( S \).

\[
\mathcal{T}(P_1, C_1) = \begin{pmatrix} X_0 & D_0 \\ -Y_0 & N_0 \end{pmatrix} \begin{pmatrix} Q_1 \\ 1 \end{pmatrix}
\]

\[
\times \begin{pmatrix} R_1 & 1 - R_1 Q_1 \end{pmatrix} \begin{pmatrix} N_0 & -D_0 \\ Y_0 & X_0 \end{pmatrix}.
\]

**Note**

\[
\begin{pmatrix} X_0 & D_0 \\ -Y_0 & N_0 \end{pmatrix} \begin{pmatrix} N_0 & -D_0 \\ Y_0 & X_0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\]

**Proof:**

\[
\mathcal{T}(P_1, C_1) = \begin{pmatrix} D_1 \\ N_1 \end{pmatrix} \begin{pmatrix} Y_0 & X_0 \end{pmatrix}.
\]

\[
= \begin{pmatrix} X_0 & D_0 \\ -Y_0 & N_0 \end{pmatrix} \begin{pmatrix} Q_1 \\ 1 \end{pmatrix}
\]

\[
\times \begin{pmatrix} R_1 & 1 - R_1 Q_1 \end{pmatrix} \begin{pmatrix} N_0 & -D_0 \\ Y_0 & X_0 \end{pmatrix}.
\]

\[
= \begin{pmatrix} X_0 & D_0 \\ -Y_0 & N_0 \end{pmatrix} \begin{pmatrix} Q_1 \\ 1 \end{pmatrix}
\]

\[
\times \begin{pmatrix} R_1 & 1 \end{pmatrix} \begin{pmatrix} N_0 & -D_0 \\ Y_0 & X_0 \end{pmatrix}.
\]

\[
= \begin{pmatrix} X_0 & D_0 \\ -Y_0 & N_0 \end{pmatrix} \begin{pmatrix} Q_1 \\ 1 \end{pmatrix}
\]

\[
\times \begin{pmatrix} R_1 & 1 - R_1 Q_1 \end{pmatrix} \begin{pmatrix} N_0 & -D_0 \\ Y_0 & X_0 \end{pmatrix}.
\]
Identification

Applying the above formulae to the closed-loop model selection phase, we may write,

\[
T(P, C_{i-1}) = T(\bar{P}_i, C_{i-1}) = (Q - Q_i) \begin{pmatrix} X_{i-1} & X_{i-1} \end{pmatrix} (Y_{i-1} \ Y_{i-1})
\]

Model identification may be seen as

\[
\min_{Q \in \mathbb{M}} \left\| T(P, C_{i-1}) - T(\bar{P}_i, C_{i-1}) \right\|
\]

where \(\mathbb{M}\) is the class of permissible \(Q_i\) parametrizing the model set. Now introducing from (5) the metric distance between model and plant,

\[
\delta_i(\bar{P}_i, P) = \left\| (D_iD_i^* + N_iN_i^*)^{-\frac{1}{2}} (Q - Q_i) (D_i^*D + N_i^*N)^{-\frac{1}{2}} \right\|_\infty
\]

we observe that closed-loop identification aims to keep \(T(P, C_{i-1}) - T(\bar{P}_i, C_{i-1})\) small. This, in turn, will keep \(Q - Q_i\) small subject to the permissible model class of \(\{Q_i\}\), which will assist in keeping \(\delta_i(\bar{P}_i, P)\) small to satisfy (5).

The interpretation of (5) is that its satisfaction is a requirement on the sufficient complexity of the model class to permit adequate closed-loop matching of the true plant with the previous controller. This is not a restriction on the allowable magnitude of variation of successive models. Constraint (6) is a limit to model variation which, if the search is permitted only over simultaneously \(C_{i-1}\)-stabilized models, is automatically satisfied.

Controller Caution

Condition (7) is a limitation on the allowable variation of successive controllers. We have

\[
\delta_i(C_i, C_{i-1}) = \left\| (Y_i^*Y_i + X_i^*X_i)^{-\frac{1}{2}} R_i (Y_i^*Y_i + X_i^*X_i)^{-\frac{1}{2}} \right\|_\infty
\]

where \(R_i\) is the Youla-Kucera parameter of \(C_i\) expressed as a variation of \(C_{i-1}\) stabilizing \(\bar{P}_i\).

While one might appeal (as was done in the case of identification condition (6)) to its automatic satisfaction through restriction of control design to the parametrized class of controllers simultaneously stabilizing both \(\bar{P}_{i-1}\) and \(\bar{P}_i\), the inclusion of a requirement for \(\delta_i(C_i, C_{i-1})\) to be less than \(\delta_{P,C_{i-1}}\) militates against this. Furthermore, the above restriction of the admissible controller does not fit well with orthodox control design approaches. It appears that the reliable procedure is to explore small adjustments to the control design, as quantified by the parameter \(R_i\).

### 4. Achieving Caution in Iterative Control

With a heritage in linear systems theory, it is standard to develop iterative schemes as variants of optimization-based approaches to modeling and control. Thus \(H_\infty\) and LQG/Least-squares methods form the basis for design. The algebraic nature of the computation of controllers and models represents a difficulty in achieving caution, since closeness relies on continuity properties which themselves are problematic. From the earlier sections, it is apparent that working directly with coprime factors might prove amenable to constraining the class of systems considered.

Detuning is a familiar method for enhancing the robust stability of otherwise algebraic design methods. Partanen's modification [11] to the Zangschene control design phase modifies the adjustable frequency weightings in the control design criteria (1).

\[
F_i(z) = \frac{\phi_i^{\frac{1}{2}}(z) + (1 - \delta)\phi_i^{\frac{1}{2}}}{\phi_i^{\frac{1}{2}}},
\]

in place of the Zangschene's variant with \(\delta = 1\). (The \(F_i\) version is similarly modified with the same \(\delta\).) This serves to diminish large-scale variations in the frequency-weightings defining successive controllers. Choosing \(\delta\) near zero diminishes the excursions of the frequency weighting compared with values of \(\delta\) near one. Combined with this, in practice Partanen found it necessary to ensure that the disturbance model \(\bar{H}\) possessed values of frequency response neither very large nor very small in magnitude. This reflects similar design variants of Schrama [12]. De Bruyne [13] has also recently established conditions for small variation in optimal controller coprime factors. Such modifications of performance optimization objective functions degrade the achievable performance but introduce some control of the variation of successive models and controllers. These have been found to provide manageable caution in practical iterative control of sugar-cane crushing mills.

For \(H_\infty\) control one may define: \(S(z) = (1 + P(z)C(z))^{-1}\), \(\bar{S}(z) = (1 + \bar{P}(z)C(z))^{-1}\), \(\bar{M}(z) = C(z)(1 + P(z)C(z))^{-1}\) and \(\bar{L}(z)\) as a bound on model error \(P - \bar{P}\). Then a standard robust control criterion is, see [9],

\[
\bar{J}(P, L(P, \bar{P}), C) = \frac{\frac{\bar{M}}{H S} - \bar{H}S}{\frac{\bar{M}}{H S}}
\]

For the control design phase performance is traded against guaranteed robust stability by increasing \(L\).
While caution appears to be adequately able to be introduced into control design, there is an apparent need to fit coprime factors in the system identification phase. Sugie [14] and Van den Hof et al. [15,16] have considered such direct coprime factor identifications. In order to capture all relevant aspects of both factors with some precision, it is important to use widespread excitation of the closed loop. Schrama [12] has examined some aspects of this design. De Bruyne has established that a white reference spectrum and controller (Gc-based) filtering is optimal for such estimation. Nevertheless, it is not clear how in introducing controller caution by criterion perturbation one might guarantee the identified models to be adequately close in order that successive optimization criteria should yield δc-close controllers. Anderson et al. [18] have explored continuity properties of LQ controllers with respect to the plant model. In order to appeal to this theory to establish that successive controllers were close, it would be necessary to require plant models to be close. That is that Qt should be small. These issues are still largely open.

5. CONCLUSION

We have examined issues arising from iterative control design in which caution must be introduced to the evolution of successive models and controllers in order that stabilizability conditions be maintained at each stage. This caution is manifested in the small adjustment (measured via their Youla-Kucera parameters) of the designed controllers as the iterations are performed. In practical terms this has the requirement of limiting the alterations made to frequency weightings and disturbance models in control design criteria. There remains much to explore connected with closed-loop caution. Not least amongst these are the issues of practical costs associated with cautious (and therefore repetitive and slow to adjust) designs on operating plant and of introduction of nonlinearities into the model class.

6. REFERENCES


ACKNOWLEDGEMENTS

The first author acknowledges the support of the Cooperative Research Centre for Robust & Adaptive Systems and of the Australian Commonwealth Government through the Cooperative Research Centres Program. The second author acknowledges the support of the Belgian Government through the Interuniversity Attraction Poles Program. The primary responsibility for the scientific content remains with the authors.