Pergamon

0005-1098(95)00082-8

Automatica, Vol. 31, No. 11, pp. 1577–1594, 1995

0005-1098/95 $9.50 + 0.00

Iterative Weighted Least-squares Identification and Weighted LQG Control Design*

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A paradigm is developed for combining successive stages of closed-loop least-squares identfication and frequency-weighted LQG model-based control design. The modelling and control design phases have objective functions related to ultimate achieved performance.

Key Words—Closed-loop identification; linear–quadratic Gaussian control; iterative control design; robust control.

Abstract—Many practical applications of control system design based on input–output measurements permit the repeated application of a system identification procedure operating on closed-loop data together with successive refinements of the designed controller. Here we develop a paradigm for such an iterative design. The key to the procedure is to account for evaluated modelling error in the control design and, equally, to let the closed-loop controller requirements determine the identification criterion. With an H_2 control problem, this is achieved by frequency weighting the linear–quadratic Gaussian (LQG) control criterion with filters that reflect the closed-loop plant/model mismatch, and by filtering the identifier signals used in a least-squares identification scheme in a logical and mutually supportive fashion.

1. INTRODUCTION

In very many practical control applications it is the case that an initial controller is applied to the process, after which on-line measurements may be taken on the closed-loop system. Further, the amount of such data is effectively unlimited. In such circumstances one may use these newly acquired closed-loop measurements to generate more appropriate (but not necessarily more complex or arbitrarily accurate) models and better feedback control laws, as opposed to, say, a once-off robust design based on a model identified in open loop, and which does not utilise process performance measurements. Our aim here is to develop a strategy for successive improvement of control laws using system data and experiments. We shall also extend robust control design methods to apply to identified plant models and conversely to focus identification on the provision of models for control design pruposes.

In performing a control design for a plant system P and disturbance model H based on models $\hat{P}$ and $\hat{H}$, it is important to separate logically the design process itself from the achieved closed loop operating on the real plant. The *design loop* is depicted in Fig. 1, and takes place on an imaginary or simulated feedback loop involving $\hat{P}, \hat{H}$ and C. The *achieved loop*, however, incorporates H and P in feedback with the controller C designed on the design loop, as is illustrated in Fig. 2. The importance of distinguishing these loops is that, while the ultimate purpose of the control design is centred on the achieved loop, the design itself takes place on the designed loop. Certainly equivalence methods operate under the principle that the design takes place on the designed loop as if $(\hat{P}, \hat{H})$ were the real plant, while robust methods endeavour to accommodate known, assumed or measured differences between (P, H) and $(\hat{P}, \hat{H})$ to ensure that features evident in the designed loop are generally preserved in the achieved loop. These features focus typically on robust stability and robust performance.

Here we shall use explicitly these two loops and analyse experimental data from each with a succession of designed controllers operating. Because we have the capacity to perform such experiments, we may use this data to develop

* Received 24 May 1994; revised 4 January 1995; received in final form 19 May 1995. This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Paul Van den Hof under the direction of Editor Torsten Söderström. Corresponding author Dr Zhuquan Zang. Tel. +61 9 351 3243; Fax +61 9 351 3244; E-mail zang@atri. curtin.edu.au.

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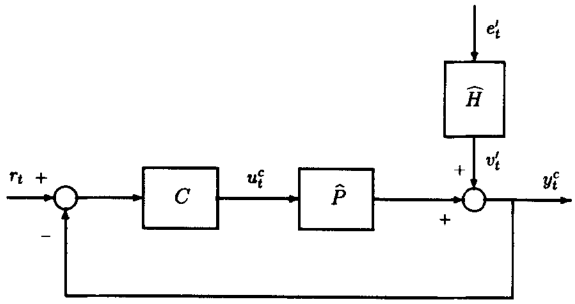

Fig. 1. Design loop.

better controller designs. This is at the heart of out iterative design approach.

In addition to adjustment of the controller design, the plant model $\hat{P}$ also may be better selected for the control purpose.† This is, in a sense, the converse question to robust control design, because it treats the issue of modifying the plant/model mismatch to permit better correspondence between the designed and achieved loops. Just as the nominal plant plus a description of the modelling error determine the robust control design, so too does the plant plus existing controller dictate desirable distributions of modelling error.

An early discussion about the interactions between model error distribution and control design can be found in a beautifully informative paper of Skelton (1989) that remained unnoticed for too long.‡ That paper raises and illustrates a number of the important questions related to the connection between model errors and control design, and it concludes on the necessity of iterative design. The interplay between identfication and control design in the context of least-squares (LS) and linear–quadratic Gaussian (LQG) criteria was first discussed by Bitmead *et al.* (1990a, b).

The question of model identification for control design has been broached recently by several authors, with a clear indication given of

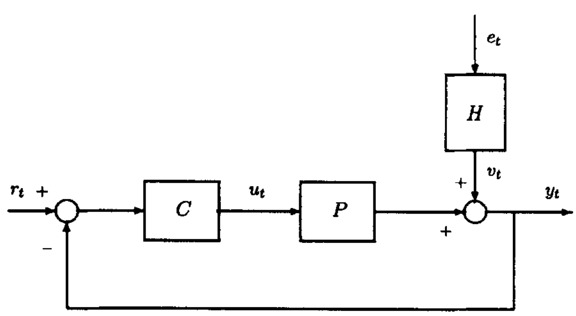

Fig. 2. Achieved loop.

† We shall abbreviate the notation of the complete plant (P, H) and model $(\hat{P}, \hat{H})$ by P and $\hat{P}$ respectively, where this does not introduce confusion.

‡ It was brought to the attention of the authors in 1993.

the advantage of collecting the data in closed loop; see e.g. Liu and Skelton (1990), Zang *et al.* (1991), Schrama (1991), Bayard *et al.* (1992), Lee *et al.* (1993), Hjalmarsson *et al.* (1994a, b), Åström (1993) and Åström and Nilsson (1994). Once again, the data available from closed-loop experiments will permit the fitting of more appropriate models, and iterative design will incorporate successive phases of closed-loop modelling and controller design.

Skelton (1989) and Schrama (1992a) have argued for the necessity of iterative control designs in the joint identfication and control design problem, mainly on the basis of bias error arguments. Hjalmarsson *et al.* (1994a) have demonstrated the superiority of such iterative schemes in the case of variance errors on the estimated transfer function models.

1.1. *Global and local problems*

If one views the controller design problem in the large then it may be considered as a search for a controller C that minimises a *global design criterion* J for the true plant P (and possibly H):

$$J_{\text{global}} = J(P, C),$$

where the objective function is minimised over all possible controllers C in some class $\mathscr{C}$. The criterion J and/or the class $\mathscr{C}$ might include some robustness measures. To conduct this design requires knowledge of P.

Since the actual plant is unavailable to us, our control design may not proceed from P. One way to resolve this problem is to introduce a model $\hat{P}$, together (possibly) with some quantification of model uncertainty, $L(P, \hat{P})$. (In classical H_∞ control design $L(P, \hat{P})$ is assumed to be god-given, while in classical LQG design certainty equivalence is used, i.e. $L(P, \hat{P}) = 0$.) In this more realistic situation the minimisation of $J(P, C)$ is replaced by the minimisation of an alternative model-based criterion $J^C(\hat{P}, L(P, \hat{P}), C)$, yielding a designed controller $\hat{C} = \hat{C}(\hat{P}, L(P, \hat{P}))$.

Applying this designed controller $\hat{C}$ in closed loop with the actual plant then yields the *achieved cost*:

$$J_{\text{global}} \triangleq J(P, \hat{C}) = J(P, \hat{C}(\hat{P}, L(P, \hat{P}))).$$

In a criterion-based control design schema, whether off-line or iterative, the performance of the scheme is to be evaluated by the eventual achieved cost, not by the designed cost. What distinguishes our approach from robust control design is the use of achieved controller performance measures. Our replacement for the infeasible direct minimisation of J_{global} with respect to C by the alternative criterion J^C, which is a function of $\hat{P}$ and its uncertainty, brings with it the requirement to provide a model $\hat{P}$ and a measure of its error L. These we choose to do by system identification with a criterion J^I also connected to the global control objective, and the definition of a modified (i.e. not certainty equivalence) LQG control design. Thus the philosophy of the paper is to perform simultaneous identification and control design. The construction of the design criteria J^C and J^I will be so that their minimization will imply good performance properties for J_{global}.

One might propose a systematic minimisation of the achieved cost J_{global} over all $(\hat{P}, \hat{C})$ in some class. However, it is in no way clear how this might be achieved. As an alternative, we pose successive *local* problems in which coordinate-wise optimization is attempted using the local criteria J^C and J^I. Fixing $\hat{P}$ and $L(P, \hat{P})$ and minimising $J_i^C(C, \hat{P}_i)$ over C yields a local control design problem. Similarly, fixing C and minimising the local identification criterion $J_i^I(C_i, \hat{P})$ over $\hat{P}$ yields a local identification problem.

We shall consider throughout this paper that we are in the practically relevant situation where P is unknown, where $\hat{P}$ is obtained by identification using real data, and where the identification is performed over a class of parametrized models that does not necessarily contain the true system: thus $\hat{P}$ may suffer from unmodelled dynamics.

1.2. *Robustness and model-based design*

As mentioned above, robustness is generally concerned with the implication of dynamical properties from the designed loop to the achieved loop. Of particular importance are *robust stability*, where the asymptotic stability of the designed system is preserved for the actual closed loop system, and *robust performance*, where some quantification of closeness of achieved and designed performance is desired. Robust performance is clearly conditional on robust stability although, as we shall see, is often also antagonistic to it.

Sufficient conditions for robust stability (for additive uncertainty) are given usually by (see e.g. Doyle *et al.*, 1992),

$$\left\| \frac{P-\hat{P}}{\hat{P}} \frac{\hat{P}C}{1+\hat{P}C} \right\|_\infty < 1, \tag{1}$$

or

$$\left\| \frac{P-\hat{P}}{P} \frac{PC}{1+PC} \right\|_\infty < 1. \tag{2}$$

Each of these formulations displays the critical interplay between the (relative) modelling error and the control design (via the complementary sensitivity function). Our successive iterative components will take cognisance of these conditions, firstly in generating a controller and secondly in providing a new model. In neoclassical robust control design the robust stability condition is treated as a constraint on the controller C. We extend its interpretation here to include the role of placing constraints on the permissible distribution of modelling errors. Some degree of plant/model mismatch is unavoidable, and the control objective must determine the preferred distribution.

Suppose that we have a true plant system with input–output relationship described by

$$\begin{aligned} y_t &= P(z)u_t + v_t, \\ v_t &= H(z)e_t, \end{aligned} \tag{3}$$

where $P(z)$ is a strictly proper rational transfer function, u_t is the input, and v_t is an unmeasurable disturbance acting on the output y_t. It is assumed here that v_t can be modelled as the output of a filter $H(z)$ driven by white noise e_t. Also we are given a parametrised model set

$$\mathcal{M} \triangleq \{\hat{P}(z, \theta), \theta \in D_\theta \subset \mathbb{R}^d\}, \tag{4}$$

together with (possibly) a fixed (non-parametrised) noise model v_t'. A particular model in that model set, driven by an input u_t^c, will produce an output signal described by

$$\begin{aligned} y_t^c(\theta) &= \hat{P}(z, \theta)u_t^c + v_t', \\ v_t' &= \hat{H}(z)e_t' \end{aligned} \tag{5}$$

for a particular value of the parameter θ, where $\hat{P}$ is a strictly proper transfer function. The noise model $\hat{H}$ is the designer's best estimate (or guess) of the actual noise model H; it is driven by white noise e_t'. ($\hat{H}$ could be included as part of the identification, but, for simplicity only, here we take it fixed.)

In this paper we consider a regulation problem, where the controller $C(z)$ is to be designed to minimise the effect of the disturbance on the plant output y_t. Our control design objective is to achieve a high disturbance rejection performance on the design loop of Fig. 1, while at the same time guaranteeing robust stability performance, i.e. achieving stability on the achieved loop of Fig. 2 as well as a disturbance rejection performance that is near the designed performance. Consider a particular measure of the achieved disturbance rejection performance $H/(1+PC)$. Notice that this objective function can be written as

$$\frac{H}{1+PC} = \frac{H}{1+PC} - \frac{\hat{H}}{1+\hat{P}C} + \frac{\hat{H}}{1+\hat{P}C}.$$

By using the double triangle inequality, as suggested by Schrama (1992b), the performance of the two loops may be linked by the following relationships:†

$$\left| \left\| \frac{\hat{H}}{1+\hat{P}C} \right\| - \left\| \left(\frac{H}{1+PC} - \frac{\hat{H}}{1+\hat{P}C} \right) \right\| \right| \leq \left\| \frac{H}{1+PC} \right\| \tag{6}$$

$$\left\| \frac{\hat{H}}{1+\hat{P}C} \right\| + \left\| \left(\frac{H}{1+PC} - \frac{\hat{H}}{1+\hat{P}C} \right) \right\| \geq \left\| \frac{H}{1+PC} \right\|. \tag{7}$$

From the above inequalities, one discerns that plant and model performance will be close if the following performance robustness measure is small:

$$J^{\text{pr}} \triangleq \left\| \left(\frac{H}{1+PC} - \frac{\hat{H}}{1+\hat{P}C} \right) \right\|. \tag{8}$$

We shall see in the sequel that joint minimisation of both these robustness measures ((1) or (2) and (8)) is not always feasible when the plant does not belong to the model set. This will have repercussions in the selection of the balance between iterative design for performance and that for the stability robustness, which will be reflected in the frequency weightings chosen for the identification and the controller design.

1.3. *Contribution*

The major contribution of this paper is the development of an iterative data-driven identification/control design schema whose control aim is to improve iteratively the achieved performance. This is accomplished by the combination of two novel features.

- The least-squares (LS) identification of a new model is performed on closed-loop data obtained on the real plant controlled by the previously computed controller, and with a deliberately selected data filter which improves model accuracy at those frequencies where robust stability and/or robust performance dictate that a better model is needed. This allows for performance enhancement at the next controller design stage or the detection of potential stability problems.
- The LQG control design uses a frequency-weighted LQG criterion, where the frequency weightings in the control design stage account for the imperfection of the estimated model as reflected in the mismatch between the actual closed-loop system and the designed (or nominal) closed-loop system. These weightings are derived from relatively coarse spectrum estimates of measured and simulated closed-loop signals. They have the effect of rendering the controller cautious in frequency bands where the data reflect a plant/model mismatch.

The combination of frequency-weighted LS identification with frequency-weighted LQG control design was initially proposed in conference papers by Bitmead and Zang (1991) and Zang *et al.* (1991) for the tracking problem and Zang *et al.* (1992) for the disturbance rejection problem. Since then, this H_2-based iterative scheme has been used, studied and extended by a number of authors, with several new variants and insights appearing; see e.g. Partanen and Bitmead (1993b, 1995), Hakvoort *et al.* (1994), Åström and Nilsson (1994) and van der Klauw *et al.* (1994). The purpose of this paper is to provide a comprehensive presentation of this iterative scheme, by building on the many insights gained since 1991, and with the addition of some novel features that take account of robust stability (as opposed to just robust performance) measures.

1.4. *Relationship to the work of others*

The concept of optimal identification design for control goes back to Gevers and Ljung (1986). The idea was pursued in Hakvoort (1990). The idea of closed-loop identification with a performance robustness enhancement data filter is not entirely new: it was advocated by Bitmead *et al.* (1990a, b) and subsequently developed by Schrama (1992b). The dual idea of incorporating model error information, obtained from data, into an LQG criterion is most certainly novel; it was first proposed by Bitmead and Zang (1991) and Zang *et al.* (1991). By combining these two ideas into our iterative scheme, we propose a combined 'robust identification/robust control' schema in which the robustness enhancement features of the two parts of the design are mutually supportive.

Successive passes through the LS filtered closed-loop identification and the frequency-weighted LQG control design using batches of fresh data will generate our iterative design. Our underlying idea is certainly not to adapt indefinitely the controller parameters, but rather to arrive in a theoretically sound way at a fixed high-performance controller in a few iterations, using data from previous controllers to drive up the achieved performance. Such a design is

† The specific norm here is somewhat immaterial.

neither truly an adaptive controller nor truly an off-line robust controller, but it combines features of both design schemes: our control law resulting is indeed shaped for robustness by the measured data and not just by *a priori* assumptions. Indeed, the notion of occasional controller refinement on the basis of observed achieved performance after initial tuning is common in practice. Being completely signal driven, our iterative scheme does not rely heavily on prior information about the unknown system, unlike similar H_∞ design schemes.

Related iterative 'control design/identification design' strategies have recently been proposed by several authors already mentioned: Liu and Skelton (1990), Bayard *et al.* (1992), Schrama (1992a, b), Hakvoort *et al.* (1994), Lee *et al.* (1993) and Åström and Nilsson (1994). A succession of surveys, all but one excellent, can be found in Gevers (1993), Bitmead (1993) and Van den Hof and Schrama (1994). We should also mention the recent result of Hjalmarsson *et al* (1994b), who have obtained a direct iterative scheme, which estimates the controller parameters directly without the intermediate model identification step; convergence of this scheme to a local minimum of the achieved cost has been proven.

The main features that distinguish the scheme proposed in this paper from the other schemes are as follows.

- Our scheme is entirely based on H_2 methods, namely linear–quadratic Gaussian (LQG) control and least-squares identification, with a consistent criterion of performance operating in both phases;
- The order of the plant model and the consequent controller is fixed, implying that no controller reduction step is necessary;
- Most importantly, the controller design explicitly uses closed-loop plant/model mismatch information derived directly from data.

1.5. *Précis*

In Section 2 we pose an artificial H_∞ iterative control design problem, which is presented to fix our ideas without too much algebraic machinery. This is an artificial problem, because it relies on the use of an H_∞ system identification procedure, including estimation of H_∞ model errors, that requires knowledge of the unknown true system transfer function. It does, however, admit a simple demonstration of a performance oriented coupled control and identification iterative strategy, because the criteria J^C and J^I may be taken identical, and it serves to motivate the more realistic H_2 (least-squares) system identification methods and the corresponding H_2 (LQG) control laws of subsequent sections. Section 3 deals with the specification of the local LQG control design phase, which focuses on the global criterion, while Section 4 presents the local least-squares identification directed towards performance robustness or stability robustness. In contrast to the H_∞ iterations of Section 2, the H_2 iterations are now feasibly data-driven, with the successive controllers being applied to the process and the collected data being used in the computation of the next model/controller couple. Section 5 contains the iterative algorithm specification incorporating the local problems and the quality assessment measurements. In Section 6 we develop a computational example and give some comments and discussions about the results obtained. Throughout this paper, all our results are derived for the simple case of single-input single-output systems and in the context of a disturbance rejection objective. The extensions to the multivariable case and to a tracking objective are reasonably straightforward but messy.

2. H_∞ ITERATIVE IDENTIFICATION AND CONTROL DESIGN

2.1. *Problem formulation*

Our purpose in this section will be to develop a formulation of a combined identification and control design methodology in the H_∞ framework that illustrates the principles underpinning our subsequent H_2 approach before proceeding to a feasible signal-based approach in later sections.

We consider the setup of Figs 1 and 2. The true plant is described by (3). The ideal but infeasible control design would be to minimise

$$J_{\text{global}} \triangleq \left\| \frac{H(z)}{1 + P(z)C(z)} \right\|_\infty$$

with respect to C. Now we introduce the model set (4), (5) and we define our additive modelling error as

$$L(P, \hat{P}) \triangleq P(z) - \hat{P}(z, \theta),$$

the control sensitivity function as

$$\hat{M}(z) \triangleq C(z)[1 + C(z)\hat{P}(z, \theta)]^{-1},$$

and our actual and designed sensitivity functions as

$$S(z) \triangleq [1 + C(z)P(z)]^{-1},$$

$$\hat{S}(z) \triangleq [1 + C(z)\hat{P}(z, \theta)]^{-1}.$$

We now define $\hat{J}(P, L(P, \hat{P}), C) \triangleq J^C \triangleq J^I$ for this problem as

$$\hat{J}(P, L(P, \hat{P}), C) = \left\| \begin{matrix} L\hat{M} \\ H\hat{S} \end{matrix} \right\|_\infty. \qquad (9)$$

This is the classical mixed sensitivity H_∞ design problem; see Doyle *et al.* (1992), where it is shown that

$$\hat{J} < 1 \Leftrightarrow J_{\text{global}} < 1, \quad \|L\hat{M}\|_\infty < 1.$$

2.2. *Problem 'solution'*

With the above remarks in mind, we propose the following iterative procedure to endeavour to solve the above minimisation problem.

Step 1: *identification initialisation.* Take

$$\hat{\theta}_0 = \arg\min_\theta \|P(z) - \hat{P}(z, \theta)\|_\infty$$

Take $\hat{P}_0(z) \triangleq \hat{P}(z, \hat{\theta}_0)$ and

$$L_0(z) \triangleq P(z) - \hat{P}_0(z)$$

as the outputs of the identification.

Step 2: *weighted* H_∞ *optimal control design.* Select

$$C_i = \arg\min_C \left\{ \left\| \begin{matrix} L_i(z)\dfrac{C(z)}{1 + C(z)\hat{P}_i(z)} \\ H(z)\dfrac{1}{1 + C(z)\hat{P}_i(z)} \end{matrix} \right\|_\infty \right\}$$

$$\triangleq \arg\min_C J_i^C(C)$$

where $J_i^C(C) \triangleq \hat{J}(\hat{P}_i, L(P, \hat{P}_i), C)$.

Step 3: *weighted* H_∞ *identification.* Select

$$\hat{\theta}_{i+1} = \arg\min_\theta \times \left\{ \left\| \begin{matrix} (P(z) - \hat{P}(z, \theta))\dfrac{C_i(z)}{1 + C_i(z)\hat{P}(z, \theta)} \\ H(z)\dfrac{1}{1 + C_i(z)\hat{P}(z, \theta)} \end{matrix} \right\|_\infty \right\}$$

$$\triangleq \arg\min_\theta J_i^I(\hat{P})$$

where $J_i^I(\hat{P}) \triangleq \hat{J}(\hat{P}, L(P, \hat{P}), C_i)$. Take $\hat{P}_{i+1}(z) \triangleq \hat{P}(z, \hat{\theta}_{i+1})$ and $L_{i+1}(z) \triangleq P(z) - \hat{P}_{i+1}(z)$ as the outputs of the identification.

Repeat. repeat Steps 2 *and* 3, *replacing i* by $i + 1$.

Remarks.

(i) As we have already said, this is an artificial solution procedure, because the identification step is not feasible, since it involves effective knowledge of the true plant $P = \hat{P} + L$ from the identification phase. However, it is appealing, because it links the successive control design and identification criterion in a logical fashion with a fixed objective.

(ii) The above construction of the iterative identification and control design leads to a monotonic decrease of the local cost functions J_i^C and J_i^I, and hence of $\hat{J}$. This can be seen from

$$J_{i+1}^C(C_i)$$

$$= J_i^I(\hat{P}_{i+1}) \triangleq \min_{\hat{P}} \hat{J}(\hat{P}, L(P, \hat{P}), C_i) \quad (10)$$

$$\leq J_i^I(\hat{P}_i) = \hat{J}(\hat{P}_i, L(P, \hat{P}_i), C_i) \quad (11)$$

$$= J_i^C(C_i) \triangleq \min_C \hat{J}(\hat{P}_i, L(P, \hat{P}_i), C) \quad (12)$$

$$\leq \hat{J}(\hat{P}_i, L(P, \hat{P}_i), C_{i-1}) = J_i^C(C_{i-1}). \quad (13)$$

(iii) The novelty of this approach, compared with the neoclassical H_∞ methods, is that the plant modelling is adjusted to reflect the control requirements and also the controller design is modified to reflect the H_∞ modelling error. This is the key to our approach versus, say, the standard methods of robust control—we use the robustness requirements to assist in the specification of desired model fits, and vice versa. When this is coupled to the capacity to perform experiments, and so derive new data, we have a design philosophy (albeit not a computational technique) for the intelligent amelioration of controller designs on the basis of closed-loop (operating) experimental design. The failing of this H_∞ approach here is its inability easily to be formulated with measured data. The exact same design procedure was proposed independently by Bayard *et al.* (1992), who arrived at an identical conclusion about the inability to perform the identification step of the design. Simulations presented in Bayard *et al.* (1992) add weight to the credibility of the scheme.

3. LOCAL LQG CONTROL DESIGN

In this section we replace the H_∞ control problem of Section 2 with an LQG design problem that is more amenable to use with data and that is better linked to the identification phase. The novelty of our H_2 approach is to account for evaluated modelling error in an LQ optimal control design. This is achieved by frequency-weighting the LQ control criterion using a filter that contains plant/model mismatch information. This deviation from the more traditional certainty equivalence LQG control design schemes is an attempt to build in robust performance features by tying local design objectives to the global criterion.

Our global objective is to design a controller C in order to minimise the following global LQ regulation performance criterion:

$$J_{\text{global}}(P, C) = \min_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} (y_t^2 + \lambda^2 u_t^2). \quad (14)$$

The certainty equivalence formulation of an LQ regulation problem is to minimise the following performance criterion $\bar{J}$:

$$\bar{J} = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} [(y_t^c)^2 + \lambda^2 (u_t^c)^2], \quad (15)$$

where u_t^c is the designed control signal and y_t^c is the output of an identified model $\hat{P}$ driven by u_t^c and a noise source v_t'; see (5). Instead of following the traditional route of minimising (15) under the constraint (5), we begin with the following frequency-weighted *local LQ criterion*:

$$J^C = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \{[F_1(z)(y_t^c)]^2 + \lambda^2 [F_2(z) u_t^c]^2\}, \quad (16)$$

where F_1 and F_2 are weighted functions (linear filters) to be chosen. We assume that a previously computed controller, say C_i, obtained on the basis of some previous model, say $\hat{P}_i$, has been operating on the actual plant and that data have been collected on this closed loop system. The minimisation of the local criterion J^C will deliver C_{i+1}. Recall that control design is performed by relying on an identified (and hence approximate) model.

By direct comparison between (16) and (14), we select $F_1(z)$ and $F_2(z)$ as

$$F_1 = \left(\frac{\Phi_y}{\Phi_{y^c}}\right)^{1/2}, \quad F_2 = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2}. \quad (17)$$

This makes the frequency-weighted regulation objective (16) become

$$J^C = \lim_{N\to\infty} \frac{1}{N} \sum_{t=1}^{N} \left\{ \left[\left(\frac{\Phi_y}{\Phi_{y^c}}\right)^{1/2} y_t^c \right]^2 + \lambda^2 \left[\left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2} u_t^c \right]^2 \right\}. \quad (18)$$

Here Φ_y, Φ_{y^c}, Φ_u and Φ_{u^c} are the spectra of the corresponding signals obtained on the actual and the simulated closed-loop systems with the previous controller, C_i. All these signals are readily available from the closed loops of Figs 1 and 2 with $r_t = 0$ and C replaced by C_i. Since $u_t = -C_i(z) y_t$ and $u_t^c = -C_i(z) y_t^c$, we observe that

$$F \triangleq F_1 = F_2 = \left(\frac{\Phi_y}{\Phi_{y^c}}\right)^{1/2} = \left(\frac{\Phi_u}{\Phi_{u^c}}\right)^{1/2}. \quad (19)$$

We comment later on the actual computation of the filter $F(z)$.

The frequency-weighted criterion (18) should be compared with (14) and interpreted as a distortion of the certainty equivalence control objective function (15) for the identified model in order to reflect the global criterion, as was motivated by the H_∞ frequency-weighted control design method derived in Section 2. We observe that the effect of the frequency weightings is to make the filtered output signal and control signal of the design loop respectively in (18) have the same spectra as the corresponding signals in the global (ideal) performance criterion J_{global} corresponding to the achieved loop, so that the controller designed by minimizing (18) effectively minimizes the global performance criterion (14).

In our iterative design scheme described above, the spectra in (18) will be replaced by low-order spectral estimates obtained from data collected on the local model and on the real plant, both operating in closed loop with the presently active (local) controller. Thus our local control design criterion J^C becomes an approximation to the global criterion J_{global}.

Besides forcing the local control objective to mimic the global one, as explained above, the effects of the frequency weightings in (18) have entirely logical and intuitive interpretations.

Special observation. If at some frequency, Φ_y is larger than Φ_{y^c}, this means that at that frequency the model fit is poor, with the consequence that the achieved disturbance rejection performance (with the presently active controller) is worse than expected from the designed system. Hence more emphasis should be put on the weighting at that frequency at the next control design stage, which is reflected by the weighting being larger than 1. If at some frequency, Φ_y is smaller than Φ_{y^c}, this also means that at that frequency the model fit is poor but in such a way that the presently active controller actually achieves a better disturbance rejection performance on the true plant than on the model. The penalty at that frequency should therefore be decreased at the next control design stage to provide scope for improvement at other frequencies. Similar astute and entirely intuitive observations can be made by the reader as regards the frequency weighting on the control.

Evaluation of F. Since the filter F must be a stable and stably invertible spectral factor of the signal spectra, it may be computed from an approximation obtained from the data. The method that we actually use is to estimate directly autoregressive models for the data signals. This permits

- a stable and stably invertible filter, since it is the ratio of AR models;
- fixed degree filters F, which prevents the controller order from increasing at each iteration;
- the avoidance of spectral factorisation.

Remarks.

(i) This application of experimental spectra in the control design phase signifies the departure of our H_2 theory from the H_∞ results where modelling errors required for the control design were not assessable. That is, the computation of approximations of these spectra is feasible while the measurement of H_∞ modelling errors is not.

(ii) This frequency-weighted LQ regulation problem can be recast as an unweighted LQ problem with modified transfer functions $\bar{P} = \hat{P}$ and $\bar{H} = F\hat{H}$. The optimal controller $\bar{C}$ resulting from this unweighted problem is identical to the optimal C of the frequency weighted problem: $\bar{C} = C$.

3.1. *Robustness considerations*

The proposed frequency-weighted LQG design is focused directly on performance enhancement. That is, its aim is to cajole an LQG design on the designed loop to deliver good performance on the achieved loop. There is no implied quality of robust stability. Indeed, the designed performance is deliberately distorted to yield achieved loop outcomes.

As the remarks above indicate, the filter F deviates from unit magnitude when the closed-loop plant and model responses differ substantially. Thus the applied frequency weighting takes maximum effect just where the model is inaccurate. For frequencies where the weighting has magnitude much different from one, and if no specific countermeasures were taken to ensure that this does not lead to a high-gain control action where the model is deficient, this might violate the robust stability criterion (1). Therefore we interpret the filtering by F as a robust performance feature of the iterative control design; for robust stability, we rely on the modelling part of the design. We shall subsequently present a stability robustness check based on a high-order approximation of Φ_u/Φ_{u^c} that can indicate when the model $\hat{P}$ is inadequate for robust control design.

4. LOCAL LEAST-SQUARES IDENTIFICATION DESIGN

As should now be evident, our thesis of model-based controller design is that the controller design and system modelling phases may jointly be used to develop mutually supportive approaches to a global controller design. We shall now present the identification component of our iterative approach and consider that at some stage of the iterations the system has been operating in feedback with a frequency-weighted LQG controller designed on the basis of a previous model. To perform the identification, it is necessary to introduce an external excitation signal r_t, which in turn will become part of the design, as our analysis will show.

4.1. *Local identification criterion—robust performance*

To derive the local identification criterion, we work under the assumption that a stabilizing controller C_i is operating. Consistently with our theme, we should select a (local) identification criterion commensurate with the global performance criterion (14) to reflect our 'modelling for closed-loop control' objective. We do this by selecting a model $\hat{P}$ that, with the same fixed controller C, yields a frequency-unweighted closed-loop performance $\bar{J}$, (15), as close as possible to the global closed-loop performance J_{global}, (14).

To show how this can be obtained, we first reformulate the global criterion (14) by using the following 2-norm definition for a vector process $\begin{pmatrix} x_t \\ y_t \end{pmatrix}$:

$$\left\| \begin{matrix} x_t \\ y_t \end{matrix} \right\|_2 \triangleq \left\{ \sum_{t=1}^{N} \left[\left(\frac{1}{\sqrt{N}} x_t \right)^2 + \left(\frac{1}{\sqrt{N}} y_t \right)^2 \right] \right\}^{1/2}. \quad (20)$$

we can then rewrite the global criterion, for finite time, as,

$$J^N_{\text{global}} = \left\| \begin{matrix} y_t \\ \lambda u_t \end{matrix} \right\|_2^2. \quad (21)$$

Our aim is to obtain a model $(\hat{P}, \hat{H})$ of (P, H) such that the closed loop with (P, H) replaced by $(\hat{P}, \hat{H})$ and driven by the same signals r_t and e_t resembles the actual closed-loop as much as possible in a measure that is determined by J_{global}. The signals y^c_t and u^c_t are those of the design loop of Fig. 1 with controller $C = C_i$, excitation $e'_t = e_t$ and reference $r_t = 0$. This is not realisable, but will provide an achievable identification criterion that is matched in this sense to J_{global}. Observe that, by using $y_t = y_t^c + (y_t - y_t^c)$, the similar expression for λu_t and the triangle inequality, we have

$$\left| \left\| \begin{matrix} y_t^c \\ \lambda u_t^c \end{matrix} \right\|_2 - \left\| \begin{matrix} y_t - y_t^c \\ \lambda(u_t - u_t^c) \end{matrix} \right\|_2 \right| \le \left\| \begin{matrix} y_t \\ \lambda u_t \end{matrix} \right\|_2 \le \left\| \begin{matrix} y_t^c \\ \lambda u_t^c \end{matrix} \right\|_2 + \left\| \begin{matrix} y_t - y_t^c \\ \lambda(u_t - u_t^c) \end{matrix} \right\|_2. \quad (22)$$

We note the similarity to the performance robustness criterion (6), (7). The first term of the upper bound is the frequency-unweighted closed-loop performance of the model $\hat{P}$, while the second term is a measure of the closed-loop model error. We shall select the second term as our local identification criterion:

$$J_N^I \triangleq \left\| \begin{matrix} y_t - y_t^c \\ \lambda(u_t - u_t^c) \end{matrix} \right\|_2^2 = \frac{1}{N} \sum_{t=1}^{N} [(y_t - y_t^c)^2 + \lambda^2 (u_t - u_t^c)^2]. \quad (23)$$

The minimisation of J_N^I over the set of models will then aim at making the designed and the actual closed loops close *where the closeness is measured in the same norm which we use to define the global LQG control criterion.* Taking the limit as $N \to \infty$, the inequalities (22) can then be rewritten as

$$|(\bar{J})^{1/2} - (J^I)^{1/2}| \le (J_{\text{global}})^{1/2} \le (\bar{J})^{1/2} + (J^I)^{1/2}, \quad (24)$$

with an obvious definition of $J^I = \lim_{N \to \infty} J_N^I$.

From the achieved and design loops with $r_t = 0$ and $e_t' = e_t$, we have

$$y_t^c = \frac{\hat{H}}{1 + \hat{P} C_i} e_t, \quad u_t^c = -\frac{C_i \hat{H}}{1 + \hat{P} C_i} e_t. \quad (25)$$

$$y_t = \frac{H}{1 + P C_i} e_t, \quad u_t = -\frac{C_i H}{1 + P C_i} e_t. \quad (26)$$

From (25) and (26), we get the following frequency-domain expression of the local identification criterion (23):

$$J^I = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1 + \lambda^2 |C_i|^2) \times \left| \frac{H - \hat{H} + C_i(H\hat{P} - \hat{H}P)}{(1 + PC_i)(1 + \hat{P}C_i)} \right|^2 d\omega. \quad (27)$$

We note that, in order to minimize J^I, we need to consider the two loops of Figs 1 and 2 driven by the same signals $r_t = 0$ and, $e_t' = e_t$ and with controller $C = C_i$. By using closed-loop data from the achieved loop this is accomplished in the standard identification algorithm, as will be seen later.

Remarks.

(i) The inequalities show that J^I is a *performance robustness measure*, just as its H_∞ counterpart in (8). A similar use of the triangle inequality in the H_∞ case led Schrama (1992b) to use the minimisation of J^{pr} in (8), with an ∞-norm, as an identification criterion.

(ii) We note that the estimated plant model $\hat{P}$ and the controller C both influence the two terms $\bar{J}$ and J^I. Thus, ideally, one should minimise the two terms jointly over the class of admissible plant models and admissible controllers. This is currently an impossible task in the case of restricted complexity models. An obvious suboptimal strategy is to make J^C small by controller design for a given plant model, and to keep J^I small by identification design for a given controller. Since J^C depends on the estimated plant model, and J^I depends on the designed controller, this strategy can only be applied in an iterative manner, using a succession of *local controller designs* and *local identification designs.*

4.2. *Closed-loop identification for performance*

The criterion J^I is definitely a non-standard identification criterion. We examine its connection with classical prediction error minimisation. We also assume that we have the capacity to apply an external excitation signal r_t to this closed-loop system; we shall explain later the importance of this external excitation for robust identification. Thus we consider that a data set has been collected on the closed-loop system of Fig. 2 in which $C = C_i$. Associated with the model (5) is the following one-step-ahead predictor:

$$\hat{y}_t(\theta) = \hat{H}^{-1}(z)\hat{P}(z, \theta)u_t + [1 - \hat{H}^{-1}(z)]y_t \quad (28)$$

(see Ljung, 1987), where $\hat{H}(z)$ is a noise-model transfer function, which we consider not to depend upon θ. The least-squares identification criterion is defined as

$$V_N(\theta) = \frac{1}{N} \sum_{t=1}^{N} [\epsilon_t^f(\theta)]^2 \triangleq \frac{1}{N} \sum_{t=1}^{N} \{D(z)[y_t - \hat{y}_t(\theta)]\}^2, \quad (29)$$

where $D(z)$ is a stable, linear data filter operating on the prediction errors $\epsilon_t(\theta) = y_t - \hat{y}_t(\theta)$.

$V_N(\theta)$ converges to a limiting value $V(\theta) =$

$\lim_{N\to\infty} E\{V_N(\theta)\}$ under reasonable conditions (Ljung, 1987). Using (28), (3) and Parseval's formula, this criterion can be written (in the frequency domain) as

$$V(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \Delta V(\omega, \theta)\left|\frac{D(e^{j\omega})}{\hat{H}(e^{j\omega})}\right|^2 d\omega, \quad (30)$$

where

$$\Delta V(\omega, \theta) \triangleq \left|\frac{[P(e^{j\omega}) - \hat{P}(e^{j\omega}, \theta)]C_i(e^{j\omega})}{1 + P(e^{j\omega})C_i(e^{j\omega})}\right|^2 \Phi_r(\omega) + \left|\frac{1 + \hat{P}(e^{j\omega}, \theta)C_i(e^{j\omega})}{1 + P(e^{j\omega})C_i(e^{j\omega})}\right|^2 \Phi_v(\omega). \quad (31)$$

For simplicity, in the sequel we shall drop the arguments $e^{j\omega}$, θ, and ω in similar frequency-domain expressions.

The above criterion gives a characterisation of the convergence point of the closed loop identification algorithm obtained by minimising (29) on the basis of closed-loop data. Note that in minimising (30) we have used the uncorrelatedness of r_t and v_t. The exertion of influence over the least-squares identification criterion is through three media:

- the choice of the data filter D;
- the design of the excitation spectrum $\Phi_r(\omega)$;
- the selection of the model structure.

If the model structure is fixed a priori then the principal means of adjustment are D and Φ_r.

We now compare J^l, defined by (27), with $V(\theta)$, defined by (30). In the case where the correct disturbance model is known, i.e. $\hat{H} = H$, a direct equivalence can be made.

Lemma 4.1. Let the signals y_t^c, u_t^c, y_t, u_t and r_t be defined by Figs 1 and 2 respectively with $C = C_i$ and $e_t' = e_t$, and let J^l be defined by (27) with $\hat{H} = H$. Then, with the reference signal spectrum chosen as

$$\Phi_r(\omega) = \gamma^2\Phi_v(\omega) = \gamma^2 |H(e^{j\omega})|^2 \quad \forall\omega \in [-\pi, \pi), \quad (32)$$

for some nonzero γ, and with the filter $D(z)$ in (30) chosen as

$$D(z, \theta) = \hat{H}(z)G(z)[1 + \hat{P}(z, \theta)C_i(z)]^{-1}, \quad (33)$$

where $G(z)$ is defined as a stable filter obtained from the factorisation problem

$$G(z)G^*(z^{-1}) = 1 + \lambda^2 c_i(z)C_i^*(z^{-1}), \quad (34)$$

we have

$$\arg\min_{\theta} J^l = \arg\min_{\theta} V(\theta), \quad (35)$$

where $V(\theta)$ is defined as in (30) and (31).

Proof. Substituting (32)–(34) into (30), we obtain

$$V(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\frac{(1 + \lambda^2 |C_i|^2)|H|^2}{|1 + PC_i|^2}\left|\frac{C_i(P - \hat{P})}{1 + \hat{P}C_i}\right|^2 \gamma^2 + \frac{(1 + \lambda^2 |C_i|^2)|H|^2}{|1 + PC_i|^2}\right] d\omega. \quad (36)$$

Since the second term of the integrand in the above expression is independent of the model to be identified, we have

$$\arg\min_{\theta} V(\theta) = \arg\min_{\theta}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{(1 + \lambda^2 |C_i|^2)|H|^2}{|1 + PC_i|^2} \times \left|\frac{C_i(P - \hat{P})}{1 + \hat{P}C_i}\right|^2 \gamma^2 d\omega\right\}. \quad (37)$$

Comparing this with (27), with $\hat{H} = H$, we obtain (35). □

Lemma 4.1 provides an algebraic connection between two criteria: the identification criterion $V(\theta)$ and the performance robustness criterion J^l. It establishes the need for excitation to make the two criteria match exactly, under the assumption $\hat{H} = H$. It also demonstrates that J^l can be minimised by minimizing an identification criterion that only depends on measurable signals u_t and y_t.

Remarks.

(i) This lemma further consolidates our claim that the choice of the local identification criterion J^l is mainly based on a robust performance consideration.

(ii) We recall that the minimisation of J^l was justified earlier on the basis of robust performance considerations using the triangle inequality. The idea is to obtain a model such that the signals in the achieved loop and the design loop are 'close' in a sense that is defined by the global regulation performance criterion. It is easy to see that the integrand of J^l is precisely the square of the difference between the actual and the designed 'disturbance to output' transfer functions, weighted by $1 + \lambda^2 |C|^2$. From a performance point of view, this is indeed an ideal choice for the identification criterion. However, this means that the identification is entirely based on signals whose spectra are determined by the disturbance process alone, thereby concentrating modelling effort for maximum disturbance rejection effect. The criterion does not incorporate any robust stability guarantees. We shall see in the next subsection how stability robustness can be injected into the design.

(iii) In an iterative design scheme the identification of the new model $\hat{P}_{i+1}$ is performed using closed-loop data with the previous controller C_i operating on the real plant. The equivalence between the criteria $V(\theta)$ and J^I has been established when a data filter D defined by (32)–(34) is used. We observe that the filter D depends on the model $\hat{P}(z, \theta)$; therefore the equivalence holds strictly only if one lets the filter be parametrised by θ. This will typically complicate the identification algorithm. An alternative, that we shall advocate in our iterative scheme described in Section 5, is to let the filter be defined by (33) with $\hat{P}$ replaced by the model $\hat{P}_i$ obtained at the previous iteration.

4.3. *Closed-loop identification for robust stability*

The sufficient conditions for robust stability are given by (1) and (2), and are distinct from those for robust performance in several important ways: they do not depend upon the disturbance process H at all, and they should be satisfied uniformly across all frequencies (because of the ∞-norm criterion). We now make the comparison between the criterion (2) and the identification objective (30).

Lemma 4.2. Let the signals y_t^c, u_t^c, y_t, u_t and r_t be defined as in Lemma 4.1 and consider the argument of the robust stability criterion (2):

$$\left|\frac{[P(e^{j\omega}) - \hat{P}(e^{j\omega})]C_i(e^{j\omega})}{1 + P(e^{j\omega})C_i(e^{j\omega})}\right|.$$

Then, with the filter $D(z)$ in (30) chosen as

$$D(z, \theta) = \hat{H}(z), \tag{38}$$

and the spectrum $\Phi_r(\omega)$ of the reference r_t chosen as

$$\Phi_r(\omega) \approx \gamma^2 \gg |\Phi_v(\omega)| \quad \forall \omega \in [-\pi, \pi), \tag{39}$$

we have

$$\arg\min_\theta V(\theta) = \arg\min_\theta \left\|\frac{[P(e^{j\omega}) - \hat{P}(e^{j\omega})]C_i(e^{j\omega})}{1 + P(e^{j\omega})C_i(e^{j\omega})}\right\|_2. \tag{40}$$

Remarks.

(i) The obvious remark is that the norm minimised in (40) is a 2-norm, while that of the criterion (2) if the ∞-norm. This is the price to be paid for using standard least-squares identification. There has been a raft of results recently dealing with ∞-norm identification methods, often utilising polynomial interpolation followed by Nehari extension. They are not yet mature enough for application here, and, further, are profligate in their use of data. Accordingly, with realistically limited data sets, we are bound to restrict attention to 2-norms, which after all are commensurate with the control performance criterion.

(ii) When describing stability robustness, we presented two candidate robustness criteria: (1), which is the more familiar from control design, and (2), which is usually not mentioned. In the framework of identification, it is this latter version of the criterion that is used, since it relates the variable component of the design, $\hat{P}$, to the fixed elements, P and C_i. Further, since it is known that C_i stabilises P during the identification experiment, it is sensible to search over those $\hat{P}$s that are also stabilised.

4.4. *Assessment of model quality for control*

We have now proposed two different experimental conditions in which closed-loop model identification might take place, depending upon the emphasis desired for the model. Our aim now is to study the assessment of the model quality for control by direct evaluation of the robust stability criterion. To do this we use a mechanism similar to that of Van den Hof and Schrama (1993) in identifying directly the closed-loop sensitivity function, but then extend this to generate an a priori robust stability test in which the robust stability of the plant with a new controller obtained from a model is evaluated, before this new controller is applied to the plant. The signals u_t and r_t generated during either one of the identification experiments described above are related by

$$u_t = \frac{C_i}{1 + PC_i} r_t - \frac{C_i}{1 + PC_i} He_t.$$

By fitting a high-order ARX model between r_t and u_t, one can therefore compute an accurate estimate of $M_i \triangleq C_i(1 + PC_i)^{-1}$. Using this estimate and the new plant model $\hat{P}_{i+1}$, the following ratio of control sensitivity functions may be calculated:

$$\psi_i(z) \triangleq \frac{M_i}{\hat{M}_i} = \frac{1 + \hat{P}_{i+1}C_i}{1 + PC_i}. \tag{41}$$

The information contained in ψ_i is easily related to our existing stability robustness condition.

Lemma 4.3. Let the signals y_t^c, u_t^c, y_t, u_t and r_t be defined as in Lemma 4.1 and let $\Phi_r(\omega)$ be selected with complete support over $\omega \in [-\pi, \pi)$. Further, consider the ratio $\psi_i(z)$ between the identified and computed control sensitivities M_i and $\hat{M}_i$ from (41). We have the following relationship between the argument of the robust stability criteria (1) and (2) and ψ_i:

$$\psi_i - 1 = \frac{(\hat{P}_{i+1} - P)C_i}{1 + PC_i}, \tag{42}$$

$$\psi_i^{-1} - 1 = \frac{(P - \hat{P}_{i+1})C_i}{1 + \hat{P}_{i+1}C_i}. \tag{43}$$

This lemma provides a mechanism to check the robust stability of the achieved loop with (P, C_i) from that of $(\hat{P}_{i+1}, C_i)$, or vice versa. However, since the stabilty of both these loops is a precondition for performing the sensitivity identifications, this does not provide much useful information. What is desired is to be able to check the robust stability of the achieved loop for some new controller K, which stabilises $\hat{P}_{i+1}$, before it is applied to the real plant P.

We have the following relationship:

$$\frac{(P - \hat{P}_{i+1})K}{1 + \hat{P}_{i+1}K} = (\psi_i^{-1} - 1)\frac{K(1 + \hat{P}_{i+1}C_i)}{C_i(1 + \hat{P}_{i+1}K)}. \tag{44}$$

The import of this relationship is that it provides a mechanism to assess directly the stability robustness of the (P, K) feedback pair using information from stable loops (P, C_i), $(\hat{P}_{i+1}, C_i)$ and $(\hat{P}_{i+1}, K)$. In our iterative scheme C_i is the presently active controller and K the new controller derived using the new $\hat{P}_{i+1}$. If the stability robustness is found lacking, the identification experiment may be re-performed with an altered excitation to adjust $\hat{P}_{i+1}$ to provide better stability robustness. It is worth reiterating, however, that (1) or (2) provide only a sufficient condition for robust stability. Further phase information would be needed to develop a necessary and sufficient test.

4.5. *Estimation of control weighting filters*

We have presented two alternative experiment designs for identification, to which we now add a third for the computation of the frequency-weighting filter used for control design.

Identification for robust performance. Here

$$D = \frac{\hat{H}G}{1 + \hat{P}C}, \quad \Phi_r = \gamma^2 \Phi_v.$$

Identification for robust stability. Here

$$D = \hat{H}, \quad \Phi_r = \gamma^2.$$

Control weighting computation. Here

$$\Phi_r = 0.$$

The aim of the control weighting filter F is to convert the designed performance measure on $\hat{P}$ to mimic the design on the actual plant P. In the disturbance rejection context here, where the excitation signal r_t diminishes achieved control performance and is injected only to effect the identification, we have the global criterion,

$$J_{\text{global}} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1 + \lambda^2 |C|^2}{|1 + PC|^2} |H|^2 \, d\omega. \tag{45}$$

The designed criterion is

$$J^C = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1 + \lambda^2 |C|^2}{|1 + \hat{P}C|^2} |\hat{H}|^2 |F|^2 \, d\omega,$$

and it is apparent that the desired F should satisfy

$$|F| = \left|\frac{1 + \hat{P}C}{1 + PC}\frac{H}{\hat{H}}\right|. \tag{46}$$

This is precisely achieved by (19) operating under the condition of no excitation, since

$$u_t = -\frac{C}{1 + PC}He_t, \quad u_t^c = -\frac{C}{1 + \hat{P}C}\hat{H}e_t'.$$

That is, the frequency-weighted LQG design actually uses data derived only from unexcited operating conditions. A variant of this scheme has been used by Partanen and Bitmead (1993b) to generate a direct control design procedure in which the identification phase is avoided altogether.

5. H_2 ITERATIVE IDENTIFICATION AND CONTROL DESIGN ALGORITHM

We are now in a position to present our iterative procedure.

Initial identification and control—initial model identification. Select $D = 1$ or any other specific non-constant filter if a priori information about the true plant is available. An experiment with r_t excitation is performed in open loop (or with a pre-existing controller in the case of an unstable plant) to yield an initial model $\hat{P}_0(z)$. Take a fixed noise model $\hat{H}$ and set $i = 0$.

Initial control design. Select $F_0(z) = 1$, i.e. uniform weighting. With plant model $\hat{P}_0(z)$, design the controller $C_0(z)$ via the minimisation of the local control criterion

$$J^C = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} [(y_t^c)^2 + \lambda^2 (u_t^c)^2].$$

Iterations of closed-loop identification and control design

Step 1: *compute identification filter D_i and select excitation signal r_t*. With $\hat{P}_i$, C_i, $\hat{H}$ and λ, compute the identification filter $D_i(z)$ and select the excitation signal r_t as follows:

- for robust performance,

$$D_i = \frac{\hat{H}}{1+\hat{P}_i C_i}(1+\lambda^2 |C_i|^2)^{1/2}, \quad \Phi_r = \gamma^2 \Phi_v;$$

- for robust stability

$$D_i = \hat{H}, \quad \Phi_r = \gamma^2;$$

or some compromise between these.† Generate closed loop data signals $\{y_t\}$ and $\{u_t\}$. Initially performance should be sought.

Step 2: *model and sensitivity identification.* Using a least-squares prediction error identification scheme with data $\{r_t\}$, $\{y_t\}$ and $\{u_t\}$ and data filter $D_i(z)$, compute $\hat{P}_{i+1}$ as the order-n model between u_t and y_t, $M_i = C_i(1+PC_i)^{-1}$ using a high-order model between r_t and u_t, and $\hat{M}_i$ as $C_i(1+\hat{P}_{i+1}C_i)^{-1}$. Compute the robust stability function $\psi_i = M_i/\hat{M}_i$.

Step 3: *plant and model experiment without excitation.* With the controller C_i acting in feedback with P and with no excitation signal, i.e. $r_t = 0$, generate closed-loop data signals $\{y_t\}$ and $\{u_t\}$. Compute the low-order AR model $A_u^{-1}(z)$ for $\{u_t\}$. With C_i acting in feedback with $\hat{P}_{i+1}$ and with $r_t = 0$, simulate closed-loop data signals $\{y_t^c\}$ and $\{u_t^c\}$. Compute the low-order AR model $A_{u^c}^{-1}(z)$ for $\{u_t^c\}$.

Step 4: *computation of LQG controller.* Compute the frequency-weighting filter

$$F_i(z) = \frac{A_{u^c}(z)}{A_u(z)}.$$

With $\hat{P}_{i+1}(z)$, $\hat{H}(z)$ and $F_i(z)$, compute the new controller $C_{i+1}(z)$.

Step 5: *robust stability test.* With $\hat{P}_{i+1}$, C_{i+1}, C_i and ψ_i, check the robust stability of the (P, C_{i+1}) loop. Compute the magnitude of the quantity

$$\phi_i(z) = \frac{(P-\hat{P}_{i+1})C_{i+1}}{1+\hat{P}_{i+1}C_{i+1}} = (\psi_i^{-1}-1)\frac{C_{i+1}(1+\hat{P}_{i+1}C_i)}{C_i(1+\hat{P}_{i+1}C_{i+1})}, \tag{47}$$

and check whether it remains bounded by one for all frequencies.

Step 6: *loop.* If the robust stability test is passed then loop to Step 1 with robust performance objective; otherwise set $C_{i+1} = C_i$ and loop to Step 1 with robust stability objective.

Other variants of the scheme are clearly possible. We have chosen to focus on disturbance rejection only, with a fixed noise model.

† A practical implementation of such a compromise on an industrial problem is discussed by Partanen and Bitmead (1995).

6. DESIGN EXAMPLE

6.1. *Computer experiment setup*

In this section we shall present simulation results to illustrate the effectiveness of our iterative weighted LS identification and weighted LQG control designed algorithm. The iterative design strategy was performed on the following example.

The true plant is chosen to be fifth-order of the form

$$y_t = P(z)u_t + H(z)e_t, \tag{48}$$

$$p(z) = \frac{z^{-1} - 1.2z^{-2} - 0.3z^{-3} + 0.156z^{-4} + 0.0845z^{-5}}{1 - 1.25z^{-1} + 0.4575z^{-2} + 0.0279z^{-3} - 0.0491z^{-4} + 0.0077z^{-5}},$$

$$H(z) = \frac{2}{1+0.6121z^{-1}}.$$

This is a stable and non-minimum-phase plant with single delay. Figure 3 shows the magnitude plot of the true noise transfer function $H(z)$; it is essentially high-frequency noise. The plant model to be identified is assumed to be third-order with a single delay, i.e. of the form

$$y_t^c = \hat{P}(z)u_t^c + v_t'. \tag{49}$$

In the whole experiment e_t and the model

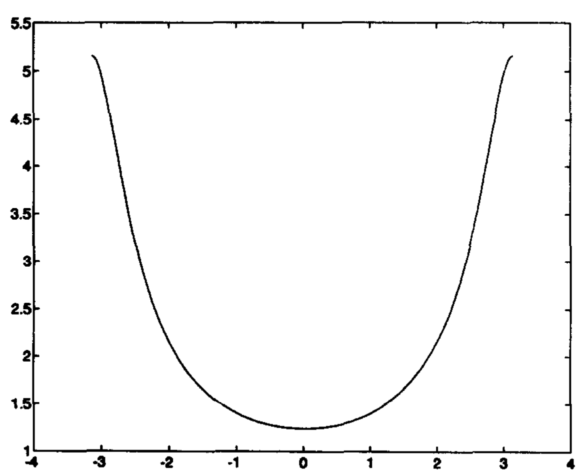

Fig. 3. Magnitude plot of the true noise model.

disturbance $v_t' = e_t'$ are taken to be independent white noise of unit variance, i.e. $\hat{H} = 1$. The LQ control criterion has $\lambda^2 = 0.1$ and the Kalman filter is designed with process noise of unit variance entering through the input channel and measurement noise of variance $\rho = 0.8$ (roughly an LQG/LTR strategy). The robust performance variant of the identification filter D is used. The excitation signal r_t used in the identification process is taken to be white noise with power spectrum $\Phi_r = 9$. The closed-loop signal spectra Φ_u and Φ_{u^c} are estimated from measured signals using third-order AR models; see Step 3 of the iterative algorithm. To compute the robust stability test function $\psi(z)$, the transfer function from r_t to u_t is estimated using a 30th-order ARX model. The closed loop identification was performed on 2048 samples per iteration, and a total of six iterations was performed.

6.2. *Simulation results*

To facilitate exposition, let us use the following notations: C_{opt} denotes the full-order LQG controller, designed on the basis of the true plant and noise model, which truly optimises J_{global}. $\hat{P}_0$ denotes the plant model identified in open loop. $\hat{P}_i$ $(i = 1, \ldots, 6)$ denotes the plant model obtained in the ith iteration of closed-loop identification. C_i $(i = 0, 1, \ldots, 6)$ denote the controllers designed based on these plant models $\hat{P}_i$ $(i = 0, 1, \ldots, 6)$ to minimise the frequency-weighted LQ criterion J^C. Corresponding to the above, we define the optimal, achieved, and designed sensitivity functions respectively as follows:

$$S_{opt}(z) \triangleq \frac{1}{1 + P(z)C_{opt}(z)}, \quad S_i(z) \triangleq \frac{1}{1 + P(z)C_i(z)},$$

$$\hat{S}_i(z) \triangleq \frac{1}{1 + \hat{P}_i(z)C_i(z)}.$$

The weighting functions used in the ith iteration of LQ control design are denoted by F_i.. The filters used in the ith iteration of closed-loop identification are denoted as D_i. The stability robustness test function estimated in the ith iteration is denoted by ϕ_i; see (47). We are now in a position to display our simulation results.

6.2.1. *The cost functions.* We first design a full-order optimal LQG controller C_{opt} for the true plant (48) and compute the optimal performance cost J_{opt}:

$$J_{opt} = 6.3789.$$

Next we perform open-loop identification to obtain a plant model $\hat{P}_0$, and then design a controller C_0 based on the model $\hat{P}_0$. Applying the controller to the true plant and to this model and, with 2048 samples of the corresponding closed-loop data, we calculate the achieved cost J_{global}, the designed cost $\bar{J}$, the frequency-weighted cost J^C, and the identification cost J^I:

$$J_{global,0} = 11.7356, \quad \bar{J}_0 = 1.2114,$$
$$J_0^C = 1.2114, \quad J_0^I = 12.5979.$$

Next, six iterations of closed-loop identification and LQG control design are performed. Corresponding to these six iterations, we obtain the following J_{global}, $\bar{J}$, J^C and J^I (where the subscripts stand for the iteration numbers):

$$J_{global,1} = 9.5360, \quad J_{global,2} = 8.5509,$$
$$J_{global,3} = 7.7685, \quad J_{global,4} = 7.0462,$$
$$J_{global,5} = 6.9403, \quad J_{global,6} = 6.8608;$$
$$\bar{J}_1 = 1.3916, \quad \bar{J}_2 = 1.4313,$$
$$\bar{J}_3 = 1.3047, \quad \bar{J}_4 = 1.2969,$$
$$\bar{J}_5 = 1.1481, \quad \bar{J}_6 = 1.1967;$$
$$J_1^C = 1.8847, \quad J_2^C = 1.7787,$$
$$J_3^C = 1.5551, \quad J_4^C = 1.3229,$$
$$J_5^C = 1.8896, \quad J_6^C = 1.7640;$$
$$J_1^I = 10.5433, \quad J_2^I = 9.6113,$$
$$J_3^I = 8.7908, \quad J_4^I = 8.1607$$
$$J_5^I = 7.8810, \quad J_6^I = 7.8601.$$

6.2.2. *The open- and closed-loop models versus the true plant.* Figure 4 shows a plot of the transfer function magnitude of the true plant P (solid line) and of the plant model $\hat{P}_0$ (dashed line) identified in open-loop, as well as that of the plant models $\hat{P}_5$ (dotted line), and $\hat{P}_6$ (dash–dotted line) obtained in the fifth and sixth

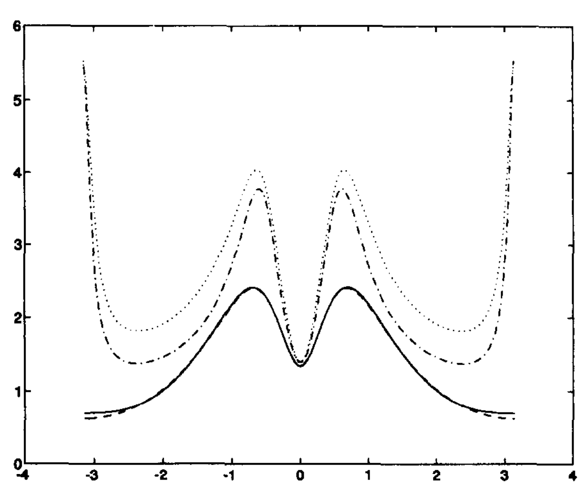

Fig. 4. Magnitude plot of the true plant transfer function and the plant models P (solid line), $\hat{P}_0$ (dashed line), $\hat{P}_5$ (dotted line) and $\hat{P}_6$ (dash–dotted line).

iterations of closed-loop identification. Further iterations of closed-loop identification produce plant model transfer functions with roughly the same magnitudes.

6.2.3. *The achieved and designed sensitivity functions.* Figure 5 shows plots of the magnitude frequency responses of the optimal sensitivity function S_{opt} and of the achieved sensitivity functions S_0, S_5 and S_6. Figure 6 shows a plot of the magnitude frequency responses of the optimal sensitivity function S_{opt} and of the designed sensitivity functions $\hat{S}_0$, $\hat{S}_5$ and $\hat{S}_6$.

6.2.4. *The identification and control weightings.* Figure 7 shows a plot of the transfer function magnitudes of the identification filters D_i obtained in the first, second, fifth and sixth iterations of plant identification and control design. Figure 8 shows a plot of the frequency response magnitudes of the control design weightings F_1 (solid line), F_2 (dashed line), F_5 (dotted line) and F_6 (dash-dotted line).

6.2.5. *The stability robustness test function.* Figure 9 shows a plot of the frequency response magnitude of the robust stability test function ϕ_1 (solid line), ϕ_2 (dashed line), ϕ_5 (dotted line) and ϕ_6 (dash-dotted line).

6.2.6. *The closed-loop impulse responses.* Figure 10 shows plots of the true closed-loop impulse response when the full-order optimal controller C_{opt} is used (solid line), the controller C_0 based on the open-loop model (dashed line) and the controller C_6 obtained at the sixth iteration (dotted line).

6.2.7. *The closed-loop output signal y_t.* Figure 11 shows plots of the closed-loop output signal y_t ($t = 1, 2, \ldots, 100$) when the full-order optimal controller C_{opt} is used (solid line), the controller C_0 based on the open-loop model (crosses) and the controller C_6 obtained at the sixth iteration (dotted line). For the reader's convenience, the relevant controllers and plant models are given as follows:

$$C_{opt}(z) = \frac{(0.0028 + 0.1002z^{-1} - 0.1229z^{-2} + 0.0154z^{-3} + 0.0136z^{-4} - 0.0018z^{-5})}{(1 - 0.2944z^{-1} - 0.5368z^{-2} - 0.0340z^{-3} + 0.0799z^{-4} + 0.0228z^{-5})},$$

$$C_0(z) = \frac{0.2872 - 0.2265z^{-1} + 0.0171z^{-2}}{1 - 0.4067z^{-1} - 0.1929z^{-2} - 0.0009z^{-3}},$$

$$C_6(z) = \frac{(0.0380 + 0.1895z^{-1} - 0.0335z^{-2} - 0.1078z^{-3} - 0.0171z^{-4} - 0.0090z^{-5})}{(1 + 0.7149z^{-1} - 0.4911z^{-2} - 0.4222z^{-3} - 0.0952z^{-4} - 0.0313z^{-5} + 0.0001z^{-6})},$$

$$\hat{P}_0(z) = \frac{0.9807z^{-1} - 0.9834z^{-2} - 0.3846z^{-3}}{1 - 1.0766z^{-1} + 0.4149z^{-2} - 0.0508z^{-3}},$$

$$\hat{P}_6(z) = \frac{1.6066z^{-1} - 0.9120z^{-2} - 1.4404z^{-3}}{1 - 0.3139z^{-1} - 0.6349z^{-2} + 0.4839z^{-3}}.$$

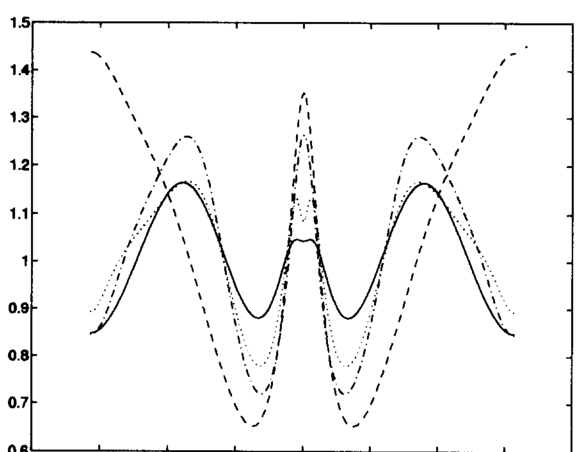
Fig. 5. Magnitude plot of S_{opt} (solid line), S_0 (dashed line), S_5 (dotted line) and S_6 (dash-dotted line).

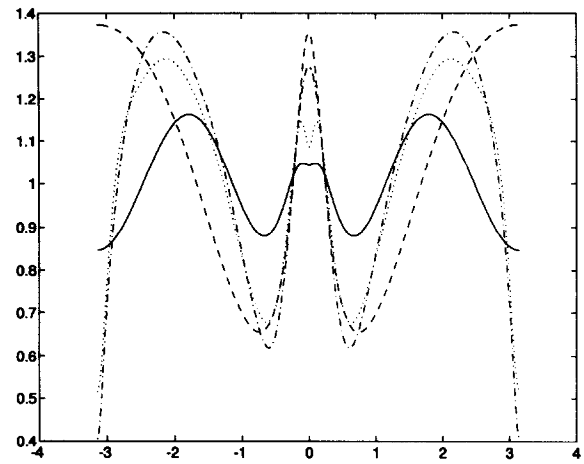
Fig. 6. Magnitude plot of S_{opt} (solid line), $\hat{S}_0$ (dashed line), $\hat{S}_5$ (dotted line) and $\hat{S}_6$ (dash-dotted line).

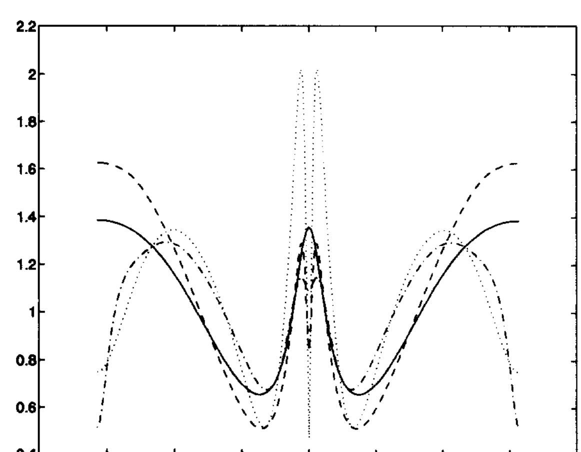
Fig. 7. Magnitude plot of the identification filter D_1 (solid line), D_2 (dashed line), D_5 (dotted line) and D_6 (dash-dotted line).

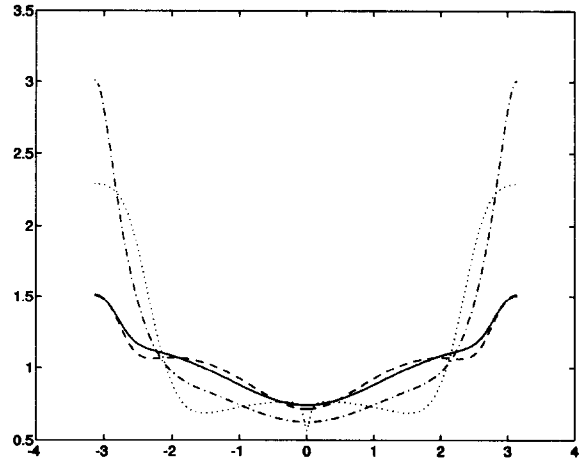

Fig. 8. Magnitude plot of the control design weighting F_1 (solid line), F_2 (dashed line), F_5 (dotted line) and F_6 (dash–dotted line).

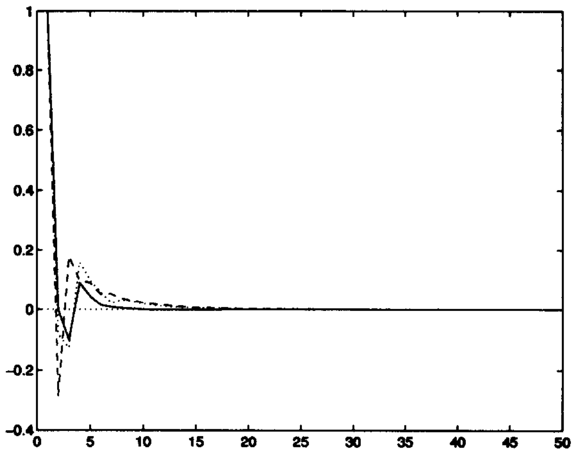

Fig. 10. Plot of the true closed-loop impulse responses using the full-order controller C_{opt} (solid line), the controller C_0 based on the open-loop model (dashed line), and the controller C_6 obtained at the sixth iteration (dotted line).

6.3. *Comments and general design guidelines*

This computer design example demonstrates several salient features of our iterative approach to plant identification and robust control design.

- J_{global} *versus* $\bar{J}, J^C$ *and* J^I. The most crucial observation is that the closed-loop performance achieved with the true plant, as measured by the global criterion J_{global}, improves from step to step. In this example most of the other local control and identification criteria are also decreasing step by step as the iterations go on, but this need not always be the case.
- *The estimated plant models.* Observe that the open-loop identified plant model $\hat{P}_0$ is apparently much closer to the true P than the subsequent closed-loop identified models. This needs to be understood in conjunction with the mismatch of the noise model and the disturbance rejection objective of the iterative scheme. Note also that, while the plant does not fit within the model set, the derived controllers do possess sufficient order to yield C_{opt}. This is due to the increased order provided by the weighting filters. We see that achieved performance is very close to the best possible within the class of controllers, which is reassuring. With more complex plants P, the same effect is evident, but the comparison to the optimal restricted complexity control is more difficult to demonstrate.
- *The achieved sensitivity.* Figure 5 clearly indicates that the effect of our iterative design is to make the achieved sensitivity small in the high-frequency range, where the high value of the noise demands this. We note that the sensitivity obtained at high frequency obtained with the controller based on the open-loop identified model gives poor disturbance rejection. Conversely, we note that this low sensitivity at high frequencies is obtained with

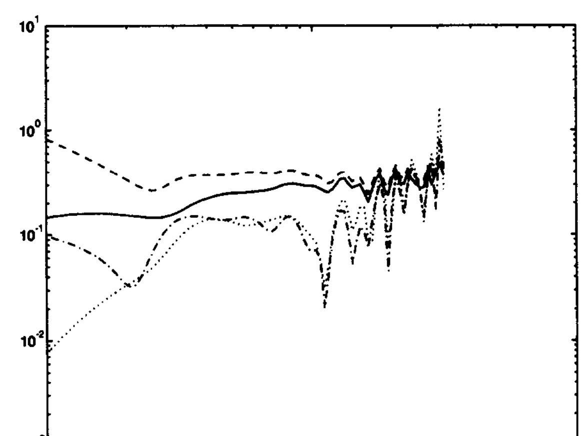

Fig. 9. Magnitude plot of the stability robustness test function ϕ_1 (solid line), ϕ_2 (dashed line), ϕ_5 (dotted line) and ϕ_6 (dash–dotted line).

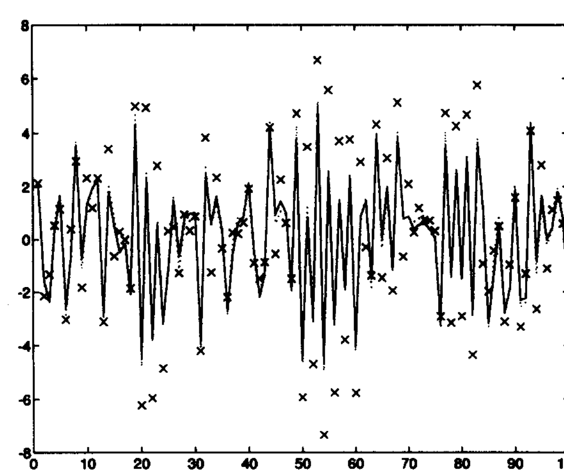

Fig. 11. Plot of 100 output data y_t collected from the closed loop with the full-order controller C_{opt} (solid line), the controller C_0 obtained from open-loop identification (crosses) and the controller C_6 obtained from closed-loop identification (dotted line).

a model $\hat{P}_6$ that appears to be much worse than the model obtained in open loop; see Fig. 4. These are typical features of these iterative controller-oriented design schemes. As is well known from H_∞ sensitivity minimisation design theory, if the given plant possesses a right-half-plane zero as in our example, trying to make the sensitivity function magnitude small over some frequency range will inevitably force it to be large elsewhere. Our simulation shows that a very good trade-off (for the achieved sensitivity minimization) can be obtained after two or three iterations. The plots of the output (Fig. 11) confirm these observations: the noise rejection achieved on the real plant with the controller C_6 is significantly better than that achieved with open-loop identification. In fact, it is almost as good as that of the optimal full-order controller.

- *The role of the filters.* We note that, as the iterations proceed, the identification filter magnitude becomes smaller at high frequency; see Fig. 7. This is mainly the effect of the designed sensitivity becoming small at those frequencies; see Fig. 6. The closed-loop plant model mismatch therefore becomes large at high frequency, and this is compensated for by the control design frequency weighting function F; see Fig. 8.
- *The excitation signal power.* In this example the ratio of the variance of the excitation signal r_t to noise v_t is slightly higher than 1. Experience has shown that choosing Φ_r too small often leads to a robust stability problem, while choosing it too large and poorly aligned with the noise spectrum deteriorates the achieved performance. The stability problem can be detected by monitoring the stability test function ϕ. Experience with this and other examples has also shown that a small violation of the 1 bound for the function ϕ at a few points usually does not produce instability of the achieved loop.
- *Number of iterations.* Our experience on a large number of examples and on real life applications† is that, for a time-invariant plant, a significant improvement in performance is obtained during the first two or three iterations, but that not much improvement is obtained by doing further iterations. *This iterative scheme should certainly not be seen as one in which iterations should be continued forever, but rather as a way of getting the benefits (in terms of performance) of a controller designed on the basis of closed-loop identification.* These benefits can (and should) be obtained in a very small number of iterative design steps. Users should be fully aware that this iterative identification/control design scheme can sometimes diverge.‡ This divergence has been observed on some simulation examples, and some reasons for such divergence have been analysed recently. The fixed points of a somewhat simpler iterative identification-and-pole-placement-control design scheme have been studied by Åström and Nilsson (1994), showing both stable and unstable fixed points. The connection with a convergent iterative controller design scheme has been established by Hjalmarsson *et al.* (1994b), giving some indication of what might go wrong in the identification-based schemes if iterations are continued forever, Let us repeat that our message is not to do so.

† The Zangscheme has been successfully applied by the authors or their colleagues to the control of an industrial sugar mill (see Partanen and Bitmead, 1995) and to the control of a flexible robot arm (see Hoffmann, 1993).

‡ Not on this example.

7. CONCLUSIONS

We have developed an iterative identify-then-control paradigm. The focus of the approach is to consider a single global control objective and then to perform an interlaced sequence of

- frequency-weighted least-squares system identifications,
- frequency-weighted LQG control designs,

each with their respective local objective functions. These criteria (embodied in the frequency weighting) for each case reflect the current local objective but are tuned to address minimization of the global objective.

Methods have been presented for the (somewhat fictitious) H_∞ case, with a guaranteed descent property but no algorithm for the implementation, and the more realistic H_2 (LQG and least-squares), case with no guarantee of descent for the achieved cost. To implement the methods requires that the identification stage provide not only a best fitting model but also a measure of model error. In the H_∞ case the provision of the complete frequency response error is an unrealistic expectation, but in the H_2 case estimates of signal spectra are used as measures of the plant/model misfit. The model error is introduced to modify the local control law specification. In a complementary fashion, the global control objective and the local controller are used to adjust the frequency weightings of the data that are fed into the identifier, operating in closed loop.

Our iterative scheme is essentially a performance enhancement scheme for LQG disturbance rejection with reduced-order models, although some robust stability safeguards can be incorporated as we have shown. A considerable amount of further work is needed to establish more detailed properties of such methods and to extend their validity fully to adaptive control. This work is ongoing but it is clearly of interest to establish the connection between the applicability of these schemes and the provision of a priori plant information. In terms of practical applications, however, the methodology advanced here goes a long way towards addressing the questions of how to adjust and improve existing controllers using current on-line experimental data.

Acknowledgements—The second author acknowledges the funding of the activities of the Cooperative Research Centre for Robust and Adaptive Systems by the Australian Commonwealth Government under the Cooperative Research Centres Program. This paper presents research results of the Belgian Programme on Interuniversity Poles of attraction initiatied by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility rests with its authors. The work has also been partially supported by Boeing (BCAC) and a research grant from the Australian Research Council.

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