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OPTIMIZING THE SETTLING TIME WITH ITERATIVE FEEDBACK TUNING [1]

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Abstract: We present a variant of the Iterative Feedback Tuning (IFT) method in which time weightings are used in the minimization criterion. A particularly useful application of this idea is when zero weightings are put on the transient phase of the step response of the system, thereby focusing on a rapid tracking of the desired reference change rather than on the shape of the transient response. By varying the size of this "zero weighting window", or mask, one can optimize the settling time of the closed-loop system. *Copyright © 1999 IFAC*

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1. INTRODUCTION

The Iterative Feedback Tuning method is a model-free technique for the optimization of the parameters of a controller of fixed structure using only signal information on the closed-loop system. The method was initially derived in (Hjalmarsson *et al.*, 1994) and has quickly proved its efficiency in both laboratory and industrial applications: see e.g. (Hjalmarsson *et al.*, 1995), (Lequin, 1997). It has also given rise to a number of extensions, notably for the tuning of nonlinear feedback loops (DeBruyne *et al.*, 1997), (Sjöberg and Agarwal, 1996), (Hjalmarsson, 1998). A recent and rather complete presentation of the theory, as well as applications to controller tuning for mechanical systems and chemical plants, can be found in (Hjalmarsson *et al.*, 1998).

In the classical formulation of IFT, as developed in (Hjalmarsson *et al.*, 1994), the linear closed loop system of Figure 1 was considered, where P is the unknown plant to be controlled, $C = [C_r, C_y]$ is a two degree of freedom controller, v is an unknown disturbance, and r, u, and y are the reference, the control signal and the output signal, respectively.

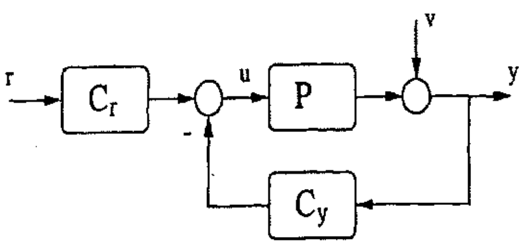

Fig. 1. Actual closed loop system

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The controller has a given structure in which some free parameters need to be tuned to achieve a desired objective. The vector of controller parameters is denoted ρ. In (Hjalmarsson *et al.*, 1994), the following quadratic criterion was adopted :

$$J(\rho) = \frac{1}{2N}\mathrm{E}\left[\sum_{t=1}^{N}(L_y\tilde{y}_t(\rho))^2 + \lambda\sum_{t=1}^{N}(L_u u_t(\rho))^2\right] \quad (1)$$

Here $\tilde{y}_t(\rho)$ is the error between the output $y_t(\rho)$ of the actual system controlled by the controller $C(\rho) = [C_r(\rho), C_y(\rho)]$ and a desired output signal y_d, $u_t(\rho)$ is the control signal, L_y and L_u are frequency weighting filters, λ expresses the relative importance of the penalty on the control signal versus the tracking error, N is the number of data points, and E stands for expected value. The output of the actual system, $y_t(\rho)$, and the control signal, $u_t(\rho)$, are explicitly shown to depend on the control parameters ρ. The main contribution of (Hjalmarsson *et al.*, 1994) was to show how to compute the minimum of this cost function with respect to these control parameters ρ without knowledge of the system, i.e. by iterative computations of the gradient $\frac{\partial J}{\partial \rho_i}$ and the use of a stochastic approximation algorithm for the update of the controller parameter vector ρ_i:

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1}\frac{\partial J}{\partial \rho}(\rho_i). \quad (2)$$

Here R_i is some appropriate positive definite matrix, typically a Gauss-Newton approximation of the Hessian of J, while γ_i is a positive real scalar that determines the step size. The sequence $\{\gamma_i\}$ must obey some constraints for the algorithm to converge to a local minimum of the cost function $J(\rho)$: see (Hjalmarsson *et al.*, 1994).

In this paper we present a variant of this criterion in which the signals $\tilde{y}_t(\rho)$ and $u_t(\rho)$ (or their frequency weighted versions) are time weighted by weightings $w_y(t)$ and $w_u(t)$, respectively. This idea was initially suggested in (Lequin, 1997). Thus, the criterion (1) is replaced by:

$$J_w(\rho) = \frac{1}{2N}\mathrm{E}\left[\sum_{t=1}^{N}w_y(t)(L_y\tilde{y}_t(\rho))^2 + \lambda\sum_{t=1}^{N}w_u(t)(L_u u_t(\rho))^2\right], \quad (3)$$

where $w_y(\cdot)$ and $w_u(\cdot)$ are any nonnegative numbers. The flexibility offered by the time weightings $w_y(t)$ and $w_u(t)$ is that they allow one to put different weightings on different parts of the time responses. A particularly interesting application, that is discussed in this paper, is when zero weightings are put on the transient response of the output response to a step change in the reference signal. Thus, in this paper the following criterion is examined:

$$J_m(\rho) = \frac{1}{2N}\mathrm{E}\left[\sum_{t=t_0}^{N}(L_y\tilde{y}_t(\rho))^2 + \lambda\sum_{t=1}^{N}(L_u u_t(\rho))^2\right]. \quad (4)$$

We say in such case that a mask of length t_0 is put on the transient response of the tracking error. The motivation for the use of such masks is as follows.

One of the frequent practical uses of controller design is to tune a controller of fixed structure (for example a PID controller) in such a way that the step response of the closed-loop system has a minimal settling time with a small overshoot. The objective in such applications is to move the output of the closed-loop system quickly from one reference value to another one; however, the particular shape of the transient response from the initial reference value to the final value is of no importance, provided that it does not have large overshoots. In addition, without knowledge of the actual system (which is a major reason for using IFT) it is not known in advance how fast a settling time can be achieved for this particular system with this particular controller structure.

By imposing the entire response of the closed-loop system through a specific choice of a desired response y_d, rather than just the endpoint of this transient response, the classical IFT criterion leads to controller parameters that realize a compromise between fitting the transient response and fitting the new reference value, even though the user does not care about the exact shape of the transient response. Instead, by imposing a mask on the transient response, the criterion will tune the controller parameters in such a way as to achieve the new desired reference value without focusing on a particular pre-imposed transient response that is perhaps not naturally achieved by the closed loop system. In other words, by imposing a mask on the transient response one does not waste the available degrees of freedom in the controller parameters on the matching of a specific and entirely arbitrary transient response. Instead one can focus these parameters entirely on achieving a fast settling time. The cost achieved after the masked interval is always smaller than when no mask is used: see Section 2 below.

These same observations can of course also be made about any model-based control design method. For example, in classical (i.e. model-based) LQG design, one often chooses a control design

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criterion that requires the output of the controlled system to track a desired response y_d, even though in applications where a setpoint change has to be made, one is not particularly interested in the specific shape of the transient response. It so happens that, with IFT, this modification of the criterion (i.e. the imposition of masks) is extremely easy to handle.

Our first contribution in this paper will be to illustrate, through a few examples, the typical advantages that can be gained by imposing zero weightings on the transient response, in terms of achieving a faster settling time. Our second and perhaps most useful contribution will be to propose a procedure whereby this settling time can be minimized even when the system is unknown, as is assumed with IFT. This procedure consists of imposing initially a rather large mask (i.e. a rather large zero-weighting time interval), and then to progressively reduce the size of the mask until oscillations start to appear in the transient response. This allows one to chooze the mask of appropriate length, and hence to design the controller that achieves the smallest settling time without oscillations. The procedure will again be illustrated with an example.

2. IFT CONTROL DESIGN WITH A FIXED MASK

It is very easy to show that a controller designed with a mask always achieves a smaller cost over the non-masked time interval than a controller designed without a mask. Let ρ^* be the parameter vector ρ that minimizes the initial criterion $J(\rho)$ of (1), and let ρ_m^* be the parameter vector ρ that minimizes the modified criterion $J_m(\rho)$ of (4) with a mask of length t_0. Assume that, in both cases, we are only concerned with the cost after the initial period t_0. Then, by the optimality principle, we obviously have

$$J_m(\rho_m^*) < J_m(\rho^*). \qquad (5)$$

This shows that, over the interval $[t_0, \infty)$, the controller $C(\rho_m^*)$ designed with a mask, always achieves a better performace than the controller $C(\rho^*)$ designed without the use of a mask.

We now compare the performances that are typically achieved for a step change in the reference, with and without a mask, with the following example. In this and the next example, the following simulation design choices were made:

- no frequency weightings were used in the IFT criterion, i.e. $L_y = L_u = 1$;
- no weight was put on the control energy, i.e. $\lambda = 0$;
- a white noise of variance $\sigma^2 = 0.0025$, filtered through a shaping filter $H(s) = \frac{1}{s+1}$, was added to the output of the closed loop system;
- an industrial PID controller structure was used in which the integral action is applied to the tracking error, but the derivative action is applied to the output signal only. In addition, a first order filter was used to limit the high frequency gain of this derivative action. These modifications of the "academic" PID controller, typical of industrial applications, result in a two degree of freedom controller as shown in Figure 1;
- the maximum step size γ_i in the iterative algorithm (2) was set at $\gamma_{max} = 1$; a line search procedure was used to compute the local minimum in the descent direction, with successive divisions of the step size by 2 until γ_i reached 2×10^{-4};
- a Gauss-Newton approximation of the Hessian was used for R_i, as recommended in (Hjalmarsson *et al.*, 1998).

2.1 *Example 1*

Consider the plant

$$P(s) = \frac{0.5s + 1}{s(8s+1)(5s+1)}$$

The objective is to tune a PID controller in order to achieve a unit step change in about 100 seconds with no overshoot. The initial regulator parameters were $K = 0.1$, $T_i = 500$ and $T_d = 2$, and the experiment time was 500 seconds. Two different criteria were minimized.

The first one was the classical IFT criterion (1) with the desired trajectory y_d being the ouput of the following reference model $T_d = \frac{1}{(10s+1)^4}$; this reference model has a settling time of about 100 seconds. After 5 iterations of the IFT scheme, the parameters had converged to $K = 0.026495$, $T_i = 32441$ and $T_d = 1.9765$, with a cost over the time interval $[100, 500]$ of $J = 4.3158 \times 10^{-4}$. Figure 2 shows the corresponding closed loop step response (in full) together with the desired response of the reference model (dotted). Observe that the required specifications concerning settling time and absence of overshoot are not met.

The second criterion was the modified criterion $J_m(\rho)$ of (4) with a mask over the first 100 time periods, i.e. $t_0 = 100$. After 7 iterations of the IFT scheme, the parameters had converged to $K = 0.017929$, $T_i = 18948$ and $T_d = 0.71413$, with a corresponding cost over the time interval $[100, 500]$ of $J = 1.9390 \times 10^{-4}$. Figure 3 shows the corresponding closed loop step response, which now meets the specifications.

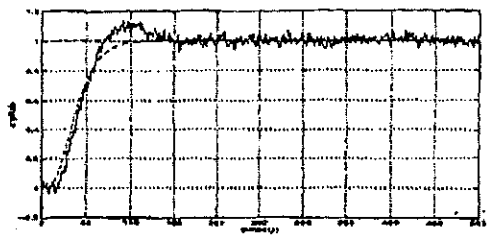

Fig. 2. Optimal closed loop step response obtained with a reference model (full), and desired response (dashed)

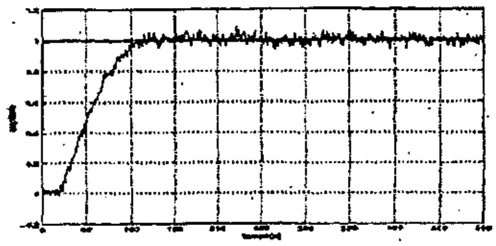

Fig. 3. Optimal closed loop step response obtained with a mask of length 100

3. IFT CONTROL DESIGN WITH A VARIABLE SIZE MASK

One of the difficulties in choosing a reference model or a reference signal is to decide what is a reasonable settling time for the system to be controlled. The same difficulty arises with the mask techniques, because the length of the mask, t_0, is the design parameter that fixes the settling time. If this length t_0 is chosen too long, then the obtained response is slower than could be achieved by the system with the chosen controller structure. If it is chosen too short, then our experience shows that an overshoot and oscillations appear in the transient response. The same phenomenon of course occurs with the choice of a reference model if the step response of the reference model is chosen too short with respect to what the controlled system is able to achieve.

Our solution to this problem, with the use of masks, is to initially use masks of rather large length and to progressively decrease the length of the mask until an overshoot begins to occur. This has shown to be extremely practical and easy to implement. We have also observed that, if such procedure terminates with a mask length t_0^*, then the corresponding closed-loop step response is typically better than what would be achieved by directly choosing a mask of length t_0^* rather than progressively decreasing the length of the mask. The assumed reason, which is in tune with the observations made in (Hjalmarsson *et al.*, 1998) about the choice of reference models, is that by progressively shortening the length of the mask (and hence the required settling time) through the iterations, one minimizes a succession of well behaved cost functions, whereas the direct minimization of the criterion with a mask of length t_0^* typically results in a criterion with multiple minima. Thus, by progressively reducing the mask length, one reduces the risk of converging to a local minimum.

The use of masks of decreasing length is illustrated by the following example.

3.1 *Example 2*

Consider the plant

$$P(s) = \frac{1}{s^2 + 0.1s + 1}$$

One wishes to tune a PID controller in order to achieve a settling time of 20 seconds for the closed loop system. The initial PID parameter values are taken as $K = 0.025$, $T_i = 2$ and $T_d = 1$. This yields the very sluggish response shown in Figure 4.

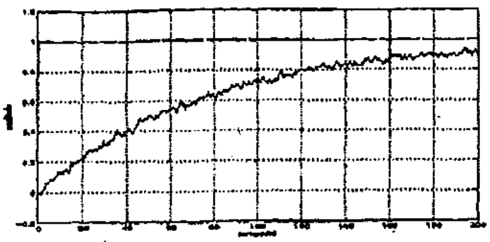

Fig. 4. Closed loop step response with initial PID parameters

The application of the classical IFT criterion with a desired response shown in dotted line in Figure 5 yields the response shown in full line on that same figure. This response is very unsatisfactory; this is in large part due to an unfortunate choice of initial parameters.

With the use of a fixed mask of length $t_0 = 20$ seconds, the minimization of the modified IFT criterion with the same initial parameters leads to the closed loop response of Figure 6. This response

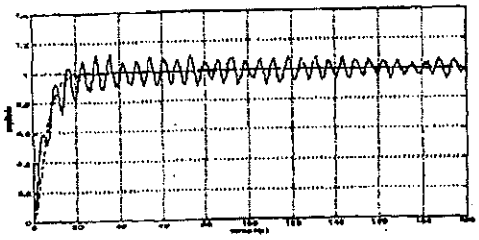

Fig. 5. Optimal closed loop step response (full) obtained with the classical IFT criterion and using the desired response (dashed)

is better than that obtained with a reference trajectory, but still very oscillatory.

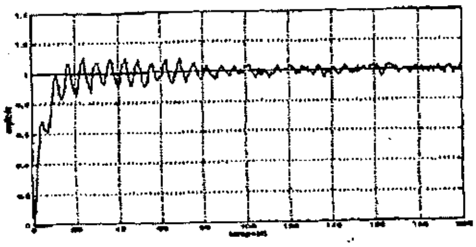

Fig. 6. Optimal closed loop step response obtained with the IFT criterion using a mask of length 20

Finally a mask of decreasing length was used, with an initial length of 80 seconds, and with the same initial parameters again. At every iteration of the IFT scheme, the length of the mask was decreased by 20 seconds, until a mask of length 20 was reached. This led to the closed loop response shown in Figure 7.

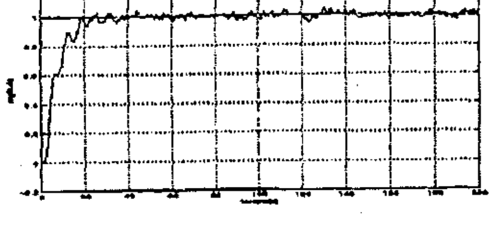

Fig. 7. Optimal closed loop step response obtained with the IFT criterion using masks of decreasing length

Observe the dramatic improvement of the response of Figure 7 vis-à-vis the response of Figure 6, even though in both cases the final criterion is identical (i.e. the criterion $J_m(\rho)$ of (4) with a mask of length $t_0 = 20$), and even though the starting values used for ρ_0 were identical in both simulations. The improvement is due to the use of masks of decreasing lengths in the last simulation, leading to a sequence of cost criteria (rather than a one-shot criterion), and to a different sequence of ρ_i parameter vectors than resulted with the direct use of a mask of length $t_0 = 20$.

4. CONCLUSION

The IFT method is a very flexible controller design tool, lending itself to variants that are typically easy to implement. Here we have studied a variant, first proposed in (Lequin, 1997), in which time weightings are added in the IFT criterion. We have shown that, by doing so, one can easily design controllers that focus better on a rapid tracking of a new reference value rather than wasting some of the degrees of freedom on the tracking of a specific transient response. In addition, we have shown that the use of masks of decreasing length allows one to easily design controllers that minimize this settling time.

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