C. 98

Proceedings of the 1998 IEEE
International Conference on Control Applications
Trieste, Italy 1-4 September 1998

FM06

ITERATIVE FEEDBACK TUNING OF A NONLINEAR CONTROLLER FOR AN INVERTED PENDULUM WITH A FLEXIBLE TRANSMISSION

Benoît Codrons[†], Franky De Bruyne[‡], Michel De Wan[†] and Michel Gevers[†]

[†] CESAME, Centre for Systems Engineering and Applied Mechanics, Université Catholique de Louvain, Bâtiment Euler, Avenue Georges Lemaître 4-6, 1348 Louvain-La-Neuve, Belgium

[‡] Department of Systems Engineering, Research School of Information Sciences and Engineering, The Australian National University, Canberra ACT 0200, Australia

E-mail: {Codrons, Gevers, DeWan}@auto.ucl.ac.be, DeBruyne@syseng.anu.edu.au

Abstract

Recently a model-free Iterative Feedback Tuning (IFT) scheme has been proposed for nonlinear plants and/or controllers; we refer the reader to [3, 8] for more details. This iterative control strategy only uses closed loop data. In this paper, we examine the applicability of IFT for the tuning of a nonlinear controller for an inverted pendulum with a flexible transmission.

1 Introduction

Recently, the authors of [6] have proposed an iterative control design method which optimizes a restricted complexity linear controller structure. The key ingredient is that an unbiased gradient of the control design criterion is computed using closed loop data. An important result is that this scheme converges to a local minimum of the design criterion under assumption of boundedness of the signals in the loop. This method, acronymed Iterative Feedback Tuning (IFT) by its inventors, has shown to give excellent results from both an experimental and industrial point of view; see e.g. [1, 2, 4, 5, 7].

Very recently, it has been shown that most of the concepts originally proposed in [6] carry over when both the process and the controller are allowed to be nonlinear. Indeed, it is shown in [3, 8] that it is still possible to estimate the gradient signals by performing experiments with almost identical reference signals. The objective of this paper is to first review the IFT method in its nonlinear setting, and to then report on its performances when applied to the tuning of a nonlinear controller for an inverted pendulum with a flexible transmission built at CESAME by the third author.

The paper is organized as follows. In Section 2 we present an overview of Iterative Feedback Tuning for nonlinear processes and/or nonlinear controllers. In Section 3, the experimental results with the inverted pendulum are presented. We conclude in Section 4.

2 Iterative controller optimization for nonlinear systems

In this section, we briefly recall the data-driven Iterative Feedback Tuning scheme that was proposed in [3] in its nonlinear setting.

Problem setting

Let us assume that the true system is the Single-Input Single-Output (SISO) nonlinear time-invariant system described by

$$\mathcal{S} : y_t = P(u_t, v_t) \tag{2.1}$$

where P is an unknown nonlinear operator. Here u_t is the control input signal, y_t is the achieved output signal and v_t is a process disturbance signal. The input signal is determined according to

$$\mathcal{C} : u_t = C(\rho, r_t, y_t) \tag{2.2}$$

where r_t is an external reference uncorrelated with v_t and the controller C is a nonlinear operator of both r_t and y_t and is parametrized by a controller parameter vector ρ, with $\rho \in \mathbb{R}^n$. In the sequel we often make an implicit use of linearizations of some nonlinear operators around their operating trajectories. We therefore require that the plant, the controller and all closed loop operators are smooth functions of the reference signal, the input signal, the output signal and the disturbance signal. We also require a high Signal-to-Noise-Ratio (SNR) and we assume that the closed loop system is stable in the Bounded-Input-Bounded-Output (BIBO) sense. For ease of notation, we from now on omit the time argument of the signals. In order to stress the dependence of the output signal of the closed loop system on the particular controller parameter vector, we denote by $y(\rho)$ the output of (2.1) in feedback with (2.2).

Let y_d be the desired closed loop response to the reference signal r. Then, the error between the achieved and the desired response is $\tilde{y}(\rho) = y(\rho) - y_d$. The control design objective is formulated as the minimization of an LQG control criterion for the reduced complexity controller (2.2), i.e. $\rho^* = \arg\min_\rho J(\rho)$ with

$$J(\rho) = \frac{1}{2} E\,[\tilde{y}(\rho)]^2. \tag{2.3}$$

The expected value is taken with respect to the probability distribution of the noise. An extension to a criterion $J(\rho)$ that includes a frequency weighting or a control penalty is straightforward.

Criterion minimization

It is standard that one can find a solution for ρ to

$$J'(\rho) = E\,[\tilde{y}(\rho)\, y'(\rho)] = 0 \tag{2.4}$$

by taking repeated steps in the negative gradient direction

$$\rho[i+1] = \rho[i] - \gamma_i\, R_i^{-1}\, J'(\rho[i]) \tag{2.5}$$

where R_i is some appropriate positive definite matrix, typically an estimate of the Hessian of J and $\{\gamma_i\}$ is a sequence of positive numbers that determines the step size. Here $\rho[i]$ denotes the controller parameter vector ρ at iteration i. As shown in [3], this problem can be tackled by replacing $J'(\rho[i])$ with an approximation based on signals measured on the closed loop system. In order to do so, estimates of the signal $\tilde{y}(\rho[i])$ and its gradient $y'(\rho[i])$ are computed using $n+2$ experiments of sufficiently long duration N, each with the controller $C(\rho[i])$.

Algorithm

We now present the algorithm for the controller optimization procedure. We refer the reader to [3] for details.

- **Step 0:** Start with a stabilizing controller $C(\rho[0])$.
- **Step 1:** With the current controller $C(\rho[i])$ in the loop, perform one experiment on the actual system with the reference signal $r_i^1 = r$. Collect the output of this experiment and denote it by y_i^1.
- **Step 2:** Compute a realization of

$$\tilde{y}(\rho[i]) = y_i^1 - y_d \tag{2.6}$$

and generate the signals

$$\tilde{r}_j(\rho[i]) = \delta C_r[\rho[i], r, y_i^1]^{-1}\; C'_{\rho_j}[\rho[i], r, y_i^1] \tag{2.7}$$

for $j = 1, \cdots, n$. Here $\delta C_r[\rho, r, y(\rho)]$ denotes the linearization of C in response to a perturbation in r around the trajectory produced by r and $y(\rho)$, while $C'_{\rho_j}[\rho, r, y(\rho)]$ denotes the partial derivative of $C(\rho)$ with respect to ρ_j.

- **Step 3:** With the current controller $C(\rho[i])$ in the loop, perform $n+1$ additional experiments using the reference signals

$$\begin{cases} r_i^2 &= r, \\ r_i^3 &= r + \mu_1\, \tilde{r}_1(\rho[i]), \\ \vdots & \vdots \quad \vdots \\ r_i^{n+2} &= r + \mu_n\, \tilde{r}_n(\rho[i]) \end{cases} \tag{2.8}$$

with the scalars μ_j chosen such that the signals $\mu_j\, \tilde{r}_j(\rho[i])$ are small for all $j = 1, \cdots, n$. Collect the outputs of these experiments and denote them by y_i^k for $k = 2, \cdots, n+2$.

- **Step 4:** Compute

$$\hat{y}'_{\rho_j}(\rho[i]) = \mu_j^{-1}(y_i^{j+2} - y_i^2) \text{ for } j = 1, \cdots, n. \tag{2.9}$$

The signal $\hat{y}'_{\rho_j}(\rho[i])$ is the j-th component of the vector $\hat{y}'(\rho[i])$ which is an estimate of $y'(\rho[i])$.

- **Step 5:** Compute

$$\hat{J}'(\rho[i]) = \frac{1}{N}\sum_{t=1}^{N} \tilde{y}(\rho[i])\, \hat{y}'(\rho[i]) \quad \text{and} \tag{2.10}$$

$$\hat{R}_i = \frac{1}{N}\sum_{t=1}^{N} \hat{y}'(\rho[i])\, [\hat{y}'(\rho[i])]^T. \tag{2.11}$$

- **Step 6:** Update the parameter vector using

$$\rho_{i+1} = \rho_i - \gamma_i^*\, \hat{R}_i^{-1}\, \hat{J}'(\rho[i]) \tag{2.12}$$

where γ_i^* is obtained by optimizing γ_i at iteration step i using a line search procedure. Go to Step 1.

Remarks

- The gradient of the controller, $C'_y(\rho, r, y(\rho))$ and the linearization $\delta C_r[\rho, r, y(\rho)]$ are used to generate the signals $\tilde{r}_j(\rho)$ using r and $y(\rho)$. The first operator could be unstable and the second operator could be non minimum phase making the calculation of the gradient infeasible. Both problems can be overcome by introducing an appropriate all-pass operator in (2.3); see [3] for further details.
- An alternative approximative procedure for the estimation of $\hat{y}'(\rho[i])$ based on an identification of the linearized closed loop system is given in [3]. Simulations have shown that this alternative procedure is especially useful in a low SNR situation.

3 Application to an inverted pendulum

In this section, we apply the results of Section 2 to tune a nonlinear controller for an inverted pendulum with a flexible transmission.

The inverted pendulum

The test case is a non-classical inverted pendulum depicted in Figure 3.1. The lower tip of the arm is fixed on a wheel that allows it to rotate around its vertical position. The (driven) wheel is linked to the driving wheel with two elastic belts. In turn, this driving wheel is actuated by a motor that is controlled with a local feedback so that its angular position is directly proportional to the applied voltage U_{in}. The job of the controller is to rotate the pendulum to a given reference by feedback of the measured angular position. Due to the physical contraints, i.e. the length of pendulum, the set up has open loop stable left and right equilibria. A PC using the VisSim package is used to control the system. The signal to be tracked by the controlled system is the reference position for the angular position of the pendulum, i.e. θ_{ref}. The controlled system output is the measured angular position of the pendulum. The closed loop system is sampled with a sampling frequency of $f_s = 100$ Hz. The vertical open loop unstable equilibrium point corresponds to $\theta_{\text{ref}} = 0$. All variables are scaled in such a way that the open loop stable left and right equilibria, respectively, correspond to $\theta_{\text{ref}} = 1$ and $\theta_{\text{ref}} = -1$.

Note that the system has nonlinear dynamics due to the nonlinear (sinusoïdal) dependence of the gravitational force and the elastic force on the angular position of the pendulum. Also, this setup suffers from nonlinear stiction forces which are induced by the potentiometer angular position measurement system.

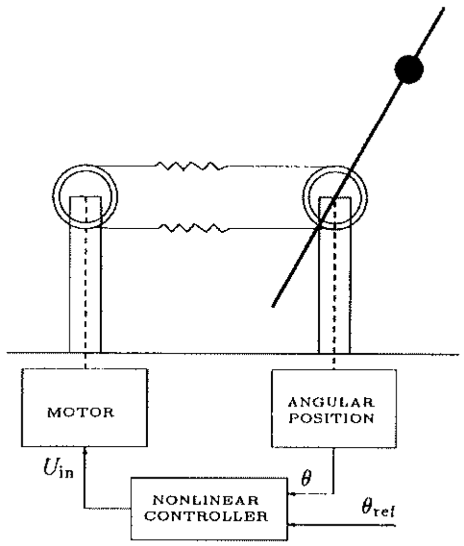

Figure 3.1: Inverted pendulum with flexible transmission

The nonlinear PID controller

The following one degree of freedom continuous time nonlinear PID controller structure was employed

$$\begin{aligned} U_{\text{in}} &= C[\rho, \theta, \theta_{\text{ref}}] \\ &= \frac{(1 + T_d\, s)\,(1 + T_i\, s)}{s\, T_i\,(1 + \delta T_d\, s)}\,[K_p\, e + k \sin(c\, e)] \end{aligned} \quad (3.1)$$

where $e = \theta - \theta_{\text{ref}}$. The intuition for the nonlinear term in (3.1) is that the controller should compensate the sinusoïdal dependence of the gravitational forces on the angular position of the pendulum. Note that for a reference $\theta_{\text{ref}} = 0$, the forces applied to the flexible transmission should be maximum when the pendulum is horizontal. Therefore, we have chosen the constant c in such a way that $\sin(c\, e) = 1$ for a horizontal pendulum and $\theta_{\text{ref}} = 0$.

As an illustrative example, we have computed the signals $\tilde{r}_j$ defined earlier for the parameter T_d. We have the following results

$$\begin{aligned} C'_{T_d}[\rho, \theta, \theta_{\text{ref}}] &= \frac{(1-\delta)(1 + T_i\, s)}{T_i\,(1 + \delta\, T_d\, s)^2}\,[K_p\, e + k \sin(c\, e)], \\ \delta C_{\theta_{\text{ref}}} &= -\frac{(1 + T_d\, s)\,(1 + T_i\, s)}{s\, T_i\,(1 + \delta\, T_d\, s)}\,[K_p + k\, c\, \cos(c\, e)], \\ \tilde{r}_{T_d} &= \frac{1}{K_p + k\, c\, \cos(c\, e)} \times \\ & \frac{(\delta - 1)\, s}{(1 + \delta\, T_d\, s)\,(1 + T_d\, s)}\,[K_p\, e + k \sin(c\, e)]. \end{aligned}$$

Here, $[K_p + k\, c\, \cos(c\, e))]^{-1}$ is a time varying gain.

The initial parameter vector was taken to be

$$\rho[0] = [\delta\ \ K_p\ \ T_d\ \ T_i\ \ k]^T = [0.15\ \ -0.05\ \ 5\ \ 1\ \ -0.45]^T.$$

This stabilizing controller was obtained after a few trials on the setup. No model of the inverted pendulum was used either during the derivation of the initial controller or during the later optimization of this controller.

The design quantities

Reference signal
The reference signal consists of a filtered step from the open loop stable right equilibrium ($\theta_{\text{ref}} = -1$) to the open loop unstable equilibrium ($\theta_{\text{ref}} = 0$) after 5 seconds followed by another filtered step of amplitude -0.22 after 35 seconds. We have used a third order Butterworth filter with cut-off frequency ω_c. We have collected 55 seconds of data per experiment, i.e. $N = 5500$.

Artificial noise
We have injected an artificial white noise signal of zero mean and variance σ^2 at the control input of the system at the first experiment of each iteration as a way of partially preventing the nonlinear stiction. Indeed, by adding a dither on the input signal, we partially manage to alleviate the steady state error due to friction effects. Several attempts without injection of artificial white noise at the control input have shown that the iterative scheme produces controllers that have more and more integral action. This is due to the inevitable static error that is present. Only marginal improvements could therefore be obtained in such cases.

Desired response
The desired response is just the reference signal delayed by a delay d. As is advised in [6], we have chosen to make conservative choices for the desired time response at the initial stages, i.e. a high enough delay d. The performance specifications were made more stringent whenever possible. Several attempts with a too small delay d,

i.e. too stringent performance specifications, at the early stages of the procedure gave only marginal improvements over the initial controller.

Engineering aspects

At each iteration i, the scalars μ_j ($j = 1, \cdots, n$) in (2.8) have been computed as follows

$$\mu_j = \frac{m}{(\max\{\tilde{r}_j(\rho[i])\})} \tag{3.2}$$

where m is a constant that can be adapted at each iteration and is used as a design parameter. Notice that the disturbing nature of the n last experiments can be diminished by decreasing m. Of course, this might be at the expense of the quality of the estimated gradient signals if the SNR is not high enough. Indeed, the choice of the parameter m is a compromise between the quality of the linearization or the disturbing nature of the experiments on the one hand and the sensitivity to noise on the other hand.

Also, we have automated the 7 experiments needed for the gradient computation as follows. During the first experiment the signals $\tilde{r}_j(\rho[i])$ and the scalars μ_j are calculated on line as described in (2.7) and (3.2). When performing the second experiment, the reference signals (2.8) necessary for the next n experiments are computed on line. During each of these n experiments, the gradient signals (2.9) are computed on line, i.e. they are readily available at the end of the 7-th experiment.

The iterations

The design parameters shown in Table 3.1 have been used at the first and the second iteration.

	Iter. 1	Iter. 2
σ^2	0.03	0.03
d	1	0.5
ω_c	0.5	0.5
m	0.5	0.25

Table 3.1: Design parameters.

Notice again that the performance specifications have been made more stringent at the second iteration by decreasing the value of the delay d. The value of m has been decreased at the second iteration in order not to excite too much the high frequency harmonics of the closed loop system. This has been done without noticeable improvement or deterioration of the estimated gradient signals.

Using the experimentally generated gradients, we have performed a line search along the descent direction. Indeed, for each γ_i in (2.5), we have performed one experiment on the setup to compute the experimental cost (2.3). This line search procedure has allowed us to select the best controller along the descent direction, thereby significantly reducing the number of iterations. Table 3.2

γ_i	Iter. 1	Iter. 2
0	0.3424	0.0444
0.5	0.2200	0.0266
1	0.0796	0.0252
1.5	0.0604	0.0242 *
2	0.0416	0.0247
2.5	0.0320 *	0.0249
3	0.0369	

Table 3.2: Experimental costs and performance improvements using a line search procedure along the descent direction.

shows that the performance has improved considerably. Indeed, the control cost has been decreased by a factor 10 at the end of the first iteration.

Figure 3.2 shows the reference signal, the desired output response at the first iteration and the achieved responses with the initial controller and with the controller obtained at the end of the first iteration.

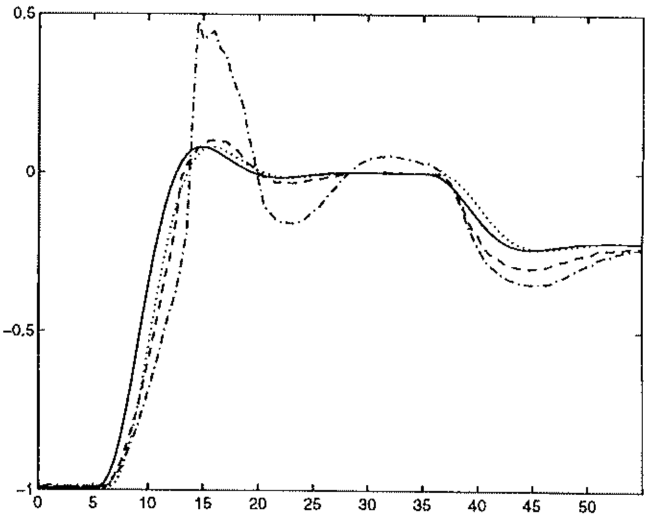

Figure 3.2: Reference signal (—), desired closed loop response at the first iteration (···) and achieved closed loop response with the initial controller (–·) and with the controller obtained at the end of the first iteration (– –).

Figure 3.3 shows the reference signal, the desired output response at the second iteration and the achieved response with the controllers obtained at the end of the first and the second iterations. As can be seen from Table 3.2 and Figure 3.3, the performance improvements have become much less spectacular.

Further iterations with more stringent performance specifications were not able to improve the achieved performance significantly and were not included in the text. The performance limitations can largely be explained by the presence of the nonlinear stiction forces.

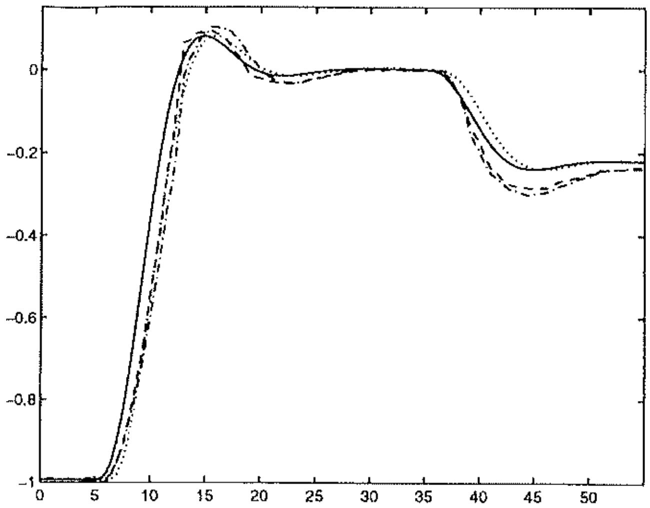

Figure 3.3: Reference signal (—), desired closed loop response at the second iteration (···) and achieved closed loop response with the controller obtained at the end of the first (-·) and the second iteration (−−).

We have converged to the following parameter vector

$$\begin{aligned}\rho[2] &= [\delta \; K_p \; T_d \; T_i \; k] \\ &= [0.1532 \;\; -0.1035 \;\; 10.2441 \;\; 1.0338 \;\; -0.5277]^T.\end{aligned}$$

As an illustrative example, the fifth experiment of the first iteration is shown in Figure 3.4, i.e. this is the experiment with the reference signal $r_1^5 = r_{1,T_d}$. As can be seen from Figure 3.4, this reference signal is not too different from the usual reference signal and is therefore not disturbing the operating conditions too much.

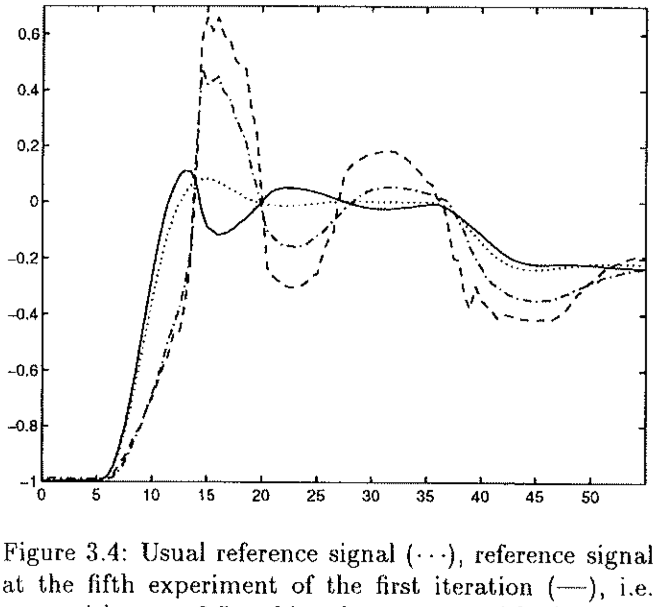

Figure 3.4: Usual reference signal (···), reference signal at the fifth experiment of the first iteration (—), i.e. r_{1,T_d} with $m = 0.5$, achieved responses with the usual reference signal (-·) and with the reference signal r_{1,T_d} (−−).

4 Conclusions

In this paper, we have applied the nonlinear extension of the IFT method proposed in [3] to tune a nonlinear controller for an inverted pendulum with a flexible transmission. We have shown using this experimental mechanical example that the IFT method in its nonlinear setting gives very satisfactory results and that most of the advantages of the IFT method in its linear setting carry over when the plant and/or the controller are allowed to be nonlinear. Indeed, the algorithm is easy to implement and the small number of design parameters contribute to make it a straightforward method to apply.

Acknowledgement: This paper presents research results of the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. The authors also wish to acknowledge the funding of this research by the US Army Research Office, Far East, the Office of Naval Research, Washington. The scientific responsibility rests with its authors.

References

[1] Codrons B. and B. Ceysens (1997). "Iterative Identificationless Control Design". *Journal A*, Vol. 38, No 1, pp. 26-30.

[2] De Bruyne F. and P. Carrette (1996). "Synthetic generation of the gradient for an iterative controller optimization method". *CD Proceedings of the European Control Conference*, Brussels, Belgium, TH-A-F-2 (52), 1997.

[3] De Bruyne F., B.D.O. Anderson, M. Gevers and N. Linard (1997). "Iterative controller optimization for nonlinear systems". *Proceedings of the 36th Conference on Decision and Control*, San Diego, USA, 3749-3754.

[4] De Bruyne F. (1997). "Iterative feedback tuning for MIMO systems". *Proceedings of the 2th International Symposium on Intelligent Automation and Control*, Anchorage, Alaska, USA.

[5] Hjalmarsson H., M. Gevers and O. Lequin (1997). "Iterative Optimal Controller Tuning Scheme: theory and applications in chemical process control". *Journal A*, Vol. 38, No 1, pp. 16-25.

[6] Hjalmarsson H., Gunnarsson S. and M. Gevers (1994). "Model-free data-driven optimal tuning of controller parameters". Technical Report LiTH-ISY-R-1680, Linköping University.

[7] Hjalmarsson H., Gunnarsson S. and M. Gevers (1995). "Model-free Tuning of A Robust Regulator for a Flexible Transmission System". *European Journal of Control*, Vol. 1, pp. 148-156.

[8] Sjöberg J. and M. Agarwal (1996). "Model-Free Repetitive Control Design for Nonlinear Systems". *Proceedings IEEE Conference on Decision and Control*, Kobe, Japan, pp. 2824-2829.