Identification and Validation for Robust Control

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Abstract. In this chapter we first review the

1 Introduction

This chapter focuses on the *interplay* between the design of the identification criterion and of the validation procedure, and the robust stability and performance of the resulting controller. The chapter elaborates on the basic ideas presented in Chapter **??**, as well as on the basic robust analysis tools presented in Chapter **??**. We shall restrict our presentation to the identification and validation of parametric model sets and - except stated otherwise - to least squares prediction error identification criteria.

For the sake of making our message as clear as possible, and even though this terminology is not universally accepted, we shall introduce the following distinction between *identification* and *validation*. We shall in this chapter call *identification* the task of constructing a nominal model G_{mod}. For a given data set, collected under specific experimental conditions, this nominal model is the direct result of the chosen model set and of the identification criterion. We shall call *validation* the task of constructing an uncertainty set $\mathcal{D}$ that contains the true system, perhaps at a certain probability level α. Very often, the construction of this uncertainty set $\mathcal{D}$ is also the result of a prediction error identification experiment. Thus, a single identification experiment could be used to construct both a nominal model G_{mod} and a validated uncertainty set $\mathcal{D}$. However, in identification for control one often wants to work with a low order model G_{mod} for control design. In such case there are good reasons to distinguish between

- an "identification" experiment for the construction of a control oriented nominal model; this is typically achieved by an identification step with a low order model structure, control-oriented experimental conditions and/or a control-oriented criterion;
- a "validation" experiment for the construction of a control oriented uncertainty set; this can be achieved by an identification step with a full order model structure, control-oriented experimental conditions and/or a control-oriented criterion.

First let us recall the main motivation for studying this interplay between identification/validation and robust control, as well as the change in strategy that has been made possible by the new insights gained on this interplay. As explained in Chapter **??** the traditional strategy in model-based robust control design was to first identify the best possible model G_{mod} and construct the most reliable uncertainty set $\mathcal{D}$ around G_{mod} on the basis of prior information and available data, and to then design a controller C that achieves closed loop stability and meets the required performance with all models in $\mathcal{D}$, and hence with the unknown true system G_0. For this scenario to be successful, a very accurate (and hence complex) model G_{mod} was typically required. This not only requires an important investment in identification and/or modelling; it also leads to unnecessarily complex controllers.

The main lesson learned from ten years of research on the interplay between identification/validation and robust control is that one can design low order model-based controllers that achieve high performance on the actual system. These low order controllers are based on low order (and hence biased) models whose bias error has been tuned for robust control. They are selected from a class of controllers $\mathcal{C}$ that achieve robust stability and robust performance with all models in an uncertainty set $\mathcal{D}$ whose shape has been tuned for robust control.

As a result, the present scenario for model-based control design resulting from these new insights can be described as follows: *On the basis of the required performance, of any knowledge of the unknown system, and of the performance achieved with the present controller (if any), design a control-oriented identification experiment to compute a low order model* G_{mod} *and an uncertainty set* $\mathcal{D}$*; then design a new controller* C *that achieves closed loop stability and meets the required performance with all models in* $\mathcal{D}$*, and hence with the unknown true system* G_0*.* If necessary, repeat this design procedure, possibly with a more demanding performance criterion. In most versions of this new scenario, one first computes a class of controllers $\mathcal{C}(G_{mod}, \mathcal{D})$ which all achieve the required performance with all models in $\mathcal{D}$; the new controller C is then chosen within this class in such a way as to have some additional nice features (e.g. low complexity).

The goal of the new scenario is to achieve the same or better performance based on models of lower complexity. The class of controllers $\mathcal{C}$ that achieve the required performance is larger because the model uncertainty set $\mathcal{D}$ is tuned towards that aim. All in all, the same or better performance is achieved with a controller that is easier to compute and of lower complexity than is possible with the traditional scenario.

In terms of the global design procedure, the main distinction between the traditional concept of robust control design and the new one is that, in the new scenario, the identification and validation steps have become part of the global control design procedure, whereas in the traditional scenario the control performance specifications played no role in the identification/validation step. In highlighting the distinction between the *identification* part of the design, which determines the nominal model G_{mod}, and the *validation* part of the design, which determines the validated uncertainty set $\mathcal{D}$, we also want to stress that the identification step focuses on the bias error distribution of the nominal model G_{mod}, whereas the validation step focuses on the variance error distribution.

The early work on the interplay between identification and robust control addressed almost exclusively the question of the nominal model. The question was: *how should the bias error of the nominal model be tuned so that the resulting controller based on this nominal model G_{mod} stabilizes the unknown true system and achieves with this system a closed loop performance that is close to the nominal closed loop performance?* In a nutshell, the answer to that question is that the nominal closed loop system (G_{mod}, C) and the actual closed loop system (G_0, C) must be "close", where the closeness is measured in a norm that is determined by the control performance criterion. A key point here is that the controller C is the to-be-designed controller, which itself depends on the model G_{mod}. This is what makes the problem difficult and results in the need for an iterative approach, in which a sequence of models and controllers are computed, achieving higher and higher performance. The difficulty, as already mentioned in Chapter **??**, is that this procedure is not guaranteed to converge: higher performance may be achieved at the cost of lower stability margins, eventually leading to instability. Thus, such schemes must include safety checks to guarantee stability. These safety checks take the form of bounds on a measure of the distance between two successive controllers. They will be dsecribed in Section 4.

Even though a lot of progress has been made all through the nineties on the development of new methods for computing uncertainty bounds on identified models, much of this work was developed independently of the control objective. Thus, the uncertainty bounds developed in this work are not "control-oriented", in that their construction does not take account of the interplay between the validated set of models $\mathcal{D}$ and the set of controllers $\mathcal{C}$ that achieve the required control performance with all models in $\mathcal{D}$.

One reason for the paucity of results on this aspect of the synergistic design problem is the difficulty of the problem. It is already hard to understand the interplay between the true system G_0, the present controller C_{id} (if any), the model G_{mod}, the designed controller C, the nominal performance $\bar{J}^{nom}$ and the achieved performance $\bar{J}^{ach}$, and to derive from this understanding the qualities that the model must possess for the nominal performance of the loop (G_{mod}, C) to be close to the achieved performance of the loop (G_0, C). It is much harder to understand the interplay between the true system G_0, the present controller C_{id} (if any), the set of validated models $\mathcal{D}$, and the set of admissible controllers $\mathcal{C}$ for which the worst case performance with all models in the validated set $\mathcal{D}$ is acceptable.

Another reason is that, even if the interplay between validation and robust control were well understood, there would not be a unanimously accepted way of formulating a "validation for robust control" problem. In this chapter, we will review two reasonable formulations of such problem, but we are fully aware that, as the insight and the technical tools evolve, better formulations will almost certainly eventuate.

The contents of this chapter is as follows. In Section 2 we first present the identification/control setup, and we introduce some basic concepts from robust optimal control. Section 3 will focus on the interplay between true system, nominal model, achieved performance and nominal performanc. This will lead to the formulation of control-oriented identification criteria, which have consequences on the shaping of the bias error distribution. In Section 4, we shall explain the need for caution in iterative model-based robust control design, and we shall show how such cautious steps can be implemented. Section 5 will discuss the connection between the model uncertainty set and the corresponding controller set. Two approaches will be discussed: one belongs to the realm of optimal experiment design with full order models, the other belongs to the realm of robust experiment design. We end up with some conclusions in Section **??**.

2 The identification/control setup and some basic formulae

For simplicity we shall consider in this chapter only single input single output (SISO) linear time-invariant (LTI) systems, and we shall limit our presentation to one-degree-of-freedom controllers. The "true system" is assumed to be represented by

$$\mathcal{S} : y_t = G_0(z)u_t + v_t, \tag{1}$$

where $G_0(z)$ is a linear time-invariant causal operator, y is the measured output, u is the control input, and v is noise, assumed to be quasistationary.

The control law is represented by:

$$u_t = C(z)[r_t - y_t] + d_t, \tag{2}$$

where r_t is the reference excitation and d_t is a possible disturbance acting on the system. The signal d_t can also be seen as an error between the computed control action and the actually applied control action.

Our generic feedback loop can thus be represented as in Figure 1. The equations of this closed loop system can be written as:

$$y_t = \frac{GC}{1+GC}r_t + \frac{G}{1+GC}d_t + \frac{1}{1+GC}v_t, \tag{3}$$

$$u_t = \frac{C}{1+GC}r_t + \frac{1}{1+GC}d_t - \frac{C}{1+GC}v_t, \tag{4}$$

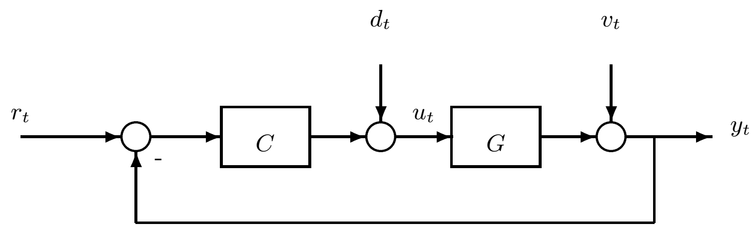

Fig. 1. The generic feedback configuration

In analysing the identification/control interplay, it will be useful to consider - and compare - the following special cases of this feedback configuration.

- Assume that, at the time of designing a new controller C, the true system G_0 is already under feedback control with a controller denoted C_{id}. We then consider the *current feedback system* (G_0, C_{id}), on which data are typically collected in order to identify a new model G_{mod} for the design of a new controller C.
- With $G = G_{init}$, $v_t = \hat{v}_t$ and $C = C_{id}$ we have, correspondingly, the *current design loop* (G_{init}, C_{id}). Here G_{init} is the present model of G_0 if available, while $\hat{v}_t = H_{init}e_t$, with e_t white noise, is the present noise model if available.
- With $G = G_{mod}$, the newly identified model I/O model, and $v_t = \hat{v}_t = H_{mod}e_t$, the newly identified noise model if any, we have the *nominal closed loop system* (G_{mod}, C), where C is the new controller designed from G_{mod}.
- The new controller C, designed from G_{mod} is applied to the true system G_0, thus generating the *achieved closed loop system* (G_0, C). If the design is robust, then the performance of this achieved closed loop system (G_0, C) must be reasonaly close to that of the nominal closed loop system (G_{mod}, C).

For the analysis of both the stability and the performance of this closed loop system $(G\ \ C)$, a key role is played by the closed loop transfer function matrix $T(G, C)$ defined as follows:

$$T(G,C) = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} \frac{GC}{1+GC} & \frac{G}{1+GC} \\ \frac{C}{1+GC} & \frac{1}{1+GC} \end{pmatrix}. \tag{5}$$

It is the transfer function matrix from the external signals r and d to the loop signals y and u.

The closed loop system (G, C) is stable (or "internally stable") if all four transfer functions in $T(G, C)$ are stable. This notion of stability guarantees that there are no unstable pole-zero cancellations in any of the paths from the external signals r, d, v to the internal signals y, u in the feedback system of Figure 1. This can be conveniently expressed in mathematical terms using the *generalized stability margin* $b(G,C)$ introduced by Vinnicombe [12]: see Chapter **??**.

The generalized stability margin of the closed loop system (G,C) is defined as

$$b_{GC} = \begin{cases} \|T(G,C)\|_\infty^{-1} & \text{if } [G\ C] \text{ is stable} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Note that this generalized stability margin takes its values between 0 and 1; the higher this value, the better the stability margin. It is also important to note that, for a given plant G, and whatever the linear controller, the generalized stability margin has a maximum value $b_{opt}(G)$ (see e.g. [16]) given by

$$b_{opt}(G) = \sup_C b(G,C) = \sqrt{1 - \| \, [N \quad M] \, \|_H^2}, \tag{7}$$

where $\| \ A \ \|_H$ is the Hankel norm of the operator A and $\{N, M\}$ is the normalized coprime factorization of G; see e.g. [17] for the definitions of these concepts.

As for the performance of the closed loop system (G,C), most of the commonly used performance criteria are defined from the following general frequency function:

$$J(G,C,W_l,W_r,\omega) = \sigma_1 \left(\overbrace{\begin{pmatrix} W_{l1} & 0 \\ 0 & W_{l2} \end{pmatrix}}^{W_l} T(G(e^{j\omega}), C(e^{j\omega})) \overbrace{\begin{pmatrix} W_{r1} & 0 \\ 0 & W_{r2} \end{pmatrix}}^{W_r} \right) \tag{8}$$

where $W_{l1}(e^{j\omega}), W_{l2}(e^{j\omega})$ and $W_{r1}(e^{j\omega}), W_{r2}(e^{j\omega})$ are frequency weights that allow one to define specific performance levels, and where $\sigma_1(A)$ denotes the largest singular value of A.

The frequency function $J(G,C,W_l,W_r,\omega)$ defines a template. Any function that is derived from J can of course also be handled. Thus, a typical optimal control problem formulation is

$$C^* = \arg\min_C ||J(G,C,W_l,W_r,\omega)||, \tag{9}$$

where $|| \quad ||$ denotes a suitable norm. See, for example, [5] where $\| \ W_l T(G,C) W_r \ \|_\infty$ is used. The corresponding optimal cost will be denoted

$$\bar{J}(G,C^*) = \ \| \ W_l T(G,C^*) W_r \ \| \ . \tag{10}$$

The choice of a diagonal structure for W_l and W_r is no loss of generality, since the four transfer functions in $T(G,C)$ can all be weighted differently. For example, a common choice for the performance measure of a closed loop system is the shape of the modulus of the frequency response of one or several of the four transfer functions defined in (5) (see [14]) or the ∞-norm of a frequency weighted version of one of these four transfer functions.

The optimal control problem formulation defined by (9) produces a controller C^* that is optimal, with respect to the control objective, for a specific system G that is assumed known. The controller C^* that is optimal for G may yield very poor performance with a system G_0 that is very close to G; worse still, it could destabilize G_0. This point was put very strongly in the much studied Schrama example [10].

The whole paradigm of robust optimal control is to formulate control design objectives that deliver a controller which has a guaranteed level of performance with all systems in a given uncertainty set $\mathcal{D}$. For the formulation of such problems, one defines a *worst case performance function.* Thus (8) is replaced by

$$J_{WC}(\mathcal{D}, C, W_l, W_r, \omega) = \tag{11}$$
$$\max_{G(z)\in\mathcal{D}} \sigma_1 \left(\overbrace{\begin{pmatrix} W_{l1} & 0 \\ 0 & W_{l2} \end{pmatrix}}^{W_l} T(G(e^{j\omega}), C(e^{j\omega})) \overbrace{\begin{pmatrix} W_{r1} & 0 \\ 0 & W_{r2} \end{pmatrix}}^{W_r} \right).$$

A robust optimal control problem can then be formulated as

$$C^* = \arg\min_C ||J_{WC}(\mathcal{D}, C, W_l, W_r, \omega)||, \tag{12}$$

where $||\ \ ||$ denotes a suitable norm. The corresponding optimal worst case cost is then denoted by

$$\bar{J}_{WC}(\mathcal{D}, C^*) = ||J_{WC}(\mathcal{D}, C^*, W_l, W_r, \omega)||. \tag{13}$$

We now show that the formulation of optimal control problems in the form of (9), based on a unique model G (rather than a set $\mathcal{D}$), results in optimal controllers that already possess an inherent stability robustness property. This is a feature of a control problem formulation that is based on the whole 2×2 transfer function $T(G, C)$ rather than on one only of its elements. As already stressed in Chapter **??** (Chapter Astrom1), the main difference is that such control problem formulation is based on more than just the loop gain $L = GC$.

Suppose that a model $G_{mod} = \frac{N_{mod}}{D_{mod}}$ of a plant is given, where the stable causal transfer functions N_{mod} and D_{mod} are the normalized coprime factors of G_{mod}: see Chapter **??**. Then, from a robust stability point of view, it makes sense to design a controller C that minimizes $||T(G_{mod}, C)||_\infty$. Indeed, as explained in Chapter **??**, the optimizing controller C^* maximizes the stability robustness to errors in the coprime factors N_{mod} and D_{mod} of the model G_{mod}. More precisely, let

$$C^* = \arg\min ||T(G_{mod}, C)||_\infty \tag{14}$$

and let $\gamma^* = ||T(G_{mod}, C^*)||_\infty$. Then the closed loop systems $(G_{mod} + \Delta G, C^*)$ are stable for all systems $G_{mod} + \Delta G = \frac{N_{mod}+\Delta N}{D_{mod}+\Delta D}$ such that

$$|| \begin{pmatrix} \Delta N \\ \Delta D \end{pmatrix} ||_\infty < \frac{1}{\gamma^*}. \tag{15}$$

Thus we observe that the maximization, with respect to a controller C, of the stability robustness to coprime factor uncertainty on a nominal model $G_{mod} = \frac{N_{mod}}{D_{mod}}$ can be formulated as the maximization of a specific version of the performance criterion defined in (9), with $W_l = W_r = I$.

3 A control-oriented nominal model

In this section we show how the control performance objective leads to the design of a control-oriented identification criterion, whose minimization delivers a control-oriented nominal model. As explained in the introduction of this chapter (see also Chapter **??** Chapter Gevers1), we assume that the model set used for identification is a restricted complexity model set $\{G(z, \theta)\}$ parametrized by some parameter vector θ, i.e. there is no θ_0 such that $G_0(z) = G(z, \theta_0)$. As a result, the model obtained by identification, $G_{mod} \stackrel{\triangle}{=} G(z, \hat{\theta})$, is necessarily biased, and the focal point of the discussion in this section is therefore on the control-oriented shaping of the bias distribution of $G(z, \hat{\theta})$.

The problem can be formulated as follows. Since we shall construct our controller C on the basis of a model G_{mod} that is biased (i.e. necessarily wrong), how should we formulate the criterion in such a way that the frequency distribution of these bias errors has the smallest possible impact on the closed loop *performance degradation*, while at the same time guaranteeing stability of the achieved closed loop system (G_0, C). By performance degradation we mean the difference between the performance of the nominal closed loop system (G_{mod}, C) and the performance of the actual closed loop system (G_0, C).

Recall first that, if we knew the true system G_0, we would compute the optimal controller as the solution of

$$C^* = \arg\min_C \parallel W_l T(G_0, C) W_r \parallel, \tag{16}$$

where $|| \quad ||$ denotes some suitable norm, such as the H_∞ norm or the H_2 norm for example. Now one can write:

$$W_l T(G_0, C) W_r = W_l T(G_{mod}, C) W_r + W_l [T(G_0, C) - T(G_{mod}, C)] W_r \tag{17}$$

By applying the triangle inequality, Schrama showed that one can then squeeze the achieved cost $\parallel W_l T(G_0, C) W_r \parallel$ between the following lower and upper bounds [10]:

$$\begin{aligned}&|\ \| W_l T(G_{mod},C)W_r \| - \| W_l[T(G_0,C) - T(G_{mod},C)]W_r \|\ | \\ &\leq ||W_l T(G_0,C)W_r|| \\ &\leq \| W_l T(G_{mod},C)W_r \| + \| W_l[T(G_0,C) - T(G_{mod},C)]W_r \| . \end{aligned} \tag{18}$$

As stated above, the ideal (but elusive) goal would be to compute the controller that minimizes the control performance objective $\bar{J}^{ach}(G_0, C)$ on the actual system. Since G_0 is unknown, this is replaced by a model-based control design, where one computes the controller C^* that minimizes the nominal performance objective:

$$C^* = \arg\min_C \| W_l T(G_{mod},C)W_r \| . \tag{19}$$

This results in the nominal cost $\bar{J}^{nom}(G_{mod}, C^*)$.

The double triangle inequality (18) shows that the achieved cost $\bar{J}^{ach}(G_0, C^*)$ will be close to the nominal cost $\bar{J}^{nom}(G_{mod}, C^*)$ if the performance degradation term

$$\bar{J}^{pr} = \| W_l[T(G_0,C^*) - T(G_{mod},C^*)]W_r \| \tag{20}$$

is small. This observation is at the basis of all "identification for control" schemes developed in the early part of the nineties. It shows that what matters for a model G_{mod} to be tuned for control design is that the closed loop transfer functions $T(G_0, C^*)$ and $T(G_{mod}, C^*)$ must be close to one another in a norm that is entirely determined by the control performance objective. Indeed, the way in which $T(G_0, C^*)$ and $T(G_{mod}, C^*)$ must be close is exactly determined by the requirement that $\bar{J}^{pr}$ must be small; this term has often been called the *performance robustness* term: see e.g. [6]. If an H_∞ norm is chosen for the control performance objective, then a model G_{mod} that is tuned for control design is one for which $\| W_l[T(G_0,C^*) - T(G_{mod},C^*)]W_r \|_\infty$ is minimized; if an H_2 norm is used in the control objective, then a good model for control should minimize $\| W_l[T(G_0,C^*) - T(G_{mod},C^*)]W_r \|_2$.

Important observations

The statements made above on the basis of the double triangle inequality require that some important observations and cautionary notes be made.

1. We note that the estimated plant model, G_{mod}, and the controller, C^*, both influence the two terms $\bar{J}^{nom}$ and $\bar{J}^{pr}$. Thus, ideally, one should minimize the two terms jointly over the class of admissible plant models and admissible controllers. This is an impossible task in the case of restricted complexity models.[1]

[1] In dual control the achieved criterion is minimized jointly over the parametrized set of plant models and corresponding controllers, but the model set is assumed to contain the true system, and the minimization leads to a tractable solution only for the very simple minimum variance control criterion: see [3].

2. On the other hand, minimizing the nominal criterion $\bar{J}^{nom}(G_{mod}, C)$ with respect to the controller for a given model G_{mod} is a classical control design task, whereas minimizing $\bar{J}^{pr}(G, C)$ with respect to a model G for a given controller C was shown to be achievable by closed loop identification. Therefore, an obvious suboptimal strategy is to make $\bar{J}^{nom}$ small by controller design for a given plant model, and to keep $\bar{J}^{pr}$ small by identification design for a given controller. Since $\bar{J}^{nom}$ depends on the estimated plant model, and $\bar{J}^{pr}$ depends on the designed controller, this strategy can only be applied in an iterative manner, using a succession of *local controller designs* and *local identification designs*:[2]
$$\min_{C} \bar{J}(G^{i}_{mod}, C) \longrightarrow C_{i+1}$$
$$\min_{G(\theta)\in\mathcal{M}} \bar{J}^{pr}(G(\theta), C_i) \longrightarrow G^{i+1}_{mod}. \tag{21}$$
This idea is at the heart of the iterative identification/controller design methods developed in the early and mid-nineties [10,11,8,15].
3. One important technical problem raised by these iterative identification and control schemes is that the reasoning developed on the basis of the double triangle equality assumes that the controller C is identical in all expressions. However, in the iterative schemes, G_{mod} is obtained by minimizing $\| W_l[T(G_0, C_i) - T(G_{mod}, C_i)]W_r \|$ where C_i is the presently operating controller, while the new controller $C_{i+1} = C^*$ is computed from this new model G_{mod}. Hence, the controller C_i that appears in the nominal performance term $\| W_l T(G_{mod}, C_i) W_r \|$ of the triangle inequality (18) is not the same as the controller C_{i+1} that appears in the robust performance term $\| W_l[T(G_0, C_{i+1}) - T(G_{mod}, C_{i+1})]W_r \|$. Thus, for the reasoning to apply, it is required that $\| W_l[T(G_0, C_{i+1}) - T(G_{mod}, C_{i+1})]W_r \|$ is very close to $\| W_l[T(G_0, C_i) - T(G_{mod}, C_i)]W_r \|$. This technical problem has generated a lot of further research, whose main result has been to introduce "caution" in the controller adjustment from C_i to C_{i+1} in such a way as to make these two quantities close to one another. A key technical tool for establishing robust stability and robust performance guarantees using caution has been the ν-gap metric introduced by Vinnicombe [12], already presented in Chapter **??** (Chapter Astrom1), and its related robust stability and robust performance results. The most obvious use of the ν-gap is to measure a distance between two plants, say G_0 and G_{mod}, and to establish stabilization of G_0 by some controller C based on stabilization of G_{mod} by C and a bound on the distance $\delta_\nu(G_0, G_{mod})$ between the two systems. However, by duality, the ν-gap can also be used to measure the distance between two successive controllers, say C_i and C_{i+1}, and to establish stabilization of

[2] The term 'local' refers to the fact that, at each iteration, the controller design (resp. the identification design) is performed on the basis of some present (i.e. local) plant model (resp. presently operating (i.e. local)) controller.

G_0 by C_{i+1} on the basis of stabilization of G_0 by C_i and a bound on the distance $\delta_\nu(C_{i+1}, C_i)$ between the two controllers. In the next section, we shall present some of these technical results and explain how they can be used to obtain prior stability guarantees when moving between successive controllers.

Before we conclude this section, we comment on the identification part of the iterative schemes. These have been formulated above as

$$G_{mod} = \arg \min_{G \in \mathcal{M}} \parallel W_l[T(G_0, C_i) - T(G, C_i)]W_r \parallel, \tag{22}$$

where $\mathcal{M} = \{G(z, \theta),\ \theta \in \mathcal{D}_\theta\}$ is a model set chosen by the user, typically a reduced order model set. Whatever the specific norm that is used, this is not a standard identification criterion. As it turns out, it has been shown that such criterion can be minimized by closed loop identification, with the controller C_i operating on the actual system. Several schemes have been proposed, for different specific choices of norm: see ...

The fact that the identification for control criteria, in iterative identification and control, turned out to be closed loop criteria with the most recent controller in the loop, was of course of major practical significance and explains why these schemes were quickly adopted in industry. For a process control engineer, it is indeed a lot more appealing to collect identification data under normal operating conditions than to have to perform special identification experiments, or worse still open the loop.

4 Caution in iterative design

We first present some results, based on the Vinnicombe ν-gap, that form the basis for many of the stability robustness guarantees that can be achieved in iterative identification and robust control design.

Proposition 1 [13]
Consider a plant G and two controllers C_1 and C_2, with C_1 stabilizing G. Then

(i) (G, C_2) is stable for all controllers C_2 satisfying $\delta_\nu(C_1, C_2) \leq \beta$ if and only if $b(G, C_1) > \beta$.
(ii) If $\delta_\nu(C_2, C_1) < 1$ then

$$\arcsin b(G, C_2) \geq \arcsin b(G, C_1) - \arcsin \delta_\nu(C_1, C_2), \tag{23}$$

and

$$\begin{aligned} \delta_\nu(C_1, C_2) \leq \|T(G, C_1) - T(G, C_2)\|_\infty &\leq \frac{\delta_\nu(C_1, C_2)}{b(G, C_1) b(G, C_2)} \\ &= \frac{\delta_\nu(C_1, C_2)}{b(G, C_1)} \|T(G, C_2)\|_\infty \end{aligned} \tag{24}$$

The importance of Part (i) of the Proposition 1 is that it provides a sufficient condition on a new controller, C_2, that guarantees its stabilization of the plant G:

$$\delta_\nu(C_2, C_1) < b(G, C_1). \tag{25}$$

The norm $\|T(G,C)\|_\infty = b^{-1}(G,C)$ is one measure of the closed-loop performance of the (G,C) loop. One upshot of these results is the intuitive property that a well-behaved controller, as measured by a small $\|T(G,C)\|_\infty$, provides greater scope for variation before striking stability or performance guarantee barriers. The following proposition is the exact dual of Proposition 1.

Proposition 2
Consider the plants G_1 and G_2, and a controller C stabilizing G_1. Then

(i) (G_2, C) is stable for all plants G_2 satisfying $\delta_\nu(G_1, G_2) \leq \beta$ if and only if $b(G_1, C) > \beta$.
(ii) If $\delta_\nu(G_1, G_2) < 1$ then

$$\arcsin b(G_2, C) \geq \arcsin b(G_1, C) - \arcsin \delta_\nu(G_1, G_2), \tag{26}$$

and

$$\delta_\nu(G_1, G_2) \leq \|T(G_1, C) - T(G_2, C)\|_\infty \leq \frac{\delta_\nu(G_1, G_2)}{b(G_1, C) b(G_2, C)} \tag{27}$$

$$= \frac{\delta_\nu(G_1, G_2)}{b(G_1, C)} \|T(G_2, C)\|_\infty$$

We also have the following result, derived in [1].

Proposition 3
Consider two stable closed loop systems (G_1, C) and (G_2, C). Then

$$|b(G_1, C) - b(G_2, C)| \leq \delta_\nu(G_1, G_2). \tag{28}$$

Hence

$$|b(G_1, C) - b(G_2, C)| \leq \delta_\nu(G_1, G_2) \leq \|T(G_1, C) - T(G_2, C)\|_\infty \leq \frac{\delta_\nu(G_1, G_2)}{b(G_1, C) b(G_2, C)}. \tag{29}$$

Expression (29) shows that the distance between the stability measures $b(G_1, C)$ and $b(G_2, C)$ is always smaller than the distance between the closed loop transfer functions $T(G_1, C)$ and $T(G_2, C)$, measured in H_∞ norm. We have shown in Section 3 that, in identification for control, one computes the new model by minimizing some norm of $\| T(G_0, C) - T(G, C) \|$, where C is the presently operating controller. If the ∞ norm is used, this implies that the stability margin of the new nominal closed loop system (G_{mod}, C) will be close to the stability margin of the actual closed loop system (G_0, C). We shall see that this is a key feature that will allow us to obtain prior stability guarantees.

4.1 Cautious controller adjustment

Consider now that, at some stage of an iterative model/controller design, we have arrived at a controller C_i that stabilizes both the true plant G_0 and the present model G_i, and that we have computed a new model G_{i+1} by minimizing some norm of $\| G_0 - G_{i+1} \|$ over a given model set. Assume also that G_{i+1} is stabilized by the present controller C_i; we shall see later how this can be guaranteed. We also consider that our control objective takes the generic form described in Section 2:

$$C^* = \arg\min_C \bar{J}(G_{i+1}, C) \quad \text{where} \tag{30}$$

$$\bar{J}(G_{i+1}, C) = \| W_l T(G_{i+1}, C) W_r \| \ . \tag{31}$$

Assume now that we have an accurate estimate $\hat{b}(G_0, C_i)$ of the stability margin $b(G_0, C_i)$ of the present closed loop system, i.e. such that

$$|\hat{b}(G_0, C_i) - b(G_0, C_i)| \leq \epsilon, \tag{32}$$

$$\Rightarrow b(G_0, C_i) > \hat{b}(G_0, C_i) - \epsilon = k\hat{b}(G_0, C_i) \text{ for some } k \in (0, 1). \tag{33}$$

Then any controller C_{i+1} such that

$$\delta_\nu(C_{i+1}, C_i) < k\hat{b}(G_0, C_i) \tag{34}$$

is guaranteed to stabilize the true plant G_0, since C_i stabilizes G_0. Thus, the new controller C_{i+1} stabilizes the true plant if the adjustment from C_i to C_{i+1} is small enough, as expressed by the constraint (34).

4.2 Estimation of a bound on $b(G_0, C_i)$

There are several ways of estimating the stability margin $b(G_0, C_i)$ together with a bound on its estimation error. One way is to compute a direct estimate of $\| T(G_0, C_i) \|_\infty = b^{-1}(G_0, C_i)$ from measurements on the actual closed loop system (G_0, C_i): see [4], where the selection of appropriate external excitation signals for this task is also discussed. The inverse of the estimate of $\| T(G_0, C_i) \|_\infty$ is then used as the estimate $\hat{b}(G_0, C_i)$ in (33), and a safety factor $k \in (0, 1)$ is introduced to account for the estimation error on $\hat{b}(G_0, C_i)$.

An alternative way is to use an H_∞ identification method for the estimation of G_{i+1} from measurements obtained on the actual closed loop system (G_0, C_i): see e.g. [7,9]. These methods also deliver a bound on the ∞-norm error, i.e. they deliver a number ϵ such that

$$\| T(G_0, C_i) - T(G_{i+1}, C_i) \|_\infty \leq \epsilon. \tag{35}$$

By Proposition 3 it then follows that

$$|b(G_0, C_i) - b(G_{i+1}, C_i)| \leq \epsilon, \tag{36}$$

$$\Rightarrow b(G_0, C_i) > b(G_{i+1}, C_i) - \epsilon = kb(G_{i+1}, C_i) \text{ for some } k \in (0, 1). \tag{37}$$

Observe that $b(G_{i+1}, C_i)$ is known, since G_{i+1} and C_i are known. One can thus use $\hat{b}(G_0, C_i) = b(G_{i+1}, C_i)$ as an estimate of $b(G_0, C_i)$ in (33), with an error bounded by ϵ, which can again be accounted for by a safety factor $k \in (0, 1)$.

To summarize, if G_{i+1} is a new model obtained from closed loop data on the present (G_0, C_i) loop such that G_{i+1} is stabilized by the present C_i and such that (35) holds, then any controller C_{i+1} such that

$$\delta_\nu(C_{i+1}, C_i) < kb(G_{i+1}, C_i), \tag{38}$$

with k defined as in (37), is guaranteed to stabilize the true plant G_0.

4.3 Construction of a stabilizing controller

The condition (34) defines a set of controllers $\{C_{i+1}\}$ that are guaranteed to stabilize G_0, but it says nothing about how to construct such controllers. In addition, most of the controllers in that set may not achieve a good performance with G_0. We now present a procedure for the construction of controllers that satisfy the stability condition (34) and at the same time achieve better performance than was achieved with the previous controller C_i.

Consider the presently operating controller C_i which stabilizes both the unknown plant G_0 and the new model G_{i+1}. Suppose that we are given a known bound on the differences between the closed loop transfer matrices

$$\|T(G_0, C_i) - T(G_{i+1}, C_i)\|_\infty \leq \epsilon. \tag{39}$$

Let $C_i = U_i V_i^{-1}$ be a right coprime factorization of C_i, with U_i and V_i stable proper transfer functions and, similarly, let $G_{i+1} = N_{i+1} D_{i+1}^{-1}$ be a right coprime factorization of the new model G_{i+1}. Then the set of all controllers C_{i+1} that stabilize the model G_{i+1} and which also stabilize all G_0 satisfying the condition (39) is given by

$$\mathcal{C} = \tag{40}$$
$$\{C(Q) \ : \ C(Q) = (U_i - D_{i+1}Q)(V_i + N_{i+1}Q)^{-1}, Q \in \mathcal{RH}_\infty, \| Q \| \leq \epsilon^{-1}\}.$$

Here Q also belongs to the set of stable proper transfer functions. This result was established in [4]. Observe that, without the constraint on the norm of Q (i.e. $\| Q \| \leq \epsilon^{-1}$), the set $\mathcal{C}$ is the set of all controllers stabilizing the new model G_{i+1}. By imposing the additional constraint on $\| Q \|$, we are limiting the distance between the old controller C_i and the new controller C_{i+1}, which is what guarantees the stabilization of the true plant G_0.

Now consider that the control objective $\bar{J}(G_{i+1}, C)$ of (30) depends in a convex manner on Q, as is the case in H_2 and H_∞ optimal control problems, and let $C^*_{i+1} = C(Q^*)$ be the solution of the unconstrained optimization problem:

$$C^*_{i+1} = C(Q^*) = \arg\min_Q \bar{J}(G_{i+1}, C(Q)). \tag{41}$$

If $\| Q^* \| \leq \epsilon^{-1}$, we can take $C_{i+1} = C^*_{i+1} = C(Q^*)$. Otherwise, consider the set:

$$C(\alpha Q^*) = (U_i - \alpha D Q^*)(V_i + \alpha N Q^*)^{-1}\}, \text{ for } \alpha \in [0,\ 1]. \tag{42}$$

All controllers in that set stabilize G_{i+1}. The value $\alpha = 0$ corresponds to the present controller C_i, while $\alpha = 1$ corresponds to $C^*_{i+1} = C(Q^*)$ which is not guaranteed to stabilize G_0. Let α^* be the largest value for which $\| \alpha Q^* \| < \epsilon^{-1}$. Then the controller $C_{i+1} = C(\alpha^* Q^*)$ is guaranteed to stabilize G_0 and, in addition, we have [2]:

$$\bar{J}(G_{i+1}, C(Q^*)) \leq \bar{J}(G_{i+1}, C(\alpha^* Q^*)) < \bar{J}(G_{i+1}, C_i) \tag{43}$$

4.4 Cautious model adjustment

We have seen above that one of the ways of imposing a cautious controller adjustment, in order to guarantee closed loop stability of the loop (G_0, C_{i+1}), is to limit the controller movement with respect to the nominal stability margin $b(G_{i+1}, C_i)$: see (38). This of course requires that the nominal loop formed of the present controller and the new model is stable. A convenient way to ensure that the identified model G_{i+1} is stabilized by the present controller is to parametrize the model set in the dual Youla parametrization as follows.

Proposition 4
Consider the present controller C_i that stabilizes both the true system G_0 and the present model G_i.

Let $C_i = U_i V_i^{-1}$ be a right coprime factorization of C_i, with U_i and V_i stable proper transfer functions and, similarly, let $G_i = N_i D_i^{-1}$ be a right coprime factorization of the present model G_i. Then the set of all models G_{i+1} that are stabilized by the controller C_i is given by

$$\mathcal{M} = \{G(R) : G(R) = (N_i - V_i R)(D_i + U_i R)^{-1}, R \in \mathcal{RH}_\infty. \tag{44}$$

The subset of such models that also satisfies the condition

$$\|T(G_i, C_i) - T(G_{i+1}, C_i)\|_\infty \leq \epsilon \tag{45}$$

is given by

$$\mathcal{M} = \tag{46}$$
$$\{G(R)\ :\ G(R) = (N_i - V_i R)(D_i + U_i R)^{-1}, R \in \mathcal{RH}_\infty, \| R \| \leq \epsilon^{-1}\}.$$

Thus, not only can one perform the identification in such a way that the new model is stabilized by the present controller C_i, but one can also impose that the new nominal closed loop system (G_{i+1}, C_i) is close to the old one, (G_i, C_i), in the sense of the condition (45).

5 Model validation for robust control design

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