Closed-loop identification with an unstable or nonminimum phase controller

Benoît Codrons [a], Brian D.O. Anderson [b] and Michel Gevers [c]

[a] *Laborelec S.C., product line 'Process Control', B-1630 Linkebeek, Belgium*
E-mail: `Benoit.Codrons@laborelec.be`

[b] *Research School of Information Sciences and Engineering, Australian National University, Canberra, ACT 0200, Australia*
E-mail: `Brian.Anderson@anu.edu.au`

[c] *Centre for Systems Engineering and Applied Mechanics (CESAME), Université catholique de Louvain, B-1348 Louvain-la-Neuve, Belgium*
E-mail: `Gevers@csam.ucl.ac.be`

Abstract

In many practical cases, the identification of a system is done in closed loop with some controller. In this paper, we show that the internal stability of the resulting model, in closed loop with the same controller, is not always guaranteed if this controller is unstable and/or nonminimum phase, and that the classical closed-loop prediction-error identification methods present different properties regarding this stability issue. With some of these methods, closed-loop instability of the identified model is actually guaranteed. This is a serious drawback if this model is to be used for the design of a new controller. We give guidelines to avoid the emergence of this instability problem; these guidelines concern both the experiment design and the choice of the identification method.

Key words: closed-loop identification, internal stability, unstable or nonminimum phase controller, control design.

1 Introduction

In this paper we address some problems that arise in closed-loop identification when the controller contains unstable poles or nonminimum phase zeros. We show that, with some of the commonly used closed-loop identification methods, the resulting nominal closed-loop system is internally unstable, even though the true system is stabilised by the same controller. A model that is not stabilised by the present controller is intrinsically flawed for the design of a better controller, in that it deprives the control designer of some of his/her most important robust stability tools for the design of this new controller.

We consider four different approaches to closed-loop identification: the indirect approach (Söderström and Stoica, 1989), the coprime-factor approach (Van den Hof and Schrama, 1995), the direct approach (Ljung, 1999) and the dual Youla method (Hansen *et al.*, 1989). We show that these closed-loop identification methods exhibit very different properties in the presence of unstable or nonminimum phase controllers. This is somewhat surprising in view of the fact that the recent analyses of the various closed-loop identification methods have tended to illustrate their similarities, rather than their differences: see (Gevers *et al.*, 2001; Forssell and Ljung, 1999). We show, e.g., that in the case of indirect closed-loop identification with an unstable controller, the choice that is most commonly made in step 1 of the procedure leads to a guaranteed unstable identified closed-loop model, while the choice of another closed-loop transfer function in step 1 produces a guaranteed stable closed-loop model. The mechanism that produces guaranteed closed-loop instability is one of near pole-zero cancellation of an unstable pole-zero pair. Our results therefore give guidelines as to the choice of a closed-loop identification method as a function of the singularities that are known to be present in the controller. In addition, they tell us where to put the external excitation as a function of these controller singularities.

The outline of the paper is as follows. In Section 2, we introduce some preliminaries about closed-loop systems and their stability margins. In Section 3 we explain why it is important to obtain a stable nominal closed-loop system in the context of iterative identification and control design. In Section 4 we show that for some of the most commonly used closed-loop identification methods the presence of an unstable or nonminimum phase controller leads to a model that is unstable in closed loop, even though the actual closed-loop system is stable. Experiment design guidelines are given to avoid this problem. In Section 5 we present the dual Youla identification method that guarantees stability of the nominal closed-loop system, even in the presence of both unstable poles and nonminimum phase zeros in the controller. We tested these results on a numerical simulation, based on the Landau benchmark (Landau *et al.*, 1995); this is presented in Section 6. Finally, conclusions are drawn in Section 7.

2 Preliminaries about closed-loop systems

We consider a Linear Time Invariant (LTI) "true" plant $G_0(z)$ in closed loop with some stabilising LTI controller $K(z)$, as depicted in Figure 1, where $u(t)$ is the input of the plant, $y(t)$ is its output, $v(t)$ is an output disturbance, while $r_1(t)$ and $r_2(t)$ are two possible sources of exogenous signals (references).

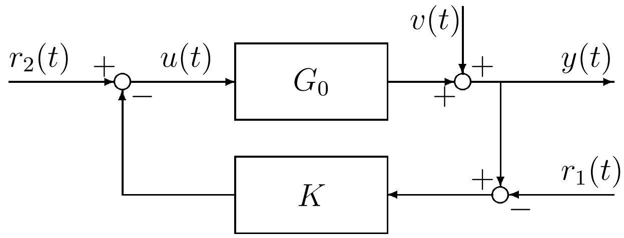

Fig. 1. Closed-loop system

The closed-loop system of Fig. 1 is described by

$$\begin{bmatrix} y(t) \\ u(t) \end{bmatrix} = T(G_0,\, K) \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} + S(G_0,\, K)v(t) \tag{1}$$

where

$$T(G_0,\, K) = \begin{bmatrix} \frac{G_0 K}{1+G_0 K} & \frac{G_0}{1+G_0 K} \\ \frac{K}{1+G_0 K} & \frac{1}{1+G_0 K} \end{bmatrix} \triangleq \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \text{ and } S(G_0,\, K) \triangleq \begin{bmatrix} T_{22} \\ -T_{21} \end{bmatrix} \cdot \tag{2}$$

The generalised stability margin $b_{G_0,K}$ of the closed-loop system is defined as

$$b_{G_0,K} \triangleq \begin{cases} \|T(G_0,\, K)\|_\infty^{-1} & \text{if } (G_0,\, K) \text{ is internally stable} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Observe that $0 \leq b_{G_0,K} < 1$. There is a maximum attainable value of $b_{G_0,K}$ over all controllers stabilising G_0 (Vinnicombe, 2000):

$$\sup_K b_{G_0,K} = \sqrt{1 - \left\| \begin{bmatrix} N_0 \\ M_0 \end{bmatrix} \right\|_H^2}. \tag{4}$$

Here $\left[\begin{smallmatrix} N_0 \\ M_0 \end{smallmatrix}\right]$ is a normalised coprime factorisation of G_0, i.e. $G_0 = N_0 M_0^{-1}$ with M_0 and N_0 in $\mathcal{RH}_\infty$, the ring of proper stable rational transfer functions, and $M_0 M_0^\star + N_0 N_0^\star = 1$ where $X^\star(e^{j\omega}) = X(e^{-j\omega})$. $\|\cdot\|_H$ denotes the Hankel norm. It should be clear that it is easier to design a stabilising controller for a system with a large $\sup_K b_{G_0,K}$ than for a system with a small one. We refer the reader to (Zhou and Doyle, 1998) for more details.

3 Why do we want a stable nominal closed-loop system?

When one identifies a model $\hat{G}$ of G_0 from data collected in closed loop with a stabilising controller K, it makes good sense to request that the nominal closed-loop system $T(\hat{G}, K)$ be stable since we know that the actual closed-loop system $T(G_0, K)$ is stable. In iterative identification and control design, the model $\hat{G}$ is used for the design of a new controller $\bar{K}$ that must achieve a better performance on the actual system G_0 than the present controller K. For the model $\hat{G}$ to be suitable for the design of such controller $\bar{K}$, it is necessary that G_0 and $\hat{G}$ be close *in a closed-loop sense*, i.e. $||T(G_0, K) - T(\hat{G}, K)||$ must be small (Gevers, 1993; Hakvoort *et al.*, 1994; Milanese and Taragna, 1999; Lee *et al.*, 1995; Skelton, 1989; Zang *et al.*, 1995). This is unlikely to be the case if one of these closed-loop systems is stable and the other is not. In addition, the stability margin, $b_{\hat{G},K}$, of the nominal system and the maximal stability margin, $\sup_K b_{\hat{G},K}$, of the model are useful tools to ascertain that a new controller $\bar{K}$, designed from $\hat{G}$, stabilises the true system G_0. If $b_{\hat{G},K} = 0$ and/or if $\sup_K b_{\hat{G},K}$ is very small, it will be much harder – if not impossible – to design a new controller with prior stability guarantees.

There exists an extensive literature on the use of such stability robustness tools in iterative identification and control design: see e.g. (Anderson *et al.*, 1998; Bitmead *et al.*, 2000; de Callafon and Van den Hof, 1997; Van den Hof *et al.*, 1995). These robust stability tools are all based on the fact that some distance measure between model and true system must be bounded above by some function of the nominal stability margin. We work here with a distance measure between systems called the ν-gap, because it is directly connected to the generalised stability margin $b_{G,K}$ defined above. The ν-gap between two plants is defined as follows:

$$\delta_\nu(G_1, G_2) = \max_\omega \kappa\left(G_1(e^{j\omega}), G_2(e^{j\omega})\right) \tag{5}$$

with

$$\kappa\left(G_1(e^{j\omega}), G_2(e^{j\omega})\right) \triangleq \frac{|G_1(e^{j\omega}) - G_2(e^{j\omega})|}{\sqrt{1 + |G_1(e^{j\omega})|^2}\sqrt{1 + |G_2(e^{j\omega})|^2}}, \tag{6}$$

subject to some winding number condition: see (Vinnicombe, 1993) for details. Observe that $0 \leqslant \delta_\nu(G_1, G_2) \leqslant 1$. The key robust stability result that relates the ν-gap to the generalised stability margin is as follows.

Proposition 1 *(Vinnicombe, 1993) Let us consider a ν-gap uncertainty set M_ν of size β and centered at a model $\hat{G}$:*

$$M_\nu = \{G \mid \delta_\nu(\hat{G}, G) \leq \beta\}. \tag{7}$$

Then, a controller K stabilising $\hat{G}$ stabilises all plants in the uncertainty region

M_ν *if and only if it lies in the controller set:*

$$\{K(z) \mid b_{\hat{G},K} > \beta\}. \tag{8}$$

If $(\hat{G}, K)$ is unstable, it may be impossible to use $\hat{G}$ for the design of a new controller $\bar{K}$ that is guaranteed to stabilise G_0. By Proposition 1, such stability guarantee is obtained for $\bar{K}$ if $\delta_\nu(G_0, \hat{G}) < b_{\hat{G},\bar{K}}$. If K is a stabilising controller achieving small closed-loop bandwidth (a controller that one would like to replace by a better one), then $b_{G_0,K}$ is typically large. Now, if $(\hat{G}, K)$ is unstable, we know by Proposition 1 that $\delta_\nu(G_0, \hat{G})$ is large because $b_{G_0,K} \leq \delta_\nu(G_0, \hat{G})$. There is a risk, therefore, that $\sup_{\bar{K}} b_{\hat{G},\bar{K}} \leq \delta_\nu\left(G_0, \hat{G}\right)$. In such case, no controller $\bar{K}$ stabilising $\hat{G}$ is guaranteed to stabilise G_0, and it may even be impossible to find a controller that stabilises both $\hat{G}$ and G_0. Our simulation example will illustrate that this indeed happens.

4 Effects of unstable poles or nonminimum phase zeros in the controller on the nominal closed-loop stability

We show that, in the presence of unstable poles or nonminimum phase zeros in the controller, some closed-loop identification schemes necessarily lead to unstable closed-loop models. We consider the identification of a model $\hat{G}$ of a plant G_0, in closed loop with a stabilising controller K as in Figure 1.

4.1 The indirect approach

The indirect approach consists in first estimating one of the four entries of the matrix $T(G_0, K)$ of (2) using appropriate combinations of measurements of $r_1(t)$ or $r_2(t)$, and $y(t)$ or $u(t)$. From this estimate, a model $\hat{G}$ is derived, using knowledge of the controller K and one of the following algebraic relations:

$$\begin{array}{ll} \hat{G}(\hat{T}_{11}) = \frac{\hat{T}_{11}}{K - K\hat{T}_{11}}, & \hat{G}(\hat{T}_{12}) = \frac{\hat{T}_{12}}{1 - K\hat{T}_{12}}, \\ \hat{G}(\hat{T}_{21}) = \frac{1}{\hat{T}_{21}} - \frac{1}{K}, & \hat{G}(\hat{T}_{22}) = \frac{1}{K}\left(\frac{1}{\hat{T}_{22}} - 1\right). \end{array} \tag{9}$$

We can thus consider four different cases, depending on which of the four entries is identified in the first step.

Theorem 1 *Consider the closed-loop setup of Figure 1. Assume that an indirect approach for closed-loop identification is used to obtain a stable estimate of one of the four closed-loop transfer functions* $\hat{T}_{ij}$ *(*$i,\ j \in \{1, 2\}$*) in step 1,* *from a finite number of noisy data samples, followed by an estimate $\hat{G}$ of G_0 using the corresponding formula in (9). Then*

(1) the nominal closed-loop system $(\hat{G}, K)$ will be unstable if K has one or more nonminimum phase zeros and $\hat{G}$ is computed from either $\hat{T}_{11}$, or $\hat{T}_{21}$, or $\hat{T}_{22}$;
(2) the nominal closed-loop system $(\hat{G}, K)$ will be unstable if K has one or more unstable poles and $\hat{G}$ is computed from either $\hat{T}_{11}$, or $\hat{T}_{12}$, or $\hat{T}_{22}$.
(3) the nominal closed-loop system $(\hat{G}, K)$ will be stable if K is minimum phase and $\hat{G}$ is computed from $\hat{T}_{21}$;
(4) the nominal closed-loop system $(\hat{G}, K)$ will be stable if K is stable and $\hat{G}$ is computed from $\hat{T}_{12}$.

Proof. We provide the proof for the case where $\hat{T}_{11}$ is estimated in step 1. The analysis of the other three cases is similar. The generalised closed-loop transfer matrix of the loop $(\hat{G}(\hat{T}_{11}), K)$ is given by

$$T(\hat{G},\, K) = \begin{bmatrix} \hat{T}_{11} & \frac{\hat{T}_{11}}{K} \\ K(1-\hat{T}_{11}) & 1-\hat{T}_{11} \end{bmatrix}. \tag{10}$$

(i) if K has a nonminimum phase zero at some z_0, then $T_{11}(z_0) = 0$: see (2). However, the estimate $\hat{T}_{11}$ has an unavoidable variance error, and possibly a bias error as well. As a result, $\hat{T}_{11}(z_0) \neq 0$, and $\frac{\hat{T}_{11}}{K}$ will be unstable because it contains an imperfect cancellation of an unstable pole-zero pair. The stability of the other entries of $T(\hat{G},\, K)$ is unaffected by nonminimum phase zeros of K;
(ii) if K has an unstable pole at some z_0, then $1-T_{11}(z_0) = 0$, but $1-\hat{T}_{11}(z_0) \neq 0$ due to estimation error. Hence, $K(1-\hat{T}_{11})$ will be unstable for the same reason of imperfect cancellation of an unstable pole-zero pair. The stability of the other entries of $T(\hat{G},\, K)$ is not affected by unstable poles of K.

□

Conclusion: As a result of this theorem, with the indirect approach one must

- identify $\hat{G}$ from $\hat{T}_{12}$ using $\{y(t),\, r_2(t)\}$ data if K has nonminimum phase zeros but no unstable poles;
- identify $\hat{G}$ from $\hat{T}_{21}$ using $\{u(t),\, r_1(t)\}$ data if K has unstable poles but no nonminimum phase zeros.

If K is both unstable and nonminimum phase, the nominal closed loop $(\hat{G},\, K)$ will be unstable whichever entry of $T(G_0,\, K)$ is identified; hence $b_{\hat{G},K} = 0$.

Remarks
1. The following additional comments can be made for the case where the

controller has poles or zeros on the unit circle when T_{11} or T_{12} is estimated in step 1; similar remarks hold for the other two cases.

(1) If K has a blocking zero at some frequency ω_0, then $T_{11}(e^{j\omega_0}) = 0$, and the output Signal-to-Noise Ratio (SNR) will be zero at this frequency and very low around it, yielding a bad estimate of T_{11} at this frequency.
(2) If K has a blocking pole at some frequency ω_0 (which will often be the case, since many controllers contain an integrator, i.e. a pole at $\omega_0 = 0\ rad/s$), then $T_{12}(e^{j\omega_0}) = 0$, and the output SNR will be zero at this frequency and very low around it, yielding a bad estimate of T_{12}, and hence of G_0, at this frequency.

2. The results of this paper, published at SYSID 2000, have led (Goodwin and Welsh, 2001) to suggest an alternative procedure for indirect identification when the controller contains a singularity. Their procedure is to prevent the occurrence of a near cancellation of an unstable pole-zero pair by imposing interpolation constraints on the closed loop transfer function estimates, thereby forcing these to be exact at the singularities of the controller.

4.2 The coprime-factor approach

The coprime-factor approach consists in first estimating the two entries of one column of $T(G_0, K)$ using measurements of $r_1(t)$, $y(t)$ and $u(t)$, or of $r_2(t)$, $y(t)$ and $u(t)$. The model $\hat{G}$ is then given by the ratio of these two entries: see (2). This method requires that the controller be LTI, *but it does not have to be known.* Two cases can be considered, depending on whether $r_1(t)$ or $r_2(t)$ is used as the excitation signal.

Theorem 2 *Consider the closed-loop setup of Figure 1. Assume that a coprime-factor approach for closed-loop identification is used to obtain a model $\hat{G}$ for G_0 from a finite number of noisy data samples, and that the estimates of the closed-loop transfer functions $\hat{T}_{ij}$ ($i, j \in \{1, 2\}$) in step 1 are stable. Then*

(1) in the case where $\hat{G}$ is computed from $\hat{G}(\hat{T}_{11}, \hat{T}_{21}) = \frac{\hat{T}_{11}}{\hat{T}_{21}}$ using measurements of $r_1(t)$, $y(t)$ and $u(t)$, the nominal closed-loop system $(\hat{G}, K)$ will be stable if and only if K has no strictly nonminimum phase zeros and $\frac{\hat{T}_{21}}{K} + \hat{T}_{11}$ has no nonminimum phase zeros;
(2) in the case where $\hat{G}$ is computed from $\hat{G}(\hat{T}_{12}, \hat{T}_{22}) = \frac{\hat{T}_{12}}{\hat{T}_{22}}$ using measurements of $r_2(t)$, $y(t)$ and $u(t)$, the nominal closed-loop system $(\hat{G}, K)$ will be stable if and only if K has no strictly unstable poles and $\hat{T}_{22} + \hat{T}_{12}K$ has no nonminimum phase zeros.

Proof. We provide a full proof for case (1) only; case (2) is very similar. The generalised closed-loop transfer matrix of $\hat{G}(\hat{T}_{11}, \hat{T}_{21})$ is given by

$$T(\hat{G}, K) = \begin{bmatrix} \frac{\hat{T}_{11}}{\frac{\hat{T}_{21}}{K}+\hat{T}_{11}} & \frac{\frac{\hat{T}_{11}}{K}}{\frac{\hat{T}_{21}}{K}+\hat{T}_{11}} \\ \frac{\hat{T}_{21}}{\frac{\hat{T}_{21}}{K}+\hat{T}_{11}} & \frac{\frac{\hat{T}_{21}}{K}}{\frac{\hat{T}_{21}}{K}+\hat{T}_{11}} \end{bmatrix} \neq \begin{bmatrix} \hat{T}_{11} & \frac{\hat{T}_{11}}{K} \\ \hat{T}_{21} & \frac{\hat{T}_{21}}{K} \end{bmatrix}. \tag{11}$$

Note that, even though the coprime-factor procedure first computes estimates of $\hat{T}_{11}$ and $\hat{T}_{21}$, the resulting estimate of the first column of $T(\hat{G}, K)$ is not $\begin{bmatrix} \hat{T}_{11} \\ \hat{T}_{21} \end{bmatrix}$. It follows from (11) that, if $\hat{T}_{11}$ and $\hat{T}_{21}$ are reasonable estimates of T_{11} and T_{21}, then these estimates must have the property that

$$\frac{\hat{T}_{21}(z)}{K(z)} + \hat{T}_{11}(z) \approx 1 \quad \forall z. \tag{12}$$

Now observe that

(i) if K has a strictly nonminimum phase zero at z_0, then (12) cannot hold at z_0. In addition, $\frac{\hat{T}_{11}}{K}$ and $\frac{\hat{T}_{21}}{K}$ will be unstable since the unstable zero of K will never be exactly cancelled by a corresponding zero in $\hat{T}_{11}$ and $\hat{T}_{21}$;

(ii) if K has a blocking zero, i.e. a zero on the unit circle at $z_0 = e^{j\omega_0}$, say, then $T_{11}(z_0) = T_{21}(z_0) = 0$, but $\hat{T}_{11}$ and $\hat{T}_{21}$ will be very poor estimates of T_{11} and T_{21} around the frequency ω_0 because of the very bad SNR around that frequency. In particular, $\hat{T}_{11}(z_0) \neq 0$ and $\hat{T}_{21}(z_0) \neq 0$. Hence

$$\left| \frac{\hat{T}_{21}(z_0)}{K(z_0)} + \hat{T}_{11}(z_0) \right| = \infty, \tag{13}$$

while (12) must hold at frequencies where the quality of the estimates is not affected by blocking zeros. This yields, for the four entries of $T(\hat{G}, K)$ in (11), $\frac{\hat{T}_{11}(z_0)}{\frac{\hat{T}_{21}(z_0)}{K(z_0)}+\hat{T}_{11}(z_0)} = T_{11}(z_0) = 0$, $\frac{\hat{T}_{21}(z_0)}{\frac{\hat{T}_{21}(z_0)}{K(z_0)}+\hat{T}_{11}(z_0)} = T_{21}(z_0) = 0$, $\frac{\frac{\hat{T}_{11}(z_0)}{K(z_0)}}{\frac{\hat{T}_{21}(z_0)}{K(z_0)}+\hat{T}_{11}(z_0)} = \frac{\hat{T}_{11}(z_0)}{\hat{T}_{21}(z_0)}$ and $\frac{\frac{\hat{T}_{21}(z_0)}{K(z_0)}}{\frac{\hat{T}_{21}(z_0)}{K(z_0)}+\hat{T}_{11}(z_0)} = 1$, which are all finite. Hence a blocking zero (and similarly an unstable pole) of K does not cause nominal closed-loop instability.

(iii) if $\frac{\hat{T}_{21}(z)}{K(z)} + \hat{T}_{11}(z)$ has a nonminimum phase zero, then $T(\hat{G}, K)$ is unstable.

To summarise, when $\hat{G}$ is computed from $\hat{G} = \frac{\hat{T}_{11}}{\hat{T}_{21}}$, the nominal closed-loop matrix $T(\hat{G}, K)$ will be stable if and only if K has no strictly nonminimum phase zeros and condition (12) holds, except at possible blocking zeros of K where (13) will hold. Condition (12) serves as a way of validating the quality of the estimates $\hat{T}_{11}$ and $\hat{T}_{21}$; it implies that $\frac{\hat{T}_{21}(z)}{K(z)} + \hat{T}_{11}(z)$ is minimum phase.

The case where $\hat{G}$ is computed from $\hat{G} = \frac{\hat{T}_{12}}{\hat{T}_{22}}$ leads to a similar analysis. In this case, we need

$$\hat{T}_{22}(z) + \hat{T}_{12}(z)K(z) \approx 1 \quad \forall z, \tag{14}$$

which cannot hold at poles of K on the unit circle. The same reasoning as before leads to the conclusion that $T(\hat{G}, K)$ will be stable in this case if and only if K has no strictly unstable poles and condition (14) holds, except at possible poles of K on the unit circle, where $|\hat{T}_{22} + \hat{T}_{12}K| = \infty$. Condition (14) serves as a way of validating the quality of the estimates $\hat{T}_{12}$ and $\hat{T}_{22}$; it also implies that $\hat{T}_{22}(z) + \hat{T}_{12}(z)K(z)$ is minimum phase. □

Conclusion: With the coprime-factor approach, one must

- identify $\hat{G}$ from $\hat{G} = \hat{T}_{11}\hat{T}_{21}^{-1}$, if K has *strictly unstable poles and no strictly nonminimum phase zeros*, using $\{y(t),\, u(t),\, r_1(t)\}$ data. Condition (12) must be checked *a posteriori*;
- identify $\hat{G}$ from $\hat{G} = \hat{T}_{12}\hat{T}_{22}^{-1}$, if K has *strictly nonminimum phase zeros and no strictly unstable poles*, using $\{y(t),\, u(t),\, r_2(t)\}$ data. Condition (14) must be checked *a posteriori*.

If K has both poles and zeros strictly outside the unit circle, the coprime-factor approach cannot be used to obtain a model $\hat{G}$ stabilised by K.

4.3 The direct approach

In the direct approach, a parametric model $\hat{G}$ of the system G_0 is directly identified using measurements of $u(t)$ and $y(t)$. Here, the stability of $T(\hat{G}, K)$ does not hinge on the cancellation of unstable poles or zeros in K. Hence, the direct approach can be used with any external excitation, $r_1(t)$ or $r_2(t)$, even if K has unstable poles and/or nonminimum phase zeros. However, there is no prior guarantee of nominal closed-loop stability.

Notice that if $K(e^{j\omega_1}) = 0$ for some frequency ω_1, and if $r_1(t)$ is used ($r_2(t) = 0$), the power spectral density of $u(t)$ will be zero at ω_1, yielding a very poor estimate $\hat{G}$ around ω_1. Therefore, $r_2(t)$ should be used to excite the system if K has a zero on the unit circle. On the other hand, a pole of K on the unit circle causes no problem with the direct method: it has a blocking effect on both $r_2(t)$ and $v(t)$, without affecting the SNR ratio of these two contributions to the output signal.

The effects of the presence of unstable poles or nonminimum phase zeros in the controller on the stability of the nominal closed-loop systems obtained by the three most commonly used closed-loop identification techniques can be summarised in the following table.

Singularities of K	Indirect				Cop.-fac.		Direct		
	r_1, y	r_2, y	r_1, u	r_2, u	r_1	r_2	r_1	r_2	$r_1 + r_2$
Strictly unstable poles	–	–	+	–	0	–	0	0	0
Unit-circle poles	–	–	+	–	0	0	0	0	0
Strictly nonmin. ph. zeros	–	+	–	–	–	0	0	0	0
Unit-circle zeros	–	+	–	–	0	0	0	0	0

Table 1
Stability of the nominal closed-loop model w.r.t. the identification method, the excitation signal and the singularities of K: stability is guaranteed if a stable model structure is used in step 1 (+); stability has to be checked *a posteriori* (0); instability is guaranteed (–). When K has several listed singularities, the most unfavourable one outclasses the others.

This table must be interpreted as follows. If K has strictly unstable poles but no nonminimum phase zeros, and if an indirect method is used, then the closed-loop system must be excited via $r_1(t)$ and $\hat{T}_{21}$ must be estimated in step 1 using a stable model structure. Nominal closed-loop stability is then guaranteed. In the same situation, if the coprime factor method is used, the closed-loop system must again be excited with $r_1(t)$; however, closed-loop stability must be checked *a posteriori*.

Concluding remarks:

- Only the indirect approach guarantees the stability of $T(\hat{G}, K)$ *a priori*, provided the correct entry of $T(G_0, K)$ is identified in step 1, and its estimate is stable. However, this method cannot be used if the controller has both unstable poles and nonminimum phase zeros.
- The coprime-factor approach cannot be used if the controller has both poles and zeros strictly outside the unit circle. Note that an integrator (or any other unit-circle singularity) in the controller causes no problem.
- The direct approach is the only method that can be used if the controller has both zeros and poles outside the unit circle. However, closed-loop stability of the resulting model can only be checked *a posteriori*.

5 Stability-preserving closed-loop identification with an unstable and nonminimum phase controller

The dual Youla parametrisation of all LTI systems that are stabilised by a given controller was introduced in (Hansen *et al.*, 1989) as a clever way of turning a closed-loop identification problem into an open-loop problem. We show that it delivers a guaranteed stable nominal closed-loop system, even when the controller is both unstable and nonminimum phase. Let G_{aux} be any system stabilised by the present controller K. Then there exist coprime factorisations $G_{aux} = \frac{N}{M}$ and $K = \frac{U}{V}$, where N, M, U and V belong to $\mathcal{RH}_\infty$, such that the Bezout identity $VM + UN = 1$ holds. Then, the set of all LTI plants stabilised by K is given by

$$\Sigma_K = \left\{ G(R) = \frac{N + VR}{M - UR} : R \in \mathcal{RH}_\infty \right\}, \tag{15}$$

with $G_0 = G(R_0)$ for some $R_0 \in \mathcal{RH}_\infty$. From the data $r_1(t)$, $r_2(t)$, $y(t)$ and $u(t)$, one can construct the auxiliary signals:

$$\bar{r}(t) = Ur_1(t) + Vr_2(t), \quad \text{and} \quad z(t) = My(t) - Nu(t). \tag{16}$$

It can then be shown that

$$z(t) = R_0\bar{r}(t) + (M - UR_0)\, v(t), \tag{17}$$

where $\bar{r}(t)$ and $v(t)$ are uncorrelated. Using the data $z(t)$ and $\bar{r}(t)$ one can estimate a stable model $\hat{R}$ of the stable 'true' dual Youla parameter R_0, from which a model $\hat{G}$ of G_0 is obtained as

$$\hat{G} = \frac{\hat{N}}{\hat{M}} \triangleq \frac{\left(N + V\hat{R}\right)}{\left(M - U\hat{R}\right)} \tag{18}$$

By construction, $\hat{G}$ is stabilised by K, whatever the possible unstable poles and nonminimum phase zeros of K.

Remarks: The factorisation of K into $\frac{U}{V}$ with U, V normalized puts some constraints on the design of $r_1(t)$ and $r_2(t)$. A low gain of one of these factors at some frequency corresponds to a high gain of the other factor at the same frequency. In addition, a blocking zero of K is a blocking zero of U, while a blocking pole of K is a blocking zero of V. As a result, the following guidelines are appropriate for the excitation signals.

- If U has a low gain at some frequencies, then $r_2(t)$ must be nonzero in order to have a good SNR and avoid a poor estimation of R_0 at these frequencies;
- If V has a low gain at some frequencies, $r_1(t)$ must be nonzero in order to have a good SNR and avoid a poor estimation of R_0 at these frequencies;

- It is always better to use both reference signals if possible;
- If K has no blocking poles or zeros, either of the two reference signals can be used even if the controller is unstable and/or nonminimum phase; this is a significant practical advantage in comparison with the other approaches.

6 Numerical illustration

We consider as 'true system' the following ARX system (with $v(t) = H_0(z)e(t)$, and $e(t)$ white Gaussian noise: see Figure 1):

$$G_0(z) = \frac{0.1028z + 0.1812}{z^4 - 1.992z^3 + 2.203z^2 - 1.841z + 0.8941}; \tag{19a}$$

$$H_0(z) = \frac{z^4}{z^4 - 1.992z^3 + 2.203z^2 - 1.841z + 0.8941}. \tag{19b}$$

It describes a flexible transmission system that was used in (Landau *et al.*, 1995) and references therein as a benchmark for testing various control design methods. We consider this system in closed loop with the feedback controller

$$K(z) = \frac{0.5517z^4 - 1.765z^3 + 2.113z^2 - 1.296z + 0.4457}{z^3\,(z-1)}.$$

This controller was obtained via an Iterative Feedback Tuning (IFT) scheme in (Hjalmarsson *et al.*, 1995); the achieved generalised stability margin with K is $b_{G_0,K} = 0.2761$, while the maximum stability margin that could be reached for this system is $\sup_{\bar{K}} b_{G_0,\bar{K}} = 0.4621$. Note that $K(z)$ has a unit-circle pole, located in $z = 1$, which will have a blocking effect on $r_2(t)$, and that $K(z)$ also has a pair of strictly nonminimum-phase complex zeros in $z = 1.2622 \pm 0.2011j$, which may pose problems if $r_1(t)$ is used alone.

We now test the various closed-loop identification techniques described in this paper [1] . Each of the models $\hat{G}$ obtained by these methods is used to compute a controller stabilising $\hat{G}$ and presenting good nominal performance (zero static error and a faster response than with the current controller K, namely a closed-loop bandwidth located between the two resonant peaks of the open-loop system, while keeping the overshoot at less than 10%). Because of the increased closed-loop bandwidth, the new controllers have smaller stability margins than the current one. The suitability of a model $\hat{G}$ for control design will be assessed by checking the sufficient stability conditions of Section 3 and the performance of the designed controller $\bar{K}$ with the true system G_0.

[1] Due to space limitation, it is not possible to show the results for all methods. The interested reader will find all results in Chapter 4 of (Codrons, 2000).

The signals $r_1(t)$ and/or $r_2(t)$, and $e(t)$ used to excite the closed-loop system $(G_0,\, K)$ for identification are chosen as mutually independent Gaussian sequences with zero mean and variances 1, 1 and 0.05, respectively. The model structures we use all contain the true system, yielding unbiased estimates, in order to show that the variance error alone is sufficient to produce the instability problems described in this paper. The control design method used with each identified model $\hat{G}$ is as follows. First, coprime-factor based $\mathcal{H}_\infty$ control design is used to compute a one-degree-of-freedom controller $\check{K}$ for an augmented nominal model $\hat{G}_{aug}(z) = W_2(z)\hat{G}(z)W_1(z)$, where W_1 and W_2 are loop shaping filters aimed at producing good nominal performance, such that $\check{K}$ stabilises $\hat{G}_{aug}$ and satisfies the following $\mathcal{H}_\infty$ constraint:

$$\left\|T(\hat{G}_{aug},\, \check{K})\right\|_\infty \leq \frac{1}{\sup_K b_{\hat{G}_{aug},K}} + 10^{-6}. \tag{20}$$

Then, a two-degree-of-freedom controller $\bar{C} = [\bar{F} \;\; \bar{K}]$, with $u = \bar{F}r_2 - \bar{K}y$, is obtained for the nominal model $\hat{G}$ by setting $\bar{K}(z) = W_2(z)\check{K}(z)W_1(z)$ and $\bar{F}(z) = W_2(1)\check{K}(1)W_1(z)$. W_1 typically contains an integrator to ensure zero static error, while W_2 is adjusted in order to reduce the effect of the second resonant mode.

The coprime-factor and Hansen schemes lead to models of $\hat{G}$ whose order is larger than that of the true G_0; as a result, these models have nearly nonminimal modes. These are then cancelled before control design, by reducing the order of $\hat{G}$ using coprime-factor balanced truncation (Codrons, 2000). The reduced-order models are denoted $\check{G}$ in the sequel, and the two-degree-of-freedom controllers computed from such models are denoted $\tilde{C} = [\tilde{F} \;\; \tilde{K}]$.

6.1 *The indirect approach*

We analyse the case where T_{12} is identified in step 1. The closed-loop system was simulated with $r_1(t)$ set to zero, and with $r_2(t)$ and $e(t)$ as stated above. Using 1000 measurements of $r_2(t)$ and $y(t)$, an output error model $\hat{T}_{12}$ with exact structure (OE[3,8,3]) was estimated, from which a model $\hat{G}$ of the plant was derived using $\hat{G} = \frac{\hat{T}_{12}}{1-K\hat{T}_{12}}$: see Figure 2.

As expected, $\hat{G}$ is a very bad estimate at low frequencies since it has a zero at the blocking pole of K. Furthermore, the three reconstructed entries of the nominal closed-loop transfer matrix all have a pole in $z = 1$, meaning that the nominal closed-loop transfer matrix is unstable ($b_{\hat{G},K} = 0$). This is because the zero at $z = 1$ of T_{12} is estimated as a zero at $z = 0.9590$ in $\hat{T}_{12}$, which does

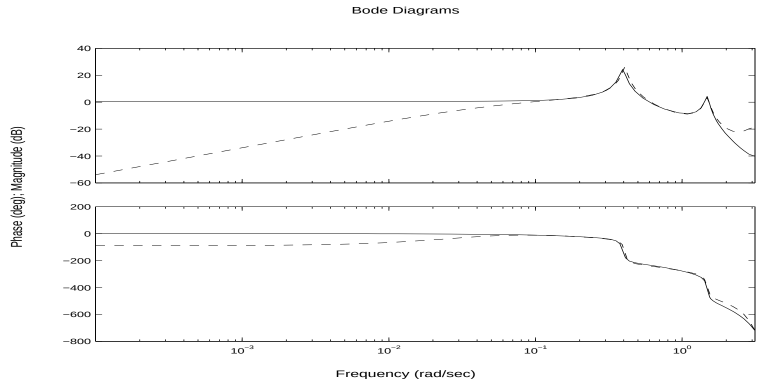

Fig. 2. Indirect identification using T_{12}: Bode diagrams of G_0 (—) and $\hat{G}$ (−−)

not cancel the integrator of K. Finally, note that

$$\delta_\nu(G_0, \hat{G}) = 1 > \sup_{\bar{K}} b_{\hat{G},\bar{K}} = 0.2389, \tag{21}$$

where the first equality results from a violation of the winding number condition (Vinnicombe, 1993). Thus, whatever controller $\bar{K}$ stabilises $\hat{G}$, there is no guarantee that this controller will also stabilise G_0. The model we have obtained here is not suitable for control design.

The following loop shaping filters were used to compute a stabilising controller:

$$W_1(z) = \left(\frac{z}{z-1}\right)^2 \qquad \text{and} \qquad W_2(z) = \frac{2.44z^2-1.017z+0.167}{z^2-0.1519z+0.7423}. \tag{22}$$

Here, a double integrator was necessary to ensure zero static error, because of the differentiator contained in $\hat{G}$. The resulting two-degree-of-freedom controller $[\bar{F} \;\; \bar{K}]$ has $\bar{K}$ of degree 14. Its performance with $\hat{G}$ and G_0 is depicted in Figure 3, where it can be seen that the nominal closed loop is stable and has good performance while the achieved closed loop is unstable. Similar results were obtained with the other three variants of the indirect method.

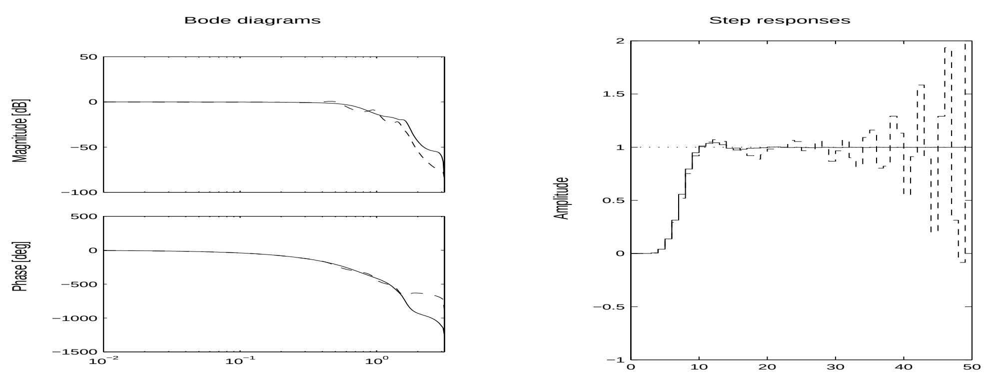

Fig. 3. Control design via indirect identification using T_{12}: Bode diagrams (left) and step responses (right) of $\frac{\bar{F}\hat{G}}{1+\bar{K}\hat{G}}$ (—) and $\frac{\bar{F}G_0}{1+\bar{K}G_0}$ (−−)

Since the controller K has a pole on the unit circle and zeros outside the unit circle, we can expect that the model obtained from $\hat{G} = \frac{\hat{T}_{11}}{\hat{T}_{21}}$ will be destabilised by K, while the model obtained from $\hat{G} = \frac{\hat{T}_{12}}{\hat{T}_{22}}$ should be stabilised by K.

6.2.1 Identification of T_{11} and T_{21}

The closed-loop system was simulated with $r_2(t) = 0$ and $r_1(t)$ and $e(t)$ as above. $\hat{T}_{11}$ was identified using an exact OE[6,8,3] model structure and 1000 samples of $r_1(t)$ and $y(t)$. $\hat{T}_{21}$ was identified using an exact OE[9,8,0] model structure and 1000 samples of $r_1(t)$ and $u(t)$. A model $\hat{G}$ of order 16 was then computed from $\hat{G} = \frac{\hat{T}_{11}}{\hat{T}_{21}}$. As expected, the nominal closed-loop transfer matrix (11) is unstable and

$$\delta_\nu(G_0, \hat{G}) = 1 > \sup_{\bar{K}} b_{\hat{G},\bar{K}} = 0.0299. \tag{23}$$

This model is again unsuitable for control design. This is confirmed by our attempt to compute a controller for G_0 using $\hat{G}$. A two-degree of freedom $\mathcal{H}_\infty$ controller $[\bar{F} \;\; \bar{K}]$ stabilising $\hat{G}$, with $\bar{K}$ of degree 22, was obtained with the following weightings:

$$W_1(z) = \frac{z}{z-1} \qquad \text{and} \qquad W_2(z) = \frac{1.09z^2 - 0.3616z + 0.2327}{z^2 - 0.3072z + 0.2683}.$$

Its performance with $\hat{G}$ and G_0 is illustrated in Figure 4. The nominal closed loop is stable, but the performance is poor due to the small value of $\sup_{\bar{K}} b_{\hat{G},\bar{K}}$ while, as expected, the achieved closed loop is unstable.

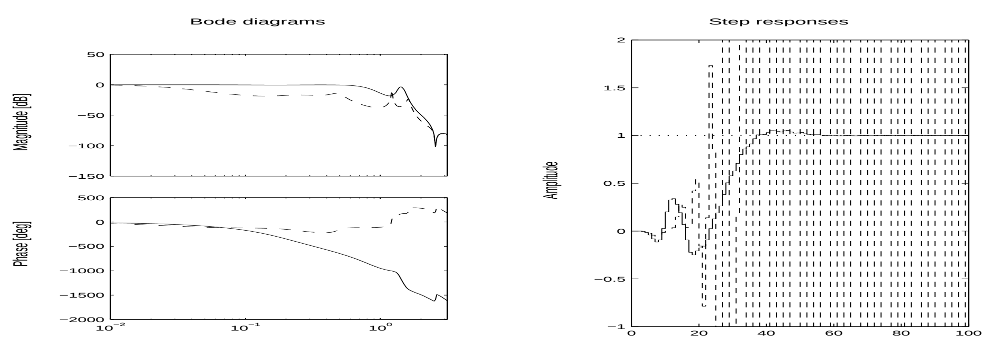

Fig. 4. Control design via coprime-factor identification using $r_1(t)$: Bode diagrams (left) and step responses (right) of $\frac{\bar{F}\hat{G}}{1+\bar{K}\hat{G}}$ (—) and $\frac{\bar{F}G_0}{1+\bar{K}G_0}$ (−−)

6.2.2 *Identification of T_{12} and T_{22}*

$\hat{T}_{12}$ was obtained as in the indirect approach, while an exact OE[6,8,0] model structure was used to estimate $\hat{T}_{22}$ from 1000 samples of $r_2(t)$ and $u(t)$. A model $\hat{G}$ of order 13 was then computed from $\hat{G} = \frac{\hat{T}_{12}}{\hat{T}_{22}}$. As predicted by the theory, Figure 5 shows that $\hat{T}_{22} + \hat{T}_{12}K \approx 1$ except near $\omega = 0$ where it goes to infinity. This is a necessary condition to ensure stability of the nominal closed-loop system when the controller contains an integrator. The nominal closed-loop system is indeed stable with $b_{\hat{G},K} = 0.2517 \approx b_{G_0,K} = 0.2761$. Furthermore, $\delta_\nu(G_0, \hat{G}) = 0.1881 < \sup_{\bar{K}} b_{\hat{G},\bar{K}} = 0.4560$, which means that $\hat{G}$ is a good model for control design.

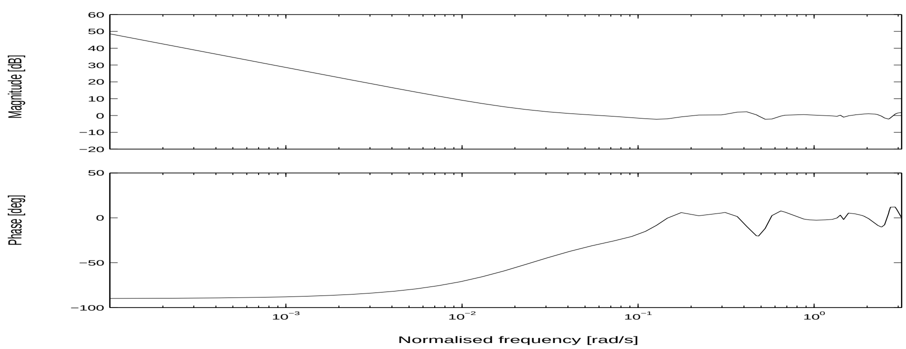

Fig. 5. Coprime-factor approach using $r_2(t)$: Bode diagram of $\hat{T}_{22} + \hat{T}_{12}K$

6.3 *The direct approach*

We consider separately excitation through r_1 and excitation through r_2, each time using 1000 samples of $u(t)$ and $y(t)$ to identify a model $\hat{G}$ with the correct structure ARX[4,2,3]. The model $\hat{G}$ obtained by direct identification with $r_2(t)$ set to zero, is stabilised by K and the achieved nominal stability margin is close to that of G_0: $b_{\hat{G},K} = 0.2751 \approx b_{G_0,K} = 0.2761$. Furthermore, $\delta_\nu(G_0, \hat{G}) = 0.0620 < \sup_{\bar{K}} b_{\hat{G},\bar{K}} = 0.4760$; hence this is a good model for control design. With $r_1(t)$ set to zero, the ν-gap between $\hat{G}$ and G_0 increased, but the obtained model was still very close to the true system and could be used to compute a satisfactory controller. See (Codrons, 2000) for details.

6.4 *The Hansen scheme*

We present the results here when r_1 only is used for excitation. For the auxiliary model G_{aux}, we chose a model that represents the plant with zero load, while G_0 represents the plant for a 50% load (Landau *et al.*, 1995):

$$G_{aux}(z) = \frac{0.2826z + 0.5067}{z^4 - 1.418z^3 + 1.589z^2 - 1.316z + 0.8864}. \tag{24}$$

G_{aux} is stabilised by K. Figure 6 shows the Bode diagrams of the normalized coprime factors U and V of K. Problems might occur when using $r_1(t)$ alone, because of the low gain of U between 0.1 rad/s and 2 rad/s.

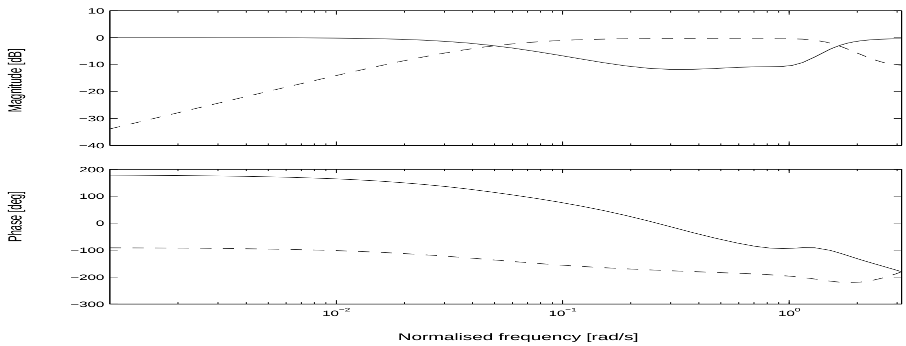

Fig. 6. Bode diagrams of the normalised coprime factors of K: U (—) and V (−−)

An output error model structure OE[14,16,3] was used for the Youla parameter $\hat{R}$. A 28th-order model $\hat{G}$ of G_0 was obtained using (18). Using coprime-factor balanced truncation, it was reduced to a model $\check{G}$ of order 4. The nominal stability margin of $\check{G}$ with the current controller is $b_{\check{G},K} = 0.2795$, and the ν-gap between the model and the true system is $\delta_\nu(G_0, \check{G}) = 0.5121 < \sup_{\bar{K}} b_{\check{G},\bar{K}} = 0.5523$. A stabilising two-degree of freedom controller was computed for $\check{G}$ using the loop shaping filters

$$W_1(z) = \frac{z}{z-1} \qquad \text{and} \qquad W_2(z) = \frac{2.411z^2 - 1.042z + 0.1741}{z^2 - 0.2037z + 0.7468}, \tag{25}$$

leading to a feedback controller $\tilde{K}$ of degree 9. The nominal and achieved closed loops have stability margins $b_{\check{G},\tilde{K}} = 0.0742$ and $b_{G_0,\tilde{K}} = 0.0558$, respectively.

7 Conclusions

In the case where an unstable or nonminimum phase controller is used during closed-loop identification, some of the classical closed-loop identification methods deliver a model that forms an unstable loop with the acting controller. As a result, standard tools used to guarantee robust stability and performance of a newly designed model-based controller become useless. For each of the studied identification methods, we have examined what happens when the controller has an unstable pole, a nonminimum phase zero, or both, leading to the conclusion that some combinations of controller singularity and identification method are an absolute no-no: they lead to a guaranteed unstable nominal closed-loop system!

The main outcome of our results is that, on the basis of the known controller singularities, the user can choose an identification method and the location of the external excitation in such a way as to guarantee stability of the nominal closed-loop system.

Acknowledgements

B. Codrons and M. Gevers acknowledge the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. B.D.O. Anderson acknowledges funding of this research by the US Army Research Office, Far East, the Office of Naval Research, Washington. The scientific responsibility rests with its authors.

References

Anderson, B.D.O., X. Bombois, M. Gevers and C. Kulcsár (1998). Caution in iterative modeling and control design. In: *IFAC Workshop on Adaptive Control and Signal Processing*. Glasgow. pp. 13–19.

Bitmead, R. R., B.D.O. Anderson, M. Gevers and L.C. Kammer (2000). Cautious controller tuning, submitted for publication. *Automatica*.

Codrons, B. (2000). Experiment design issues in modelling for control. Technical report. Ph.D. thesis. Université Catholique de Louvain.

de Callafon, R.A. and P.M.J. Van den Hof (1997). Suboptimal feedback control by a scheme of iterative identification and control design. *Mathematical Modelling of Systems* **3**(1), 77–101.

Forssell, U. and L. Ljung (1999). Closed-loop identification revisited. *Automatica* **35**, 1215–1241.

Gevers, M. (1993). Towards a joint design of identification and control?. In: *Essays on Control: Perspectives in the Theory and its Applications, Birkhauser, New York* (H.L. Trentelman and J.C. Willems, Eds.). Birkhauser. New York. pp. 111–151.

Gevers, M., L. Ljung and P.M.J. Van den Hof (2001). Asymptotic variance expressions for closed-loop identification. *Automatica* **37**(5), 781–786.

Goodwin, G.C. and J.S. Welsh (2001). Singularity issues associated with closed loop identification. In: *Model Identification and Adaptive Control* (G.C. Goodwin, Ed.). Springer. pp. 27–52.

Hakvoort, R.G., R.J.P. Schrama and P.M.J. Van den Hof (1994). Approximate identification with closed-loop performance criterion and application to LQG feedback design. *Automatica* **30**(4), 679–690.

Hansen, F., G. Franklin and R. Kosut (1989). Closed-loop identification via the fractional representation: Experiment design. *Proc. ACC*.

Hjalmarsson, H., S. Gunnarsson and M. Gevers (1995). Model-free tuning of a robust regulator for a flexible transmission system. *European Journal of Control* **1**(2), 148–156.

Landau, I.D., D. Rey, A. Karimi, A. Voda and A. Franco (1995). A flexible transmission system as a benchmark for robust digital control. *European Journal of Control* **1**(2), 77–96.

Lee, W.S., B.D.O. Anderson, I.M.Y Mareels and R.L. Kosut (1995). On some key issues in the windsurfer approach to adaptive robust control. *Automatica* **31**(11), 1619–1636.

Ljung, L. (1999). *System Identification: Theory for the User, 2nd Edition*. Prentice-Hall. Englewood Cliffs, NJ.

Milanese, M. and Michele Taragna (1999). SM identification of model sets for robust control design. In: *Robustness in Identification and Control* (A. Garulli, A. Tesi and A. Vicino, Eds.). Vol. 245 of *Lecture Notes in Control and Information Sciences*. Springer-Verlag. pp. 17–34.

Skelton, R.E. (1989). Model error concepts in control design. *Int. Journal of Control* **49**(5), 1725–1753.

Söderström, T. and P. Stoica (1989). *System Identification*. Prentice-Hall International. Hemel Hempstead, Hertfordshire.

Van den Hof, P.M.J. and R.J.P. Schrama (1995). Identification and control - closed-loop issues. *Automatica* **31**, 1751–1770.

Van den Hof, P.M.J., R.J.P. Schrama, R.A. de Callafon and O.H. Bosgra (1995). Identification of normalized coprime plant factors from closed-loop experimental data. *European Journal of Control* **1**, 62–74.

Vinnicombe, G. (1993). Frequency domain uncertainty and the graph topology. *IEEE Trans Automatic Control* **AC-38**, 1371–1383.

Vinnicombe, G. (2000). *Uncertainty and Feedback - H_∞ loop-shaping and the ν-gap metric*. Imperial College Press.

Zang, Z., R.R. Bitmead and M. Gevers (1995). Iterative weighted least-squares identification and weighted LQG control design. *Automatica* **31**(11), 1577–1594.

Zhou, K. and J.C. Doyle (1998). *Essentials of Robust Control*. Prentice Hall, New Jersey.