Towards a joint design of identification and control ?*

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Abstract

This paper aims at introducing the reader to the various issues that arise in the development of a coherent methodology for the development of robust control design on the basis of models identified from data. When a reduced complexity model is identified with the purpose of designing a robust controller, the model is just a vehicle for the computation of a controller. The design of the identification and of the controller must be seen as two parts of a joint design problem. The central message of this paper is to show that the global control performance criterion must determine the identification criterion. This leads to non standard identification criteria, which can be minimized by appropriate experimental set-ups.

1 Introduction

The intensive work that is presently going on in the general area of identification in connection with robust control design finds its origin in the awareness, among people from both the identification and the robust control community, that a wide gap exists between the premises on which robust control design is built and the tools and results that 'classical' identification theory is able to deliver. (By 'classical' I mean the theory as it existed at the end of the nineteen eighties, and which can be found in classical textbooks such as [Lju87] or [SS89].) To understand how such gap has materialized, a historical retrospect is perhaps appropriate.

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A historical perspective

Until 1960 and the publication of Kalman's celebrated work on state space models [Kal60], control design was for the most part based on graphical techniques using Bode plots and Ziegler-Nichols charts. The controllers that were built were PI and PID controllers; they were not model based. Robustness concepts were incorporated in the design techniques in the form of gain and phase margins.

The major impact of Kalman's work was perhaps not so much the introduction of state space models, but the replacement of graphical design techniques by *model based certainty equivalence* control design. Linear Quadratic Gaussian (LQG) control and model reference control became major new design techniques. Parametric models became the central focus of attention, and it is therefore natural that the development of parametric identification techniques followed that of model based control design on its heels. One of the important early papers on parametric identification is that of Åström and Bohlin [ÅB65] which introduced many of the formal concepts that dominated identification theory for about 20 years.

The Achilles' heel of the model based control era of the sixties and seventies was the certainty equivalence principle. Except for the gain and phase margins naturally inherited by LQG controllers, the model based control design methods did not lend themselves easily to tractable design methods that would incorporate plant model uncertainty descriptions. Perhaps for that reason (or would it be sheer laziness?), for a long time identification theorists focused on questions of convergence and efficiency of parameter and transfer function estimates in the case when the true system was contained in the model set, rather than on the effect on controller performance of plant/model errors due to undermodeling.

It is interesting to observe that the first analyses of the *interplay between identification and control design* were produced during this certainty equivalence era. Motivated by problems in communications, Fel'dbaum introduced the concept of *dual control*, in which he showed that, when controlling a system whose parameters are unknown, the control effort must pursue the dual goal of "investigating" and "directing" [Fel60]. Fel'dbaum then called "investigation risk" the loss in the achieved performance due to the fact that the control is not optimal in view of obtaining information about the system (resulting in a subsequent suboptimal control action), and he called "action risk" the loss in the achieved performance due to the fact that the control causes a deviation from the best achievable state.

The ideas of Fel'dbaum inspired Åström and Wittenmark [ÅW71], who addressed the problem of *combined identification and control* in the context of exact modeling (i.e. the true system is in the model set) using a linear regression (ARX) model structure. They considered the direct minimization of both a one-step and a multi-step minimum variance regulation criterion. In the case of an exact model structure, the unknown parameters and their

covariance matrix can be added to the state, the problem can be re-framed as an optimal control problem for an augmented system, and solved (at least in principle) using dynamic programming. Even though the model errors in [ÅW71] are only noise-induced errors - and not bias errors due to unmodeled dynamics - the contribution of Åström and Wittenmark is significant for the following reasons:

- it showed, probably for the first time and albeit in the context of exact model structure, that the combined identification and control problem can be formulated as the minimization of a global control performance criterion, leading in simple cases to a simple and computable solution;
- it produced an optimal solution to the minimum variance regulator problem with unknown parameters in the form of a *cautious controller* with reduced gains;

The publication of Zames' paper [Zam81] marked the start of the model-based H_∞ robust control design period that dominated most of the eighties. The key technical result that made it all possible was the availability of the Youla-Kucera parametrization of all stabilizing controllers for a given plant model. The structured singular value (or μ-synthesis) approach later extended worst case control design from the realm of unstructured uncertainties (i.e. error bounds on the transfer function model) to that of structured uncertainties (i.e. error bounds on parameters). It is interesting to observe that these robust control design methods have been developed entirely in the context of models that are not data-based.[1] As the robust control design methods invaded the literature (if not the world of process control applications), the pressure grew to apply these methods on models identified from real data. This required the development of identification techniques able to deliver error bounds in the case of undermodeling and, more importantly, it required an understanding of the interactions between control design and identification design in this restricted complexity modeling situation.

In the identification community - and for the reasons explained above - very few attempts had been made to quantify errors on estimated transfer functions due to the use of restricted complexity model sets. In fact, at the end of the 'classical era' of the development of identification theory (say, 1987), the only useful result available on the error due to undermodeling (also called the 'bias error') was an implicit characterization of the convergence point of the parameter estimation algorithm [WL86]. While very useful for design, this integral formula did not give a clue as to a bound on the error between true and estimated transfer functions.

As for the interaction between identification and control in the case of restricted complexity models, a small step had been made in [GL86], where,

[1] Having just presented a sketch of the robust control design steps, John Doyle, one of the key contributors to robust control theory, recently asked a bunch of identifiers: "Can you guys tell me how data can improve our designs in any way?" [Doy92]

for certain control performance criteria, it was shown that the performance degradation due to errors in the identified model can be minimized by performing the identification in closed loop and with an appropriate data filter. Since the 'appropriate data filter' is itself a function of the optimal controller, and hence of the unknown model, the result can only be applied in practice by replacing the optimal filter by an approximation, thus leading to the presently popular iterative control and identification design methods. The result of [GL86] was the first instance in which closed loop identification was shown to be helpful, rather than something to be avoided at all cost. The necessity of performing closed loop identification when the model is to be used for control design has since been recognized as a key element in the successful application of identification for control, as we shall see in Section 5.

State of the art at the end of the eighties

At the end of the eighties, the state of affairs concerning the connection between identification and control can thus be characterized as follows:

- robust control control design tools were being developed at a rapid pace, under the assumption that prior hard bounds were available on transfer function errors or parameter errors, or a combination of these;
- with the exception of a small school of thought that had developed "bounded error identification methods", mainstream identification theory had almost come to a halt. It was able to deliver sophisticated models and techniques, but was unable to quantify the errors on identified transfer function models in the frequency domain, as required by the H_∞ robust control design theory;
- some preliminary but scant results were available on optimal design of the identification when the model is of restricted complexity and is to be used for control design [GL86]; these early results were based on H_2 performance measures, and made no connection yet with the new theories of H_∞ robust control;
- except for the case of exact model structure (see [ÅW71]), no results were available on the interplay between identification and control, or a fortiori on their combined design.

It is thus clear that a rather huge gap had developed at the end of the eighties between the tools and assumptions of robust control design and the techniques that identification theory had produced. The most obvious manifestation of this gap, and the one that has triggered most of the present research activity, was the realization that robust control theory requires a priori *hard bounds* on the model error, whereas classical identification theory

delivers at best *soft bounds*[2] in the case where the system is in the model set and no bounds at all in the case of undermodeling.

Nowadays, one has come to realize that the great 'hard-versus-soft' bound debate is not the real issue. An identification and control design method that leads to a closed loop system that is stable with probability .99 is of course just as acceptable as an H_∞-based design that leads to a 'guaranteed stable' closed loop, but that is based on prior error bounds that cannot always be verified. However, the main focus of research - and by far - is still on trying to produce identification methods that allow for the computation of uncertainty bounds, whether hard or soft. This is of course a most pertinent scientific pursuit: whatever the eventual objective, it is unreasonable to deliver a model to a user without a statement about its quality. However, if the objective of the identification is to design a robust controller, then the most important issue is probably not the estimation of uncertainty bounds on the identified model, but the design of a control-oriented identification or, even better, the synergistic design of identification and control.

This leads me to suggest that the new and fashionable research area that deals with the interconnection of identification and control can be subdivided into three areas, that correspond to three aspects of the identification and robust control design problem:

1. Estimation of uncertainty bounds on identified models;
2. Identification *for* robust control design;
3. The combined (synergistic) design of the identification and control.

The point of this classification is to stress that the goals pursued in these three areas of research are quite different.

A brief review of the present research efforts.

For the moment, the mainstream approach seems to be 'Perform the identification with a method that allows the computation of error bounds on the estimated model, then design a robust controller using that model and its bounds'. The main focus of research is therefore on the estimation of error bounds, whether hard or soft, and novel identification techniques are being produced for the sake of delivering such bounds. The problem is that identification methods whose sole merit is to deliver accurate error bounds on restricted complexity models may well produce nominal models that are ill-suited for robust control design: that is, the frequency distribution of the model error may be such that they lead to poor closed loop performance. Examples have been given in [Sch92a,b].

[2]By soft bounds we mean confidence intervals or ellipsoids in the probabilistic sense.

The idea of the second line of research above is to develop identification methods that will produce models whose uncertainty distribution, over the frequency range, allows for high performance robust control design. Thus these models should have low uncertainty where closed loop control specifications require this, but they may have large uncertainty in frequency bands where this does not imperil closed loop stability or penalize closed loop control performance. Results are now available for the tuning of the identification method towards such objectives: the identification must be performed in closed loop with an appropriate data filter. These results rely on an understanding of the interactions between identification, robust stability and robust performance. As already mentioned, an early result leading to the idea of closed loop identification was [GL86]; however, that result was based on performance degradation ideas and the authors failed to consider the connections with the newly emerging robust control theory. It is probably fair to say that the book by Bitmead et al [BGW90] gave a major impetus to this second line of research (as well as the third one). There, the robust stability and robust performance criteria of H_∞ control design were used as the key ingredient for an understanding of the identification/control interactions, in an adaptive control framework. Using H_∞-based prescriptions as a guide (or an excuse), a design was proposed using a combination of Least Squares identification and LQG/LTR control design. In the scheme of [BGW90], the identification design takes account of the robust control requirements through the data filters and the prescription of closed loop identification. Thus, this scheme fits in the framework of identification *for* control. It was followed by other results, using different identification techniques and/or different control design schemes, which all came to similar conclusions. The Delft school played and continues to play a major role in the progress of control-oriented identification and approximation: [Hak90], [HSV92], [Sch91], [Sch92a], [Sch92b].

The third line of research is to combine the identification and the control design in a mutually supportive way, from the point of view of robust stability and/or robust performance. Even though the objective might seem overly ambitious and elusive, some results are now emerging. They all take the form of iterative schemes in which a succession of identification and control design steps are performed, leading to more and more performant control systems. The identification steps are performed in closed loop using data obtained with the last controller operating on the actual plant. The control design steps use the most recently identified plant model. The different schemes vary in the identification criteria and techniques, the model structures that are used, the control design criteria, the way in which the model uncertainty is or is not used in the control design step, the way in which the performance requirements are increased or not between successive design steps: see [BYM92], [LAKM92], [LS90], [PB93], [Sch92a], [Sch92b], [ZBG91], [ZBG92]. Even though the specific identification and control design techniques vary between these schemes, they all have in common a

succession of performance enhancement designs. The idea of redesigning controllers using closed loop data collected on the plant in order to improve performance is what process control engineers naturally tend to do. The merit of the recent research is to develop systematic and theoretically justified procedures to achieve this performance enhancement.

Conclusion for an introduction

To summarize, most of the present focus of research is on the estimation of error bounds. This problem is not only of independent interest, but it is also an important step towards the design of robust controllers based on identified models. However, the key ingredient for the successful application of robust control design methods to identified models is not so much the computation of error bounds, but it is to *let the global control performance criterion dictate what the identification criterion should be, and to design the controller in a way that takes account of data-based information about the plant/model mismatch.* This idea will be the focal point of this paper.

To achieve this objective requires a better understanding of the interconnections between closed loop identification and control design. Thus, this paper will focus on these interconnections. In Section 2, we introduce the concepts of *optimal loop*, *design loop* and *actual loop*, and we present the robust stability and robust performance constraints that are at the heart of the identification and control interplay. This interplay leads to three questions that correspond to the three research areas delineated above. These are briefly discussed in Section 3. In Section 4, we give a brief review of prediction error identification theory in open and closed loop, with the aim of displaying the role of the experimental set-up and the design variables in the properties of the identified model. These results are used in Section 5 to show how the identification criterion can be shaped to become a performance robustness criterion, which is itself tuned by the global control design criterion. This result serves as an inspirational source for the more ad-hoc iterative design schemes that are described in Section 6. These iterative schemes are the presently available alternatives to the combined design of identifier and controller in the form of a global - but for the moment elusive - optimization problem. The formulation of this combined problem raises many deep and challenging open questions that will undoubtedly occupy numerous researchers in the years to come.

2 The identification/control interplay

Our first task is to demonstrate how identification and robust control interact in a model based control design procedure in which the model is constructed from data collected on the process.

To make things perfectly clear[3] about the set-up to which our ensuing developments apply, we shall make the following assumptions.

Assumptions

1. The true plant will be assumed to be representable as follows,

$$y_t = P(z)u_t + v_t, \tag{1}$$

   where $P(z)$ is a scalar strictly proper rational transfer function, u_t is the input, v_t is an unmeasurable disturbance acting on the output y_t.

2. Prior knowledge about the system may have helped the designer to select a parametric model structure or may have given him insight about the achievable bandwidth, but the information about the dynamics of the process is assumed to be derived from data collected on the process. Hence no prior model or approximation of $P(z)$ is available.

3. The exact model structure is assumed to be unknown, but we shall consider that - possibly after some initial analyses including plant data information - the designer has set his or her eyes on a certain parametrized model set,

$$\mathcal{M} \stackrel{\triangle}{=} \{\widehat{P}(z,\theta), \qquad \theta \in D_\theta \subset R^d\}, \tag{2}$$

   together (possibly) with a noise model v_t', where $\widehat{P}$ is a strictly proper transfer function. Thus, the model structure estimation will not be part of our discussion.

4. The true plant is not contained in the model structure, i.e. there exists no value of θ for which $\widehat{P}(z,\theta) = P(z)$ for almost all z.

There are, of course, control design procedures that are based not on data, but on a prior model of the system, perhaps with some knowledge or estimate about its quality. However, the situation described by our assumptions is typical of process control applications, in which such prior knowledge is usually not available and in which the model is necessarily data-based.

[3] As Richard Nixon used to say.

Thus, the task that we consider here is to design a feedback controller on the basis of a model that is to be identified from data taken on the process, using the available techniques of robust control design, given that whatever the identification exploits that are accomplished by the designer, the model will not be able to represent the true system accurately.

A typical control design type situation is that the designer has a global control performance criterion, $J_{global} \stackrel{\triangle}{=} J(P,C)$, in mind. For example, in an H_∞ design, one might like to minimize the following control performance criterion,

$$J_{H_\infty} \stackrel{\triangle}{=} J(P,C) \stackrel{\triangle}{=} ||W(z)\frac{1}{1+P(z)C(z)}||_\infty, \tag{3}$$

over a class of admissible controllers, where $W(z)$ is a weighting that reflects performance specifications and $\frac{1}{1+P(z)C(z)}$ is the sensitivity of the actual feedback system. In an LQG framework, J_{global} could take the form

$$J_{LQG} \stackrel{\triangle}{=} J(P,C) \stackrel{\triangle}{=} \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N}[(y_t - r_t)^2 + \lambda u_t^2], \tag{4}$$

where the signals y_t, r_t and u_t are, respectively, the output signal, the tracking signal, and the to be designed control signal, and where λ is a positive weighting factor that reflects the respective importance given to the tracking error and the control effort.

These criteria cannot be minimized because the first one depends explicitly on the unknown $P(z)$, while the second depends on $P(z)$ through the dynamic relationship that links r_t, u_t and y_t. Instead, one designs a controller on the basis of an estimate $\widehat{P}(z,\hat{\theta})$ of $P(z)$, which we shall in this paper consider to have been obtained from plant data by identification. Thus, one has to design both an identification method (taking into account that the model set is of restricted complexity) and a model-based control design procedure, possibly taking account of plant/model error information. In analysing the interplay between the identification and the control parts of the design, it will prove useful to consider the three feedback loops represented in Figures 1, 2 and 3.

The optimal loop of Figure 1 contains the true system in feedback with the optimal controller $C^{opt}(z)$. This optimal controller depends on the unknown true system $P(z)$, and can therefore not be computed. The design of the controller $C(z)$ is conceptually performed on the basis of the nominal (or design) loop of Figure 2, in which the true plant is replaced by an identified model $\widehat{P}(z,\hat{\theta})$. The actual feedback loop of Figure 3 contains the true system $P(z)$ and the designed controller $C(z)$.

The reasons for drawing attention to these three figures is that much of the discussion about the interplay between identification and robust control is based on a comparison between these loops.

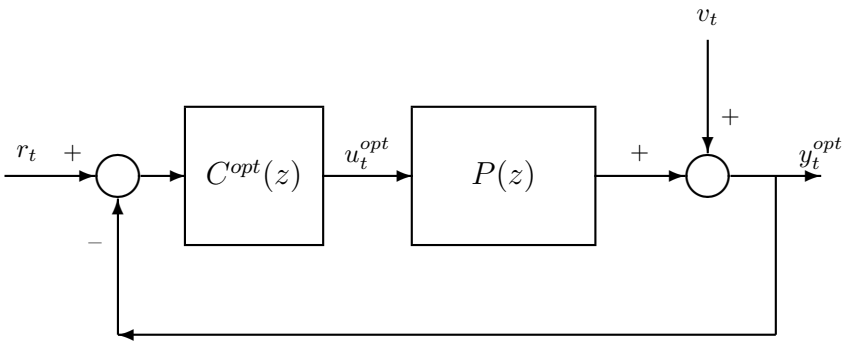

Figure 1: Optimal feedback loop

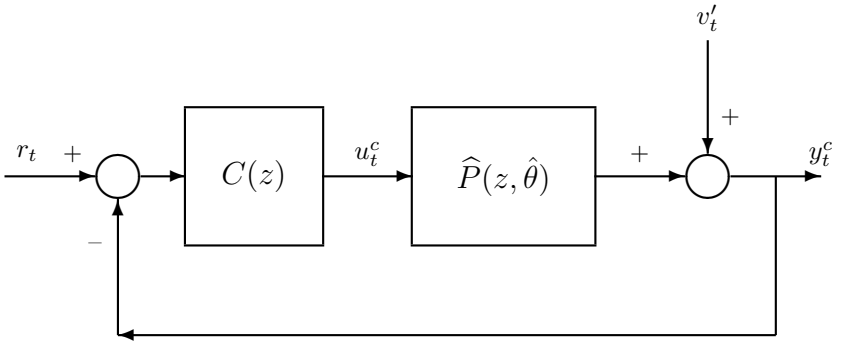

Figure 2: Nominal (or design) feedback loop

- The three loops are all driven by the same reference signal, r_t, while the noise signal, v_t', in Figure 2 is an estimate of the actual noise source v_t.
- Ideally, one would like the identification and control design to be such that the performance achieved by the designed controller on the actual system is as close as possible to that achieved by the optimal controller. That is, one would like the loops of Figures 1 and 3 to be 'close to one another' in some sense. Since C^{opt} is unknown, it is usually impossible to use the closeness of these two loops as a design criterion. Instead, one compares the loops of Figures 2 and 3.
- One has to make sure that the controller $C(z)$ designed for the loop of Figure 2 stabilizes the actual loop of Figure 3: this is the concept of *robust stability.*
- The controller designed for the design loop must also produce on the actual loop an achieved performance that is not too different from the designed performance: this is the concept of *robust performance.*

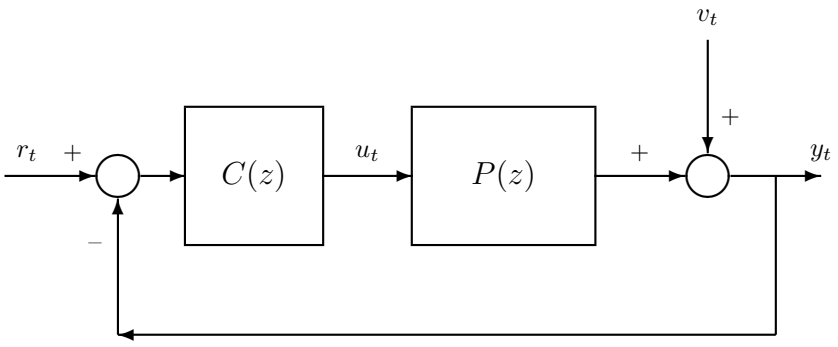

Figure 3: Actual (or achieved) feedback loop

- Thus, in robust analysis one wants the loops of Figures 2 and 3 to be 'close to one another' in some sense.
- Note that a comparison between the signals u_t^{opt}, u_t^c and u_t, as well as between y_t^{opt}, y_t^c and y_t, gives information about the mismatch between the corresponding closed loop systems. This is exploited in the scheme of Zang et al. [ZBG91] that we describe in Section 6.

In the classical robust stability and robust performance analyses, it is assumed that the 'nominal model' $\widehat{P}$ is obtained from prior information and/or modeling techniques, and that the plant/model error is either experimentally based or god-given. The key point is that the choice of $\widehat{P}$ is typically not a part of the control design procedure. Here, with a model $\widehat{P}(z,\hat{\theta})$ that is estimated from data within a set of $\mathcal{M} = \{\widehat{P}(z,\theta)\}$ of candidate models, the robust stability and robust performance requirements hinge both on the identification design and on the control design. *The model is only used as a vehicle to compute a high performance controller, and therefore it need not necessarily be a good open loop model of the plant.*

Without rederiving any of the theory, we now briefly summarize some fundamental formulae about robust stability and robust performance insofar as they clearly exhibit the identification/control interplay.

Robust stability of a unity feedback loop

We first introduce some notations. With $P(z)$ the true plant and $\widehat{P}(z,\hat{\theta})$ an estimated model, we define the additive plant/model error as

$$L(z,\hat{\theta}) = P(z) - \widehat{P}(z,\hat{\theta}). \tag{5}$$

We denote the designed mixed sensitivity function as,

$$\widehat{M} \stackrel{\triangle}{=} C(z)[1 + C(z)\widehat{P}(z,\hat{\theta})]^{-1}, \tag{6}$$

the designed sensitivity function as,

$$\widehat{S}(z) \triangleq [1 + C(z)\widehat{P}(z,\hat{\theta})]^{-1}, \tag{7}$$

and the achieved sensitivity function as,

$$S(z) \triangleq [1 + C(z)P(z)]^{-1}. \tag{8}$$

We now consider the two feedback loops of Figures 2 and 3, with some fixed estimated model $\widehat{P}(z,\hat{\theta})$ in the nominal loop. One simple version of a robust stability result for such unity feedback loops is as follows.

Assume that $P(z)$ and $\widehat{P}(z,\hat{\theta})$ have the same number of unstable poles, and that the designed loop of Figure 2 is internally stable. Then the controller $C(z)$ will stabilize all plants $P(z)$ for which the following inequality holds:

$$||L(z)\widehat{M}(z)||_\infty < 1. \tag{9}$$

This inequality can be more explicitly restated as follows.

$$\left| [P(e^{j\omega}) - \widehat{P}(e^{j\omega},\hat{\theta})] \times \frac{C(e^{j\omega})}{1 + \widehat{P}(e^{j\omega},\hat{\theta})C(e^{j\omega})} \right| < 1 \quad \forall\omega. \tag{10}$$

We make the following observations concerning this inequality.

- The term $P - \widehat{P}$ represents the plant/model error and is essentially determined by the identification part of the design. Notice that the distribution of the model error in an identified model is strongly influenced, among other things, by the input spectrum.[4] Therefore, in closed loop identification (and, in particular, in adaptive control) the controller also influences the frequency distribution of this plant/model error.

- The right hand fraction in (10) is the mixed sensitivity function of the nominal loop. Thus, for an estimated plant model and with a designed controller, this frequency dependent quantity is entirely known. We note that this quantity is therefore influenced by both the identification and the control design.

This inequality exhibits the interplay between identification and control design as far as robust stability is concerned. In classical robust control design, it is interpreted as a constraint on the controller to be designed for a given plant/model error bound: the controller must provide for a small value of the mixed sensitivity function where the plant/model error is large. In our joint identification and control design, it can alternatively

[4]For those readers not too familiar with identification theory, this point will be made amazingly obvious in Section 4.

be interpreted as putting constraints on the identification: the plant/model error must be small in frequency bands where the mixed sensitivity of the designed closed loop system is large.

Robust performance of a unity feedback loop

In terms of performance, there are various ways of defining the robustness of a controller design. Let us assume first that the control design is based on the minimization of some global performance criterion, $J(P, C)$, as illustrated above by two examples. In order to discuss the performance achieved by a controller designed on the basis of a reduced complexity model, it is useful to introduce the following concepts.

If the plant were known exactly, then the minimization of $J(P, C)$ over the class of admissible full order controllers would result in an optimal controller, C^{opt}, to which there corresponds an *optimal cost*, denoted $J^{opt} \triangleq J(P, C^{opt})$. This is the cost obtained for the loop of Figure 1.

The controller is effectively designed on the basis of a nominal model, $\widehat{P}(z, \hat{\theta})$, which we shall denote $\widehat{P}$ for short, and possibly on the basis of information about a bound on the model error $L(e^{j\omega}) = P(e^{j\omega}) - \widehat{P}(e^{j\omega})$. We denote the corresponding controller $\widehat{C}(z)$. The *designed cost* is then defined as $J^{des} \triangleq J(\widehat{P}, \widehat{C})$. It is the cost obtained on the design loop of Figure 2.

The quantity that really matters is not the optimal cost nor the designed cost, but the *achieved cost*, i.e. the cost achieved by the designed controller on the actual plant: $J^{ach} \triangleq J(P, \widehat{C})$. Thus, one measure of the performance robustness of an identification/control design is the comparison between J^{des} and J^{ach}. This comparison expresses how 'close' the loops of Figures 2 and 3 are, *as measured in terms of the global control performance criterion.* Notice that J^{ach} can be either larger or smaller than J^{des}.

To be more specific, we consider the two performance criteria suggested above. Consider first the H_∞ design criterion (3), and let $\widehat{C}$ be a controller designed on the basis of $\widehat{P}$ and (possibly) some known or assumed bound on the error, $L(e^{j\omega})$. We can then write:

$$W\frac{1}{1+P\widehat{C}} = W\frac{1}{1+\widehat{P}\widehat{C}} + [W(\frac{1}{1+P\widehat{C}} - \frac{1}{1+\widehat{P}\widehat{C}})].$$

By expressing successively each one of the three terms above as the sum (or difference) of the other two, and by applying the triangle inequality to each of these three expressions, Schrama showed that one can squeeze $J^{ach} \triangleq ||W\frac{1}{1+P\widehat{C}}||_\infty$ between the following lower and upper bounds [Sch92a]:

$$\left|\; ||W\frac{1}{1+\widehat{P}\widehat{C}}|| - ||W(\frac{1}{1+P\widehat{C}} - \frac{1}{1+\widehat{P}\widehat{C}})||\;\right|$$

$$\leq ||W\frac{1}{1+P\widehat{C}}||$$
$$\leq ||W\frac{1}{1+\widehat{P}\widehat{C}}|| + ||W(\frac{1}{1+P\widehat{C}} - \frac{1}{1+\widehat{P}\widehat{C}})||. \quad (11)$$

Thus, the achieved cost is bounded above by the sum of the designed cost, $J^{des} = ||W\frac{1}{1+\widehat{P}\widehat{C}}||_\infty$, and the H_∞ norm of the weighted difference between the sensitivity of the actual loop (Figure 3) and that of the design loop (Figure 2). We call this second term $J^{pr}_{H_\infty}$, because it is a performance robustness measure:

$$J^{pr}_{H_\infty} \stackrel{\triangle}{=} ||W(\frac{1}{1+PC} - \frac{1}{1+\widehat{P}C})||_\infty. \quad (12)$$

$J^{pr}_{H_\infty}$ expresses the performance error that results from applying the controller $\widehat{C}$, designed for $\widehat{P}$, to the true plant P. With these notations, we can rewrite the inequalities (11) in a more suggestive way:

$$|J^{des} - J^{pr}| \leq J^{ach} \leq J^{des} + J^{pr}. \quad (13)$$

The inequalities show that, if $J^{pr}_{H_\infty}$ is very small, the controller designed for $\widehat{P}$ achieves almost the same performance on the true plant.

Consider now the LQG criterion (4), and let $\widehat{C}$ again denote a controller computed on the basis of some nominal model together (possibly) with plant/model error information. Using the triangle inequality again, we show that the square root of J^{ach} can be squeezed between a lower and an upper bound. To do this, we first rewrite the LQG criterion as the square of a vector norm. We denote

$$J^N_{LQG} = J^N(P,C) = \frac{1}{N}\sum_{t=1}^{N}[(y_t - r_t)^2 + \lambda u_t^2]. \quad (14)$$

We now introduce the following vector 2-norm for a vector process $\begin{pmatrix} x_t \\ y_t \end{pmatrix}$:

$$\left\| \begin{array}{c} x_t \\ y_t \end{array} \right\|_2 \stackrel{\triangle}{=} (\sum_{t=1}^{N}[(\frac{1}{\sqrt{N}}x_t)^2 + (\frac{1}{\sqrt{N}}y_t)^2])^{1/2}. \quad (15)$$

Redefining α as the positive square root of λ, $\alpha \stackrel{\triangle}{=} (\lambda)^{1/2}$, we can then rewrite the criterion as,

$$J^N_{LQG} = \left\| \begin{array}{c} y_t - r_t \\ \alpha u_t \end{array} \right\|_2^2 \quad (16)$$

Now, consider the signals defined in the loops of Figures 2 and 3, and observe that

$$\begin{pmatrix} y_t - r_t \\ \alpha u_t \end{pmatrix} = \begin{pmatrix} y^c_t - r_t \\ \alpha u^c_t \end{pmatrix} + \begin{pmatrix} y_t - y^c_t \\ \alpha(u_t - u^c_t) \end{pmatrix}$$

Therefore, by repeated use of the triangle inequality again, we have

$$\begin{aligned}
&\left| \left\| \begin{array}{c} y_t^c - r_t \\ \alpha u_t^c \end{array} \right\|_2 - \left\| \begin{array}{c} y_t - y_t^c \\ \alpha(u_t - u_t^c) \end{array} \right\|_2 \right| \\
&\leq \left\| \begin{array}{c} y_t - r_t \\ \alpha u_t \end{array} \right\|_2 \\
&\leq \left\| \begin{array}{c} y_t^c - r_t \\ \alpha u_t^c \end{array} \right\|_2 + \left\| \begin{array}{c} y_t - y_t^c \\ \alpha(u_t - u_t^c) \end{array} \right\|_2 \qquad (17)
\end{aligned}$$

For the same reasons as above, we denote

$$\begin{aligned}
J^{pr,N} &\triangleq \left\| \begin{array}{c} y_t - y_t^c \\ \alpha(u_t - u_t^c) \end{array} \right\|_2^2 \\
&= \frac{1}{N} \sum_{t=1}^{N} [(y_t - y_t^c)^2 + \lambda(u_t - u_t^c)^2]. \qquad (18)
\end{aligned}$$

$J^{pr,N}$ expresses the performance error that results from applying the LQG controller $\widehat{C}$, designed for $\widehat{P}$, on the actual plant P. Taking the limit for $N \to \infty$, the inequalities (17) can then be rewritten as

$$|(J^{des})^{1/2} - (J^{pr})^{1/2}| \leq (J^{ach})^{1/2} \leq (J^{des})^{1/2} + (J^{pr})^{1/2}, \qquad (19)$$

with an obvious definition for $J^{pr} = \lim_{N\to\infty} J^{pr,N}$. The upper bound in (19) had been obtained in [ZBG91] by a more complicated argument using the Hölder inequality, and served as the basis for the iterative design scheme to be described in Section 6.

We note that $J^{ach} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [(y_t - r_t)^2 + \lambda u_t^2]$, where y_t and u_t are the signals in the actual loop when the controller $\widehat{C}$ is applied to that loop, while y_t^c and u_t^c are the signals in the design loop when the same controller $\widehat{C}$ is used.

The inequalities (11) and (17) call for the following observations.

Comments

1. In both cases, the inequalities show that the achieved performance is bounded above by the sum of the designed performance, and a term expressing the performance error between the two closed loops, in a measure that is determined by the global control performance criterion. The achieved cost will be close to the designed cost, provided J^{pr} is much smaller than the designed cost. Thus, J^{pr} is a *robust performance criterion*, hence the notation. In the H_∞ case, $J^{pr}_{H_\infty}$ is indeed the classical robust performance criterion. In the LQG case, minimizing $J^{pr,N}$ corresponds to making the errors between the corresponding signals in the two loops small in the sense defined by the

LQG control performance criterion. In each case, J^{pr} will be small if the actual loop and the design loop are "close to one another in the appropriate sense".

2. Both inequalities show that in order to make the achieved cost small, one should minimize the designed cost (this is what the control design classically does), and at the same time one should keep the difference between the two closed loops small, again in the norm determined by the global control performance criterion $J_{glob} = J(P, C)$.

3. We note that the estimated plant model, $\widehat{P}$, and the controller, $\widehat{C}$, both influence the two terms J^{des} and J^{pr}. Thus, ideally, one should minimize the two terms jointly over the class of admissible plant models and admissible controllers. This is an impossible task in the case of restricted complexity models.[5] On the other hand, minimizing J^{des} with respect to the controller for a given model $\widehat{P}$ is in both cases a classical control design task, whereas J^{pr} expresses in both cases a distance between two closed loop transfer functions, in the appropriate measure. Therefore, an obvious suboptimal strategy is to make J^{des} small by controller design for a given plant model, and to keep J^{pr} small by identification design for a given controller. Since J^{des} depends on the estimated plant model, and J^{pr} depends on the designed controller, this strategy can only be applied in an iterative manner, using a succession of *local controller designs* and *local identification designs*:[6]

$$\begin{aligned} \min_{C} J(\widehat{P}_i, C) \quad &\longrightarrow \quad \widehat{C}_{i+1} \\ \min_{P(\theta)\in\mathcal{M}} J^{pr}(P(\theta), \widehat{C}_i) \quad &\longrightarrow \quad \widehat{P}_{i+1}. \end{aligned} \tag{20}$$

This idea is at the heart of the iterative identification/controller design methods that we discuss in Section 6.

4. In classical robust control design, a unique nominal model is given a priori together with error bounds, and the problem is restricted to designing a controller. When the model is obtained from data, as discussed in this paper, we observe that it is natural to design the controller such as to minimize the designed cost, and to design the model such as to minimize the performance robustness criterion: the term J^{pr} becomes our local identification criterion. This is certainly

[5] We recall that in [ÅW71] the achieved criterion is minimized jointly over the parametrized set of plant models and corresponding controllers, but the model set is assumed to contain the true system, and the minimization leads to a tractable solution only for the very simple minimum variance control criterion.

[6] The term 'local' refers to the fact that, at each iteration, the controller design (resp. the identification design) is performed on the basis of some present (i.e. local) plant model (resp. presently operating (i.e. local)) controller.

a non standard identification criterion: in (11) it is the H_∞ norm of the frequency weighted difference between the sensitivities of the two loops; in (17) it is a weighted Least Squares criterion of the difference between the corresponding signals in the two loops. We make the important observation that **these identification criteria are entirely determined by - and hence consistent with - the global control performance criteria**. This is what we meant in the introduction by stating that in a combined identification/control design *the control performance criterion should dictate what the identification criterion should be.*

5. It remains to be seen whether these nonstandard identification criteria can actually be minimized over a class of admissible models: this is the object of Section 5. It also remains to examine the properties of the iterative schemes that have only been sketched above. This is the object of Section 6.

To get a better understanding of the constraints imposed on the estimated plant model by the performance robustness requirements, we elaborate on the term J^{pr} of the inequalities (11) and (17). For (11) we get, straightforwardly,

$$J^{pr}_{H_\infty} \triangleq ||W(\frac{1}{1+PC} - \frac{1}{1+\widehat{P}C})||_\infty = ||W\frac{(P-\widehat{P})C}{(1+\widehat{P}C)(1+PC)}||_\infty. \quad (21)$$

The computations for (17) are more complicated. An LQG control design minimizing (4) leads to a two-degree-of-freedom controller as shown in Figure 4, where n_t is the input to the reference model, $r_t = R(z)n_t$: see e.g. [BGW90].

Assume now that we take $v'_t \equiv 0$ in the corresponding design loop: see Figure 5. It is shown in [ZBG91] that the second right hand term of (17) can then be expressed as follows.

$$\begin{aligned} J^{pr}_{LQG} &\triangleq \lim_{N\to\infty}\frac{1}{N}\sum_{t=1}^{N}[(y_t - y^c_t)^2 + \lambda(u_t - u^c_t)^2] \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\frac{|(P-\widehat{P})C_1|^2(1+\lambda|C_2|^2)}{|(1+PC_2)(1+\widehat{P}C_2)|^2}\Phi_n \right. \\ &\qquad \left. + \frac{(1+\lambda|C_2|^2)}{|1+PC_2|^2}\Phi_v \right\} d\omega \end{aligned} \quad (22)$$

In these expressions, y_t and u_t are defined on the actual LQG controlled plant of Figure 4, while y^c_t and u^c_t are defined from the design LQG loop of Figure 5, and Φ_n and Φ_v are the spectra of the signals n_t and v_t, respectively. Note that the two loops are driven by the same external signal, n_t, but that the loop of Figure 5 is noise free. Assuming that the spectrum of

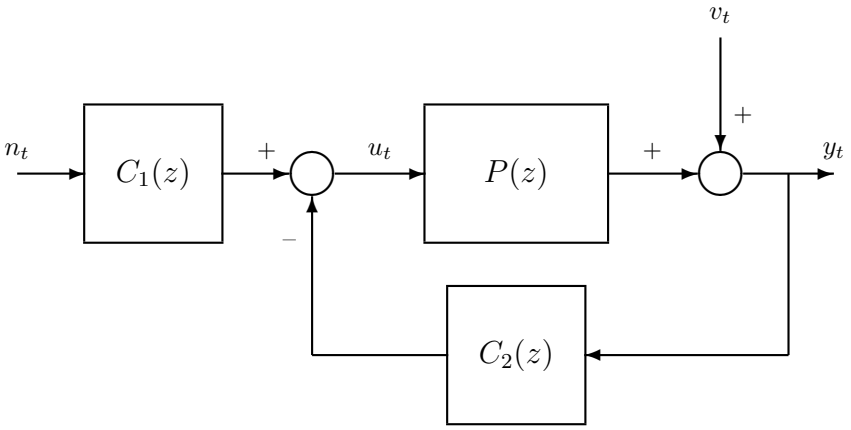

Figure 4: Actual LQG controlled system

the input to the reference model, n_t, dominates that of the noise, v_t, within the passband of the closed loop system, then the model fit obtained by the minimization of this Least Squares criterion will be essentially determined by the first term.

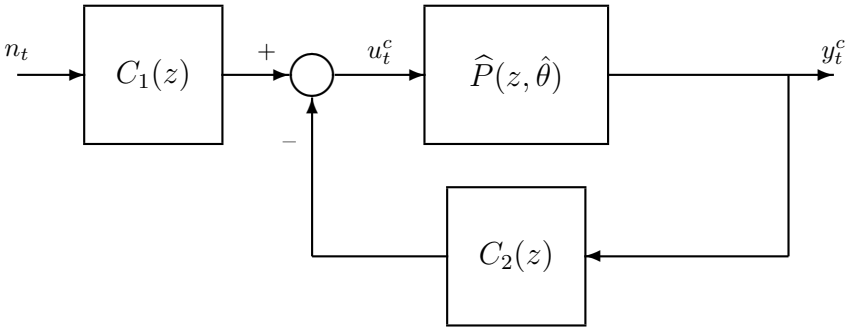

Figure 5: Nominal (or design) LQG controlled system

In both cases (H_∞ and LQG design) we observe that the model fit that is imposed by the minimization of J^{pr} is one in which the error between $P(e^{j\omega})$ and $\widehat{P}(e^{j\omega})$ must be made small in a frequency weighted sense, where the frequency weighting contains the product of the actual and designed sensitivities. Thus, the model errors must be small where these

sensitivities are large, and in particular around the crossover frequency of the actual and the designed closed loop systems. Comparing with the robust stability criterion (10), we note that we have an additional weighting by the sensitivity function of the *actual* closed loop system. In Section 5, we examine how these performance robustness criteria can be minimized by identification design.

3 Questions raised by the interplay

The expressions of the previous section have shown that the control design and the identification design are closely intertwined. For example, the robust stability inequality (10) shows that the model error can be large in frequency areas where the mixed sensitivity function of the designed closed loop system is small, but must be small where this sensitivity function is large. Similarly, the robust performance considerations have shown that what really matters for performance is not the error between the open loop transfer functions $P(z)$ and $\widehat{P}(z)$, but the error between the closed loop transfer functions (or, equivalently, the closed loop sensitivities) of the achieved and designed loops of Figures 3 and 2.

These expressions, and the interplay they reveal, raise several questions, leading to several research topics.

1. Can we estimate the model error $P(e^{j\omega}) - \widehat{P}(e^{j\omega})$, or a bound on this error, for every ω, when that model has been obtained by identification on the basis of input/output data information? This is the question of *model error quantification*.

2. Can we design the identification in such a way that

   - the controller designed from that model will stabilize the actual plant,
   - and the performance robustness criterion J^{pr} is minimized in (21) or (22)?

   This is the question of *identification for control*.

3. Can we jointly optimize the identification and the control design, in order to maximize the achieved control performance? This is the question of *synergistic identification and control design*.

As stated in the introduction, the first question is where most of the research effort is presently concentrated. Although most of this work is said to be 'motivated' by robust control, the control design or the control performance criterion is rarely mentioned in the papers that deal with 'identification for robust control'. The effort is spent on trying to develop

identification methods whose sole merit is to deliver error bounds, without much attention paid to the question of whether the identified model is good for the control design problem at hand. Our analysis above has clearly indicated that the identification method must take account of the control objective. *Given that with a restricted complexity model set a certain amount of plant/model mismatch is inevitable, the control objective must determine the distribution of this plant/model error.* The point of the remark above is to stress that the identification methods will be useful for robust control design only if they do take account of the control objective.

The quantification of the error on identified models is of course a very important objective in itself: it has historically been a trademark of engineers that they should be able to produce an evaluation of the quality of the product that they deliver. In addition, the estimation of the error on an identified model is indeed an important ingredient for robust control design. However, it is not central to the design of the identification for robust control or to the identification/control interplay. Therefore we do not attempt here to cover the huge amount of literature that is published on this problem. Let it suffice to say that the methods are essentially distinguished by the following ingredients.

- The type of prior knowledge that is assumed about the unmodeled dynamics and the noise: this prior knowledge can be in the form of stochastic descriptions or hard bounds, it can be unstructured or parametric.
- The form in which the data are assumed to be injected in the algorithms, such as time series data, or Fourier transform estimates.
- The methods that are used to propagate the prior uncertainty using data information. Examples are least squares, recursive least squares with bounding ellipsoids, or worst case constraints leading to large size linear programming problems.
- The norms, criteria and algorithms that are used to formulate the approximation problems: H_∞, H_2, l_1, etc.

It is fair to say that, even though the objective is to arrive at bounds on transfer function errors, the imagination of our research community to get there is essentially unbounded.

Recent surveys of available methods can be found in [WL92], [Tem93], [MV91], [GK92], [Gev91]. Finally, we like to mention that [GGN92] is probably the only contribution in which the prior assumptions on unmodeled dynamics and noise are of a qualitative rather than a quantitative nature: the prior uncertainty descriptions are parametric functions whose parameters are subsequently estimated from the data.

4 What can identification do for you?

We shall from now on adopt a performance enhancement objective. We have seen in Section 2 that a reasonable scheme for the minimization of the *achieved performance* by the combined design of identification and control is to perform a succession of model-based controller designs and controller-based identification designs: for a given (local) plant model, a designed control criterion, J^{des}, is minimized over the class of admissible controllers, and with a given (local) controller operating on the plant, an identification criterion, J^{pr}, is minimized over the set of plant models. This odd-looking identification criterion consists of a measure of the difference between the actual and the design loop, this measure being derived from and compatible with the global control performance criterion.

We now examine whether these bizarre control-performance-based identification criteria can indeed be minimized by classical identification methods. We shall do the development for the LQG criterion (an H_2 control criterion) because, as we will show, the corresponding criterion J^{pr}_{LQG} can be naturally connected to a prediction error identification criterion (an H_2 identification criterion). Similarly, the H_∞ criterion $J^{pr}_{H_\infty}$ naturally leads to an H_∞ identification criterion.

Least squares prediction error identification

We first recall the basic ingredients of prediction error identification. Remember that the true plant is assumed to be representable by (1). We consider that the model set takes the form

$$y_t = P(z,\theta)u_t + H(z)e_t. \tag{23}$$

Here $P(z,\theta)$ is a proper rational transfer function parametrized by some real vector θ, e_t is a zero mean white noise sequence, while $H(z)$ is, for simplicity, assumed to be some fixed noise model chosen by the user. From the model set (23) it is easy to write the one-step ahead prediction for y_t:

$$\hat{y}_{t|t-1}(\theta) = H^{-1}(z)P(z,\theta)u_t + [1 - H^{-1}(z)]y_t. \tag{24}$$

The one-step ahead prediction error is

$$\begin{aligned} \epsilon_t(\theta) &\stackrel{\triangle}{=} y_t - \hat{y}_{t|t-1}(\theta) \\ &= H^{-1}(z)[(P(z) - P(z,\theta))u_t + v_t]. \end{aligned} \tag{25}$$

In Least Squares prediction error identification, the estimation of the parameter vector θ on the basis of N input-output data is obtained by minimizing the sum of the squares of the prediction errors $\{\epsilon_t(\theta), t = 1, \ldots, N\}$. However, for reasons that will soon become transparent, it is often desirable to minimize a frequency weighted sum or, equivalently, to filter the errors

by some stable filter with transfer function $D(z)$. We denote by $\epsilon_t^f(\theta)$ the filtered errors :

$$\epsilon_t^f(\theta) \stackrel{\triangle}{=} D(z)\epsilon_t(\theta). \tag{26}$$

Least-squares prediction error identification amounts to estimating θ that minimizes

$$V_N(\theta) \stackrel{\triangle}{=} \frac{1}{N}\sum_{t=1}^{N}[\epsilon_t^f(\theta)]^2. \tag{27}$$

The parameter estimate is then defined as

$$\hat{\theta}_N = \arg\min_{\theta\in D_\theta} V_N(\theta), \tag{28}$$

where D_θ is a predefined set of admissible values. The parameter vector $\hat{\theta}_N$ then defines an estimated input-output model $P(z,\hat{\theta}_N)$.

Under reasonable conditions on the data and the model structure (see [Lju87]), $\hat{\theta}_N$ converges as $N \to \infty$ to

$$\theta^* = \arg\min_{\theta\in D_\theta} \bar{V}(\theta), \tag{29}$$

where

$$\bar{V}(\theta) = \lim_{N\to\infty} EV_N(\theta). \tag{30}$$

If the data are a realization of a stationary stochastic process, then $\bar{V}(\theta) = E[\epsilon_t^f(\theta)]^2$, the variance of the filtered prediction errors. Expressing these filtered prediction errors as a function of the 'true system' and the model transfer functions, and using Parseval's identity,

$$E[\epsilon_t^f(\theta)]^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Phi_{\epsilon^f}(\omega)d\omega,$$

allows one to obtain an expression for the frequency distribution of the asymptotic model error. To make this exercise useful, we shall successively derive the expressions of ϵ_t^f in the case of open loop and closed loop identification.

The filtered prediction error, ϵ_t^f, can be written, using (25) and (26),

$$\epsilon_t^f(\theta) = D(z)H^{-1}(z)[(P(z) - P(z,\theta))u_t + v_t]. \tag{31}$$

Identification in open loop

Assume first that the data have been collected while the process operates in open loop. In such case, the signals u_t and v_s are uncorrelated for all t and s. It then follows from (31) that,

$$\begin{aligned}\bar{V}(\theta) \stackrel{\triangle}{=} \quad E[\epsilon_t^f(\theta)]^2 = \tfrac{1}{2\pi}\textstyle\int_{-\pi}^{\pi}\{|P(e^{j\omega}) - P(e^{j\omega},\theta)|^2\Phi_u(\omega) + \Phi_v(\omega)\} \\ \times\tfrac{|D(e^{j\omega})|^2}{|H(e^{j\omega})|^2}d\omega\end{aligned} \tag{32}$$

Since $\hat{\theta}_N$ converges to θ^*, and $\theta^* \stackrel{\triangle}{=} \arg\min_{\theta \in D_\theta} \bar{V}(\theta)$, this integral expression gives an implicit characterization of the model $P(e^{j\omega}, \theta^*)$ to which $P(e^{j\omega}, \hat{\theta}_N)$ will converge if the number of data tends to infinity. In other words, it gives an implicit characterization of the asymptotic bias error.

The expression (32) shows that, when identification is performed on data collected in open loop operation, $P(e^{j\omega}, \hat{\theta}_N)$ converges to that model within the model set that minimizes a frequency weighted integral of the square error between the true transfer and the model transfer function, with a frequency weighting $\frac{\Phi_u(\omega)|D(e^{j\omega})|^2}{|H(e^{j\omega})|^2}$. With our assumption of a fixed noise model (i.e. $H(z)$ is θ-independent), the convergence point of $\hat{\theta}_N$ is independent of the actual noise distribution. It depends on the noise model $H(e^{j\omega})$, but only through the combined weighting $\frac{\Phi_u|D|^2}{|H|^2}$.

The formula (32) is useful because it shows that, in the situation where some restricted complexity model structure has been chosen for $P(z, \theta)$, one can still manipulate the frequency distribution of the plant/model error to a certain extent by playing with the design variables Φ_u, D and H. Since the whole interplay between identification and robust control design is based on obtaining a frequency distribution of the plant/model error that satisfies performance constraints, we will come back to this design issue and examine, on the basis of formula (32), whether open loop identification with the required choices of Φ_u, D and H can help us obtain robust performance. But first, we derive a similar expression for the frequency distribution of the plant/model error in the case of closed loop identification.

Identification in closed loop

We now consider that the data have been collected on the true system when some controller was operating, and we compute the expression of the filtered one step ahead prediction error. To make our derivation more general, and because we shall return to the LQG controller in Sections 5 and 6, we consider that the system operates under a two-degree-of-freedom controller as shown in Figure 4. First we compute ϵ_t^f. Substituting the expression for u_t derived from Figure 4 into (31) yields:

$$\epsilon_t^f(\theta) = \frac{D(z)}{H(z)[1 + P(z)C_2(z)]}[(P(z) - P(z,\theta))C_1(z)n_t + (1 + \widehat{P}(z,\theta)C_2(z))v_t]. \tag{33}$$

To get a better insight into some properties of closed loop identification, we shall further assume that the noise v_t on the true system can be modeled as $v_t = H_0(z)e_t$, where e_t is zero mean white noise. The expression for $\epsilon_t^f(\theta)$ can then be rewritten as (dropping the dependence on z for simplicity of

notation):[7]

$$\epsilon_t^f(\theta) = \frac{D}{H(1+PC_2)}[(P-\widehat{P}(\theta))C_1 n_t + (H_0(1+\widehat{P}(\theta)C_2) - H(1+PC_2))e_t] + De_t. \tag{34}$$

When identification is performed on closed loop data collected on the process operating under a two-degree-of-freedom controller, the estimate $\hat{\theta}_N$ converges to the minimum of the cost function $\bar{V}(\theta) \stackrel{\triangle}{=} E[\epsilon_t^f(\theta)]^2$. From (33) we get, using Parseval's theorem:

$$\begin{aligned} E[\epsilon_t^f(\theta)]^2 &= \frac{1}{2\pi}\int_{\pi}^{\pi} \left\{ \left| \frac{(P(e^{j\omega}) - \widehat{P}(e^{j\omega},\theta))}{1+P(e^{j\omega})C_2(e^{j\omega})} \right|^2 |C_1(e^{j\omega})|^2 \Phi_n(\omega) \right. \\ &\quad \left. + \left| \frac{1+\widehat{P}(e^{j\omega},\theta)C_2(e^{j\omega})}{1+P(e^{j\omega})C_2(e^{j\omega})} \right|^2 \Phi_v(\omega) \right\} \times \frac{|D(e^{j\omega})|^2}{|H(e^{j\omega})|^2} d\omega \end{aligned} \tag{35}$$

An interesting alternative expression of the minimizing value of $\bar{V}(\theta)$ can be obtained using (34). Since the last term in (34) is independent of θ, the minimization of $\bar{V}(\theta)$ with respect to θ is equivalent with the minimization of[8]

$$\begin{aligned} V^*(\theta) &= \frac{1}{2\pi}\int_{\pi}^{\pi} \left\{ \left| \frac{(P - \widehat{P}(\theta))}{1+PC_2)} \right|^2 |C_1|^2 \Phi_n(\omega) \right. \\ &\quad \left. + \left| \frac{H_0(1+\widehat{P}(\theta)C_2) - H(1+PC_2)}{1+PC_2} \right|^2 \sigma_e^2 \right\} \times \frac{|D|^2}{|H|^2} d\omega \end{aligned} \tag{36}$$

The expressions (35) and (36) describe in an implicit way the asymptotic distribution of the error between the true system $P(e^{j\omega})$ and the estimated model $\widehat{P}(e^{j\omega},\hat{\theta})$ when the identification is performed on data collected in closed loop using a reduced complexity model set. The following remarks are worth making.

- The model fit is definitely influenced by the controller: the component C_1 shapes the spectrum $|C_1|^2\Phi_n$ that enters the loop, while the component C_2 exerts its influence through the sensitivity $\frac{1}{1+PC_2}$ of the actual closed loop. The weighting on both terms of the integrand will be large where this sensitivity is large, namely around the crossover frequency of the closed loop system.

[7] This alternative calculation results from an insightful discussion with R. Hakvoort and P. Van den Hof.

[8] We drop the dependence of the transfer functions on $e^{j\omega}$ here to simplify notation.

- External excitation is definitely needed for closed loop identification. The model $\widehat{P}$ will approximate P (in the frequency weighted sense determined by the formula (35)) only if the reference signal spectrum $|C_1|^2\Phi_n$ that enters the loop dominates the noise spectrum Φ_v within the closed loop bandwidth.

- Without external reference, the model will attempt to approximate the inverse of the controller, $C_2^{-1}(e^{j\omega})$.

- The data filter $D(e^{j\omega})$ can again be used to shape the fit globally.

- The expression (36) shows that, even if the model set $\mathcal{M} = \{\widehat{P}(z,\theta)\}$ is able to represent the true system $P(z)$, closed loop identification using a direct prediction error method will lead to a biased estimate of $P(z)$ if the noise model is incorrect, that is if $H(z) \neq H_0(z)$. This is a serious drawback which has led Hansen [Han89] and Schrama [Sch92a] to propose an alternative indirect scheme that transforms the closed loop identification problem into an open loop scheme.

The Hansen-Schrama scheme

We present the Hansen scheme, as modified by Schrama, to perform closed loop identification using open loop methods. The scheme is based on the idea that if a compensator $C(z)$ stabilizes the plant $P(z)$, then $P(z)$ can be represented in the Youla parametrization of all plants stabilized by the compensator $C(z)$: this is the dual of the classical Youla parametrization. Thus, consider the loop of Figure 3, and assume that the noise v_t can be modeled as $v_t = H(z)e_t$, with $H(z)$ a rational transfer function and e_t white noise of zero mean. Let P_0 be an auxiliary model (possibly an estimate of P) such that the closed loop (P_0, C) is stable, and let P_0 and C have right coprime factorizations $P_0 = N_0(D_0)^{-1}$ and $C = N_c(D_c)^{-1}$, respectively, where N_0, D_0, N_c, D_c are all stable transfer functions. It can then be shown [Sch92a] that the feedback system of Figure 3, with $v_t = H(z)e_t$, is stable if and only if $[H \quad P]$ has a right coprime factorization of the form

$$[H \quad P] = [D_c S \quad N_0 + D_c R] \begin{bmatrix} I & 0 \\ N_c S & D_0 - N_c R \end{bmatrix}^{-1}, \tag{37}$$

where $R(z)$ and $S(z)$ are stable transfer functions. We note, in particular, that $P(z) = (N_0 + D_c R)(D_0 - N_c R)^{-1}$. For future use, we define:

$$N^a \triangleq N_0 + D_c R \qquad D^a \triangleq D_0 - N_c R. \tag{38}$$

Using this coprime factor representation of P and H, the feedback system of Figure 3 can be redrawn as in Figure 6.

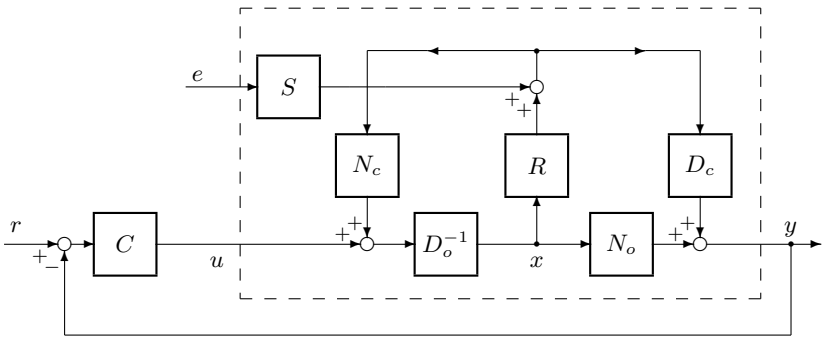

Figure 6: Coprime factor representation of P and H.

Now it is easy to show that the signal x_t of Figure 6 can be reconstructed from u_t and y_t through

$$x_t = (D_0 + CN_0)^{-1}(u_t + Cy_t). \tag{39}$$

In addition, it follows immediately from the figure that $u_t + Cy_t = r_t$. Therefore, x_t is uncorrelated with e_t, and hence with the noise v_t.

There are a number of ways to extract an estimate of the plant from this coprime factor representation. They are all based on the observation, immediately derived from (38) and the block-diagram, that

$$\begin{pmatrix} u_t \\ y_t \end{pmatrix} = \begin{pmatrix} D^a \\ N^a \end{pmatrix} x_t + \begin{pmatrix} -N_c S \\ D_c S \end{pmatrix} e_t. \tag{40}$$

Thus, the signal vector $(u \quad y)^T$ is expressed as the output of an unknown system driven by the known signal x_t, plus a noise term that is uncorrelated with x_t. These equations serve as a basis for the identification of the transfer functions N^a and D^a, thus yielding an estimate $\widehat{P} = \widehat{N}^a(\widehat{D}^a)^{-1}$ of the unknown plant P. Notice that, unlike the estimate obtained by direct identification (see (36)), the present estimate is not biased by the incorrectness of the noise model. Finally, we mention that this coprime factor representation has also been used by Zhu et al. [ZS92] for the identification of closed loop systems using spectral estimates.

We have briefly presented the bare essentials of prediction error identification theory, and we have given a frequency domain characterization of the criteria that are minimized by prediction error methods when identification is performed in open loop and in closed loop. These characterizations can also be seen as an implicit description of the frequency distribution of the asymptotic model error. We have also shown how the bias effects introduced by the noise in closed loop identification can be circumvented.

5 Identification for control

We now examine how the identification theory of the previous section can be used to tune the identification criterion towards the satisfaction of a control performance criterion, as suggested in Section 2. We thus return to the LQG design problem posed in Section 2. Ideally, one would like to design the identification in such a way that the difference between the cost achieved using the model-dependent controller $\widehat{C}$ on the real plant P achieves a performance, J^{ach}, that is as close as possible to J^{opt}.

Given that the global control performance criterion is given by J_{LQG} (see (4)), this strategy should, in principle, be pursued by designing an identification criterion that minimizes

$$V_N^{opt}(\theta) = \frac{1}{N}\sum_{t=1}^{N}[(y_t - y_t^{opt})^2 + \lambda(u_t - u_t^{opt})^2]. \tag{41}$$

For the case of minimum variance control ($\lambda = 0$), this was precisely the experiment design criterion adopted in [GL86], and it led to the conclusion that the identification should be performed in closed loop under minimum variance control. Although an exact implementation of this result is of course impossible because the minimum variance controller is a function of the unknown plant, it suggests an iterative design strategy in which the identification is pursued under feedback control with a minimum variance controller computed from the present model estimate. Adaptive minimum variance control is a fast implementation of this idea.

In the minimum variance control case, an optimal experiment design can be derived because the controllers C^{opt} and $\widehat{C}$ are an explicit function of the system and model parameters, respectively. The same is true for model reference control: see [Lju87]. In our present LQG design problem, the controller is a nonlinear function of the plant, and this seems to rule out a design based on a comparison of the optimal loop and the achieved loop. Thus, we return to a comparison between the achieved and the design loop, equipped with (or reinforced by) the inequalities (19). It was suggested in Section 2 that one could minimize the first term of the upper bound, J^{des}, by control design (this is a standard LQG control design problem) and the second term by identification design. The question was raised as to whether the non-standard criterion (18) can be minimized by identification techniques.

We first recall that the criterion (18) has been reformulated in the frequency domain as (22), and we now compare (22) with the prediction error criteria (32) and (35) for open loop and closed loop identification, respectively. It is immediately obvious that the criterion (22) cannot be minimized as a result of open loop identification. However, it can be minimized by a classical least squares prediction error method provided,

- the identification is performed in closed loop;

- with a data filter $D(z)$ obtained as the solution of

$$|D(z,\theta)|^2 = \frac{|H(z)|^2(1+\lambda|C_2(z)|^2)}{|1+\widehat{P}(z,\theta)|^2}. \tag{42}$$

We thus have the following remarkable result, that was derived in [ZBG91].

'Remarkable?' result (denoted R?R)

Assume that some two-degree-of-freedom controller $[C_1(z), C_2(z)]$ operates on the true system as in Figure 4, and on a simulated design loop containing a θ-dependent model set $\widehat{P}(z,\theta)$ as in Figure 5. Let the two loops be driven by the same reference signal source, n_t. Then the criterion

$$J_{LQG}^{pr,N} = \frac{1}{N}\sum_{t=1}^{N}[(y_t - y_t^c)^2 + \lambda(u_t - u_t^c)^2], \tag{43}$$

which expresses the 'distance' between the two loops in a measure that is determined by the global LQG criterion, can be minimized over this model set by a classical least squares prediction error method, provided the identification is performed in closed loop with the data filter $D(z,\theta)$ defined by (42).

This result is remarkable because it is a priori not obvious that the criterion (43), which is in fact a 'control performance error criterion', can be made identical to a classical prediction error criterion, given the proper experimental set-up and the proper data filter. This equivalence allows one to estimate the model $\widehat{P}(z,\hat{\theta})$ that minimizes (43) using standard identification algorithms. We now make a few comments about this result.

Comments

1. One of the important consequences of result R?R is that, if one identifies a model for control design, then the identification should be performed in closed loop. The intuition behind this result is that a model will be good for control design if its closed loop properties are close to the closed loop properties of the actual system under the same feedback control, that is if the closed loop transfer functions of the loops of Figure 3 and 2 are close. This is the message of the inequalities (11) and (17).

2. The problem is that the controller that should ideally operate during the collection of data for identification is precisely the optimal controller that is to be designed from the identified model. This is a classical case of the design method biting its own tail, and is the main reason for introducing iterative design methods. In these iterative designs, each control design step is followed by an identification

design step, and vice versa. The rationale of these iterative designs, in the light of our present identification analysis, is that, even if the presently acting controller, say $[C_{1,i}, C_{2,i}]$, is not the optimal one in the class of admissible reduced complexity controllers, it will produce a frequency distribution of the input signal, $\frac{|C_{1,i}|^2}{|1+PC_{2,i}|^2}\Phi_n$, that will force the next identified model, say $\widehat{P}_{i+1}(z,\hat{\theta})$, to have closed loop properties close to those of the true system.

3. A number of other approaches to the 'identification for control' problem have also led to the conclusion that the identification should be performed in closed loop and with appropriate data filters, and this is now widely recognized as a key ingredient for the success of control design based on reduced complexity identified models.

   - In [Han89] (see also [HFK89]) the idea of [GL86] has been extended from an H_2 measure of the performance degradation between the optimal and the actual system to an H_∞ measure of the difference between the closed loop transfer functions of these two feedback systems.
   - In [BGW90] the H_∞ stability and performance robustness constraints detailed in Section 2 were used to motivate the use of closed loop prediction error identification in the context of modeling for LQG control design. Indeed, a comparison between the robust performance criterion (21) and the closed loop identification criterion (35) shows that, by proper choice of the data filter $D(z)$ and by closed loop identification, the robust performance expression $|\frac{(P-\widehat{P})C}{(1+PC)(1+\widehat{P}C)}|$ can be made small in an H_2 sense. Even though the H_2 minimization of this quantity cannot guarantee a bound on its H_∞ norm, the idea of making that quantity small within the bandwidth of the closed loop system to enhance performance robustness was advocated.
   - The Delft group, Hakvoort, Schrama, and Van den Hof, have brought important contributions to the identification for control problem (see e.g. [Hak90], [Sch91], [Sch92a], [Sch92b], [SvdH92]). In his remarkable thesis [Sch92a], Schrama examined many facets of the 'identification for control' and 'iterative identification and control design' problems. In particular, he demonstrated convincingly with a dramatic simulation example that an open loop model that would pass all standard model validation tests can result in a disastrous controller. Conversely, the best model for control design can be so poor as an open loop model of the plant that it would fail most classical model validation tests.
   - The equivalence between a performance robustness criterion and a prediction error criterion established in [ZBG91] for an LQG

control criterion has been extended to a pole placement control design by Åström [Åst93]. For this particular control design, he shows that the 'control performance error' (i.e. the error between the outputs y and y^c of the loops of Figures 3 and 2) can be made identical to the 'identification error' (i.e. the filtered prediction error) by performing closed loop identification with a specific data filter.

- Mäkilä and Partington have advocated closed loop identification, both as a way of identifying open loop unstable but stabilizable systems [MP92a] and as a procedure for enhancing control performance robustness when the model is used for control design [MP92b]. In [MP92a] H_∞ identification of the closed loop transfer function is performed, while parameter bounding techniques using l_∞-stable coprime factor descriptions are used in [MP92b].
- Liu and Skelton [LK90] pointed to the need for closed loop identification and proposed an iterative design scheme using Skelton's q-Markov Cover models.

We have demonstrated in this section that the performance robustness criterion J^{pr}_{LQG}, defined for some given controller operating both on the plant and on the model, can be minimized over a set of parametrized models by least squares prediction error identification in closed loop with an appropriate data filter. A similar conclusion can be drawn for the minimization of $J^{pr}_{H_\infty}$.

We now examine methods for the synergistic identification and control design that are based on the iterative minimization of J^{des} by controller manipulation and of J^{pr} by model manipulation.

6 Iterative identification and control design

Suppose that, for some plant P to be controlled, a preliminary analysis has led to the choice of some model set, $\mathcal{M} = \{\widehat{P}(z,\theta), \theta \in D_\theta\}$, and some control performance criterion, $J(P,C)$, with the property that the minimization of $J(\widehat{P},C)$ with respect to C, for any $\widehat{P} \in \mathcal{M}$, uniquely determines a designed controller, $\widehat{C} = \widehat{C}(\widehat{P})$. The choice of an adequate model set and of an adequate control objective is of course very much part of the control design. In particular, one must choose a control performance objective that is compatible with the achievable closed loop bandwidth, etc. However, we adopt these assumptions (or play this game) to illustrate the central features of the synergistic design problem. Ideally, the problem of joint identification and control could then be reformulated as a parameter estimation problem as follows:

$$\min_{\theta \in D_\theta} J(P, \widehat{C}(\widehat{P}(\theta))). \tag{44}$$

To make things concrete, assume for example that $J(P, C)$ is the LQG criterion (4) and that $\mathcal{M}$ is a parametrized set of third order output error models with a parameter vector $\theta \in D_\theta = \mathbb{R}^6$. The optimal control problem has thus been turned into an identification problem.

The direct minimization of such a global control performance criterion over a set of restricted complexity models is typically intractable, but one way to attack the problem is to perform a succession of local identification steps and local control design steps in an iterative way. Several motivations can be given to rationalize such iterative procedures.

- A feasibility motivation: the intractable joint optimization problem alluded to above is replaced by a sequence of tractable closed loop identification problems with fixed controller, and controller design problems with a given model.

- A theoretical motivation: the triangle inequalities (11) and (19) are one way of giving theoretical credibility to the the idea that by performing small controller changes that minimize J^{des} for a given $\widehat{P}$, followed by small model changes that minimize J^{pr} for a given $\widehat{C}$, then this iterative procedure may tend to jointly minimize the upper bound on the achieved cost. For the moment, no hard results are yet available.

- A practical motivation: in the process control industry, it is common practice to design a controller, let it operate for a while, collect data on the controlled process, and then use these data to perform a new design in order to ameliorate performance. The novel contribution of the iterative design schemes is to provide a systematic and theoretically justified framework to perform these successive designs. Thus, they should really be seen for what they are, namely performance enhancement schemes.

There are many variants to the iterative design schemes - and we shall discuss some of them - but the basic idea is as follows.

1. **Step** 0: Identify an open loop model, $\widehat{P}_0$, from input-output data, and design a controller, $\widehat{C}_0$, that stabilizes both the true plant P and the estimated model $\widehat{P}_0$. Apply this controller to the plant and collect new input-output data.

2. **Step** i: Using the closed loop data collected on the plant while the controller $\widehat{C}_{i-1}$ operates, identify a new model $\widehat{P}_i$ by minimizing a local identification criterion. Using this new identified model $\widehat{P}_i$, design a new controller $\widehat{C}_i$ that stabilizes both P and $\widehat{P}_i$, by minimizing a local control design criterion. Apply this controller to the plant and collect new input-output data.

3. **Step** ∞: Do not iterate until convergence (who would?), for two good reasons: convergence has not been proved for any of these schemes, and which practical control engineer would want to redesign his controllers every day anyway?

In [ZBG91] such a procedure was proposed, first for the case where the global criterion is a classical H_∞ criterion, then for the case of an even more classical LQG criterion. In the case of an H_∞ criterion, the iteration of identification steps and control design steps can be formulated, and it was proved that the achieved performance criterion decreases at every step. However, no feasible algorithm is presently available for the H_∞ identification step of this joint design. The same idea of iterative minimization of an H_∞ criterion was developed independently in [BYM92], where the exact same conclusion was reached, a rather fortunate coincidence.

We therefore turn to the LQG criterion, and present the algorithm known in the process control industry as the Zangscheme.[9]

The Zangscheme[10]

The Zangscheme of [ZBG91] uses the LQG criterion (4) as the global criterion to be minimized. To simplify the notation, we shall often use the vector notation $C(z)$ to denote the two-degree-of-freedom controller $C_1(z), C_2(z)$. Thus, $C(z) \stackrel{\triangle}{=} [C_1(z) \;\; C_2(z)]$.

Consider now that we are at i-th iteration of the design (see above), and that the two-degree-of-freedom controller $\widehat{C}_{i-1}$, designed at the previous iteration, is operating on the real plant P. Thus, $C(z)$ is replaced by $\widehat{C}_{i-1}(z)$ in the loop of Figure 4, and N data y_t and u_t are being collected on that actual closed loop system.

The identification step is performed by minimizing the local prediction error identification criterion,

$$J^{id,N} = \frac{1}{N} \sum_{1}^{N} [D_i(z,\theta)\epsilon_t(\theta)]^2, \tag{45}$$

over the model set $\mathcal{M}$, where $\epsilon_t(\theta) \stackrel{\triangle}{=} y_t - \hat{y}_t(\theta)$, and where the data filter is computed from

$$|D_i(z,\theta)|^2 = \frac{|H(z)|^2(1+\lambda|\widehat{C}_{2,i-1}(z)|^2)}{|1+P(z,\theta)|^2}. \tag{46}$$

The criterion is 'local' only through its dependence on the present controller, $\widehat{C}_{2,i-1}$, acting in the loop. As shown in Section 5, this local identification

[9] ... and better known in Australia and Belgium as the Zangstuff.

[10] We present here a slightly improved version of the Zangscheme, taking account of modifications introduced by Partanen and Bitmead [PB93] and by the author: progress just cannot be stopped.

criterion is identical to the robust performance criterion J^{pr} of LQG defined in (18), where the signals u_t^c and y_t^c are those that would be generated by applying the same signal n_t to the loop of Figure 5 controlled by the same controller $\widehat{C}_{i-1}$ and with no noise input. The new model that results from the identification step is denoted $\widehat{P}_i(z)$:

$$\widehat{P}_i(z) = \arg \min_{P(z,\theta)\in\mathcal{M}} J^{id,N} \tag{47}$$

Comments

We note that the data filter depends on the model that is being identified. There are several ways to cope with this problem.

- The first solution is to replace the unknown $\widehat{P}(z,\theta)$ in the data filter by the most recent estimate, $\widehat{P}_{i-1}(z)$; this is the solution proposed in [ZBG91]. However, the equivalence between J^{pr} and J^{id} breaks down with this solution, as pointed out by Hakvoort and Van den Hof [HV93].

- A better solution is to let the filter $D_i(z)$ be θ-dependent. This effectively corresponds to solving a prediction error problem with a modified model structure. It will, however, typically lead to a more complicated minimization problem for $J^{id,N}$.

We now turn to the i-th control design iteration. The certainty equivalence control design criterion would be to minimize the following performance criterion J^{des}:

$$J^{des} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \{(y_t^c - r_t)^2 + \lambda (u_t^c)^2\}, \tag{48}$$

where u_t^c is the designed control signal and y_t^c is the output of the identified model, $\widehat{P}_i$, driven by u_t^c.

Instead of following the certainty equivalence route of minimizing (48), the Zangscheme performs a controller design that takes account of the present plant/model uncertainty in the following way.

- A closed loop simulation is performed with the controller $\widehat{C}_{i-1}(z)$ acting on the present plant model $\widehat{P}_i(z)$. The actual closed loop system of Figure 4, and the simulation loop of Figure 5, with the same controller $\widehat{C}_{i-1}(z)$, are driven by the same signal n_t[11], thus generating the signals u_t and y_t, respectively u_t^c and y_t^c.

- With these experimental and simulated data sets, low order (typically third order AR models) are fitted to the signals $(y - r)$, u, $(y^c - r)$

[11] In addition, the actual system is also driven by the noise source v_t.

and u^c, thus yielding spectral estimates $\widehat{\Phi}^{\frac{1}{2}}_{y-r}(z)$, $\widehat{\Phi}^{\frac{1}{2}}_{u}(z)$, $\widehat{\Phi}^{\frac{1}{2}}_{y^c-r}(z)$ and $\widehat{\Phi}^{\frac{1}{2}}_{u^c}(z)$.

- The following frequency weighted *local control criterion* is minimized to compute $C_i(z)$:

$$J^c = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \{[F_1(z)(y_t^c - r_t)]^2 + \lambda [F_2(z) u_t^c]^2\}, \qquad (49)$$

  where F_1 and F_2 are weighting functions (linear filters) that are chosen as the following ratios of the estimated spectra:

$$F_1 = \left(\frac{\widehat{\Phi}_{y-r}}{\widehat{\Phi}_{y^c-r}} \right)^{1/2}, \quad F_2 = \left(\frac{\widehat{\Phi}_{u}}{\widehat{\Phi}_{u^c}} \right)^{1/2}. \qquad (50)$$

We comment that all the signals necessary for the computations of the filters $F_1(z)$ and $F_2(z)$ are readily available at every iteration step. The effect of the frequency weightings is to make the filtered tracking error signal and control signal, respectively, in (49) have the same spectra as the corresponding signals in the global (ideal) performance criterion J_{LQG} of (4). Thus, the frequency weightings are a distortion of the certainty equivalence criterion that takes account of plant/model mismatch in order to reflect the global criterion. The plant/model mismatch information is injected in the design on the basis of signal information only.

Besides forcing the local control objective to mimic the global one, as explained above, the effects of the frequency weightings in (49) have entirely logical and intuitive interpretations. If at some frequency Φ_{y-r} is larger than Φ_{y_c-r}, it means that at that frequency the model fit is poor with the consequence that the achieved tracking performance (with the presently active controller) is worse than expected from the designed system. Hence, more emphasis should be put on the tracking penalty at that frequency at the next control design stage, which is reflected by the weighting being larger than 1. If at some frequency Φ_{y-r} is smaller than Φ_{y^c-r}, it also means that at that frequency the model fit is poor, but in such a way that the presently active controller actually achieves a better tracking performance on the true plant than on the model. The emphasis on the tracking penalty at that frequency should therefore be decreased at the next control design stage to provide scope for improvement at other frequencies. Similar astute and entirely intuitive observations can be made by the reader as regards the frequency weighting on the control.

The i-th iteration of the Zangscheme (at least in one of its variants) thus involves the following steps:

- Apply the controller $\widehat{C}_{i-1}(z)$ to the true plant $P(z)$ with an external reference r_t, as in Figure 4, to generate a data set $\{y_t, u_t\}$ of length N.

- Compute the data filter $D_i(z,\theta)$ using (46).
- With the data set $\{y_t, u_t\}$ identify $\widehat{P}_i(z,\hat{\theta})$ using $D_i(z,\theta)$.
- Perform a closed loop simulation, driven by the same external reference r_t, with the controller $\widehat{C}_{i-1}(z)$ acting upon the plant model $\widehat{P}_i(z,\hat{\theta})$, as in Figure 5, to generate a data set $\{y_t^c, u_t^c\}$ of length N.
- With the data sets from the experiment and the simulation, identify AR models of $(y-r),\ u,\ (y^c-r),\ u^c$ to yield $\widehat{\Phi}^{\frac{1}{2}}_{y-r}(z)$, $\widehat{\Phi}^{\frac{1}{2}}_{y^c-r}(z)$, $\widehat{\Phi}^{\frac{1}{2}}_{u}(z)$ and $\widehat{\Phi}^{\frac{1}{2}}_{u^c}(z)$.
- Calculate the frequency weightings F_1 and F_2 using (50).
- Design a new frequency weighted feedback controller $\widehat{C}_i(z)$ based on $\widehat{P}_i(z,\hat{\theta})$ and the identified signal spectra.

A number of other variants have been proposed [PB93], [HSV92], and a large number of simulations have been performed (see [ZBG91]). The disturbance rejection properties of the Zangscheme have been examined in [ZBG92]. The simulations typically exhibit an improvement of the *achieved cost* during the first three or four iterations, with no significant improvement thereafter. They also show that the model that is obtained after these few iterations can be very different from the best open loop model: this last finding is corroborated by the simulations performed with all other iterative design methods, again confirming that the best model for control design is definitely not the best open loop model.

Alternative iterative design schemes

We now briefly describe the key features of some of the other iterative identification and control design schemes. They are all based on trying to make the design loop of Figure 2 close to the achieved loop of Figure 3 in some sense.

- In Liu and Skelton [LS90], the q-Markov Cover theory of Skelton et al., which is a method for model reduction, is used in the identification step to identify a model of the closed loop system with the previously designed controller operating in the loop. Since the controller is known, a model of the open loop plant can be derived. A minimum energy controller with output variance constraint is used in the control design step.
- In [BYM92] Bayard et al. formulate an H_∞ robust performance criterion for the joint optimization of control design and identification design. A relaxation algorithm is proposed for solving the joint optimization problem, based on alternating between curve fitting and

control design steps. This strategy yields a monotonically improving achieved performance, but is presently not implementable. A numerical example using an approximate implementation, for which no descent property can be proved, shows the usefulness of the approach.

- The Delft group has done a thorough analysis of the iterative design scheme. Although several alternative methods and variants have been examined, the basic approach in [Sch92a], [Sch92b] and [SV92] is to use, as global performance measure, an H_∞ norm of the feedback system linking the exogenous signals to the $(u \quad y)^T$ vector. This encompasses most H_∞ control design criteria. The triangle inequality (11) is used to justify the iterative design schema. Both the control and the identification design are performed using coprime stable factor representations of the plant model and the controller, as explained in Section 4. This use of coprime factor representations guarantees that the designed controller is optimally robust against perturbations of the coprime factors, as shown by Vidyasagar [Vid85]. The closed loop identification step is based on the open loop scheme detailed in Section 4, using the auxiliary signal x_t defined in Figure 6. For lack of a satisfactory H_∞-algorithm, the minimization of the performance robustness criterion $J^{pr}_{H_\infty}$ in the identification step is replaced by an H_2 minimization. The control design step is a certainty equivalence minimization of the nominal H_∞ criterion. One interesting feature of the Schrama scheme is that it can 'predict' the achieved performance; this feature is used to update the performance requirements as the closed loop model becomes better.

- The idea of improving the performance requirements as the closed loop model becomes closer to the actual closed loop system is central to the philosophy of the scheme developed by Lee et al. [LAKM92], who call this idea the windsurfer approach to adaptive control. The techniques used by Lee et al. are based on the Hansen representation of the closed loop system, and are therefore close to those of Schrama et al., but the global objective is different. It is formulated as the minimization of the H_∞ norm of the difference between the achieved closed loop transfer function and that of a reference model. The emphasis is put on how to update the reference model (i.e. the performance specifications) as the model and the controller improve. The identification step, which is still not fully resolved, attempts to estimate the stable factor R in the representation (38), rather than the factors N^a and D^a in the Schrama scheme.

7 Conclusions

Two o'clock in the morning. What more can I say? The joint design of identification and control is a fresh field, ripe with ideas and probably a few misconceptions as well. Some convergent streams are emerging from the array of different approaches that have been applied to the identification and control design problem. These streams and guidelines bear the names 'iterative designs', 'closed loop identification', 'control-determined identification criterion', 'appropriate data filter'. However, the evidence is still circumstantial, the methods ad hoc, and the hard proofs scarce. So far, it appears that the benefits to be drawn from this research area are more attuned towards the development of performance enhancement design schemes for industrial process control than they are towards the fine-tuning of controllers that need to stabilize high performance aircraft.

However, beneath the surface lie fascinating and deep theoretical questions that have information theoretic significance going back to the dual control ideas of Fel'dbaum, and enough hard mathematical and control-theoretic questions to keep a few generations of PhD students busy.

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