Linear smoothing of FRF for aicraft engine vibration monitoring

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Abstract

The problem of aircraft engine condition monitoring based on vibration signals is addressed. To do so, we compare two estimators of the Frequency Response Function of an aircrat engine which input is its shaft angular position and which output is an accelerometric signal that measures vibrations. It is shown that this problem can be seen as a smoothing problem, and that linear kernel smoothing such as Gaussian Process Regression allows the computation of the FRF.

1 Introduction

We tackle the issue of monitoring the behavior of an aircraft engine from the point of view of measured vibrations. Abnormal level or odd pattern of vibrations may be the consequence of mechanical or sensor malfunction, both of dramatic importance for engine manufacturers and airline operators.

The aim of our work is to develop Damage Detection and Condition Monitoring algorithms for aicraft engines. To do so, we model the engine as an input/output system where the input is the shaft angular position, considered as a periodic excitation, and the output is the instant vibration level. Most information concerning this input/output relationship are embedded in the Frequency Response Functions (FRF) G:

$$G(j\omega_k) = \frac{Y(k)}{U(k)} \tag{1}$$

where $U(k)$ and $Y(k)$ are respectively the Fourier transforms of the shaft angular position, and of the instant vibration level at frequency ω_k. Fig. 4 gives an example of such an FRF plot.

To allow Damage Detection, we propose to estimate confidence intervals around the FRF estimates. If for one or several frequencies the FRF value crosses the limits, then a mechanical fault is likely to occur.

The originality of this work lies first in the input/output point of view that we propose, between the shaft position and accelerometric data. Furthermore we suggest a particular smoothing technique, namely Gaussian Process Regression (GPR) which have been under considerable attention in the Machine Learning community. The aim of this regression is to estimate the FRF, when records belonging to several “normal” -at least labelled as such by experts- engines operating at the same regime are available. Finally we compute confidence bounds for the FRF estimate.

Remark that we limit the scope of this work in several respects: first we deal with SISO[1] systems only. Then we disregard important topics such as the presence of nonlinear distortions. We hold the excitation to be periodic only, while important random contributions related to combustion could be taken into account. Other technical features will be mentionned in conclusion.

Section 2 relates our article to previous works in several fields. Section 3 makes precise the classical measurement and estimation model used in the litterature. Section 4 aircraft engine and measured data.

2 Related work

This article is related to Condition Based Monitoring (CBM) framework. CBM for industrial machines has been attracting increasing attention over the years in both academic and industrial areas. According to [24] it consists in four main steps: data acquisition, feature extraction, feature selection, and decision-making. The first two steps rely on mechanical modeling or rotor dynamics [16], noise and vibration phenomena in rotating machines [13, 24] and data analysis [18]. The latter builds on the general tools and methods developped in signal processing [15, 2], statistical signal estimation and detection [10, 22, 28], learning theory, change detection [1], fault detection and isolation [7]. Aircraft CBM deals with many problems such as structural health monitoring. It treats Engine Health Monitoring (EHM) as a special case [27, 9]. As a subtopic of EHM, vibrations monitoring in engines addresses the following issues: rotor/stator contact [19], rotor unbalance, blade defects [11], bearing [17] and gearings defects [30].

Some authors take a probabilistic stand on CBM and study the fluctuations of the vibration spectrum of aircraft engines. For example [4, 5] take advantage of extreme value theory to detect novelty from spectral or time-frequency data.

In the Control and System Identification litterature [12, 20] and in Modal Analysis [14], focus is put more particularly on the FRF, while its non-parametric estimation remains an active field of research [29, 21]. The statistical properties of the estimators of FRF is a topic of deep interest [20, 2.4-6], because of their role in subsequent parametric identification of plants. In order to reduce their variance, it is common practice to average the FRF estimator over several blocks, in order to cancel the various perturbations due to measurement noise of inputs and/or outputs.

From a statistical point of view, such averaging is related to nonparametric function regression. More precisely, estimating the trend of a response measurement (e.g. $Y(k)$) as a function of several predictors (e.g. $U_1(k), \ldots, U_n(k)$) in a nonparametric way, such that the estimate is less variable than the predictors, is called smoothing [8, 2.1]. Among many available (scatterplot, running-mean, kernel smoothers), we choose Gaussian Process Regression (GPR) [25], which is an instance of kernel smoothing. GPR is rooted in Bayesian statistics, and enables to add physically meaningful constraints to the regressed function (see 3.2, Appendix C). GPR has several interesting features, such as the simplicity of the underlying theory or its prediction capacity in areas where data is scarce, which could help to artificially increase the spectral resolution.

In Section 3 the measurement model is discussed, taking into account the various aspects just evoked (condition monitoring, FRF estimation, non-parametric smoothing) .

3 FRF measurement, noise model and statistical properties of estimators

To measure the FRF, the mechanical system under study is first excited by the shafts, each producing periodic excitations. More precisely, two shafts - the low pressure (LP) and high-pressure(HP) - rotate at different speeds (see Section 4) around the same axis.

[1] Single Input Single Output.

Since accelerometric data are resampled with respect to one shaft only (LP or HP), we will consider one source of excitation only and treat the other as noise. Such noise is represented as actutator noise in Fig. 1, and results in an signal noted $u(t)$ in the time domain. The mechanical system is excited by $u(t)$ and outputs the response $y(t)$ which is the sum of the pure response to u and of a noise term. Various preprocessing steps such as antialiasing and digitization are included, before the Discrete Fourier Transform, implemented as an FFT. With the notations of [20] we have by linearity :

$$Y(k) = Y_0(k) + N_Y(k) \tag{2}$$

$$U(k) = U_0(k) + N_U(k) \tag{3}$$

where k corresponds to frequency $f_k = \frac{k}{NT_s}$, N and T_s being respectively the length of recording and T_s the sampling period. $N_U(k)$ and $N_Y(k)$ are the noise contributions and have been shown to be complex circular normally distributed in a large number of situations (see Appendix A).

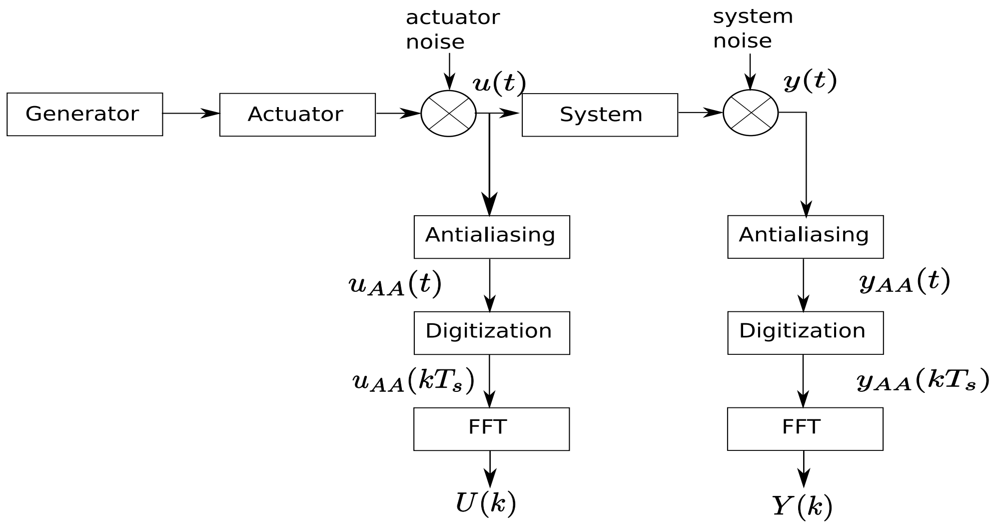


Figure 1: FRF measurement (adapted from [20]).

In Sections 3.1 and 3.2 we state the expressions of a classical estimator of FRF, and of its GPR counterpart. In both it is admitted that the rotation speed of the motor is periodic, with negligible fluctuations, so that we consider the situation of *periodic excitation FRF measurement*, as opposed to random excitation measurement (see [20, 2.6]). When explicitly available we write down the statistical properties of the estimators.

3.1 Maximum likelihood estimator of FRF under periodic excitation

As stated above, the estimator of the unknown FRF $G_0(j\omega_k) = \frac{Y_0(k)}{U_0(k)}$ is $G(j\omega_k) = \frac{Y(k)}{U(k)}$. The error analysis that reveals the statistical properties of this estimator is standard material, taken mainly from [20].

Provided that the signal was appropriately preprocessed (including band-limiting filtering), that the disturbing noises N_U and N_Y follow complex spherical normal laws and that $|N_U(k)/U_0(k)| < 1$ (see Appendix B), then the estimator is unbiased.

Similarly, under the same type of hypothesis, the variance of G can be approximated. These results are summarized as follows :

$$E[G(j\omega_k)] = G_0(j\omega_k) \tag{4}$$

$$Var[G(j\omega_k)] = |G_0(j\omega_k)|\Big(\frac{\sigma_Y^2(k)}{|Y_0(k)|^2} + \frac{\sigma_U^2(k)}{|U_0(k)|^2} - 2\mathrm{Re}(\frac{\sigma_{YU}^2(k)}{Y_0(k)\bar{U}_0(k)})\Big) \tag{5}$$

where $\sigma_U^2(k)$, $\sigma_Y^2(k)$ and $\sigma_{YU}^2(k)$ are the variances and covariance of the disturbing noise components.

Variance reduction by cyclic averaging
The material of this paragraph is taken from [20, 2.5]. In order to reduce the variance of the estimator depicted above, cyclic averaging is applied. This means that successive data blocks taken from the same long recording are averaged. Cyclic averaging needs the number of samples to be an integer multiple of a given integer, the period length. This can be achieved with the help of a keyphasor, but is insured here by computer resampling [23, 6].

Under the hypotheses that the disturbing noise $N_u(k), N_y(k)$ is independent of the undisturbed signals $U_0(k), Y_0(k)$ (which may not be true in closed loop contexts), that the M input-output data blocks $u^{[l]}(n), y^{[l]}(n),\ l = 1, 2, \ldots, M$ come from independent experiments where the noise contributions have finite moments and are independent, then the averaged estimator is unbiased:

$$\hat{G}_{ML}(j\omega_k) = \frac{\hat{Y}(k)}{\hat{U}(k)} \tag{6}$$

$$E[\hat{G}_{ML}(j\omega_k)] = G_0(j\omega_k) \tag{7}$$

where

$$U^{[l]}(k) = DFT(u^{[l]}(1:n)) \tag{8}$$

$$Y^{[l]}(k) = DFT(y^{[l]}(1:n)) \tag{9}$$

$$\hat{U}(k) = \frac{1}{M}\sum_{l=0}^{M-1} U^{[l]}(k) \tag{10}$$

$$\hat{Y}(k) = \frac{1}{M}\sum_{l=0}^{M-1} Y^{[l]}(k) \tag{11}$$

The variance of the estimator can be approximated by:

$$Var[\hat{G}_{ML}(j\omega_k)] \sim \frac{|\hat{G}_{ML}(j\omega_k)|^2}{M}\Big(\frac{\hat{\sigma}^2_Y(k)}{|Y_0(k)|^2} + \frac{\hat{\sigma}^2_U(k)}{|U_0(k)|^2} - 2\mathrm{Re}(\frac{\hat{\sigma}^2_{YU}(k)}{Y_0(k)\bar{U}_0(k)})\Big) \tag{12}$$

where $\hat{\sigma}^2_U(k), \hat{\sigma}^2_Y(k), \hat{\sigma}^2_{YU}(k)$ stand for sample variances and covariance, which definitions are given in Appendix B. It can be noticed that this term is divided by M, which was the aim of averaging. Nevertheless, the above results depend on the validity of the hypotheses, and should be treated with care. Unfortunately, it depends on Y_0 and U_0, that are most of the time unknown. This means that another way to estimate the variance of the estimator may be necessary.

3.2 GPR estimation of FRF

As remarked in the Introduction, the variance reduction in Section 3.1 can be thought of as a nonparametric smoothing [8, 2], with the constraint that abscissa are taken in the discrete set of frequencies $\{f_k\}_{k\in[1,n]}$ defined in Section 3. In this article we explicitly choose a smoother of a special kind -namely a Gaussian Process Regressor, that belongs to the category of kernel smoothing regressions.

The mathematical problem is stated as follows: let $y = f(\mathbf{x}) + \varepsilon$ be a real or complex function of $\mathbf{x}$ taken in a vector space, subject to a random perturbation. In our case y and $\mathbf{x}$ can be associated respectively to $G(j\omega_k)$ and to the frequency. We are given a finite sample $\{(y_i, \mathbf{x}_i)\}_{i=1,\ldots,n}$. Then we want to estimate the probability law of the noiseless FRF in points $\{(\mathbf{x}_{*i})\}_{i=1,\ldots,n_*}$ that can be different from $\{\mathbf{x}_i\}$. Technically, what is looked for is the conditional law evaluated in $\mathbf{x}_*$, which is noted (see [25]):

$$\mathbf{f}_*|X_*, \mathbf{y}, X \tag{13}$$

where $\mathbf{y} = (y_1, \ldots, y_n)^T,\ X = (x_1, \ldots, x_n)^T, X_* = (x_{*1}, \ldots, x_{*n_*})^T$. In order to compute $\mathbf{f}_*|X_*, \mathbf{y}, X$ we first choose a prior[2] :

$$\begin{bmatrix} \mathbf{f} \\ \mathbf{f}_* \end{bmatrix} \sim \mathcal{N}\Big(\mathbf{0}, \begin{bmatrix} K(X,X) & K(X,X_*) \\ K(X_*,X) & K(X_*,X_*) \end{bmatrix}\Big)$$

where $K(X, X_*)$ is a $n \times n_*$ matrix called the Gram matrix. The value of K depends on the choice of the kernel that parametrizes the Gaussian process. Admitting that when $\mathbf{x}$ et $\mathbf{y}$ are jointly Gaussian random vectors such that:

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \sim \mathcal{N}\Big(\begin{bmatrix} \mathbf{m}_x \\ \mathbf{m}_y \end{bmatrix}, \begin{bmatrix} A & C \\ C^T & B \end{bmatrix}\Big)$$

then (cf [25, A.2]) :

$$\mathbf{x}|\mathbf{y} \quad \sim \quad \mathcal{N}(\mathbf{m}_x + CB^{-1}(\mathbf{y} - \mathbf{m}_y), A - CB^{-1}C^T) \tag{14}$$

Consequently the conditionnal law of $\mathbf{f}_*$ is (see [25, 2.2]):

$$\begin{aligned} \mathbf{f}_*|X_*, \mathbf{y}, X \quad \sim \quad & \mathcal{N}\Big(K(X_*,X)(K(X,X) + \sigma^2 I)^{-1}\mathbf{y}, \\ & K(X_*,X_*) - K(X_*,X)(K(X,X) + \sigma^2 I)^{-1} K(X,X_*)\Big) \end{aligned} \tag{15}$$

where $K(X,X), K(X_*,X_*), K(X_*,X)$ are covariance matrices and where σ_n^2 is the variance of the observation noise. In the particular case where we estimate f_* at the same frequencies as those in the measured samples, then $X = X_*$ and:

$$\begin{aligned} \mathbf{f}|\mathbf{y}, X \quad \sim \quad & \mathcal{N}\Big(K(X,X)(K(X,X) + \sigma^2 I)^{-1}\mathbf{y}, \\ & K(X,X) - K(X,X)(K(X,X) + \sigma^2 I)^{-1} K(X,X)\Big) \end{aligned} \tag{16}$$

Computational shortcuts

The computation of the mean and covariance of the estimator needs the inversion of a large matrix. For example, with the available data, $n = 8192$ and four engines are considered at the same time. Since matrix inversion needs $0(n^3)$, the exact computation is impossible in reasonable time. Consequently, we had to compute approximate results. In this particular case, it turns out that the length-scale of the data is very short. The corresponding Gram matrix is then sparse, and sparse linear solvers were used to shrink computation time. This will be made precise in following articles.

Model parameters

The result of the regression is fully determined by the Gram matrix K, the noise level σ, and the measured data. K depends on a kernel function, that usually depends on a small number of parameters. They are estimated during a calibration phase called “model selection” in the Machine Learning litterature. Details are given in Appendix C.

Error analysis

The convergence of $\mathbf{f}_*|X_*, \mathbf{y}, X$ to the regression function $E[y|\mathbf{x}]$ is established in [25, 7], using equivalent kernels. The variance of the estimator is directly given by the estimated noise variance, obtained by model selection. Quantity that are equivalent to the bias can be computed in the Bayesian context of GPR, such as the risk, or expected loss [25, 7.2]. This will be examined in further works.

From the point of view of linear smoothers, bias and variance are defined but are hard to compute. General expressions can be written for linear smoothers [8, 3.4.2,3.8], but approximations are not available in our particular case, up to our knowledge.

[2] As discussed in [25, 2.7], it is not mandatory to choose zero mean .

4 System and data

A turbofan whose structure is presented by Fig. 2 is considered. Air from the outside enters an intake, then is successively compressed by the low-pressure (LP) and high-pressure (HP) compressors. Compressed air passes to a combustion chamber, where it is mixed with fuel and burnt. Both compressors are powered by turbines located at the rear of the engine, which transmit their energy to the compressors through two contra-rotating shafts, the low-pressure (LP) shaft and the high-pressure (HP) shaft. Although turbofan condition monitoring can be achieved in various ways, we take the stand to focus on vibrations monitoring in this work. Two accelerometers provide vibration measurements at a constant 51 kHz frequency. Since compressors and turbines are fan-like components made of a varying number of blades mounted on the shafts, it is expected that their motion entails vibrations at frequencies which are multiples of shaft speeds.

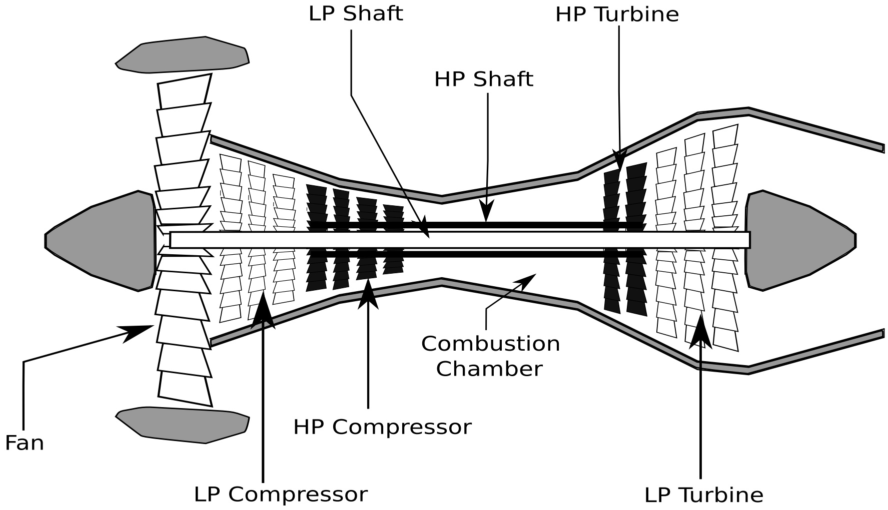
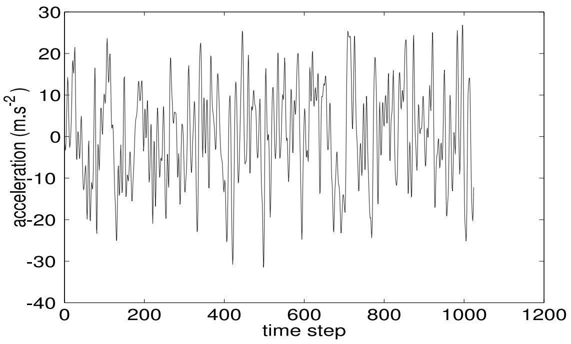
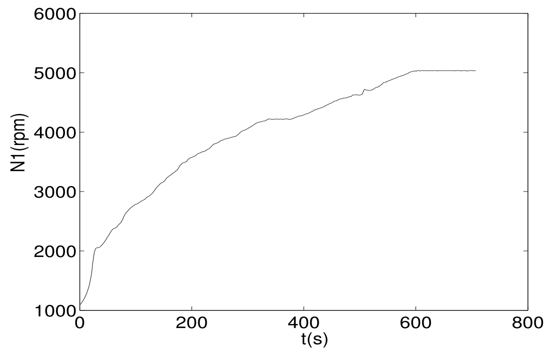


Figure 2: Turbofan engine. Simplified diagram of fan, low-pressure and high-pressure compressors and turbines attached to their respective shafts.

The recordings under study were provided by the Health Monitoring Department of SNECMA[3] and correspond to a dual-shaft turbofan mounted on a testbench, that undergoes a continuous acceleration during several minutes. They include raw vibration outputs of an accelerometer sampled at 51kHz, as well as LP and HP shaft angular velocity computed from raw keyphasor data and sampled at 6.25 Hz. Sample time series are plotted in Fig. 3. Four recordings are currently used in this study.



(a)



(b)

Figure 3: (a) Raw accelerometric data; (b) LP shaft angular velocity in rpm.

[3]http://www.snecma.fr

In order to apply cyclic averaging -so that the signal is composed of an integer number of periods- we resample the vibration measurements [23, 6] with 1024 samples per period.

Four subsamples whith length $n = 8192$ are extracted from the whole signal vectors for regime values close to 2000 rpm . Because of the relative slow increase of the regime, when can assume stationarity of vibration signals.

5 Results

The main result of this section is the fact that the model selection step and the Gaussian Process Regression were succesfully computed in reasonable time, given the fact that the dimension of the data is $n = 4 \times 8192$, which is very large for usual GPR problems as explained in Section 3.2.

We suppose that the parameters K and σ of the Gaussian Process have been adequately learnt from the data thanks to model selection (see Appendix C).

Then we compute the mean $\mathbf{f}_* | X_*, \mathbf{y}, X$ and the covariance of the FRF estimator, that allow to plot confidence interval. Fig. 4(top) shows a detail of both, on a limited order domain. Fig. 4(bottom) also shows the four individual FRF that were used for regression.

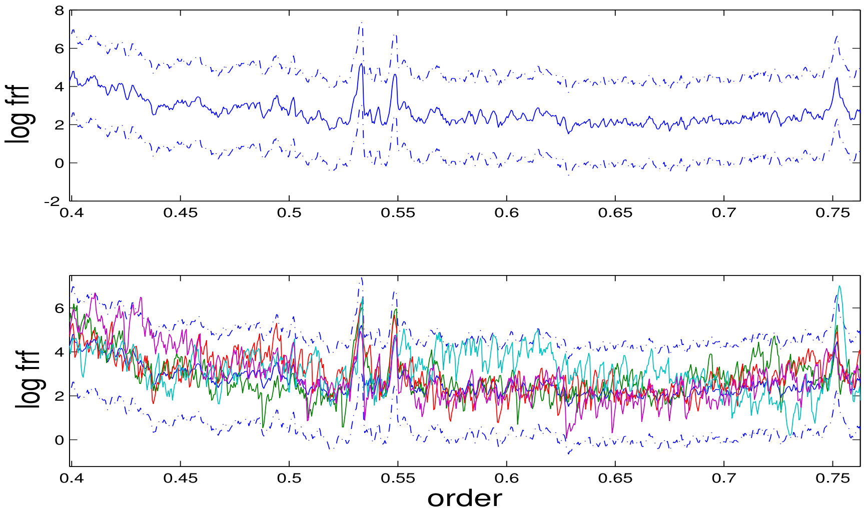


Figure 4: FRF in the order domain (top) predicted mean and associated confidence interval (bottom) superimposed FRF of all engines.

We notice that the FRF is almost everywhere included in the confidence interval, which is consistent with the fact that the four engines are labelled as normal by the expert.

6 Conclusions and perspectives

In this article we have raised the issue of using FRF to perform nonparametric Damage Detection in aircraft engine with vibration data. We have presented classical tools and more recent ones, and confronted their theoretical advantages and drawbacks. Preliminar results are available: we show that Gaussian Process Regression can be used to estimate the mean spectrum and associated confidence intervals, in spite of the size of the sample ($n = 4 \times 8192$). Thorough examination on experimental grounds is necessary, specifically concerning the estimation of the bias and variance of the two compared estimators.

Future works will concern mainly:

- the evaluation of the capacity of these tools to perform damage detection. Though this is our aim, the current article has focused mainly on estimation tools. We expect to meet specific difficulties because of the scarceness of fault data, related to the very high reliability of aircraft engines. This was addressed for example by [5].
- the assessment of the computed FRF. Indeed there are many facts that could be used to check the validity of the FRF, such as the knowledge of the geometry of the compressor, the number of blades, etc…
- the validity of hypotheses given the available data: for example it is assumed that the FFT of the noise contribution is circular complex normal. This should be tested, following [26]. The uniformity of the data, i.e. the fact that all vibration recordings belong to healthy engines, at the same regime, should be verified. Reciprocally, the presence of outliers could be automatically discovered. This is related to model selection, and to the problem of “multiple task learning”.
- the comparison between smoothers: the analytical expression of bias and variance of the ML estimator are available, but hard to estimate from data. The results given by nonparametric techniques such as bootstrap must be used to compare the empirical variance of ML and GPR estimators of the mean FRF. Other aspects such as the computational load and the convergence speed of estimators might be of interest, if easily available.
- the evaluation of other smoother: one main drawback of GPR is the fact that observation noise variance does not depend on $\mathbf{x}$. Some authors have extended GPR to heteroskedastic noise (see [25, 9.3]). Linear smoothers can also cope with the dependence to $\mathbf{x}$ (see “weighted smoothers”, [8, 3.2]).
- the relation with system identification in the frequency domain: non parametric FRF estimation is meant to be a mere subroutine of system identification? The quality of estimates determines that of system identification. Can we measure the advantage of linear smoothing with respect to the system identification that occurs after FRF estimation ?
- various generalisations:
  1. so far accelerometric data have been resampled with respect to LP shaft.
  2. can the estimation be extended to MIMO systems ?
  3. can outlier frequencies be detected and discarded ?
  4. what is the influence of nonlinear distortions on FRF estimates ?
  5. can random excitation be taken into account, to take into account the combustion noise ?

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A Noise contribution in the frequency domain

Following [3] we admit that the r-component vector $\mathbf{X}$ is circular complex normal with mean μ_X and covariance Σ_{XX} if the vector $[Re(\mathbf{X}); Im(\mathbf{X})]$ is distributed as:

$$\mathcal{N}\left(\begin{bmatrix} Re(\mu_X) \\ Im(\mu_X) \end{bmatrix}, \frac{1}{2}\begin{bmatrix} Re(\Sigma_{XX}) & -Im(\Sigma_{XX}) \\ Im(\Sigma_{XX}) & Re(\Sigma_{XX}) \end{bmatrix}\right)$$

It is shown (see [3, 4.4], [20, 14.16]) that the Fourier coefficients of a stationary time series with finite moments converge asymptotically to a circular complex normal law whith independent components. In [26] the quality of the normal approximation for small samples is assessed.

B Bias and variance of FRF estimators

The following Taylor series converges provided that $|N_U(k)/U_0(k)| < 1$, i.e. if the Signal to Noise Ratio (SNR) is large enough. Then it follows :

$$G(j\omega_k) = \frac{Y(k)}{U(k)} \tag{17}$$

$$= G_0(j\omega_k)\frac{1 + N_y(k)/Y_0(k)}{1 + N_U(k)/U_0(k)} \tag{18}$$

$$\sim G_0(j\omega_k)\Big(1 + \frac{N_y(k)}{Y_0(k)}\Big)\Big(1 + \frac{N_U(k)}{U_0(k)}\Big) \tag{19}$$

When cyclic averaging is added, the following expressions are needed:

$$\hat{\sigma}^2_U(k) = \frac{1}{M-1}\sum_{l=1}^{M} |U^{[l]}(k) - \hat{U}(k)| \tag{20}$$

$$\hat{\sigma}^2_Y(k) = \frac{1}{M-1}\sum_{l=1}^{M} |Y^{[l]}(k) - \hat{Y}(k)| \tag{21}$$

$$\hat{\sigma}^2_{YU}(k) = \frac{1}{M-1}\sum_{l=1}^{M} (Y^{[l]}(k) - \hat{Y}(k))\overline{(U^{[l]}(k) - \hat{U}(k))} \tag{22}$$

C Model selection for GPR

Model selection is aimed at estimating the hyperparameters of the covariance function which itself determines the Gram matrix K. The most popular covariance function is the squared exponential which expression is:

$$k(x_p, x_q) = \sigma_f^2 \exp\Big(-\frac{1}{2l^2}(x_p - x_q)^2\Big) \tag{23}$$

where σ_f is the signal energy and l the characteristic length-scale of the signal. Let X and $X*$ be vectors of points where k is evaluated :

$$K(X, X*) = (k(x_i, x_j*))_{1\le i\le m, 1\le j\le n} \tag{24}$$

The adequation between the data and the model is quantified by the marginal likelihood whose expression is:

$$\log p(\mathbf{y}|X) \quad = \quad -\frac{1}{2}\mathbf{y}^T(K+\sigma_n^2 I)^{-1}\mathbf{y} - \frac{1}{2}\log|K+\sigma_n^2 I| - \frac{n}{2}\log(2\pi) \tag{25}$$

where σ^n is the noise level.

Given the data sample $\{(\mathbf{x}_i, y_i)\}_{i\in[1,n]}$, the marginal likelihood is numerically optimized with respect to hyperparameters (σ_n, σ_f, l), thanks to appropriate algorithms such as the conjugate gradient [25, 5].