©Digital Stock 1996

Iterative Feedback Tuning: *Theory and Applications*

Many control objectives can be expressed in terms of a criterion function. Generally, explicit solutions to such optimization problems require full knowledge of the plant and disturbances, and complete freedom in the complexity of the controller. In practice, the plant and the disturbances are seldom known, and it is often desirable to achieve the best possible performance with a controller of prescribed complexity. For example, one may want to tune the parameters of a PID controller in order to extract the best possible performance from such simple controller.

The optimization of such control performance criterion typically requires iterative gradient-based minimization procedures. The major stumbling block for the solution of this optimal control problem is the computation of the gradient of the criterion function with respect to the controller parameters: it is a fairly complicated function of the plant and disturbance dynamics. When these are unknown, it is not clear how this gradient can be computed.

Within the framework of restricted complexity controllers, previous attempts at achieving the minimum of a control performance criterion have relied on the availability of the plant and disturbance model, or on the estimation of a full order model of these quantities, see [22] and [32]. Alternatively, reduced order controllers can be obtained from a full-order controller followed by a controller reduction step [1].

This article presents research results of the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. The scientific responsibility rests with its authors. *Hjalmarsson is with the Dept. of Signals, Sensors and Systems, Royal Institute of Technology, S-100 44 Stockholm, Sweden. Gevers is with CESAME, Louvain University, B-1348 Louvain-la-Neuve, Belgium. Gunnarsson is with the Dept. of Electrical Engineering, Linköping University, S-581 83, Linköping, Sweden. Lequin is with Solvay S.A., rue Solvay 39, B-5190, Jemeppe-sur-Sambre, Belgium.*

 0272-1708/98/$10.00©1998IEEE

In the context of controllers of simple structure for unknown systems, such as PID controllers, some schemes have been proposed for the direct tuning of the controller parameters. These schemes are based on achieving certain properties for the closed loop system that are found to be desirable in general. These properties can then be translated into constraints on the Nyquist plot (or the Ziegler-Nichols plot) of the controlled system. We refer the reader to [2] for a representative of this family of methods.

Recently, so called iterative identification and control design schemes have been proposed in order to address the problem of the model-based design of controller parameters for restricted complexity controllers, see, e.g., [8], [24], [35], [39], and [40]. These schemes iteratively perform plant model identification and model-based controller update, with the successive controllers being applied to the actual plant. Behind these schemes is the notion that closed loop experiments with the presently available controller should generate data that are "informative" for the identification of a model suited for a new and improved control design, and that controllers based on models that are better and better tuned towards the control objective should achieve increasingly higher performance on the actual system. See [9]–[11] for a presentation of these ideas.

So far, there are very few hard results to support these expectations, except for the ideal (but unrealistic) situation where full-order models (and hence full-order controllers) are used. Following up on the early results of [12], it has been shown in [18] that, for that situation, closed loop identification with a specific controller in the loop yields an estimated controller that achieves the best possible performance on the actual system. In addition, an iterative identification and control design scheme has been proposed that approaches these ideal experimental conditions.

In the case of low-order controllers, there are reported successes, including experimental and industrial ones, of the above-mentioned iterative identification-based controller design schemes [31], but there are also examples where these schemes are known to diverge. Most importantly, with the exception of some examples analyzed in [3], there is no analysis of the performance properties of the closed loop systems to which such schemes converge in the cases where they do so. In [21] it was shown that such iterative identification-based control design schemes do not converge to a controller that minimizes the control performance criterion, except possibly for full order models and controllers. This has also been pointed out in [27].

It is the analysis of [21], and our attempt to understand the convergence/divergence properties of the iterative identification and control design scheme of [3] based on a simple model reference control design, that led us to the idea of reformulating the iterative identification and control design scheme as a parameter optimization problem, in which the optimization is carried directly on the controller parameters, thereby abandoning the identification step altogether. This approach is of course analogous to direct adaptive control, the main difference being that here the complexity of the controller need in no way be related with that of the system; in fact, the major application field of our method here is for the optimal tuning of low order controllers.

In the combined identification/control design schemes, the model is only used as a vehicle towards the achievement of the minimization of a control performance objective. An obvious alternative is to directly optimize the control performance criterion over the controller parameters. However, as stated above, earlier attempts at minimizing the control performance criterion by direct controller parameter tuning had stumbled against the difficulty of computing the gradient of this cost criterion with respect to the controller parameters.

The contribution of [19] was to show that an unbiased estimate of this gradient can be computed from signals obtained from closed loop experiments with the present controller operating on the actual system. For a controller of given (typically low-order) structure, the minimization of the criterion is then performed iteratively by a Gauss-Newton based scheme. For a two-degree-of-freedom controller, three batch experiments are to be performed at each step of the iterative design. The first and third simply consist of collecting data under normal operating conditions; the only real experiment is the second batch which requires feeding back, at the reference input, the output measured during normal operation. Hence the acronym Iterative Feedback Tuning (IFT) given to this scheme. For a one-degree-of-freedom controller, only the first and third experiments are required. No identification procedure is involved. A closely related idea of using covariance estimates of signals obtained on the closed loop system to adjust the controller parameters in the gradient direction was used in an adaptive control context by Narendra and coworkers some 30 years ago, see [29] and [30]. Another related method, in which state-feedback is considered, is presented in [23]. In other optimization-based approaches that have appeared in an adaptive control context, the gradient of the criterion was obtained through the estimation of a full-order model of the plant, see, e.g., [38].

As in any numerical optimization routine, a variable step size can be used. This allows one to control the rate of change between the new controller and the previous one. This is an important aspect from an engineering perspective. Furthermore, a variable step size is the key to establishing convergence of the algorithm under noisy conditions. With a step size tending to zero appropriately fast, ideas from stochastic averaging can be used to show that, under the condition that the signals remain bounded, the achieved performance will converge to a (local) minimum of the criterion function as the number of data tends toward infinity. This appears to be the first time that convergence to a local minimum of the design criterion has been established for an iterative restricted complexity controller scheme.

An altogether different approach to controller design without a model is the concept of 'unfalsified control' proposed in [34]. The scheme proposed in that paper, which applies to noise-free systems, consists in successively eliminating controllers from a prior set of candidate controllers, on the ground that these controllers could not meet the required performance specifications when fed with data collected on the system.

The optimal IFT scheme of [19] was initially derived in 1994 and presented at the IEEE CDC 1994. Given the simplicity of the scheme, it became clear (and not just to the authors) that this new scheme had wide-ranging potential, from the optimal tuning of simple PID controllers to the systematic design of controllers of increasing complexity that have to meet some prespecified specifications. In particular, the IFT method is appealing to process control engineers because, under this scheme, the controller parameters can be successively improved without ever opening the loop. In addition, the idea of improving the performance of an already operating controller, on the basis of closed loop data, corresponds to a natural way of thinking. Finally, in many process control applications the main objective of the controller design is to achieve disturbance rejection. With the IFT scheme the tuning of the controller parameters for disturbance rejection is driven by the disturbances themselves.

Since 1994, much experience has been gained with the IFT scheme.

- It has been shown to compare favourably with identification-based schemes in simulation examples: see [19].
- Its accuracy has been analyzed in [17].
- It has been successfully applied to the flexible transmission benchmark problem posed by I.D. Landau for ECC95, where it achieved the performance specifications with the simplest controller structure [20].
- It has been tested on the flexible arm of the Laboratoire d'Automatique de Grenoble [7], on a ball-on-beam system [6], for the temperature control of a water tube, and for the control of a suspended plate [28].
- It has been adapted to linear time-invariant MIMO systems [16].
- It has been shown to handle time varying, and in particular periodically time-varying, systems [13].
- It has been applied by the chemical multinational Solvay S.A. to the tuning of PID controllers for a number of critical control loops for which opening the loop or creating limit cycles for PID tuning was not allowed: temperature control in furnaces, in distillation columns, flow control in evaporators, etc. The performance improvements achieved by applying the IFT scheme to the existing PID loops have been rather striking (this is discussed below).

Common to many of the processes in these applications is that they exhibit some kind of nonlinear behaviour and, even if IFT was developed for linear time-invariant systems, it seems to also perform well on many nonlinear systems. The reasons for this and the conditions required from nonlinear systems for IFT to perform well have been analyzed in [14].

Our objective in this article is to first present the IFT scheme, and to then review performances achieved by the scheme at the S.A. Solvay, where it was used for the optimal tuning of PID controllers on a number of control loops, and on a DC-servo with backlash. We shall leave aside the connections with identification-based schemes and all other technicalities that might be of interest to theoretically inclined researchers but that would otherwise distract the reader from the essential ideas of the scheme and its potential applications.

The article is organized as follows. In the next section we present the design criterion and following that we show how this criterion can be minimized using experimental data. The section after that ("Convergence") presents the main convergence result. The next three sections deal with implementation issues, the major design choices, and some practical engineering aspects. Two "Applications" sections follow. The first discusses how the method was used in the tuning of PID controllers on several chemical processes at S.A. Solvay, while the second shows how IFT performs when applied to the tuning of a linear controller for a nonlinear DC-servo system with backlash. Finally, some conclusions are offered.

The Control Design Criterion

We consider an unknown true system described by the discrete time model

$$y_t = G_0 u_t + v_t \tag{1}$$

where G_0 is a linear time-invariant operator, $\{v_t\}$ is an unmeasurable (process) disturbance and t represents the discrete time instants. We shall consider here, for future analysis purposes, that $\{v_t\}$ is a zero mean weakly stationary (see, e.g., [26]) random process, but this assumption will be relaxed in the convergence discussion.

We consider that this system is to be controlled by a two-degrees-of-freedom controller:

$$u_t = C_r(\rho) r_t - C_y(\rho) y_t \tag{2}$$

where $C_r(\rho)$ and $C_y(\rho)$ are linear time-invariant transfer functions parametrized by some parameter vector $\rho \in \mathbf{R}^{n_\rho}$, and $\{r_t\}$ is an external deterministic reference signal, independent of $\{v_t\}$. It is possible for $C_r(\rho)$ and $C_y(\rho)$ to have common parameters. A block-diagram of the closed loop system is represented in Fig. 1.

Whenever signals are obtained from the closed loop system with the controller $C(\rho) \triangleq \{C_r(\rho), C_y(\rho)\}$ operating, we will indicate this by using the ρ-argument; on the other hand, to ease the notation we will from now on omit the time argument of the signals. Thus, $y(\rho), u(\rho)$ will denote, respectively, the output and the control input of the system (1) in feedback with the controller (2).

Let y^d be a desired output response to a reference signal r for the closed loop system. This response may possibly be defined as the output of a reference model T_d, i.e.,

$$y^d = T_d r\ , \tag{3}$$

but for the control design method that will be developed later the knowledge of the signal y^d is sufficient. The error between the achieved and the desired response is

$$\tilde{y}(\rho) \triangleq y(\rho) - y^d = \left(\frac{C_r(\rho)G_0}{1 + C_y(\rho)G_0} r - y^d \right) + \frac{1}{1 + C_y(\rho)G_0} v. \tag{4}$$

When a reference model (3) has been defined this error can also be written as

$$\tilde{y}(\rho) = \left(\frac{C_r(\rho)G_0}{1 + C_y(\rho)G_0} - T_d \right) r + \frac{1}{1 + C_y(\rho)G_0} v. \tag{5}$$

This error consists of a contribution due to incorrect tracking of the reference signal r and an error due to the disturbance.

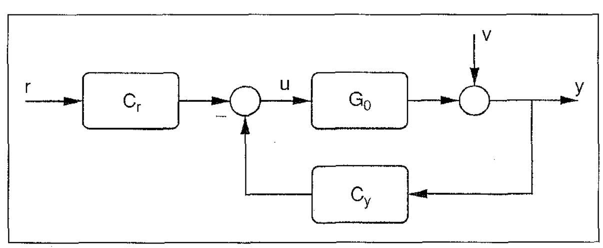


Fig. 1. Block diagram of the closed loop system.

For a controller of some fixed structure parametrized by ρ, it is natural to formulate the control design objective as a minimization of some norm of $\tilde{y}(\rho)$ over the controller parameter vector ρ. We will consider the following quadratic criterion:

$$J(\rho) = \frac{1}{2N} \mathrm{E}\left[\sum_{t=1}^{N} (L_y \tilde{y}_t(\rho))^2 + \lambda \sum_{t=1}^{N} (L_u u_t(\rho))^2 \right], \tag{6}$$

but any other differentiable signal-based criterion can be used. In (6) $\mathrm{E}[\cdot]$ denotes expectation with respect to the weakly stationary disturbance v. A time-weighting can also be introduced in the criterion; this has been found to be a very efficient way to minimize the settling time at setpoint changes (see the section "Design Choices").

The optimal controller parameter ρ is defined by

$$\rho^* = arg \min_{\rho} J(\rho). \tag{7}$$

The objective of the criterion (6) is to tune the process response to a desired deterministic response of finite length N in a mean square sense. The first term in (6) is the frequency weighted (by a filter L_y) error between the desired response and the achieved response. The second term is the penalty on the control effort which is frequency weighted by a filter L_u. The filters L_y and L_u can of course be set to 1, but they give added flexibility to the design. As formulated, this is a model reference problem with an additional penalty on the control effort. With $T_d \equiv 1$ this becomes an LQG problem with tracking.

With $T_0(\rho)$ and $S_0(\rho)$ denoting the achieved closed loop response and sensitivity function with the controller $\{C_r(\rho), C_y(\rho)\}$, i.e.,

$$T_0(\rho) = \frac{C_r(\rho)G_0}{1 + C_y(\rho)G_0} \tag{8}$$

$$S_0(\rho) = \frac{1}{1 + C_y(\rho)G_0}, \tag{9}$$

and given the independence of r and v, $J(\rho)$ can be written as

$$J(\rho) = \frac{1}{2N} \sum_{t=1}^{N} \left\{ L_y (y_t^d - T_0(\rho) r_t) \right\}^2 + \frac{1}{2} \mathrm{E}\left[\left\{ L_y S_0(\rho) v \right\}^2 \right] + \lambda \frac{1}{2N} \mathrm{E}\left[\sum_{t=1}^{N} (L_u u_t(\rho))^2 \right]. \tag{10}$$

The first term is the tracking error, the second term is the disturbance contribution, and the last term is the penalty on the control effort.

In the case where a reference model $y^d = T_d r$ is used, the problem setting has close connections with model reference adaptive control (MRAC): see, e.g., [4]. MRAC is based on the minimization of a criterion of the same type as (6) with respect to the controller parameters. To carry out the minimization it is necessary to have an expression for the gradient of this criterion with respect to the controller parameters. As will be seen below, this gradient depends on the transfer function of the unknown closed loop plant. The MRAC solution to this minimization problem is then, essentially, to replace the true closed loop plant by the reference model in the gradient computation. The novel contribution of the IFT approach [19] was to show that, in contrast to the MRAC approach, the gradient can be obtained entirely from input-output data collected on the actual closed loop system, by performing one special experiment on that system. Thus, no approximations are required here to generate the gradient.

Criterion Minimization

We now address the minimization of $J(\rho)$ given by (6) with respect to the controller parameter vector ρ for a controller of prespecified structure. We shall see later how the method can be adapted to handle controllers of increasing complexity. To facilitate the notation we shall in this section assume that $L_y = L_u = 1$. In the section "Implementation Issues" we show how the frequency filters can be incorporated. It is evident from (4) that $J(\rho)$ depends in a fairly complicated way on ρ, on the unknown system G_0 and on the unknown spectrum of v.

To obtain the minimum of $J(\rho)$ we would like to find a solution for ρ to the equation

$$0 = \frac{\partial J}{\partial \rho}(\rho) = \frac{1}{N} \mathrm{E}\left[\sum_{t=1}^{N} \tilde{y}_t(\rho) \frac{\partial \tilde{y}_t}{\partial \rho}(\rho) + \lambda \sum_{t=1}^{N} u_t(\rho) \frac{\partial u_t}{\partial \rho}(\rho) \right]. \tag{11}$$

If the gradient $\partial J / \partial \rho$ could be computed, then the solution of (11) would be obtained by the following iterative algorithm:

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \frac{\partial J}{\partial \rho}(\rho_i). \tag{12}$$

Here R_i is some appropriate positive definite matrix, typically a Gauss-Newton approximation of the Hessian of J, while γ_i is a positive real scalar that determines the step size. The sequence γ_i must obey some constraints for the algorithm to converge to a local minimum of the cost function $J(\rho)$: see [19].

As stated, this problem is intractable since it involves expectations that are unknown. However, such problem can be solved by using a stochastic approximation algorithm of the form (12) such as suggested by Robbins and Monro [33], provided the gradient $\partial J / \partial \rho(\rho_i)$ evaluated at the current controller can be replaced by an unbiased estimate. In order to solve this problem, one thus needs to generate the following quantities:

1. the signals $\tilde{y}(\rho_i)$ and $u(\rho_i)$;
2. the gradients $\partial \tilde{y} / \partial \rho(\rho_i)$ and $\partial u / \partial \rho(\rho_i)$;
3. unbiased estimates of the products $\tilde{y}(\rho_i) \partial \tilde{y} / \partial \rho(\rho_i)$ and $u(\rho_i) \partial u / \partial \rho(\rho_i)$.

The computation of the last two quantities has always been the key stumbling block in solving this direct optimal controller parameter tuning problem. The main contribution of [19] was to show that these quantities can indeed be obtained by performing experiments on the closed loop system formed by the actual system in feedback with the controller $C_r(\rho_i), C_y(\rho_i)$. We now explain how this can be done.

Output related signals

From (4) it is clear that $\tilde{y}(\rho_i)$ is obtained by taking the difference between the achieved response from the system operating with the controller $C(\rho_i)$ and the desired response. As for

$\partial\tilde{y}/\partial\rho(\rho)$, we first note $\partial\tilde{y}/\partial\rho(\rho) = \partial y/\partial\rho(\rho)$. We then have the following expression

$$\begin{aligned}\frac{\partial y}{\partial\rho}(\rho) &= \frac{G_0}{1+C_y(\rho)G_0}\frac{\partial C_r}{\partial\rho}(\rho)r - \frac{C_r(\rho)G_0^2}{(1+C_y(\rho)G_0)^2}\frac{\partial C_y}{\partial\rho}(\rho)r \\ &\quad - \frac{G_0}{(1+C_y(\rho)G_0)^2}\frac{\partial C_y}{\partial\rho}(\rho)v \\ &= \frac{1}{C_r(\rho)}\frac{\partial C_r}{\partial\rho}(\rho)T_0(\rho)r - \frac{1}{C_r(\rho)}\frac{\partial C_y}{\partial\rho}(\rho)([T_0]^2(\rho)r \\ &\quad + T_0(\rho)S_0(\rho)v). \end{aligned} \tag{13}$$

In this expression the quantities $C_r(\rho)$, $\partial C_r/\partial\rho(\rho)$ and $\partial C_y/\partial\rho(\rho)$ are known functions of ρ which depend on the parametrization of the (restricted complexity) controller, while the quantities $T_0(\rho)$ and $S_0(\rho)$ depend on the unknown system and are thus not computable. Therefore, unless an accurate model of the system is assumed to be available (assumption which we shall not make), the signal $\partial y/\partial\rho(\rho)$ can only be obtained by running experiments on the actual closed loop system.

Now observe that the last two terms in (13) involve a double filtering of the signals r and v through the closed loop system. More precisely, notice that

$$[T_0]^2 r + T_0 S_0 v = T_0 y.$$

Therefore, (13) can be rewritten as

$$\begin{aligned}\frac{\partial y}{\partial\rho}(\rho) &= \frac{1}{C_r(\rho)} = \left[\frac{\partial C_r}{\partial\rho}(\rho)T_0(\rho)r - \frac{\partial C_y}{\partial\rho}(\rho)T_0(\rho)y\right] \\ &= \frac{1}{C_r(\rho)}\left[\left(\frac{\partial C_r}{\partial\rho}(\rho) - \frac{\partial C_y}{\partial\rho}(\rho)\right)T_0(\rho)r + \frac{\partial C_y}{\partial\rho}(\rho)T_0(\rho)(r-y)\right].\end{aligned} \tag{14}$$

The last term in (14) can be obtained by substracting the output signal from one experiment on the closed loop system from the reference, and by using this error signal as reference signal in a new experiment. This observation leads us to suggest the following procedure.

In each iteration i of the controller tuning algorithm, we will use three experiments, each of duration N, with the fixed controller $C(\rho_i) \triangleq \{C_r(\rho_i), C_y(\rho_i)\}$ operating on the actual plant. Two of these experiments (the first and third) just consist in collecting data under normal operating conditions; the second is a real (i.e. *special*) experiment. We denote N-length reference signals by r_i^j, $j=1,2,3$, and the corresponding output signals by $y^j(\rho_i)$, $j=1,2,3$. Thus we have

$$r_i^1 = r,\ y^1(\rho_i) = T_0(\rho_i)r + S_0(\rho_i)v_i^1, \tag{15}$$

$$r_i^2 = T_0(\rho_i),\quad y^2(\rho_i) = T_0(\rho_i)(r - y^1(\rho_i)) + S_0(\rho_i)v_i^2, \tag{16}$$

$$r_i^3 = r,\ y^3(\rho_i) = T_0(\rho_i)r + S_0(\rho_i)v_i^3. \tag{17}$$

Here v_i^j denotes the disturbance acting on the system during experiment j at iteration i. These disturbances can be assumed to be mutually independent since they come from different experiments, provided the length N of one experiment is large compared to the correlation time of the disturbances. These experiments yield an exact realization of $\tilde{y}(\rho_i)$:

$$\tilde{y}(\rho_i) = y^1(\rho_i) - y^d, \tag{18}$$

while

$$est\left[\frac{\partial y}{\partial\rho}(\rho_i)\right] \triangleq \frac{1}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial\rho}(\rho_i) - \frac{\partial C_y}{\partial\rho}(\rho_i)\right)y^3(\rho_i) + \frac{\partial C_y}{\partial\rho}(\rho_i)y^2(\rho_i)\right] \tag{19}$$

is a perturbed version (by the disturbances v_i^2 and v_i^3) of $\frac{\partial y}{\partial\rho}(\rho_i)$. (Here and in the sequel, "$est[\partial y/\partial\rho]$" denotes the estimate of $\frac{\partial y}{\partial\rho}$.) Indeed by comparing (19) with (14), using (15)-(17), it is seen that

$$\begin{aligned}& est\left[\frac{\partial y}{\partial\rho}(\rho_i)\right] = \\ & \frac{\partial y}{\partial\rho}(\rho_i) + \frac{S_0(\rho_i)}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial\rho}(\rho_i) - \frac{\partial C_y}{\partial\rho}(\rho_i)\right)v_i^3 + \frac{\partial C_y}{\partial\rho}(\rho_i)v_i^2\right].\end{aligned} \tag{20}$$

Two things are worth observing. First, the disturbance generated in the first experiment is not a nuisance. The output of the first experiment is used in (18) to create an exact version of the signal $\tilde{y}(\rho_i)$ which is used in the criterion J: see (4). Secondly, the output of the first experiment (with the disturbance) is exactly what is needed as reference signal in the second experiment to generate an estimate of $\frac{\partial y}{\partial\rho}$: compare (16) with the second term of (13). The only nuisances that are introduced are the disturbance contributions from the second and third experiments.

Input Related Signals

It is possible to use the measurements of the process input generated from the three experiments using the reference signals (15)-(17) to generate an estimate of the sensitivity function $\frac{\partial u}{\partial\rho}(\rho_i)$. From

$$u(\rho) = \frac{C_r(\rho)}{1+C_y(\rho)G_0}r - \frac{C_y(\rho)}{1+C_y(\rho)G_0}v = S_0(\rho)[C_r(\rho)r - C_y(\rho)v]$$

and

$$\frac{\partial S_0}{\partial\rho}(\rho) = -\frac{1}{C_r}T_0(\rho)S_0(\rho)\frac{\partial C_y}{\partial\rho}(\rho)$$

it follows that

$$\begin{aligned}\frac{\partial u}{\partial\rho}(\rho) &= S_0(\rho)\left[\frac{\partial C_r}{\partial\rho}r - \frac{\partial C_y}{\partial\rho}(\rho)[T_0(\rho)r + S_0(\rho)v]\right] \\ &= S_0(\rho)\left[\frac{\partial C_r}{\partial\rho}r - \frac{\partial C_y}{\partial\rho}(\rho)y\right] \\ &= S_0(\rho)\left[\frac{\partial C_r}{\partial\rho}(\rho) - \frac{\partial C_y}{\partial\rho}(\rho)r + \frac{\partial C_y}{\partial\rho}(\rho)(r-y)\right].\end{aligned} \tag{21}$$

The experiments with reference signals defined as in (15)-(17) give the following input signals

$$u^1(\rho_i) = S_0(\rho_i)[C_r(\rho_i)r - C_y(\rho_i)v_i^1], \tag{22}$$

$$u^2(\rho_i) = S_0(\rho_i)[C_r(\rho_i)(r - y^1(\rho_i)) - C_y(\rho_i)v_i^2], \tag{23}$$

$$u^3(\rho_i) = S_0(\rho_i)[C_r(\rho_i)r - C_y(\rho_i)v_i^3]. \tag{24}$$

Thus, $u^1(\rho_i)$ is a perfect realization of $u(\rho_i)$,

$$u(\rho_i) = u^1(\rho_i), \tag{25}$$

while

$$est\left[\frac{\partial u}{\partial \rho}(\rho_i)\right] \triangleq \frac{1}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right)u^3(\rho_i) + \frac{\partial C_y}{\partial \rho}(\rho_i)u^2(\rho_i)\right] \tag{26}$$

is a perturbed version of $\frac{\partial u}{\partial \rho}(\rho_i)$. Indeed a comparison of (26) with (21) shows that

$$est\left[\frac{\partial u}{\partial \rho}(\rho_i)\right] = \frac{\partial u}{\partial \rho}(\rho_i) - \frac{C_y(\rho_i)S_0(\rho_i)}{C_r(\rho_i)}\left[\frac{\partial C_y}{\partial \rho}(\rho_i)v_i^2 + \left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right)v_i^3\right]. \tag{27}$$

An estimate of the gradient

With the signals defined in the preceding subsections, an experimentally based estimate of the gradient of J can be formed by taking

$$est\left[\frac{\partial J}{\partial \rho}(\rho_i)\right] = \frac{1}{N}\sum_{t=1}^{N}\left(\tilde{y}_t(\rho_i)\, est\left[\frac{\partial y_t}{\partial \rho}(\rho_i)\right] + \lambda u_t(\rho_i)\, est\left[\frac{\partial u_t}{\partial \rho}(\rho_i)\right]\right). \tag{28}$$

For a stochastic approximation algorithm to work, it is required that this estimate be unbiased, that is we need:

$$\mathrm{E}\left\{est\left[\frac{\partial J}{\partial \rho}(\rho_i)\right]\right\} = \frac{\partial J}{\partial \rho}(\rho_i). \tag{29}$$

The key feature of our construction of $est[\partial J / \partial\rho(\rho_i)]$, and also the motivation for the third experiment, is that this unbiasedness property holds. It would indeed be tempting to use the data from the first experiment instead of the third one in (19) and (26), but then (29) would not hold because the error between $est[\partial y / \partial\rho(\rho_i)]$ and $\frac{\partial y}{\partial \rho}(\rho_i)$ would be correlated with $\tilde{y}(\rho_i)$, and the error between $est[\partial u / \partial\rho(\rho_i)]$ and $\frac{\partial u}{\partial \rho}(\rho_i)$ would be correlated with $u(\rho_i)$.

Initial Conditions

The above derivations of the gradient assume that the initial conditions of the plant and the controllers are the same in all experiments. However, if the experiment interval N is sufficiently large these transient effects can be neglected.

The Algorithm

We now summarize the algorithm.

Algorithm With a controller $C(\rho_i) = [C_r(\rho_i), C_y(\rho_i)]$ operating on the plant, generate the signals $y^1(\rho_i), y^2(\rho_i), y^3(\rho_i)$ of (15)–(17), the signals $u^1(\rho_i), u^2(\rho_i), u^3(\rho_i)$ of (22)–(24) and compute $\tilde{y}(\rho_i)$, $est[\partial y / \partial\rho(\rho_i)]$, $u(\rho_i)$ and $est[\partial u / \partial\rho(\rho_i)]$ using (18), (19), (25) and (26). Let the next controller parameters be computed by:

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} est\left[\frac{\partial J}{\partial \rho}(\rho_i)\right] \tag{30}$$

where $est[\partial J / \partial\rho(\rho_i)]$ is given by (28), where γ_i is a sequence of positive real numbers that determines the step size and where R_i is a sequence of positive definite matrices that are, for example, given by (33). Repeat this step, replacing i by $i+1$.

Nonlinear systems

It has been shown [16], [5], [36] that for nonlinear feedback systems, the true gradient should be generated by the linear time-varying system that is obtained by linearizing the nonlinear system around the system trajectory under normal operating conditions. In [16] it is argued that this time-varying linearized system can be approximated by the nonlinear system itself, hence that *exactly* the same procedure as derived above can be applied to nonlinear systems as well. For further details, see [16]. An application of IFT to controller optimization for a nonlinear system, a DC-servo with backlash, is presented in the penultimate section.

Convergence

In this section we state exact conditions for which the controller parameters updated with the Algorithm converge to the set of stationary points of the criterion (6). Following the formal result is a discussion of its interpretation.

Let D be a convex compact subset of $\mathbf{R}^{n_\rho}$. We introduce the following assumptions on the noise, the controller, the closed loop system and the step sizes of the algorithm, respectively.

V1) In any experiment, the signal sequence v_t, $t = 1, \ldots, N$ consists of zero mean random variables which are bounded: $|v_t| \le C$ for all t. The constant C and the second order statistics of v_t are the same for all experiments, while sequences from different experiments are mutually independent.

C1) There exists a neighbourhood O to D such that the controller $C(\rho)$ is two times continuously differentiable w.r.t ρ in O.

C2) All elements of the transfer functions $C_r(\rho)$, $C_y(\rho)$, $\frac{\partial C_r}{\partial \rho}(\rho)$, $\frac{\partial C_y}{\partial \rho}(\rho)$, $\frac{\partial^2 C_r}{\partial \rho^2}(\rho)$, $\frac{\partial^2 C_y}{\partial \rho^2}(\rho)$ have their poles and zeros uniformly bounded away from the unit circle on D.

S1) The linear time-invariant closed loop systems represented by (1) and (2) are stable and have all their poles uniformly bounded away from the unit circle on D.

A1) The elements of the sequence γ_i satisfy $\gamma_i \ge 0$ and $\sum_{i=1}^{\infty}\gamma_i = \infty$.

A2) The elements of the sequence γ_i satisfy $\sum_{i=1}^{\infty}\gamma_i^2 < \infty$.

Theorem Consider the Algorithm. Assume that V1, C1, C2, S1, A1 and A2 hold. Suppose that R_i is a symmetric matrix which is generated by the experiments at iteration i and satisfies $\frac{1}{\delta}I \geq R_i I \geq \delta I$ for some $\delta > 0$. Then

$$\lim_{i\to\infty}\rho_i = D_c \triangleq \{\rho : J'(\rho) = 0\} \qquad \text{w.p.1} \tag{31}$$

on the set $A = \{\rho_i \in D \ \forall i\}$.

The proof of the theorem can be found in [15]. The basic requirement for convergence is that the signals remain bounded throughout the iterations, since the result only applies to the set A introduced in the Theorem.

The power of the theorem is that there are no assumptions on the properties of the system other than linearity and time-invariance. The same holds for the controller: the complexity of the controller is arbitrary and the result thus applies to simple PID controllers as well as to more complex ones.

It is also important to notice that even though the disturbances have to have the same second order statistics from experiment to experiment, it is *not* necessary that the disturbances are stationary during one experiment.

Implementation Issues

In this section we briefly comment on some aspects of the implementation of the scheme.

Non-minimum phase or unstable controllers

Notice that the computation of $est[\partial y / \partial\rho(\rho_i)]$ in (19) and $est[\partial u / \partial\rho(\rho_i)]$ in (26) requires the filtering with the inverse of C_r. If C_r is non-minimum phase, as may happen, this is not feasible with a causal stable filter. A similar situation occurs if the gradients of C_y and/or C_r are unstable. However, since the data are collected batch-wise the gradient can be computed by a non-causal stable filter which, modulo transients, gives the gradient. An alternative way is to extend L_y and L_u with an all-pass frequency weighting filter L_a, which leaves the objective function $J(\rho)$ of (6) unchanged. This procedure is equivalent to non-causal filtering for large experiment intervals N. We illustrate the procedure for the case of a non-minimum phase C_r.

Let $C_r(\rho_i)$ be factorized as

$$C_r(\rho_i) = \frac{n_u n_s}{d},$$

where the factor n_u

$$n_u = \Pi_{k=1}^{m}(1 - z_k q^{-1})$$

contains all the unstable zeros and nothing else. At iteration i let L_a be the following all-pass filter

$$L_a \stackrel{def}{=} \frac{n_u}{n_u^*},$$

where

$$n_u^* = \Pi_{k=1}^{m}(z_k - q^{-1}).$$

Then

$$L_a\left\{est\left[\frac{\partial y}{\partial\rho}(\rho_i)\right]\right\} = \frac{d}{n_u^* n_s}\left[\left(\frac{\partial C_r}{\partial\rho}(\rho_i) - \frac{\partial C_y}{\partial\rho}(\rho_i)\right)y^3(\rho_i) + \frac{\partial C_y}{\partial\rho}(\rho_i)y^2(\rho_i)\right] \tag{32}$$

which is stable. Thus, if L_y and L_u both contain L_a it is possible to compute the gradients. If necessary, this filtering operation should be performed at each iteration.

Modification of the Search Direction

There are many possible choices for the matrix R_i in the iteration (12). The identity matrix gives the negative gradient direction. Another interesting choice is

$$R_i = \frac{1}{N}\sum_{t=1}^{N}\left(est\left[\frac{\partial y_t}{\partial\rho}(\rho_i)\right] est\left[\frac{\partial y_t}{\partial\rho}(\rho_i)\right]^T + \lambda\, est\left[\frac{\partial u_t}{\partial\rho}(\rho_i)\right] est\left[\frac{\partial u_t}{\partial\rho}(\rho_i)\right]^T\right), \tag{33}$$

for which the signals are available from the experiments described above. This will give a biased (due to the disturbance in the second experiment) approximation of the Gauss-Newton direction. It is the authors' experience that this choice is superior to the pure gradient direction.

One Degree of Freedom Controllers

In the case where the simplified controller structure $C_r = C_y \triangleq C$ is used, i.e.,

$$u = C(\rho)(r - y),$$

the algorithm simplifies because the third experiment becomes unnecessary. Indeed, it follows immediately from expressions (14), (19), (21), and (26) that the first term in all these expressions is zero. Therefore, in the case of a one degree of freedom controller, the first two experiments are run with the same reference signals as indicated in (15) and (16), and the gradient estimates are obtained as special cases of (19) and (26):

$$est\left[\frac{\partial y}{\partial\rho}(\rho_i)\right] = \frac{1}{C(\rho_i)}\frac{\partial C}{\partial\rho}(\rho_i)y^2(\rho_i) \tag{34}$$

$$est\left[\frac{\partial u}{\partial\rho}(\rho_i)\right] = \frac{1}{C(\rho_i)}\frac{\partial C}{\partial\rho}(\rho_i)u^2(\rho_i). \tag{35}$$

These are perturbed estimates of the actual gradients:

$$\begin{aligned} est\left[\frac{\partial y}{\partial\rho}(\rho_i)\right] &= \frac{\partial y}{\partial\rho}(\rho_i) + \frac{1}{C(\rho_i)}\frac{\partial C}{\partial\rho}(\rho_i)S_0(\rho_i)v_i^2 \\ est\left[\frac{\partial u}{\partial\rho}(\rho_i)\right] &= \frac{\partial u}{\partial\rho}(\rho_i) - \frac{\partial C}{\partial\rho}(\rho_i)S_0(\rho_i)v_i^2. \end{aligned} \tag{36}$$

Disturbance Rejection Problem

The disturbance rejection problem is a special case of the one degree of freedom controller. The controller can be optimally tuned using iterations consisting of the same two experiments as just described in which the reference signal is put to zero. Thus, do two experiments with reference signals

$$r_i^1 = 0, \tag{37}$$

$$r_i^2 = -y_1(\rho_i). \tag{38}$$

Then take (34) and (35) as gradient estimates. Observe that, in the disturbance rejection case, the tuning of the controller parameter vector is entirely driven by the disturbance signal. This is in contrast with all identification-based iterative controller tuning schemes, where identifiability requires the injection of a sufficiently rich reference signal even in a disturbance rejection framework.

Disturbance Attenuation

As noted earlier, (20) contains an undesirable perturbation from the disturbances in the second and third experiments. Even though the influence of these disturbances is partly averaged out when $est[\partial J / \partial\rho(\rho_i)]$ is formed, it is, of course, of interest to make this perturbation as small as possible. One way to decrease the influence is to increase the signal-to-noise ratio in these experiments. Let W_i^j, $j = 2,3$ be two stable and inversely stable filters and replace (16) and (17) by

$$r_i^2 = W_i^2(r - y^1(\rho_i)), \;\; r_i^3 = W_i^3 r \tag{39}$$

respectively, and replace $y^j(\rho_i)$ in (19) by $[W_i^j]^{-1} y^j(\rho_i)$. Then

$$est\left[\frac{\partial y}{\partial\rho}(\rho_i)\right] = \frac{\partial y}{\partial\rho}(\rho_i)$$
$$+\frac{S_0(\rho_i)}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial\rho}(\rho_i) - \frac{\partial C_y}{\partial\rho}(\rho_i)\right)[W_i^3]^{-1}v_i^3 + \frac{\partial C_y}{\partial\rho}(\rho_i)[Wi^2]^{-1}v_i^2\right]. \tag{40}$$

Thus, for frequencies where W_i^j has a gain larger than one, the influence of the nuisance disturbances is decreased. For further details, see [17].

Design Choices

As should be apparent from the Algorithm, the IFT scheme is quite simple. Apart from the choice of step-size, the only thing that the user has to be concerned with is the choice of a criterion. This choice is usually not critical either and can be done in a simple way based on the observed signals of the system during normal operation. However, there are some fallacies to be avoided when the criterion is specified and these will be discussed in this section. In addition to this, we discuss various design choices which allow the user to inject prior information or to translate time or frequency domain performance specifications in the framework of the minimization of a LQG-like performance index.

The criterion in the frequency domain

It is important to realize that the properties of the resulting feedback system depend entirely on the criterion function. It is thus only indirectly through the user controlled quantities, such as reference signal, reference model, controller structure and the different weightings, that quantities like the sensitivity function, closed loop response, stability margins etc. can be influenced. The exact relation between these quantities is of course very complicated but some insight can be gained by studying expressions of the criterion in the frequency domain.

Assuming that a reference model $y^d = T_d r$ is used, via Parseval's formula (10) can be transformed into

$$J(\rho) \approx \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{|T_0(\rho) - T_d|^2|L_y|^2\Phi_{rr} + |L_y|^2\,|S_0(\rho)|^2\Phi_{vv}\right.$$
$$\left. + \lambda|L_u|^2\Phi_{uu}(\rho)\right\}d\omega$$
$$= \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{[|T_0(\rho) - T_d|^2|L_y|^2 + \lambda|L_u|^2|\,S_0(\rho)|^2|C_r(\rho)|^2]\Phi_{rr}\right.$$
$$\left. + [|L_y|^2 + \lambda|L_u|^2|C_y(\rho)|^2]|\,S_0(\rho)|^2\Phi_{vv}\right\}d\omega \tag{41}$$

where Φ_{rr}, Φ_{vv} and $\Phi_{uu}(\rho)$ are the spectra of r, v and u, respectively. (The approximation is due to the finite number of samples. Φ_{rr} and one term of Φ_{uu} are linespectra originating from the finite sequence $\{r_t\}_{t=1}^{N}$.)

From (41), it is obvious that at frequencies where the spectrum of r dominates that of v, and provided $\lambda|L_u|^2$ is small compared to $|L_y|^2$, the controller parameters will converge to a value that makes the closed loop response T_0 close to the reference model T_d. Hence, the user can control the closed loop response by proper choices of the design parameters $\sqrt{\lambda}\; L_u$, L_y, Φ_{rr} and T_d.

It is important that the external signals be sufficiently rich since otherwise the criterion may have a long valley where it is close to a minimum, which implies that the problem is ill conditioned. In such cases one may even have destabilizing controllers which are (close to) optimal which clearly is undesirable. Note from (41) that, in contrast with what happens with all schemes based on identification, this excitation condition need not necessarily be a requirement on the external reference signal r. In the case of a disturbance rejection objective, the first term in (41) is zero, and the tuning of the controller parameters will be driven by the noise; thus, Φ_{vv} need to be sufficiently rich. This confirms our observation following (38).

The Sensitivity Function

It is also clear from (41) that the sensitivity S_0 will be small at frequencies where the disturbance spectrum Φ_{vv} is large or where $\lambda|L_u|^2$ is large compared to $|L_y|^2$. Since in general the user cannot influence the disturbances, we consider various ways in which the user may influence the sensitivity function.

One-degree-of-freedom controllers. When $C_r = C_y = C$, the closed loop response T_0 and the sensitivity S_0 are complementary, i.e., $T_0 + S_0 = 1$. Hence, any manipulation of the closed loop response also influences the sensitivity function. In fact, by setting $T_d = 1$, the expression (41) for the criterion becomes

$$J(\rho) \approx \frac{1}{4\pi}\int_{-\pi}^{\pi}\left(|L_y|^2 + \lambda|L_u|^2|C(\rho)|^2\right)|\,S_0(\rho)|^2(\Phi_{rr} + \Phi_{vv})d\omega. \tag{42}$$

This expression shows clearly how the sensitivity function can be manipulated by proper choice of the design parameters

$\sqrt{\lambda}L_u$, L_y and Φ_{rr}: observe that the reference spectrum may be used to control the sensitivity function in exactly the same way as a disturbance does.

Two-degree-of-freedom controllers. In the case of a two degree of freedom controller, there is no direct connection between the closed loop response T_0 and the sensitivity function S_0. Still, Equation (41) shows how the sensitivity can be manipulated to a certain extent through the design parameters $\sqrt{\lambda}L_u$, L_y and Φ_{rr}. Alternatively, one can add synthetic disturbances at the input or output of the process during the first experiment to manipulate the sensitivity, as has been done in [7] and [20].

Ensuring Integral Action

To suppress low frequency disturbances and to ensure correct static gain, it is standard to include a fixed integrator in a controller. Under certain conditions it may happen that the free part of the controller tries to cancel the fixed integrator. The reason for this is that an integrator improves the performance at low frequencies but deteriorates the performance at high frequencies since it adds a phase lag which decreases the stability margin. Hence, whether or not an integrator improves the overall performance depends on whether or not the benefit at low frequencies outweighs the disadvantages at high frequencies. For a given set-up it may happen that the optimal choice is to select the free controller parameters such that the integrator is cancelled.

In practice this phenomenon becomes a much more severe problem than just the problem that the integrator is cancelled. The reason is that the zero in the controller that tries to cancel the integral action is close to one. However, due to finite data and disturbances, it will be either slightly less or slightly larger than one and, depending on this, the static gain of the controller will be either positive or negative. Thus, in such a situation one risks ending up with a controller which destabilizes the system due to incorrect sign of the static gain.

The key to avoiding the cancellation problem is to set-up the criterion in such a way that integral action is necessary in the controller. The basic rule is to ensure that the low frequency band plays a significant role in the criterion. However, how much emphasis is needed in the low frequency band depends on the low frequency behavior of the system G_0 that is going to be controlled. For one degree of freedom controllers, simple actions like the inclusion of frequency weighting filters (see below) and/or the use of a reference signal with sufficient power at low frequencies are usually sufficient when the static gain of the system G_0 is low. Another possibility is to use time dependent weighting factors in the criterion; see the subsection on minimization of the settling time below. However, as will be shown below, this is not enough if the system G_0 has high gain at low frequencies, for example, when G_0 itself contains an integrator. Controllers with two degrees of freedom also require special attention.

Consider the case where the open loop system G_0 contains an integrator, i.e., the static gain is infinite. Assume also, for simplicity, a one degree of freedom controller $C_r = C_y = C(\rho) = C_{free}(\rho)C_{fix}$ consisting of a free part $C_{free}(\rho)$ and a fixed part C_{fix} which is an integrator. Furthermore, assume that a reference model $y^d = T_d r$ with static gain 1 is used. To focus the attention on the low frequency behavior, finally assume that a step reference signal is used and that the length of the experiment is much longer than the time constant of the system.

Suppose now that the disturbance is so small that it can be neglected and that the input u is not penalized in the criterion (10). Then it is clear that by choosing C_{free} such that

$$C_{free}(e^{i\omega}) \approx \frac{T_d(e^{i\omega})}{1-T_d(e^{i\omega})} C_{fix}^{-1}(e^{i\omega}) G_0^{-1}(e^{i\omega}) \tag{43}$$

at low frequencies, the criterion (10) becomes almost zero and C_{free} is thus close to the overall optimal controller. Notice that since, by assumption, the reference model T_d has static gain 1, the factor $\frac{T_d}{1-T_d}$ contains an integrator. Hence, since both C_{fix} and G_0 contain integrators, (43) implies that the gain of C_{free} should be small at low frequencies. But this is equivalent to saying that C_{free} should (at least approximately) cancel the integrator in C_{fix}. Hence, cancellation of the integral action will occur even if the low frequency band is emphasized in this case. Intuitively, the reason is of course that integral action in the controller is superfluous since the only static property that enters in the criterion is the static gain of the closed loop and the integrator in the system ensures that this gain is correct. This is a situation where the control engineer has erroneously put an integrator in the regulator where it is not needed. IFT is not designed to check the validity of the chosen regulator structure or to automatically correct the control engineer's errors; however, a monitoring of the parameters (or better of the regulator zeros) during the iterations will warn the operator that he or she has made a poor design. Similar arguments can be applied to systems which do not contain an integrator but which have high static gain.

If it is desired that the controller should contain an integrator when the system has high low frequency gain, additional measures must be taken to further emphasize the low frequency band. A very efficient way is to add a synthetic low frequency input load disturbance such as, e.g., a step. Then the criterion contains a term which will be small only if

$$\frac{G_0(e^{i\omega})}{1+C(e^{i\omega})G_0(e^{i\omega})}$$

is small at low frequencies; this is possible only if the controller C has large static gain.

Another possibility is to add a ramp-like synthetic output load disturbance. Using the initial value theorem, the steady state error becomes

$$\lim_{z\to 1} \frac{1}{1+C(z)G_0(z)} \frac{z}{z-1}$$

and for this to become small the static gain of C has to be large. From the discussion on the sensitivity function above it follows that an equivalent way of doing this is to set the reference model to $T_d = 1$ and use a ramp-like reference.

When the controller has two degrees of freedom, the correct closed loop static gain can be obtained by proper choice of the precompensator which implies that from a tracking point of view, the integrator does not play any role. Hence, unless low frequency disturbances are present, there is a risk of ending up with the free controller trying to cancel the integral action regardless of the low frequency properties of G_0. Adding synthetic low frequency input disturbances will counteract this when the static gain of G_0 is high, c.f. the discussion above. When the static gain of G_0 is low, it suffices to add low frequency output disturbances.

In this situation another possibility is to use a controller structure for which cancellation is impossible. One can, for example, use the non-interacting form

$$C(\rho) = \frac{\alpha}{1-q^{-1}} + \bar{C}(\rho)$$

where α is a preset fixed constant and $\bar{C}(\rho)$ is the part of the controller which contains the tunable parameters and which does not contain any integrator.

Adjusting the Reference Model or Reference Trajectory

The choice of reference model or of reference trajectory is perhaps the key design decision. This is where the user can increase the speed of convergence of the algorithm significantly by injecting prior information (if any) about quantities like the delay of the system or the achievable closed loop bandwidth. If the initial controller gives bad performance, it can be quite tricky to find the optimal controller, that is, the surface of the criterion can be very rough, thus allowing only small steps in each iteration. However, it is the authors' experience that the problem is simplified by starting with an objective that is easier to achieve (lower bandwidth) and then successively increasing the bandwidth as the achieved performance is increased. The easiest method of implementation of this principle is not to use a reference model, but rather to draw the new desired reference trajectory y^d as a small modification (i.e., a small improvement) over the last achieved output response y. This has close ties with the so called windsurfing approach [24] to iterative control design.

Minimizing the Settling Time

The criterion (6) is well suited when the objective is to follow a specific reference trajectory, but is not so appropriate if the objective is to change the output from one setpoint to another one. Indeed, in such case the goal is typically to reach the new setpoint with a minimum settling time, and one does not care about the transient trajectory, provided it does not produce too much overshoot. By constraining the output to follow some particular reference trajectory y^d during the transient, one puts too much emphasis on the transient phase of the response at the expense of the settling time at the new setpoint value.

One easy way to cope with this situation is to add nonnegative weighting factors w_t^y and w_t^u to each element of $\tilde{y}_t$ and u_t in the criterion (6):

$$J(\rho) = \frac{1}{2N} \mathrm{E}\left[\sum_{t=1}^{N} w_t^y (L_y \tilde{y}_t(\rho))^2 + \lambda \sum_{t=1}^{N} w_t^u (L_u u_t(\rho))^2\right]. \qquad (44)$$

ISTE or ITSE criteria [2] can be obtained by using time-dependent weighting factors. The simplest way to obtain a satisfactory closed loop response to a desired setpoint change is to set the weighting factors w_t^y to zero during the transient period and to one afterwards. Often it is not known a priori how much time is required to achieve a setpoint change without overshoot. In such case, one can perform the IFT iterations by initially applying zero weights w_t^y over a long transient period, and then gradually reducing the length of this "zero weight window" until oscillations start occuring. This is an application to the change of a required setpoint change of the recommendation in the previous subsection that the closed loop bandwidth be gradually increased. The idea of using zero weight windows for the minimization of the settling time problem was initially proposed by Lequin [25]. This procedure has been extensively experimented with in [37] for the case of Iterative Feedback Tuning of PID controllers designed for setpoint changes. It has led to an automatic and efficient procedure for the selection of the time-weighted cost function in the case of IFT applied to setpoint changes.

Frequency Weighting

In Section 3 the algorithm has been derived under the assumption $L_y = L_u = 1$, for simplicity. In the general case we obtain the following.

$$\tilde{y}(\rho_i) = L_y(y^1(\rho_i) - y^d) \qquad (45)$$

is a realization of $\tilde{y}(\rho_i)$, and the gradient signal is obtained by the filtering operation

$$est\left[\frac{\partial y}{\partial \rho}(\rho_i)\right] \triangleq \frac{L_y}{C_r(\rho_i)}\left[\left(\frac{\partial C_r}{\partial \rho}(\rho_i) - \frac{\partial C_y}{\partial \rho}(\rho_i)\right) y^3(\rho_i) + \frac{\partial C_y}{\partial \rho}(\rho_i) y^2(\rho_i)\right]. \qquad (46)$$

Thus, a frequency weighting of the output is obtained by simply filtering all output signals through L_y. By the same arguments, a frequency weighting on u is obtained by filtering the input signals from the three experiments (22)–(24) through L_u.

The frequency weighting filters can be used to focus the attention of the controller on specific frequency bands in the input and/or output response of the closed loop system, for example, to suppress undesirable oscillations in these signals. Conversely, they can be used as notch filters in the frequency bands where the measurement noise dominates. They can also be used to meet specific frequency domain performance specifications, such as constraints on the sensitivities. The use of these filters has been illustrated in the benchmark application described in [20].

Controller Complexity Modification

The method has been described as one in which successive adjustments are being made to the controller parameter vector of a controller of fixed complexity. However, it is straightforward to extend the complexity of the controller at any given iteration if the parametrization of the new one is an extension of the old one. This is useful if one realizes that the current controller is incapable of achieving the desired objective even after convergence to its optimal value. This idea has also been illustrated in [20].

Interactive Controller Update

The step size can be used to control how much a controller changes from one iteration to another. Before actually implementing a controller it is possible to compare the Bode plots of the new controller with the previous ones to see whether they are reasonably consistent. If one doubts whether it will work or not one has the possibility of decreasing the step size and/or of extending the experiment so as to reduce the effects of the disturbances in the gradient calculation. The situation is quite comforting: one is backed up by the knowledge that for a small enough step size and large enough data set one will always go in a descent direction of the criterion. The step size can also be optimized along the gradient direction by line search optimization.

Prediction of the New Control Performance

In addition to plotting the Bode plot of a new controller, one can also predict its effect on the closed loop response and on the achieved cost using a Taylor series expansion. To see this, we denote

$$\Delta\rho_i = \rho_{i+1} - \rho_{i.} \tag{47}$$

Using Taylor series expansions, we have the following predictions:

$$\tilde{y}_t(\rho_{i+1}) \approx \tilde{y}_t(\rho_i) + est\left[\frac{\partial \tilde{y}_t}{\partial \rho}(\rho_i)\right]^T \Delta\rho_i \tag{48}$$

$$u_t(\rho_{i+1}) \approx u_t(\rho_i) + est\left[\frac{\partial u_t}{\partial \rho}(\rho_i)\right]^T \Delta\rho_i \tag{49}$$

$$J(\rho_{i+1}) \approx J(\rho_i) - \gamma_i \; est\left[\frac{\partial J}{\partial \rho}(\rho_i)\right]^T R_i^{-1} \; est\left[\frac{\partial J}{\partial \rho}(\rho_i)\right], \tag{50}$$

with R_i defined by (33). The last expression follows from (30) and is valid as long as R_i is a good approximation of the Hessian of $J(\rho_i)$. A comparison of $\tilde{y}_t(\rho_{i+1})$ with $\tilde{y}_t(\rho_i)$, of $u_t(\rho_{i+1})$ with $u_t(\rho_i)$, and of $J(\rho_{i+1})$ with $J(\rho_i)$ can help the user decide whether the step size that has led to the new controller was appropriate or not. In the section after the next one, we shall illustrate on an industrial application how the predicted performance compares with the performance that was actually achieved with the new controller.

On-Line Considerations

The second experiment is the only *special purpose experiment*, in that it uses a different reference signal than the desired one, namely $r - y^1$. This experiment reinjects into the closed loop system a signal, y^1, that contains noise, thereby producing an output, denoted y^2, that contains the sum of two noise contributions. However, note that the contribution from the disturbance v_i^2 is exactly as under normal operating conditions. As for the contribution from the disturbance in the first experiment, $T_0(\rho_i)S_0(\rho_i)v_i^1$, it is essentially a bandpass filtered version of the normal disturbance contribution $S_0(\rho_i)v_i^1$ and should normally be small since (at least for a one degree of freedom controller) $S_0 + T_0 = 1$.

There are cases, however, where the additional noise injected in the reference input during the second experiment causes unacceptable behaviour in some of the states or even in the output of the system during that experiment. This has been observed, for example, in mechanical applications with flexible structures, where the noise present in the reference input during the second experiment caused excessive vibrations. This problem essentially arises during the initial iterations of the controller tuning, that is, before the improvements in achieved controller performance outweigh the deterioration due to the noisy reference signal.

One way to address this problem is to use the filtering idea (39) presented earlier. For example, by setting W_i^2 to a constant less than 1, the excitation will be decreased. The penalty for this is that the influence of the disturbance will be amplified. An alternative way is to replace, in the initial iterations, the data-driven computations of the gradient of the cost criterion by an estimate of this gradient based on an identified model of the closed loop system. As soon as the improvement in closed loop performance achieved by the successive controllers outweighs the degradation due to the second experiment, one can then switch to the data-driven (i.e., IFT-based) computation of the gradient. This idea of using identified models during the initial iterations has been proposed and studied in [6].

Applications In The Chemical Industry

The IFT scheme has been applied by the chemical multinational Solvay S.A. for the optimal tuning of PID controllers operating on a range of different control loops. In each of these loops, PID controllers were already operating. Important performance improvements were achieved using the IFT method, both in tracking and in regulation applications. The reductions in variance achieved after a few (typically 2 to 6) iterations of the algorithm range from 25% in a flow regulation problem in an evaporator, to 87% in a temperature control problem for the tray of a distillation column, with other applications involving temperature control in furnaces. Here we present the results obtained on two such control loops. The first one is a temperature regulation problem for a tray of a distillation column, while the second illustrates the application of the algorithm to a setpoint modification problem in the flow of an evaporator.

The PID Controller

The same controller has been used in both loops. It differs slightly from standard PID in the following aspects:

- The derivative action is calculated on y and not on the control error.
- In order to limit the gain of the controller at high frequencies when the derivative action is used, a first order filter is applied to y before any calculation. The time constant of this filter is chosen as $Td/8$, Td being the derivative time constant.

The PID controller must therefore be considered as a 2-degree-of-freedom controller with common parameters.

Temperature Regulation in a Distillation Column

This first industrial application is a temperature regulation problem in a tray of a distillation column. The PID regulator parameters were iteratively tuned using the IFT scheme, with the following design choices: Gauss-Newton direction, step-size $\gamma_i = 1 \forall i$, control weighting $\lambda = 0$, sampling period of 15 seconds, $r_d = y_d = 0$ during 5 hours. The deadtime and the time constants of the process were unknown.

Fig. 2 presents temperature deviations with respect to setpoint in a tray of a distillation column, over a 24-hour period, first with the original tuning, then with the PID controller obtained after 6 iterations of the new scheme. Fig. 3 shows the corresponding histograms of these deviations over two-week periods. The control error has been reduced by 70%.

In Fig. 4 we show the Bode plots of the two-degree-of-freedom controller (C_r, C_y) before optimal tuning (full line), after three iterations of the IFT algorithm (dashed line) and after six iterations (dotted line). The gain was too low and the derivative action underused.

As mentioned in the section on design choices, an estimation of the new cost J can be made at the end of each iteration using a Taylor series expansion. Table 1 shows, for the six iterations, the cost J calculated with the first experiment as well as the predicted value with the new controller parameters. The prediction is good except for the second iteration which was perturbed by an abnormal disturbance.

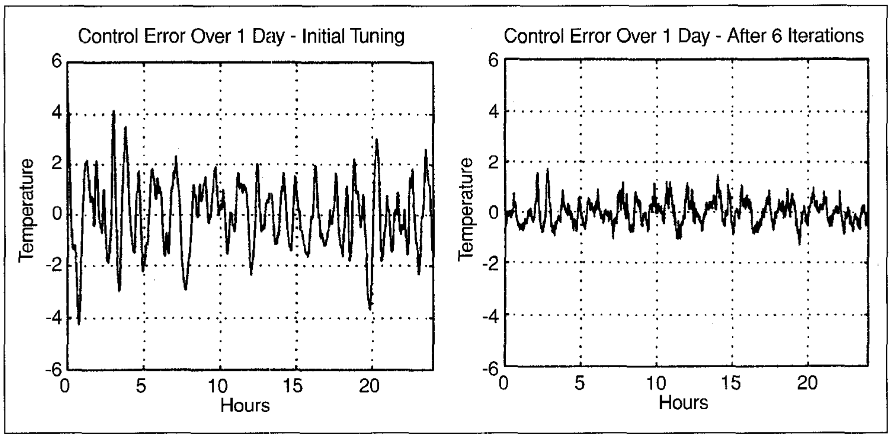


Fig. 2. Control error over a 24-hour period before optimal tuning and after six iterations of the IFT algorithm.

Flow control of an evaporator

In this case, the objective was to increase the tracking performance of the control loop during changes of production rate. We chose a two-phase reference signal r_d: a ramp of three minutes followed by a constant value of 12 minutes. The other design choices were: Gauss-Newton direction, step-size $\gamma_i = 1 \forall i$, control weighting $\lambda = 0$, sampling period of two seconds, $y_d = \frac{1}{1+20s} r_d$. The process had an apparent deadtime of 30 seconds, but the time constants were unknown.

The top part of Fig. 5 shows the closed loop response during the transient (first five minutes of the experiments) with the initial tuning and after three iterations. The bottom part represents a histogram of the corresponding tracking error $y_d - y$.

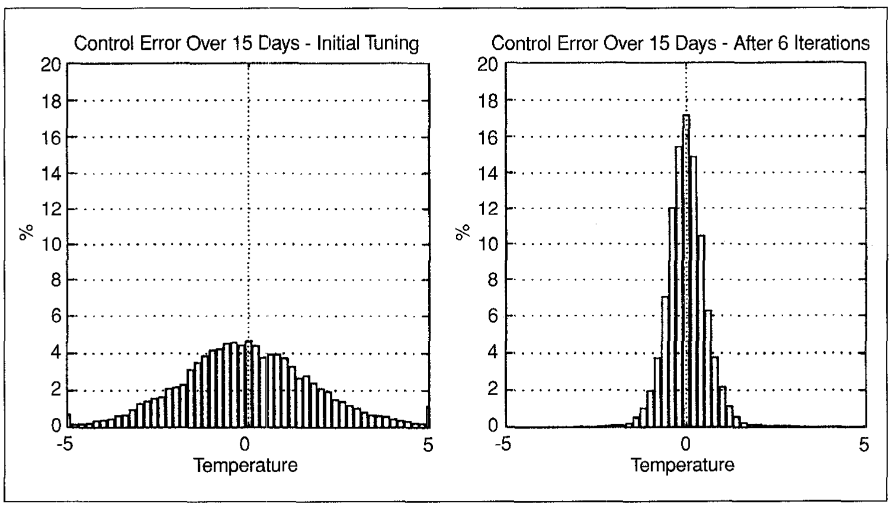


Fig. 3. Histogram of control error over two-week period before optimal tuning and after six iterations of the IFT algorithm.

Fig. 6 represents the control error over a five-day period. The dispersion has been reduced by more than 25%.

Application to a DC-servo with Backlash

In this section we will report on the experience with IFT when applied to a DC-servo motor which exhibits backlash. The experimental system consists of an Industrial Emulator, Model 220, manufactured by Educational Control Products Inc and is shown in Fig. 7.

A block diagram of the feedback system is shown in Fig. 8. The sampling frequency is 25 Hz. The controller has the structure $C = \rho_1 + \rho_2 q^{-1} + \rho_3 q^{-2} + \rho_4 q^{-3}$ followed by an integrator. Hence, the controller is a one parameter extension of a PID controller. Since the actuator is limited to ±10 V, an anti-windup compensation is included. However, the experimental conditions were chosen such that saturation was rare. The output y is the angular position of the load.

Table 1: Calculated and predicted cost

Iteration	Cost (measured)	Next cost (predicted)
1	0.80	0.36
2	1.00	0.59
3	0.57	0.35
4	0.37	0.18
5	0.22	0.15
6	0.14	0.11

The purpose is to obtain a controller which rejects the input load disturbance n given in Fig. 9. In accordance with this, the control criterion is taken to be

$$J = \frac{1}{N}\sum_{t=1}^{N}\left(y^2(t) + u^2(t)\right)$$

and the reference signal r in Fig. 8 was set to zero. The main nonlinearity comes from a backlash in the idler pulleys. The initial controller

$$C_0 = \frac{4.89 - 7.28q^{-1} + 2.66q^{-2} + 1.96q^{-3}}{1 - q^{-1}},$$

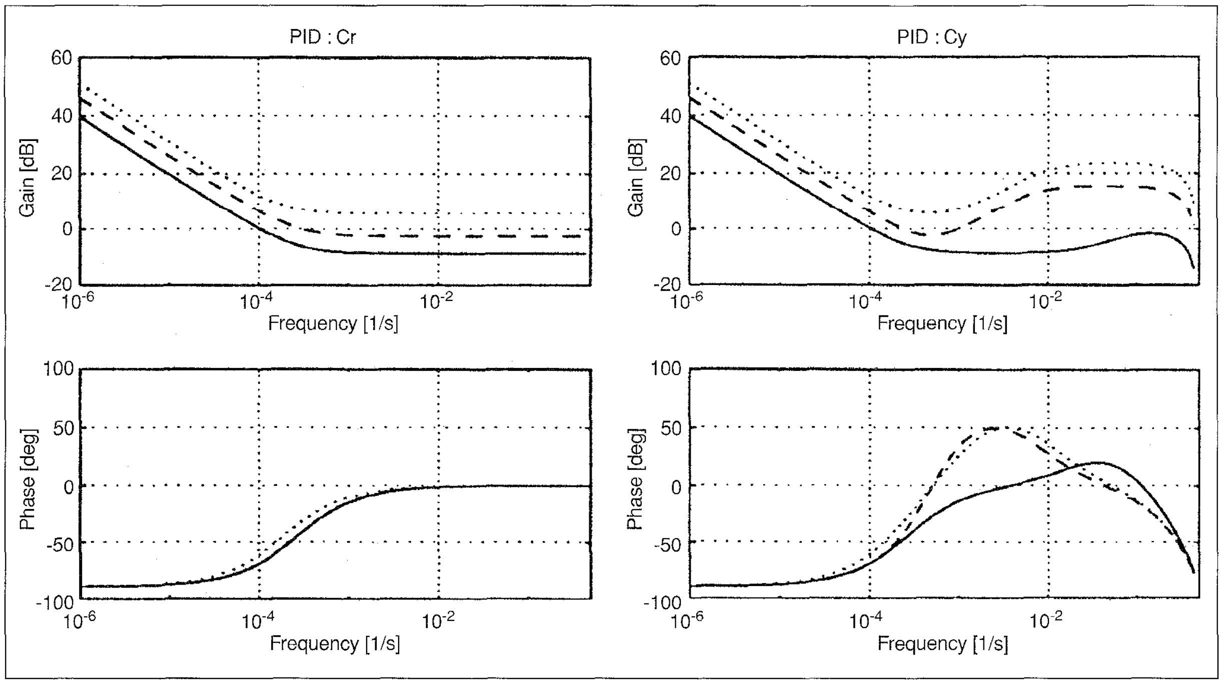


Fig. 4. Bode diagram of the two-degree-of-freedom controller before tuning (full), after three iterations (dashed) and after six iterations of the algorithm (dotted).

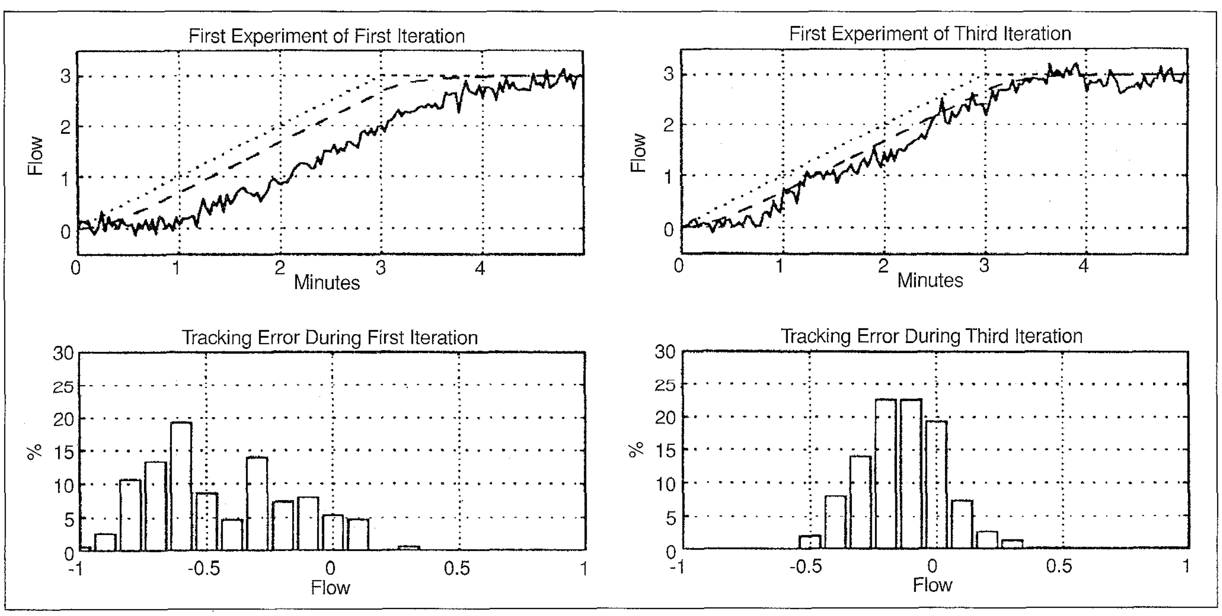


Fig. 5. Evaporator: reference signal r (dotted), desired response y^d (dashed) and closed loop response (full) during first experiment of first and third iteration, with corresponding histograms.

whose Bode diagram can be found in Fig. 12, provides good closed loop performance for the system without backlash. However, with backlash the system is in a limit cycle, see Fig. 9, with this controller.

Each experiment lasted 30 s and six iterations were performed using the Gauss-Newton direction with step-size $\gamma = 1$. In Fig. 10 the evolution of the criterion function is shown and Fig. 11 shows the output with the final controller

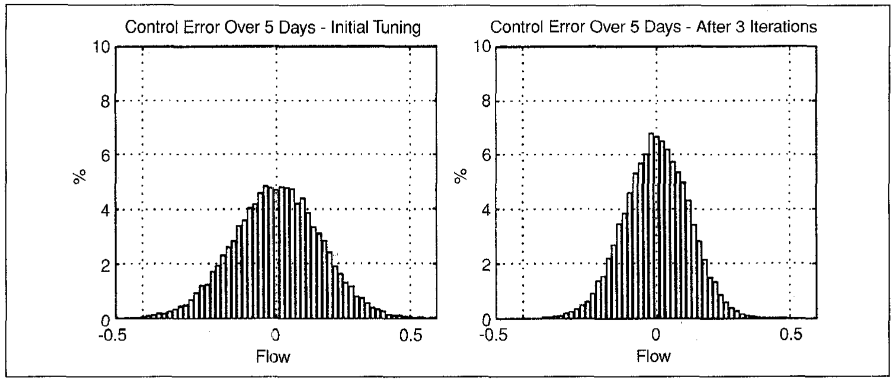


Fig. 6. Evaporator: histogram of the control error over a five-day period, with initial tuning and after three iterations.

$$C_6 = 5.46 - 9.25q^{-1} + 4.23q^{-2} + 0.28q^{-3} .$$

The improvement is quite striking. Hence, even for this system which has a non-smooth nonlinearity, IFT performs well, a quite remarkable result. The Bode diagrams of the initial controller and the final controller can be compared in Fig. 12.

Final Discussion

In this article we have examined an optimization approach to iterative control design. The important ingredient is that the gradient of the design criterion is computed from measured closed loop data. The approach is thus not model-based. The scheme converges to a stationary point of the design criterion under the assumption of boundedness of the signals in the loop.

From a practical viewpoint, the scheme offers several advantages. It is straightforward to apply. It is possible to control the rate of change of the controller in each iteration. The objective can be manipulated between iterations in order to tighten or loosen performance requirements. Certain frequency regions can be emphasized if desired.

This direct optimal tuning algorithm is particularly well suited for the tuning of the basic control loops in the process industry, which are typically PID loops. These primary loops are often very badly tuned, making the application of more advanced (for example, multivariable) techniques rather useless. A first requirement in the successful application of advanced control techniques is that the primary loops be tuned properly. This new technique appears to be a very practical way of doing this, with an almost automatic procedure. The application of the method at Solvay, of which we have presented a few typical results here, certainly appears promising.

In comparison with available methods for the tuning of PID controllers, IFT requires typically more data and experiments. However, it offers several advantages: the achieved responses are typically faster than those obtained with other model-free methods based on Nyquist (or Ziegler-Nichols) plot considerations; the control objective is clearly expressed, thereby giving the control engineer a confidence for the tuning of critical loops that he cannot have with some commercially available loop tuners that behave more like "dark grey box" systems (in the words of one control engineer). Perhaps in the long run IFT will prove to have its major potential for the tuning of nonlinear controllers or controllers applied to nonlinear systems,

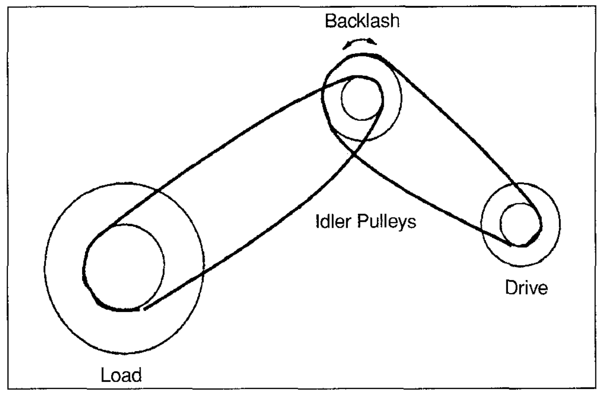


Fig. 7. DC-servo.

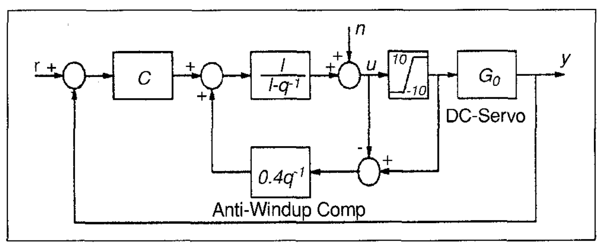


Fig. 8. Block diagram of feedback system with DC-servo.

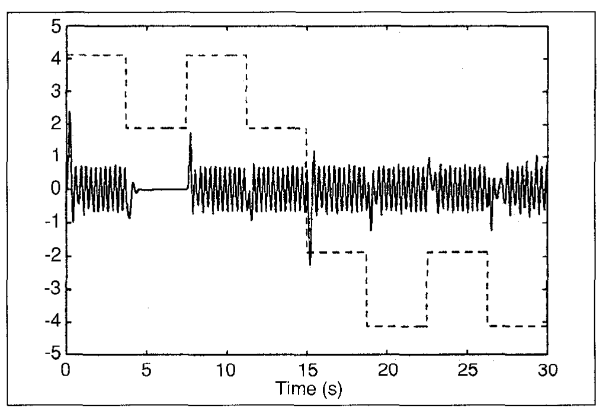


Fig. 9. Solid line: Output from DC-servo with the initial controller C_0. Dashed line: The input disturbance n.

for which preliminary analyses and applications seem to indicate great potential.

As a final remark, we should like to emphasize that, even though the industrial applications that we have presented in this article pertain to the tuning of industrial PID controllers, the method is by no means limited to the optimization of PID controllers.

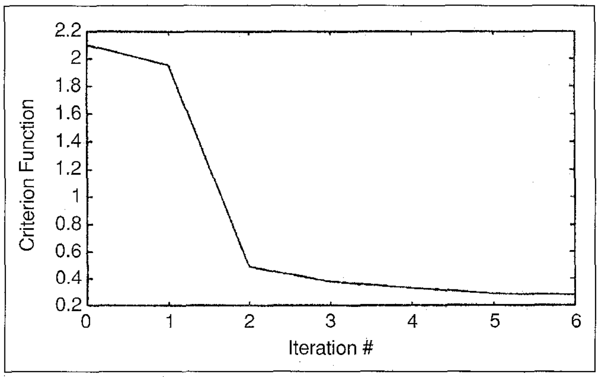


Fig. 10. Criterion function for the DC-servo application.

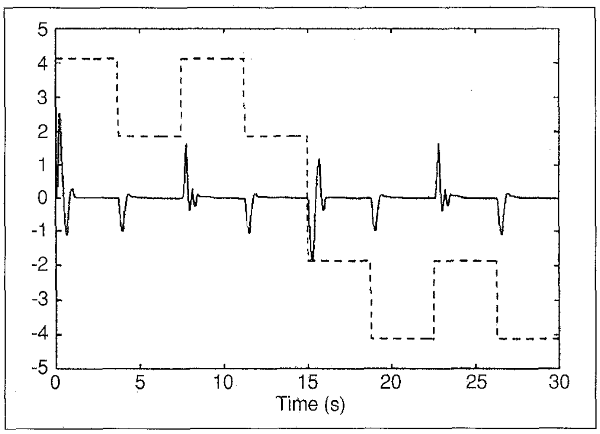


Fig. 11. Solid line: Output from DC-servo with the final controller C_6. Dashed line: The input disturbance n.

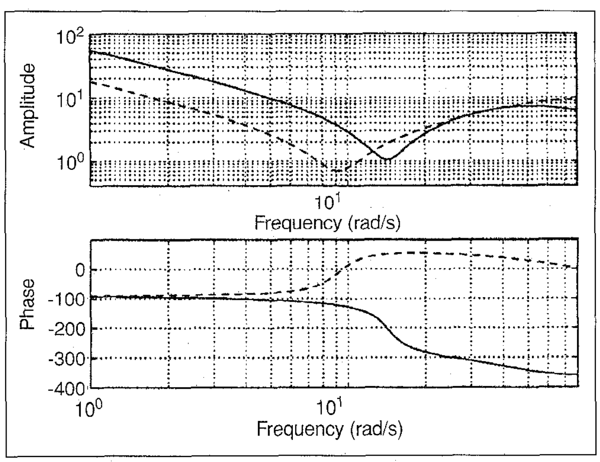


Fig. 12. Bode diagrams for controllers in DC-servo application. Solid line: Initial controller C_0. Dashed line: Final controller C_6.

References

[1] B.D.O. Anderson and Y. Liu, "Controller reduction: concepts, and approaches," *IEEE Trans. Automatic Control*, vol. 34, pp. 802-812, Aug. 1989.

[2] K.J. Åström and T. Hägglund, *PID Controllers: Theory, Design and Tuning, 2nd ed,*. North Carolina: Instrument Society of America, 1995.

[3] K.J. Åström and J. Nilsson, "Analysis of a scheme for iterated identification and control," *Proc. IFAC Symp. Identification*, pp. 171-176, 1994.

[4] K.J. Åström and B. Wittenmark, *Adaptive Control*. Reading, Massachusetts: Addison-Wesley, 1989.

[5] F. De Bruyne, B.D.O. Anderson, M. Gevers, and N. Linard, "Iterative controller optimization for nonlinear systems," in *Proc. 36th IEEE Conf. Decision and Control*, 1997, pp. 3749-3754.

[6] F. De Bruyne and P. Carrette, "Synthetic generation of the gradient for an iterative controller optimization method," *CD-ROM of 4th European Control Conference*, 1997, Paper THA-F2.

[7] B. Ceysens and B. Godrons, "Synthèse itérative de controleurs sans identification," M.S. thesis, Université Catholique de Louvain, 1995.

[8] R.A. de Callafon and P.M.J. Van den Hof. "Suboptimal feedback control by a scheme of iterative identification and control design," *Mathematical Modelling of Systems*, vol. 3(1), pp. 77-101, 1997.

[9] P.M.J. Van den Hof and R.J.P. Schrama, "Identification and control—closed-loop issues," *Automatica*, vol. 31, pp. 1751-1770, Dec. 1995.

[10] M. Gevers, "Towards a joint design of identification and control?" *Essays on Control: Perspectives in the Theory and its Applications*, eds. H. L. Trentelman and J. C. Willems, pp. 111-151, 1993.

[11] M. Gevers, "Identification for control," *Proc. 5th IFAC Symposium Adaptive Control and Signal Processing*, pp. 1-12, 1995.

[12] M. Gevers and L. Ljung, "Optimal experiment designs with respect to the intended model application," *Automatica*, vol. 22, pp. 543-554, 1986.

[13] H. Hjalmarsson, "Model-free tuning of controllers: Experience with time-varying linear systems," *Proc. 3rd European Control Conference*, pages 2869-2874, 1995.

[14] H. Hjalmarsson, "Control of nonlinear systems using iterative feedback tuning," *Proc. American Control Conf. 98, 1998* (to appear).

[15] H. Hjalmarsson, "Performance analysis of iterative feedback tuning," submitted to *Automatica*, 1998.

[16] H. Hjalmarsson and T. Birkeland, "Iterative Feedback Tuning of linear time-invariant MIMO systems," submitted to *CDC 98*, 1998.

[17] H. Hjalmarsson and M. Gevers, "Frequency domain expressions of the accuracy of a model-free control design scheme," *Proc. 11th IFAC Symp. on System Identification*, vol. 1, pp. 135-140, 1997.

[18] H. Hjalmarsson, M. Gevers, and F. De Bruyne, "For model-based control design, closed-loop identification gives better performance," *Automatica*, vol. 32, pp. 1659-1673, 1996.

[19] H. Hjalmarsson, S. Gunnarsson, and M. Gevers, "A convergent iterative restricted complexity control design scheme," *Proc. 33rd IEEE CDC*, pp. 1735-1740, 1994.

[20] H. Hjalmarsson, S. Gunnarsson, and M. Gevers, "Model-free tuning of a robust regulator for a flexible transmission system," *European J. Control*, vol. 1, pp. 148-156, 1995.

[21] H. Hjalmarsson, S. Gunnarsson, and M. Gevers, "Optimality and sub-optimality of iterative identification and control design schemes," *Proc. American Control Conf.*, vol. 4, pp. 2559-2563, 1995.

[22] A.J. Isaksson and S.F. Graebe, "Model reduction for design of digital PID controllers," *Proc. 3rd European Control Conf.*, vol. 3, pp. 2191-2196, 1995.

[23] Y. Kawamura, "Direct synthesis of LQ regulator from inner products of response signals," *Proc. 11th IFAC Symp. System Identification*, vol. 4, pp. 1717-1722, 1997.

[24] W.S. Lee, B.D.O. Anderson, R.L. Kosut, and I.M.Y Mareels, "A new approach to adaptive robust control," *Int. J. Adaptive Control and Signal Processing*, vol. 7, pp. 183-211, 1993.

[25] O. Lequin, "Optimal closed loop PID tuning in the process industry with the Iterative Feedback Tuning scheme," *CD-ROM of European Control Conference*, Paper TH-A-H6, 1997.

[26] L. Liung, *System Identification: Theory for the User.* Englewood Cliffs, NJ: Prentice-Hall, 1987.

[27] L. Ljung and T. Söderström, *Theory and Practice of Recursive Identification.* Cambridge, Massachusetts: MIT Press, 1983.

[28] D. Molenaar, "Model-free data-driven optimal tuning of controller parameters: A practical guide with application examples," Dep. Elec. Eng., Linköping University, Sweden, Tech. Rep. LiTH-ISY-R-1722 1995.

[29] K. S. Narendra and L.E. McBride, "Multiparameter self-optimizing systems using correlation techniques," *IEEE Trans. Automatic Control*, pp. 31-38, 1964.

[30] K. S. Narendra and D.N. Streeter, "An adaptive procedure for controlling undefined linear processes," *IEEE Trans. Automatic Control*, pp. 545-548, Oct. 1964.

[31] A.G. Partanen and R.R. Bitmead, "The application of an iterative identification and controller design to a sugar cane crushing mill," *Automatica*, vol. 31, pp. 1547-1563, 1995.

[32] P. Persson and K.J. Åström, "Dominant pole design—a unified view of PID controller tuning," *Selected Papers from the 4th IFAC Symposium*, pp. 377-382, 1993.

[33] H. Robbins and S. Monro, "A stochastic approximation method," *Ann. Math. Stat.*, vol. 22, pp. 400-407, 1951.

[34] M.G. Safonov and T.C. Tsao, "The unfalsified control concept and learning," *IEEE Trans. Automatic Control*, vol. 42(6), pp. 843-847, June 1997.

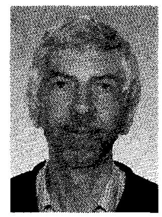
[35] R.J.P. Schrama and P.M.J. Van den Hof, "Iterative identification and control design: A three step procedure with robustness analysis," *Proc. ECC*, pp. 237-241, 1993.

[36] J. Sjöberg and M. Agarwal, "Model-free repetitive control design for nonlinear systems," *Proc. 35th Conference on Decision and Control*, pp. 2824-2829, Dec. 1996.

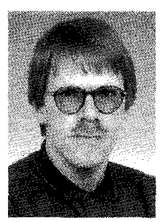
[37] L. Triest, "Etude des paramètres de synthèse dans le réglage itératif optimal d'un régulateur PID," Tech. Rep., Final year undergraduate project, CESAME, Université Catholique de Louvain, Louvain La Neuve, Belgium, 1997.

[38] E. Truesson and L. Ljung, "Adaptive control based on explicit criterion minimization," *Automatica*, vol. 21, pp. 385-399, 1985.

Håkan Hjalmarsson received the M.S. degree in Electrical Engineering in 1988, and the Licentiate degree and the Ph.D. degree in Automatic Control in 1990 and 1993, respectively, all from Linköping University, Sweden. He is an Associate Professor in the Department of Signals, Sensors and Systems, Royal Institute of Technology, Sweden. He is also an Associate Editor for *Automatica*. His research interests are system identification, signal processing and automatic tuning of controllers.



Michel Gevers obtained an EE degree from the Université Catholique de Louvain, Belgium, in 1968, and a Ph.D. from Stanford University, California, in 1972. He is now Professor and President of CESAME (Centre for Engineering Systems and Applied Mechanics) at the Université Catholique de Louvain in Louvain la Neuve, Belgium. He is the coordinator of the Belgian Interuniversity Pole on Modeling, Identification, Simulation and Control of Complex Systems, funded by the Federal Ministry of Science. His main present research interests are in system identification and its interconnection with robust control design. Professor Gevers is a fellow of the IEEE, and a Distinguished Member of the IEEE CSS.



Svante Gunnarsson was born in Tranas, Sweden, in 1959. He received the M.Sc. degree in 1983 and the Ph.D. degree in automatic control, all from Linköping University, Sweden. He is currently employed at the Division of Automatic Control, Linköping University. His research interests are recursive identification, iterative tuning of regulators and robotics, in particular application of iterative learning control.

Olivier Lequin received the Ingénieur degree in 1982 from the Faculté, Polytechnique de Mons, Belgium. In 1983, he joined the chemical multinational SOLVAY S.A. to work initially as an analyst in a computer center and later as a control engineer in charge of process control optimization. His main interests are in the optimal use of (classical) control algorithms in digital control systems in order to solve industrial process control problems.