Model Reference Control Design by Prediction Error Identification

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Abstract: This paper studies direct (non-iterative) data-based method for Model Reference (MR) control design. It shows that the optimal controller can be obtained as the solution of a Prediction Error (PE) identification problem that directly estimates the controller parameters through a reparametrization of the input-output model. The standard tools of Prediction Error Identification can thus be used to analyze the statistical properties (bias and variance) of the estimated controller. It also shows that, for MR control design, direct and indirect data-based methods are essentially equivalent.

Keywords: Data-driven controller tuning, direct controller tuning, prediction error identification, identification for control

1. INTRODUCTION

In the past two decades, a number of data-based control design methods have been proposed (Hjalmarsson et al., 1994, 1998; Campi et al., 2002; Karimi et al., 2004), where a parametrized controller structure is chosen a priori, and the controller tuning is based directly on input and output data collected on the plant without the use of a model of this plant. Some of these methods, like Iterative Feedback Tuning (Hjalmarsson et al., 1994, 1998) and Correlation-based Tuning (CbT) (Karimi et al., 2004) are iterative in nature: the optimal controller is obtained as a sequence of controllers that operate on the actual plant, and experimental data are collected on the corresponding sequence of closed-loop plants. Other methods are direct (or one-shot): they directly estimate the controller on the basis of one sequence of input-output data: Virtual Reference Feedback Tuning (VRFT) (Campi et al., 2002) and a non-iterative version of CbT (Karimi et al., 2007) are representative of this class.

The Virtual Reference Feedback Tuning (VRFT) method was first proposed in (Campi et al., 2002). It uses controller structures that are linearly parametrized, and it was shown that in such case the optimal Model Reference (MR) controller can be estimated very simply as the solution of a Least Squares problem for noise free data. In the case of noisy data, the VRFT method has been looked upon as an identification problem, but with inputs that are corrupted by measurement noise. The most common way to circumvent the problems in the application of VRFT to noisy data is to apply instrumental variable methods for the parameter estimation. The bias problem can thus be resolved, but at the expense of a loss in efficiency.

Unstable or non-minimum phase plants also cause serious problems for the application of VRFT as pointed out in (Sala and Esparza, 2005a,b), where some ad-hoc methods were discussed to alleviate these difficulties. An excellent analysis of the VRFT method in the case of noisy data can be found in (van Heusden et al., 2011): the authors discuss the similarities and differences between the controller estimation that takes place in VRFT and the model estimation in prediction error identification, and they illustrate the bias problems that arise in VRFT due to the fact that the inputs to the controller estimation problem are noisy and that the controller and noise model have common parameters. They also point to the difficulty caused by the fact that the controller parameters appear in the noise model.

In this paper we examine a completely different approach, which is based on the PE identification of the inverse of a part of the controller. The idea of identifying the inverse of the controller has been suggested in (Sala and Esparza, 2005a), but without any detail about its implementation. In (van Heusden et al., 2011) the authors have briefly discussed this idea again, but mainly to show its difficulties and shortcomings, such as possible unstable pole-zero cancellations and the presence of controller parameters in the resulting noise model.

We develop a new approach to the data-based identification of the inverse of the controller in which the input-output model of the system is replaced from the outset by an equivalent input-output description involving only parameters that are functions of the optimal MR controller

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parameters. Unlike the VRFT scheme and most other data-based control design methods, the controller structure is not limited to being linear in the parameters. With this new parametrization, the estimation of the controller parameters is now put in a completely standard open-loop PE identification problem. As a result, the optimal reference controller can be identified without bias using open-loop data provided the model structure chosen for the controller includes the optimal controller, even if the noise model is incorrect. In the non-ideal case where the controller structure does not include the optimal controller, the standard results from PE identification can be used to characterize the bias of the resulting controller.

A consequence of our embedding of the controller estimation problem in a standard PE identification framework is that a complete statistical analysis of the estimated controller is provided. Another interesting outcome of our analysis is that we show that the direct data-driven estimation of the MR controller by PE identification of the controller inverse is essentially equivalent to PE identification of the plant model followed by control computation through an algebraic equation. Thus we show that for MR control, direct and indirect design methods are essentially equivalent and that the opposition between direct data-based control and identification for control is essentially meaningless.

The paper is organized as follows. Definitions and the problem formulation are presented in Section 2. Section 3 presents the reparametrization of the input-output model using controller parameters that will be identified by Prediction Error identification. Section 4 presents the proposed controller identification method, while the design choices and the properties of the estimate are explored in Section 5. Section 6 illustrates the application of the proposed method and the design choices through a simulated example. Conclusions are presented at the end of the paper.

2. PRELIMINARIES

Consider a linear time-invariant discrete-time single-input-single-output process

$$y(t) = G_0(z)u(t) + v(t) = G_0(z)u(t) + H_0(z)e(t), \quad (1)$$

where z is the forward-shift operator, $G_0(z)$ is the process transfer function, $u(t)$ is the control input, $H_0(z)$ is the noise model, and $e(t)$ is zero mean white noise with variance σ_e^2. Both transfer functions, $G_0(z)$ and $H_0(z)$, are rational and causal.

The task is to tune the parameter vector ρ of a linear time-invariant controller $C(z,\rho)$ in order to achieve a desired closed-loop response. We assume that this controller belongs to a given user specified controller class $\mathcal{C}$ such that $C(z,\rho)G_0(z)$ has positive relative degree for all $C(z,\rho) \in \mathcal{C}$; equivalently, the closed loop is not delay-free. The control action $u(t)$ can be written as

$$u(t) = C(z,\rho)(r(t) - y(t)), \quad (2)$$

where $r(t)$ is a reference signal, which is assumed to be quasi-stationary and uncorrelated with the noise, that is $\bar{E}[r(t)e(s)] = 0 \ \forall t, s$, and $\bar{E}[f(t)] \triangleq \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} E[f(t)]$ with $E[\cdot]$ denoting expectation (Ljung, 1999). The system (1)-(2) in closed loop becomes

$$y(t,\rho) = T(z,\rho)r(t) + S(z,\rho)v(t)$$
$$T(z,\rho) = \frac{C(z,\rho)G_0(z)}{1 + C(z,\rho)G_0(z)} = C(z,\rho)G_0(z)S(z,\rho)$$

where we have now made the dependence on the controller parameter vector ρ explicit in the output signal $y(t,\rho)$.

Model Reference control design consists of specifying a "desired" closed-loop transfer function $M(z)$, which is known as the *reference model*, and then solving the following optimization problem

$$\min_{\rho} J^{MR}(\rho) \quad (3)$$
$$J^{MR}(\rho) \triangleq \bar{E}\left[((T(z,\rho) - M(z))r(t))^2\right]. \quad (4)$$

The *optimal controller* is defined as $C(z,\rho^{MR})$ with

$$\rho^{MR} = \arg\min_{\rho} J^{MR}(\rho).$$

We assume that the user can collect a batch of data from the process (1)

$$Z^N = [u(1),\ y(1), \ldots, u(N),\ y(N)].$$

His/her task is then to estimate the *optimal parameters* of the controller $C(z,\rho^{MR})$ from these data.

3. REPARAMETRIZATION OF THE INPUT-OUTPUT MODEL

Analyzing (4) we see that if the *ideal controller*

$$C_d(z) \triangleq \frac{M(z)}{G_0(z)(1 - M(z))} \quad (5)$$

were used in the closed loop then the objective function (3) would evaluate to zero. It is not the goal of this paper to discuss the choice of the reference model $M(z)$; this is the topic of many textbooks on control design. We shall assume that this choice is appropriate for the system $G_0(z)$ and that, as a consequence, the controller $C_d(z)$ is realizable as a causal filter.

The core idea of the method is to rewrite the input-output system (1) as a function of the reference model and the controller by inverting the relation (5), i.e.

$$G_0(z) = \frac{1}{C_d(z)}\frac{M(z)}{1 - M(z)}. \quad (6)$$

The task will be to identify $C_d(z)$ from input-output data using an appropriate parametrization for the set of controllers $\mathcal{C} = \{C(z,\rho),\ \rho \in \mathbb{R}^d\}$. Now it is often the case that one imposes some fixed part in the controller, which therefore need not be identified. We call $C^F(z)$ this fixed part, so that the ideal controller can be written as $C_d(z) = C_d^I(z)C^F(z)$ and its model as

$$C(z,\rho) = C^I(z,\rho)C^F(z). \quad (7)$$

This yields a new description for $G_0(z)$:

$$G_0(z) = \frac{1}{C_d^I(z)} \times \frac{M(z)}{C^F(z)(1 - M(z))}. \quad (8)$$

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We now define the filtered input signal

$$\tilde{u}(t) \triangleq \frac{1}{C^F(z)} \frac{M(z)}{1-M(z)} u(t), \qquad (9)$$

and the transfer functions

$$\tilde{C}_d(z) \triangleq \frac{1}{C_d^I(z)} \text{ and } \tilde{C}(z,\theta) \triangleq \frac{1}{C^I(z,\rho)}, \qquad (10)$$

so that

$$C_d(z) = \frac{1}{\tilde{C}_d(z)} C^F(z) \text{ and } C(z,\rho) = \frac{1}{\tilde{C}(z,\theta)} C^F(z). \quad (11)$$

The input-output system (1) is then equivalent with

$$\mathcal{S}_c : \; y(t) = \tilde{C}_d(z)\tilde{u}(t) + H_0(z)e(t), \qquad (12)$$

for which the following parametric model structure can be chosen

$$\mathcal{M}_c : \; y(t,\theta) = \tilde{C}(z,\theta)\tilde{u}(t) + H(z,\theta)e(t). \qquad (13)$$

Here $\tilde{C}_d(z)$ is the portion of the inverse of the *ideal controller* that we want to identify, $\tilde{C}(z,\theta)$ is a parametric model structure for $\tilde{C}_d(z)$ and $H(z,\theta)$ is a parametric model structure for $H_0(z)$. The model structures $\tilde{C}(z,\theta)$ and $H(z,\theta)$ may have either common or disjoint parameters, as we shall discuss in Section 5.

Before we describe some properties of the input-output model (13), let us illustrate by an example how the parameter vector θ is related to the controller parameter vector ρ one wants to obtain. Suppose the chosen controller class $\mathcal{C}$ is a class of PID controllers, given by

$$C(z,\rho) = \frac{\rho_1 z^2 + \rho_2 z + \rho_3}{z^2 - z}.$$

From (7), we have

$$C^I(z,\rho) = \rho_1 z^2 + \rho_2 z + \rho_3, \quad C^F(z,\rho) = \frac{1}{z^2 - z}$$

and then $\tilde{C}(z,\rho) = \frac{1}{\rho_1 z^2 + \rho_2 z + \rho_3}$. It is usual in system identification that the highest term of the denominator is unitary. If we divide the numerator and denominator by ρ_1 and if we use the change of variables $\theta_1 = \frac{1}{\rho_1}$, $\theta_2 = \frac{\rho_2}{\rho_1}$ and $\theta_3 = \frac{\rho_3}{\rho_1}$ we get

$$\tilde{C}(z,\theta) = \frac{\theta_1}{z^2 + \theta_2 z + \theta_3}.$$

Clearly, if a parameter vector $\hat{\theta}$ is identified, the controller parameters are recovered as $\hat{\rho} = \frac{1}{\hat{\theta}_1}[1 \;\; \hat{\theta}_2 \;\; \hat{\theta}_3]$.

Properties of the reparametrization

We now examine some properties of the input-output description (12) and of the model (13) that will be used to identify the optimal controller. It is important to stress first that (12) and (1) are two equivalent descriptions of the input-output map.

We examine the connections between the poles and zeroes of the system $G_0(z)$, the reference model $M(z)$ and the optimal controller $C_d(z)$. From (5) we have that

$$C_d(z) = \frac{d_{G_0}(z) n_M(z)}{n_{G_0}(z)(d_M(z) - n_M(z))}, \qquad (14)$$

where $n_F(z)$ denotes the numerator of a transfer function $F(z)$ and $d_F(z)$ its denominator. We observe that in our identification problem of this optimal controller, $n_M(z)$ and $d_M(z)$ are known quantities (imposed by the designer) while $n_{G_0}(z)$ and $d_{G_0}(z)$ are unknown.

On the basis of (14) we can make the following observations.

- The transfer functions $\tilde{C}_d(z)$ and $\tilde{C}(z,\theta)$ can always be made causal, by adding powers of z in their denominator, if necessary. Corresponding terms are then also added in the denominator of $C^F(z)$ so that they will cancel in forming $C_d(z)$ and $C(z,\rho)$.
- The zeroes of $C_d(z)$ are a combination of the poles of $G_0(z)$ and of the zeroes of $M(z)$, whereas its poles contain the zeroes of $G_0(z)$. As a result, the transfer function $\tilde{C}_d(z)$ in (12) will be unstable if $G_0(z)$ contains unstable poles. If that is the case, one will have to resort to an ARX or ARMAX model structure for the identification of the model (13) since such model structures allow the identification of unstable systems. This means that in such case, also the noise model will need to be estimated even if open-loop data are used.
- If the process $G_0(z)$ has a non-minimum phase zero, it must be included as a zero of the reference model $M(z)$; this is one of the basic constraints of MR control. Violating this requirement may lead to an unstable closed-loop system. One data-based procedure for detecting and estimating non-minimum phase zeroes has been described in (Campestrini et al., 2011). Assuming that this precaution has been taken, it follows that $C_d(z)$ will be stable.
- It follows from (14) that there are several possibilities to factor $C_d(z)$ into a fixed part and a part to be identified. Since $M(z)$ is known, one extreme possibility is to incorporate $\frac{n_M(z)}{d_M(z)-n_M(z)}$ in the fixed part $C^F(z)$ of the parametric controller structure (7). By doing this, one gets $\tilde{u}(t) = u(t)$ and $\tilde{C}_d(z) = G_0(z)$ in (12), and the problem is then reduced to the identification of $G_0(z)$ and the computation of the controller via (5) in which $G_0(z)$ is replaced by the identified $G(z,\hat{\theta})$. In other words, this amounts more to identification for control than to direct control design.
- If the reference model $M(z)$ has been chosen to produce zero steady-state error, then $d_M(z) - n_M(z)$ contains a zero at $z = 1$. The other extreme, therefore, is to choose $C^F(z)$ as an integrator and to let the "free part" $C_d^I(z)$ of $C_d(z)$ contain all other zeroes and poles of the right hand side of (14).
- In between these two extreme choices, the fixed part $C^F(z)$ may contain any fraction of the known transfer function $\frac{n_M(z)}{d_m(z)-n_M(z)}$, in which case the parametric model structure $\tilde{C}(z,\theta)$ will take the form

$$\tilde{C}(z,\theta) = \frac{n_G(z,\theta) d_P(z,\theta)}{d_G(z,\theta) n_P(z,\theta)}, \qquad (15)$$

  where n_P and d_P are the fractions of n_M and d_M, respectively, that the user has chosen to put in $\tilde{C}(z,\theta)$.

In Section 5 we shall discuss how this freedom in the selection of the fixed part of the desired controller impacts on the properties of the identified controller. Let

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us mention in passing that the observations made above show that, fundamentally, there is not a lot of difference between indirect and direct controller design in the case of MR control design, given that in such case the difference essentially amounts to a choice of parametrization. This is due to the algebraic link that exists between $G_0(z)$ and $C_d(z)$ in this control design methodology.

4. THE CONTROLLER IDENTIFICATION METHOD

In the previous section we have shown that, for MR control design, the controller estimation problem can be reduced to the identification problem of $\tilde{C}_d(z,\theta)$ in (12)-(13). We can thus use standard PE identification methods to estimate the controller, and the whole body of results on the statistical properties of the PE estimates are available to us for the statistical analysis of the estimated controller.

From N measured input-output data one constructs the data vector

$$Z_c^N = [\tilde{u}(1),\ y(1), \ldots, \tilde{u}(N),\ y(N)]$$

by passing the input data through the filter (9). The estimate $\hat{\theta}_N$ is then $\hat{\theta}_N = \arg\min_\theta \sum_{t=1}^N \varepsilon^2(t,\theta)$ where

$$\varepsilon(t,\theta) \triangleq y(t) - \hat{y}(t|t-1,\theta), \tag{16}$$

and

$$\hat{y}(t|t-1,\theta) = H^{-1}(z,\theta)\tilde{C}(z,\theta)\tilde{u}(t) + \left[1 - H^{-1}(z,\theta)\right] y(t).$$

Using (7) and (10), the estimated optimal controller is then obtained by

$$C(z,\hat{\rho}_N) = \frac{1}{\tilde{C}(z,\hat{\theta}_N)} C^F(z). \tag{17}$$

5. DESIGN CHOICES AND ESTIMATE PROPERTIES

Since the estimation of the optimal MR controller has been reduced to a PE identification problem, all properties of the PE identification theory apply. However, given the properties already mentioned in Section 3 of the relationship (14) between $C_d(z), G_0(z)$ and $M(z)$, the relative freedom in the choice of the fixed part $C^F(z)$ of the controller, and the choice of model structures to be used in (13), we shall in this section discuss some important design choices of the method and the corresponding properties of the resulting controller.

An important observation is that, since the object of interest is the optimal controller only, and not the input-output model, the identification of $H_0(z)$ is of no interest, which may simplify the identification problem as we shall discuss here. Unless otherwise specified, we shall consider that we are in the practical situation where the true noise model is unknown and where no knowledge is available about the structure of $H_0(z)$.

To structure the analysis, we shall distinguish between two situations, the case where the optimal controller $C_d(z)$ defined in (5) is in the controller set $\mathcal{C} = \{C(z,\rho)\}$ defined by (7), and the case where it is not.

5.1 The ideal case: $C_d(z) \in \mathcal{C}$

We first consider the ideal case where the optimal controller is in the parametrized controller class, i. e.

$$\exists\ \theta_0 \ \text{ such that } \ \frac{1}{\tilde{C}(z,\theta_0)} C^F(z) = C(z,\rho_0) = C_d(z).$$

We shall assume that input data $u(t)$ have been chosen such that the filtered inputs $\tilde{u}(t)$ are sufficiently rich with respect to the chosen model structure $C(z,\rho)$ (see (Gevers et al., 2009)). The following results then follow straightforwardly from standard PE identification theory (Ljung, 1999) applied to (13).

- If the data are collected in open loop, and if $\tilde{C}_d(z)$ and $H(z,\theta)$ have disjoint parameters, then $\hat{\theta}_N \to \theta_0$, i.e. with $C(z,\rho)$ defined by (11) we have
$$C(z,\hat{\rho}_N) \to C_d(z).$$
- The same convergence result holds if the data are collected in closed loop, and if in addition $\exists\ \theta_0$ such that $H(z,\theta_0) = H_0(z)$.

Discussion on design choices

- When the optimal controller is in the controller set and the data are collected in open loop, we thus see that the procedure converges to the optimal controller regardless of any knowledge on the noise model. In the absence of any information about the spectral properties of the noise, the easiest solution is therefore to take $H(z,\theta) = 1$, i.e. an Output Error model. If the true noise model $H_0(z)$ is more complex, then including a noise model that is able to represent $H_0(z)$ will improve the variance of the controller estimate, but its asymptotic bias will be zero in any case.
- One of the design choices is the selection of $C_F(z)$ in the factorization of the controller $C(z,\rho)$: see (11). As stated in Section 3, one extreme possibility is to incorporate $\frac{M(z)}{1-M(z)}$ in $C_F(z)$, the other extreme being to leave all parameters of $C(z,\rho)$ free, i.e. to take $C_F(z) = 1$ or $C^F(z) = \frac{1}{z-1}$. In the latter case, the vector ρ - and hence also the vector θ parametrizing $\tilde{C}(z,\theta)$ will contain more parameters than in the former case, resulting in a larger variance for the estimated parameters. This will be illustrated in Section 6; it actually follows directly from the expression (11), which shows that the identified $\tilde{C}(z,\theta)$ includes the model parameters plus additional parameters appearing in $n_P(z,\theta)$ and $d_P(z,\theta)$ which are in fact known.
- When the data are collected in closed loop, it follows from PE identification theory that the estimate of the controller will be biased if an incorrect noise model is used. A range of procedures exist to reduce or eliminate this bias (Ljung, 1999).

We conclude from this subsection that direct data-based MR control design is essentially no different from identification based control design based on the same data. It includes more degrees of freedom in the sense that there is a certain amount of freedom in the choice of the appropriate parametrization. However, the inclusion of more than the minimal number of parameters in $\tilde{C}(z,\theta)$

results in a larger variance than would be obtained by identification of $G_0(z)$ followed by control design via (5).

5.2 The non-ideal case: $C_d(z) \notin \mathcal{C}$

In the non-ideal case, the identification procedure is identical to the ideal case, but the properties of the resulting controller estimate are different. These properties again follow from standard PE identification theory.

With open-loop data, the parameter vector θ in (13) converges to the best possible parameter in the set $\{\tilde{C}(z,\theta)\}$. In other words

$$\hat{\theta}_N \longrightarrow \theta^* = \arg\min_{\theta} \bar{V}(\theta) \tag{18}$$

where

$$\bar{V}(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{|H(e^{j\omega},\theta)|^2}\left\{|\tilde{C}_d(e^{j\omega}) - \tilde{C}(e^{j\omega},\theta)|^2 \Phi_{\tilde{u}}(\omega) + \Phi_v(\omega)\right\} d\omega, \tag{19}$$

where $\Phi_v(\omega)$ is the noise spectrum and $\Phi_{\tilde{u}}(\omega)$ is the spectrum of $\tilde{u}(t)$:

$$\Phi_{\tilde{u}}(\omega) = \frac{|M(e^{j\omega})|^2}{|C^F(e^{j\omega})(1 - M(e^{j\omega}))|^2}\Phi_u(\omega).$$

Since there does not exist a parameter vector θ_0 for which $\tilde{C}_d(z) = \tilde{C}(z,\theta_0)$, the resulting estimate will converge to a biased $C(z,\theta^*)$ and the bias will be governed by the expression (19). In the extreme (but simplest case) where $C^F(z)$ has been chosen as $C^F(z) = \frac{M(z)}{1-M(z)}$ this amounts again to identification of a reduced order model for $G_0(z)$ followed by computation of a reduced order controller for $C_d(z)$, as stated earlier.

6. ILLUSTRATIVE EXAMPLE

In this section, we present an example that illustrates the design choices for the definition of the fixed part of the controller and the properties that result for these different design choices. We consider that the system is described by (1) with

$$G_0(z) = \frac{0.5}{z - 0.9}, \quad H_0(z) = \frac{z}{z - 0.3}, \tag{20}$$

where the white noise variance is $\sigma_e^2 = 0.1$. The desired reference model with zero steady-state error is chosen as:

$$M(z) = \frac{0.16z}{(z - 0.6)^2}.$$

The optimal controller is then calculated from (5) as

$$C_d(z) = \frac{0.32z(z - 0.9)}{(z - 1)(z - 0.36)} = \frac{(0.32z - 0.288)z}{(z - 1)(z - 0.36)}.$$

6.1 The ideal case: $C_d(z) \in \mathcal{C}$

We first select a controller structure $\mathcal{C} = \{C(z,\rho)\}$ that is able to represent the ideal $C_d(z)$. It is clear from (7) that at least two choices are possible for $C^F(z)$:

$$C_1^F(z) = \frac{z}{z - 1} \quad \text{or} \quad C_2^F(z) = \frac{0.16z}{(z - 1)(z - 0.36)}$$

With the first choice we get $C_1^I(z,\rho) = \frac{\rho_1 z + \rho_2}{z + \rho_3}$, while with the second choice we get $C_2^I(z,\rho) = \rho_1 z + \rho_2$. This corresponds to $\tilde{C}_1(z,\theta) = \frac{\theta_1 z + \theta_2}{z + \theta_3}$ for the first choice, and to $\tilde{C}_2(z,\theta) = \frac{\theta_1}{z + \theta_2}$ for the second choice. Thus we observe that the first choice leads to the estimation of 3 parameters, while the second leads to the estimation of only 2 parameters which are in fact the parameters of $G_0(z)$, as explained in Section 3. In order to compare the properties of the estimates corresponding to these two choices, we have identified both under the following experimental conditions.

We applied a PRBS signal with amplitude ± 1 as input signal of an open-loop experiment, and we collected 1000 samples of input data and output data on the process. The identification was done using the function `oe` of the Matlab toolbox `ident`, which means that $H(z,\theta) = 1$. 500 Monte Carlo runs were realized. With the choice $C_1^F(z)$, we obtained the mean value for the parameter vector as

$$\hat{\theta}_m = [3.12395 \ \ -1.12590 \ \ -0.90010]^T,$$

for which the corresponding controller is given by

$$C_1(z,\hat{\rho}_m) = \frac{(0.32024z - 0.28824)z}{(z - 1)(z - 0.36014)}.$$

with variance

$$Cov_1(\hat{\rho}) = 1 \times 10^{-5} * \begin{bmatrix} 4.24366 & -3.77763 & 8.57356 \\ -3.77763 & 3.40404 & -6.98214 \\ 8.57356 & -6.98214 & 30.23981 \end{bmatrix}. \tag{21}$$

With the fixed part of the controller as $C_2^F(z)$, which means that we identify $G(z,\theta)$ and then estimate $C(z,\rho)$, the mean value of the parameter estimate $\hat{\theta}$ is

$$\hat{\theta}_m = [0.49962 \ \ -0.90007]^T,$$

for which the corresponding controller is given by

$$C_2(z,\hat{\rho}_m) = \frac{(0.32030z - 0.28830)z}{(z - 1)(z - 0.36)},$$

with variance

$$Cov_2(\hat{\rho}) = 1 \times 10^{-5} * \begin{bmatrix} 1.90181 & -1.88754 \\ -1.88754 & 1.88200 \end{bmatrix}. \tag{22}$$

As expected, both estimates are unbiased. It follows immediately from (21) and (22) that $Cov_2(\hat{\rho}) < Cov_1(\hat{\rho})$, and hence the precision of the estimates ρ_1 and ρ_2 is better with the choice $C_2^F(z)$ than with $C_1^F(z)$. This is also confirmed by Figure 1, where we plot a graph that shows $\hat{\rho}_1$ and $\hat{\rho}_2$ obtained at each Monte Carlo run, for both designs. Grey circles represent the first two parameters of $C_1^I(z,\hat{\rho})$ while black dots represent the parameters of $C_2^I(z,\hat{\rho})$. This confirms that the best statistical properties are obtained with the indirect method where we only estimate the two parameters of $G(z,\theta)$.

6.2 The non-ideal case: $C_d(z) \notin \mathcal{C}$

Consider now that we choose freely a controller class of PI controllers, which does not contain the optimal controller $C_d(z)$. With the same reference model, we expect a biased controller that should make the closed-loop transfer function as close as possible to the desired one, that is $M(z)$. The PI controller class is given by

$$C(z,\rho) = \frac{\rho_1 z + \rho_2}{z - 1},$$

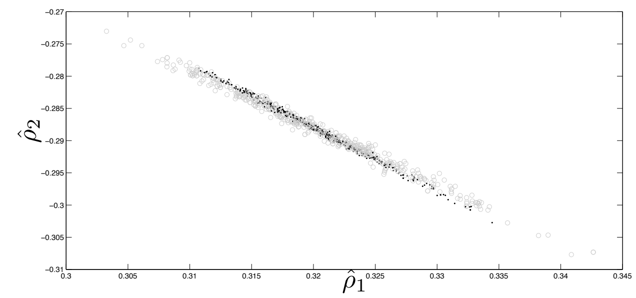

Fig. 1. Grey circles: first two parameters of $C_1^I(z, \hat{\rho})$; black dots: parameters of $C_2^I(z, \hat{\rho})$ estimated through 500 Monte Carlo runs using the proposed method for an OE model.

for which we have chosen the simplest fixed part

$$C^F(z) = \frac{1}{z-1} \quad \text{and} \quad C^I(z) = \rho_1 z + \rho_2.$$

The same experiment was performed, and again the identification was done using the function `oe` of the Matlab toolbox `ident`. After 500 Monte Carlo runs we obtained, for the mean controller

$$C(z, \hat{\rho}_m) = \frac{0.45186z - 0.40095}{(z-1)},$$

which is biased, with variance

$$Cov_{PI}(\hat{\rho}) = 1 \times 10^{-5} * \begin{bmatrix} 5.20536 & -5.23462 \\ -5.23462 & 5.28728 \end{bmatrix}.$$

Notice that the variance is bigger than the ones obtained in both designs for the ideal case, that is, (21) and (22). Figure 2 presents the step responses of the closed loop obtained with one of the PI controllers and the reference model. Note that we obtained a closed-loop response that is very close to the desired one, despite the fact that a PI controller is not the ideal one for the chosen reference model.

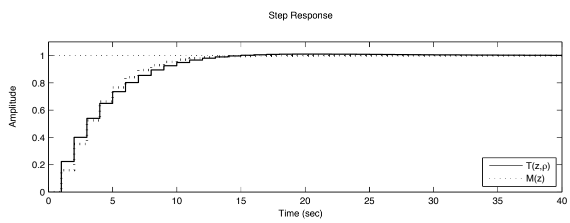

Fig. 2. Step response of the closed-loop system with the controller $C(z, \hat{\rho}) = \frac{0.44768z - 0.39665}{z-1}$ obtained from one of the 500 Monte Carlo runs and the desired reference model $M(z)$.

7. CONCLUSIONS

A direct data-based method used to identify the optimal MR controller has been presented. The method consists in solving a PE identification problem, where the inverse of the ideal controller is identified from data collected on the system. Open loop or closed loop data may be used, but they lead to different properties of the controller estimate. Unlike some other data-based designs, this method does not require that the class of controllers is linear in the parameters.

In developing this method in detail, we have made it clear that this "direct" data-based controller design method for MR control, is essentially equivalent to PE identification of the plant followed by control calculation through the MR design equation. A number of design choices can be made in the reparametrization of the input-output model, and with some design choices the method may depart from indirect identification. However, our analysis has shown that for MR control, indirect and direct data-based control design are essentially equivalent, and that the variance obtained with indirect design is actually smaller than with direct design. In doing so, we believe this paper has added useful insight into data-based MR design methods.

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