ITERATIVE CONTROLLER OPTIMIZATION FOR NONLINEAR SYSTEMS[1]

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Abstract

Recently, a data-driven model-free control design method has been proposed in [4, 6]. It is based on the minimization of a control criterion with respect to the controller parameters using an iterative gradient technique. In this paper, we extend this method to the case where both the plant and the controller can be nonlinear. It is shown that an estimate of the gradient can be constructed using only signal based information. It is also shown that by using open loop identification techniques, one can obtain a good approximation of the gradient of the control criterion while performing fewer experiments on the actual system.

Keywords: Nonlinear optimal control, estimation

1 Introduction

The iterative model-free control optimization method that has appeared in [4, 6] has been shown to give good results from both an experimental and industrial point of view: see e.g. [1, 2, 5, 8]. This scheme is based on an iterative tuning of the controller parameter vector along the gradient direction of a control performance criterion. The key contribution of [4, 6] was to show that an unbiased estimate of this gradient can be constructed from filtered versions of the signals measured on the closed loop system. The construction of this gradient requires a "special experiment" in which a finite record of the output of the closed loop system is recycled at the reference input of that system.

In [2], the previous method has been modified so as to allow its applicability when the second experiment that is described in [4, 6] causes unacceptable behaviour. This has been achieved by replacing the estimation of the gradient, as described in [4, 6], by an identification-based gradient estimate. A second contribution of this paper was to show experimentally that the iterative controller tuning procedure also gives good results for systems containing nonlinear dynamics and nonlinear friction.

In this paper, we formalize and extend these results to the case when both the controller and the plant can be nonlinear. Our main contribution is to show that it is possible to obtain an estimate of the gradient by recycling the filtered output of the actual closed loop system at the reference input of some simulated closed loop system. This simulated loop is the feedback interconnection of the linearization of both the plant and the controller around their respective trajectory, i.e. it is computed by linearization of the closed loop operator around the trajectory produced by the reference signal. It is shown that, although this linearized version of the closed loop operator can not be obtained explicitly, it is still possible to estimate the gradient signals by performing additional experiments with almost identical reference signals. The total number of experiments per iteration is equal to the number of controller parameters, n, plus two, i.e. one experiment for the generation of the error signal and $n+1$ experiments for the estimation of the n gradient signals. The $(n+1)$-th experiment in the estimation of the n gradient signals is needed to make sure that the noise-induced perturbation in the gradient signals and the error signal are uncorrelated.

It is also shown that an approximate estimate of the gradient of the control criterion can be obtained by performing just two experiments on the closed loop system, resulting in the identification of a model of the linearization of the closed system around its operating trajectory using open loop identification techniques.

A previous contribution to iterative controller optimization for nonlinear systems has very recently appeared in [7]. Although the derivations are different, the experiment-based algorithm that is outlined here shows obvious similarities with the one described in [7].

In Section 2, we extend the method described in [4, 6] to the nonlinear case. In Section 3, we present some numerical simulations. We conclude in Section 4.

2 Controller optimization for nonlinear systems

In this section, we extend the results of [4, 6] to the nonlinear setting of Figure 2.1.

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Problem setting

Let us assume that the true system is the Single-Input Single-Output (SISO) nonlinear time-invariant system described by

$$\mathcal{S} : y_t = P(u_t, v_t) \tag{2.1}$$

where P is an unknown nonlinear operator. Here u_t is the control input signal, y_t is the achieved output signal and v_t is a process disturbance signal. The input signal is determined according to

$$\mathcal{C} : u_t = C(\rho, r_t, y_t) \tag{2.2}$$

where r_t is an external reference which we assume to be quasi-stationary [9] and where the controller C is a nonlinear operator of both r_t and y_t and is parametrized by a controller parameter vector ρ, with $\rho \in \mathbb{R}^n$. In the sequel we often make use of linearizations of some nonlinear operators around their operating trajectories. We therefore require that the plant, the controller and all closed loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. We refer the reader to [3] for more details on such smoothness assumptions and a full treatment of the linearization problem. We also require a high Signal-to-Noise-Ratio (SNR) and we assume that the closed loop system in Figure 2.1 is stable in the Bounded-Input-Bounded-Output (BIBO) sense. For ease of notation, we from now on omit the time argument of the signals. We denote by $y(\rho)$ the output of (2.1) in feedback with (2.2) as shown in Figure 2.1.

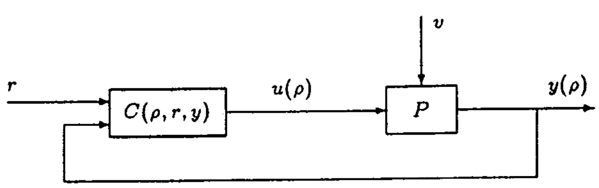

Figure 2.1: The nonlinear control loop

Let y_d be the desired closed loop response to the reference signal r. Then, the error between the achieved and the desired response is $\tilde{y}(\rho) = y(\rho) - y_d$. The control design objective is formulated as the minimization of an LQG control criterion for the reduced complexity controller (2.2), i.e. we have that $\rho^* = \arg\min_\rho J(\rho)$ with

$$J(\rho) = \frac{1}{2} E\,[L\,\tilde{y}(\rho)]^2 \tag{2.3}$$

where L is some causal BIBO operator. To facilitate notations, we assume $L = 1$. For more general criteria, see [4, 6].

Criterion minimization

The problem that is addressed now is the minimization of (2.3) with respect to the controller parameter vector ρ. It is standard that one can find a solution for ρ to

$$0 = J'(\rho) = E\,[\tilde{y}(\rho)\,\tilde{y}'(\rho)] \tag{2.4}$$

by taking repeated steps in the negative gradient direction

$$\rho[i+1] = \rho[i] - \gamma_i\, R_i^{-1}\, J'(\rho[i]) \tag{2.5}$$

where R_i is some appropriate positive definite matrix, typically an estimate of the Hessian of J and $\{\gamma_i\}$ is a sequence of positive numbers that determines the step size. Here $\rho[i]$ denotes the controller parameter vector ρ at iteration i.

In order to obtain an estimate of $J'(\rho[i])$, estimates of the signal $\tilde{y}(\rho[i])$ and its gradient $\tilde{y}'(\rho[i])$ are needed. By definition, $\tilde{y}(\rho[i])$ is obtained by taking the difference between the achieved output response $y(\rho[i])$ of the system with the most recent controller $C(\rho[i])$ and the desired response y_d. We now show that it is also possible to obtain an estimate of the gradient signal $\tilde{y}'(\rho[i])$ when both the plant P and the controller C are nonlinear. Note that $\tilde{y}'(\rho[i]) = y'(\rho[i])$. Let us first consider the following equations

$$y(\rho) = P(u(\rho), v), \quad u(\rho) = C(\rho, r, y(\rho)). \tag{2.6}$$

If one of the parameters, say ρ_j, is perturbed by a small $\delta\rho_j$, we obtain

$$\begin{aligned} y(\rho_1, \cdots, \rho_j + \delta\rho_j, \cdots, \rho_n) &= P[u(\rho_1, \cdots, \rho_j + \delta\rho_j, \cdots, \rho_n), v] \\ &\simeq P[u(\rho) + u'_{\rho_j}(\rho)\,\delta\rho_j, v] \\ &\simeq P(u(\rho), v) + \partial P_u(u(\rho), 0)\, u'_{\rho_j}(\rho)\,\delta\rho_j \\ &= y(\rho) + \partial P_u(u(\rho), 0)\, u'_{\rho_j}(\rho)\,\delta\rho_j \end{aligned} \tag{2.7}$$

where $\partial P_u(u(\rho), 0)$ is the linearization of P in response to a perturbation in u around the trajectory produced by $u(\rho)$ and by $v = 0$. Note that $\partial P_u(u(\rho), 0)$ can not be computed explicitly since P is an unknown operator. The derivative of $u(\rho)$ with respect to ρ_j is denoted $u'_{\rho_j}(\rho)$ and it is the j-th component of the vector $u'(\rho)$. It is straightforward to see that (2.7) yields

$$y'_{\rho_j}(\rho) = \partial P_u(u(\rho), 0)\, u'_{\rho_j}(\rho) \tag{2.8}$$

where $y'_{\rho_j}(\rho)$ is defined in a similar fashion as $u'_{\rho_j}(\rho)$. A similar reasoning yields

$$u'_{\rho_j}(\rho) = C'_{\rho_j}(\rho, r, y(\rho)) + \partial C_y(\rho, r, y(\rho))\, y'_{\rho_j}(\rho) \tag{2.9}$$

where $\partial C_y(\rho, r, y(\rho))$ is the linearization of C in response to a perturbation in y around the trajectory produced by r and by $y(\rho)$. The partial derivative of $C(\rho)$ with respect to ρ_j is denoted by $C'_{\rho_j}(\rho, r, y(\rho))$. Since $C(\rho)$ is a known operator, $\partial C_y(\rho, r, y(\rho))$ and $C'_{\rho_j}(\rho, r, y(\rho))$ can actually be computed analytically. Let us define the signals

$$\bar{r}_j(\rho) = C'_{\rho_j}(\rho, r, y(\rho)) \quad \text{for} \quad j = 1, \cdots, n. \tag{2.10}$$

Note that these can be reconstructed from the measurable signals $y(\rho)$ and r, at least if $C'_{\rho_j}(\rho, r, y(\rho))$ is a BIBO stable operator. The contrary case is commented upon below. It now easily follows that $u'(\rho)$ and $y'(\rho)$ can be obtained by simulating the loop in Figure 2.2 with the reference signals $\bar{r}_j(\rho)$. The stability of the previous loop follows from the smoothness assumption on the nonlinear closed loop operator and its BIBO stability.

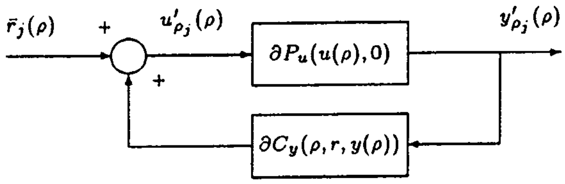

Figure 2.2: Generation of $u'_{\rho_j}(\rho)$ and $y'_{\rho_j}(\rho)$

As we show in the sequel, it is possible somehow to obtain the linearizations of the nonlinear closed loop operators T and S as defined by the control loop of Figure 2.1,

$$y(\rho) = T(\rho, r, v), \quad u(\rho) = S(\rho, r, v), \tag{2.11}$$

around their respective operating trajectories. Recall that, by assumption, T and S are BIBO stable operators. Define $\partial T_r(\rho, r, 0)$ and $\partial S_r(\rho, r, 0)$ as the linearizations of T and S in response to a perturbation in r around the trajectory produced by r and $v = 0$. Note that $\partial T_r(\rho, r, 0)$ and $\partial S_r(\rho, r, 0)$ are time-varying operators. Indeed, the linearizations of the closed loop operators T and S at a given time depend on the state of the system at that particular time and it is obvious that this state changes with time; see [3].

Using the input-output relations (2.11) and with d_r a small signal, we obtain the following equations

$$\begin{aligned} y(\rho) + d_y(\rho) &= T(\rho, r + d_r, v), \\ u(\rho) + d_u(\rho) &= S(\rho, r + d_r, v), \\ d_y(\rho) &\simeq \partial T_r(\rho, r, 0)\, d_r, \\ d_u(\rho) &\simeq \partial S_r(\rho, r, 0)\, d_r. \end{aligned} \tag{2.12}$$

Equivalently, using the input-output relations (2.6), we have

$$\begin{aligned} y(\rho) + d_y(\rho) &= P(\rho, u(\rho) + d_u(\rho), v), \\ u(\rho) + d_u(\rho) &= C(\rho, r + d_r, y(\rho) + d_y(\rho)), \\ d_y(\rho) &\simeq \partial P_u(u(\rho), 0)\, d_u(\rho), \\ d_u(\rho) &\simeq \partial C_r(\rho, r, y(\rho))\, d_r \\ &\quad + \partial C_y(\rho, r, y(\rho))\, d_y(\rho). \end{aligned} \tag{2.13}$$

Here $\partial C_r(\rho, r, y(\rho))$ denotes the linearization of C in response to a perturbation in r around the trajectory produced by r and by $y(\rho)$. The remarks that were made about $\partial C_y(\rho, r, y(\rho))$ apply also for $\partial C_r(\rho, r, y(\rho))$, i.e. $\partial C_r(\rho, r, y(\rho))$ can actually be computed analytically. It is now straightforward to see that, by equivalence between the relations (2.12) and (2.13), the gradient signals generated using Figure 2.2 can equivalently be generated using Figure 2.3.

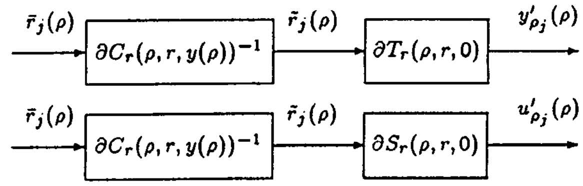

Figure 2.3: Generation of $u'_{\rho_j}(\rho)$ and $y'_{\rho_j}(\rho)$

Let us define the signals

$$\begin{aligned} \tilde{r}_j(\rho) &= \partial C_r(\rho, r, y(\rho))^{-1}\, C'_{\rho_j}(\rho, r, y(\rho)) \qquad (2.14) \\ &= \partial C_r(\rho, r, y(\rho))^{-1}\, \bar{r}_j(\rho) \quad \text{for } j = 1, \cdots, n. \end{aligned}$$

We assume that $\partial C_r(\rho, r, y(\rho))^{-1}$ is stable. The contrary case is commented upon below.

Suppose we do a first experiment on the actual system with the usual reference signal r. Then, the closed loop response is given by

$$y_1(\rho) = T(\rho, r, v_1)$$

where v_1 is the noise realization during the first experiment. Using a high SNR assumption and a smoothness assumption on T, we have that

$$y_1(\rho) \simeq T(\rho, r, 0) + \partial T_v(\rho, r, 0)\, v_1. \tag{2.15}$$

Here, $\partial T_v(\rho, r, 0)$ denotes the linearization of the closed loop operator T in response to a perturbation in v around the trajectory produced by r and by $v = 0$.

Suppose now that we do a second experiment on the system with a slightly perturbed reference signal $r + d_r$. Denote the corresponding output by $y_2(\rho)$. Then, using the small signal assumption on d_r and a smoothness assumption on T, we have that

$$\begin{aligned} y_2(\rho) &= T(\rho, r + d_r, v_2) && (2.16) \\ &\simeq T(\rho, r + d_r, 0) + \partial T_v(\rho, r, 0)\, v_2 && (2.17) \\ &\simeq T(\rho, r, 0) + \partial T_r(\rho, r, 0)\, d_r \\ &\qquad + \partial T_v(\rho, r, 0)\, v_2 && (2.18) \\ &= y_1(\rho) + \partial T_r(\rho, r, 0)\, d_r \\ &\qquad + \partial T_v(\rho, r, 0)\, (v_2 - v_1) && (2.19) \end{aligned}$$

where v_2 is the disturbance during the second experiment. Rewriting the previous equation, we obtain that

$$y_2(\rho) - y_1(\rho) \simeq \partial T_r(\rho, r, 0)\, d_r + \partial T_v(\rho, r, 0)\, (v_2 - v_1).$$

Experimental generation of the gradient

At each iteration step i of the controller parameter tuning, we use $n + 2$ experiments with the fixed controller $C(\rho[i])$. The corresponding reference signals are

$$\begin{cases} r_i^1 &= r, \\ r_i^2 &= r + \mu_1\, \bar{r}_1(\rho[i]), \\ \vdots & \vdots \quad \vdots \\ r_i^{n+1} &= r + \mu_n\, \bar{r}_n(\rho[i]), \\ r_i^{n+2} &= r \end{cases} \tag{2.20}$$

with the scalars μ_j chosen such that the signals $\mu_j\, \tilde{r}_j(\rho[i])$ are small for all $j = 1, \cdots, n$. The corresponding output and disturbance signals are, respectively, denoted by y_i^k and v_i^k, for $k = 1, \cdots, n+2$. We have the following results

$$\begin{aligned} \tilde{y}(\rho[i]) &= y_i^1 - y_d \\ &\simeq T(\rho, r) - y_d + \partial T_v(\rho[i], r, 0)\, v_i^1, \quad (2.21) \\ \tilde{y}'_{\rho_j}(\rho[i]) &= \mu_j^{-1}\,(y_i^{j+1} - y_i^{n+2}) \quad (2.22) \\ &\simeq \partial T_r(\rho[i], r, 0)\, \tilde{r}_j(\rho[i]) \\ &\quad + \mu_j^{-1}\, \partial T_v(\rho[i], r, 0)\,(v_i^{j+1} - v_i^{n+2}) \quad (2.23) \end{aligned}$$

for $j = 1, \cdots, n$. Thus $\tilde{y}'_{\rho_j}(\rho[i])$ is a noisy estimate of $y'_{\rho_j}(\rho[i])$. The signal in (2.22) is the j-th component of the vector $\tilde{y}'(\rho[i])$ which is an estimate of $y'(\rho[i])$. Note that the noise terms in $\tilde{y}'(\rho[i])$ are uncorrelated with the noise of $\tilde{y}(\rho[i])$. This is the motivation of the $(n+2)$-th experiment.

Identification-based generation of the gradient

An alternative procedure for the estimation of $y'_{\rho_j}(\rho[i])$ is as follows. At iteration step i, we use two experiments with reference signals

$$r_i^1 = r \quad \text{and} \quad r_i^2 = r + d_r \qquad (2.24)$$

where d_r is some small perturbation signal designed by the user. We have the following results

$$\begin{aligned} \tilde{y}(\rho[i]) &= y_i^1 - y_d, \quad (2.25) \\ d_i &= y_i^2 - y_i^1 \\ &\simeq \partial T_r(\rho[i], r, 0)\, d_r \\ &\quad + \partial T_v(\rho[i], r, 0)\,(v_i^2 - v_i^1). \quad (2.26) \end{aligned}$$

Since d_r and $\partial T_v(\rho[i], r, 0)\,(v_i^2 - v_i^1)$ are uncorrelated it is possible to obtain an "averaged" estimate of $\partial T_r(\rho[i], r, 0)$ by open loop identification of a linear time invariant model between the reconstructable signals d_r and d_i. The use of an independently parametrized model structure[1] ensures that the estimate $\widehat{\partial T}_i$ of $\partial T_r(\rho[i], r, 0)$ is a stable transfer function; see [9]. One can therefore compute estimates of the gradient signals $y'_{\rho_j}(\rho[i])$ using

$$\tilde{y}'_{\rho_j}(\rho[i]) = \widehat{\partial T}_i\, \tilde{r}_j(\rho[i]), \qquad (2.27)$$

the j-th component of the estimate $\tilde{y}'(\rho[i])$ of $y'(\rho[i])$.

Estimation of the gradient and of the Hessian

The signals defined previously are now used to compute an estimate of the gradient of J

$$\hat{J}'(\rho[i]) = \frac{1}{N} \sum_{t=1}^{N} \tilde{y}(\rho[i])\, \tilde{y}'(\rho[i]). \qquad (2.28)$$

[1] The input-output dynamics and the noise dynamics are independently parametrized.

A good (but biased) estimate of the Hessian is obtained using

$$\hat{R}_i = \frac{1}{N} \sum_{t=1}^{N} \tilde{y}'(\rho[i])\, [\tilde{y}'(\rho[i])]^T. \qquad (2.29)$$

The iterative algorithm is now obtained by replacing $J'(\rho[i])$ by $\hat{J}'(\rho[i])$ and R_i by $\hat{R}_i$ in (2.5).

Algorithm

We now present the algorithm for the iterative controller optimization procedure:

- **Step 0:** Start with a stabilizing controller $C(\rho[0])$.
- **Step 1:** With the controller $C(\rho[i])$ in the loop, perform one experiment on the actual system.
- **Step 2:** Compute a realization of $y(\rho[i]) - y_d$ and generate the signals $\tilde{r}_j(\rho[i])$ as shown in (2.14).
- **Step 3:**
  Experimental generation: perform $n+1$ additional experiments as shown in (2.20).
  Identification-based generation: perform one additional experiment as shown in (2.24).
- **Step 4:**
  Experimental generation: compute estimates $\tilde{y}'_{\rho j}(\rho_i)$ of the gradients $y'_{\rho j}(\rho_i)$ as shown in (2.22).
  Identification-based generation: identify a model $\widehat{\partial T}_i$ of $\partial T_r(\rho[i], r, 0)$ and compute estimates $\tilde{y}'_{\rho j}(\rho_i)$ of the gradients $y'_{\rho j}(\rho_i)$ as shown in (2.27).
- **Step 5:** Compute $\hat{J}'(\rho[i])$ and $\hat{R}_i$ as shown in (2.28) and (2.29).
- **Step 6:** Update the parameter vector using

  $$\rho_{i+1} = \rho_i - \gamma_i\, \hat{R}_i^{-1}\, \hat{J}'(\rho[i]) \qquad (2.30)$$

  and goto Step 1.

Remarks

- In some cases, the approximation involved in the identification-based scheme might be very crude and it might be worthwhile investigating the use of a time-varying model structure for the identification of the linearization of the complementary sensitivity operator; we refer the reader to [9] for further details. However, noiseless simulations have shown that, at least for step-like reference signals, the gradients signals generated by both procedures, experimental or identification-based with a time invariant model structure, are very similar.
- The gradient of the controller, $C'_y(\rho, r, y(\rho))$ and the linearization $\partial C_r(\rho, r, y(\rho))$ are used to generate the signals $\tilde{r}_j(\rho)$, see (2.14), using r and $y(\rho)$.

The first operator could be unstable and the second operator could be non minimum phase making the calculation of the gradient infeasible. In the linear case, both problems can be overcome by choosing the stable operator L in (2.3) in an appropriate way. See [4, 6], for more details. It may be possible to apply similar ideas in the nonlinear case based on nonlinear all-pass filters; we refer the reader to [10] for further details.

- The controller optimization procedure converges to a local minimum of the design criterion under conditions of bounded signals, i.e. it is assumed that stability is preserved while iterating. This is a reasonable assumption since the step size γ_i can be used effectively to control how much the controller is allowed to change per iteration. We refer the reader to [6] for a convergence analysis which appears to applicable also in the nonlinear case.

3 A numerical illustration

In this section, we discuss how the identification-based generation of the gradient can be applied for the tuning of a nonlinear controller for a nonlinear plant.

The nonlinear system

The nonlinear plant is described by

$$y(t) = a\,y(t-1) + b\,y(t-2) + c\,y(t-1)u^3(t-1) + d\,y^2(t-1) + e\,u(t-1) + v(t) \qquad (3.1)$$

where $v(t)$ is white noise of variance $\sigma^2 = 0.0001$. Here, we have taken the following plant parameters

$$a = 1.5, b = -0.7, c = 0.7, d = 0.1 \text{ and } e = 0.1. \qquad (3.2)$$

The nonlinear controller

The goal is to tune the nonlinear controller

$$u(t) = \rho_1\,e(t) + \rho_2\,e(t-1) + \rho_3\,u(t-1) + \rho_4\,y(t)e(t) \quad (3.3)$$

according to (2.3). Here $e(t) = y(t) - r(t)$ defines the tracking error at time t. The nonlinear controller in (3.3) can be rewritten in the format of (2.2);

$$u(t) = \frac{1}{1-\rho_3\,q^{-1}}\left[\rho_1\,e(t) + \rho_2\,e(t-1) + \rho_4\;y(t)e(t)\right]$$

where q^{-1} is the backward shift operator. As an illustrative example we have computed the signal $\bar{r}_j(\rho)$ defined in (2.10) for the parameter ρ_3

$$\bar{r}_3(\rho,t) = \frac{u(\rho,t-1)}{(1-\rho_3\,q^{-1})}.$$

It is straightforward to see that the linearization of C in response to a perturbation in r around the trajectory produced by r and by $y(\rho)$ is given by

$$\partial C_r(\rho, r, y(\rho)) = -\frac{[\rho_1 + \rho_4\,y(\rho,t)] + \rho_2\,q^{-1}}{1-\rho_3\,q^{-1}} \qquad (3.4)$$

where $\rho_1 + \rho_4\,y(\rho,t)$ has to be considered as a time-varying parameter. It follows that

$$\bar{r}_3(\rho,t) = -\frac{u(\rho,t-1)}{[\rho_1 + \rho_4\,y(\rho,t)] + \rho_2\,q^{-1}}. \qquad (3.5)$$

The initial controller parameter vector was taken to be

$$\rho[0] = [\rho_1 \;\; \rho_2 \;\; \rho_3 \;\; \rho_4] = [-1 \;\; 0.4 \;\; 0.6 \;\; 0]^T. \qquad (3.6)$$

Notice that the initial controller is linear. The important fact is that this initial controller stabilizes the actual system (3.1) in the BIBO sense.

The design quantities

The reference signal consisted of the sum of three upward steps and a downward step of amplitude 0.5 delayed in time and filtered with a Butterworth filter of cut-off frequency 0.05.

For the desired response, we chose a similar signal, except that the amplitudes of the second, the third and the fourth step were, respectively, 0.6, 0.7 and 0.6. The reference signal, the desired closed loop response and the closed loop response achieved by the initial controller are shown in Figure 3.1.

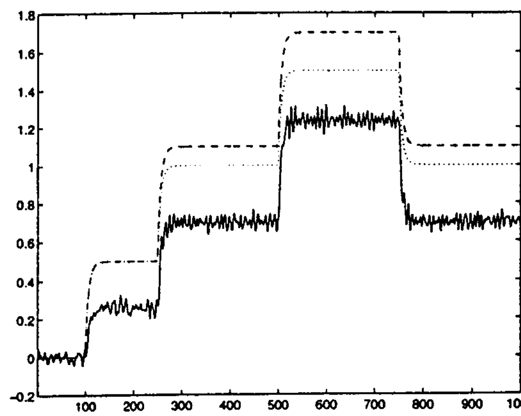

Figure 3.1: Reference signal ($\cdots$), desired closed loop response (−−) and achieved closed loop response with the initial controller (—).

The gradient of y with respect to ρ was estimated on the basis of closed loop identification, i.e. we have used an identification-based generation of the gradient signal. To generate the second experiment, we have perturbed the reference signal with a signal d_r chosen as a white noise sequence of variance 0.001 filtered with a first order Butterworth filter of cut-off frequency ω_c. In the first stages of the optimization, we have used a low cut-off frequency ω_c in order not to excite high frequency harmonics. We have increased the cut-off frequency ω_c as the bandwidth of the closed loop system increased with the iterations.

Throughout, we have used an Output Error (OE) model structure of order n to identify the estimate of the linearized closed loop system, $\partial T_r(\rho, r, 0)$, using 1000 data samples per experiment. As stated above $\partial T_r(\rho, r, 0)$ is actually time-varying. Here we have identified a unique time invariant model of $\partial T_r(\rho, r, 0)$ for the whole data set. We have used the line search procedure outlined in [2] to optimize the step size γ_i of the parameter updating equation (2.5) at each iteration step. The design parameter ω_c, the step size γ_i, the model order n and the achieved cost are shown in Table 3.1.

Iteration	γ_i	ω_c	n	Cost
0				0.1435
1	0.25	1	1	0.0086
2	0.36	3	3	0.0018
3	0.93	3	5	0.0011
4	2.02	3	5	0.0010

Table 3.1: Design parameter, step size, model order and achieved cost at each iteration.

As in [2], the line search procedure has allowed us to select the best controller parameters along the descent direction, thereby significantly reducing the number of iteration steps. Table 3.1 shows that the performance has improved considerably. Figure 3.2 shows the reference signal, the desired response and the achieved closed loop response with the final nonlinear controller. Notice that we have improved both the disturbance rejection and the tracking of the desired closed loop response.

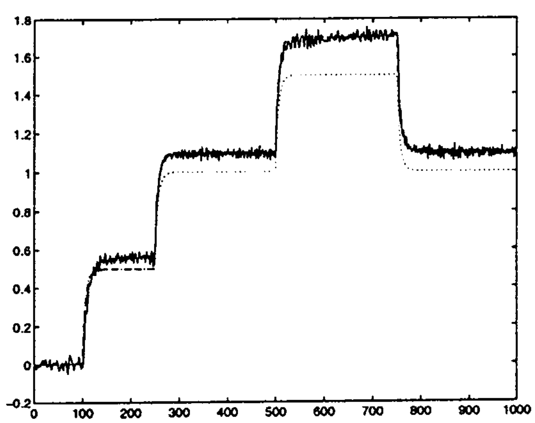

Figure 3.2: Reference signal $(\cdots)$, desired closed loop response $(--)$ and achieved closed loop response with the final nonlinear controller $(—)$.

Further iterations with the nonlinear controller structure in (3.3) were not able to improve the achieved cost. We have obtained the following controller parameters:

$$\rho[4] = [-2.4436 \;\; 2.2287 \;\; 1.0207 \;\; 0.1110]^T.$$

The parameter ρ_4 that characterizes the nonlinear part of the controller is small but it has its importance for control performance. Indeed, for comparison purposes, we have tuned the linear controller obtained by setting $\rho_4 = 0$. The control cost achieved with the best linear controller is 0.0017. This figure has to be compared with 0.0010 with the best nonlinear controller.

4 Conclusions

In this paper, we have presented a nonlinear extension of the controller optimization method proposed in [4, 6]. It is shown that one can obtain an estimate of the gradient either experimentally using $n+2$ experiments on the actual system or by doing just two experiments per iteration on the actual system and using open loop identification techniques. A numerical simulation example with an identification-based generation of the gradient has shown to give very satisfactory results.

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