An Empirical Taxonomy of Start-Up Firms Growth Trajectories

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Introduction

Over the past decades, new and small firm growth has received considerable attention from researchers and policy-makers around the world. New firms have been identified as engines of growth, innovation and wealth creation (Audretsch and Thurik, 2000; Birch, 1981; Davidsson, 1995; Davidsson Lindmark and Olofsson, 1998; Levie, 1997; OECD, 1994, 1998, 2002; Storey, 1994; Welbourne, 1997). Indeed empirical evidence indicates that only a small proportion of firms accounts for a significant percentage of new job creation. Those firms often accelerate the development of new technologies and products that play a fundamental role in the prosperity of many countries (Birch, Haggetty and Parsons, 1997; Julien et al., 2001; Storey, 1997). New firms are therefore a key element in regional economic development and represent as such an interesting research subject.

Despite their importance to regional development, knowledge about new firm growth is still scattered (Davidsson and Wiklund, 2000; Delmar, 1997) and little knowledge is available regarding how firms grow and perform over time (Geroski, 2001). This can be partly attributed to methodological problems, such as the difficulties experienced in identifying entrepreneurial firms. For example, to this concern Gibb and Davies (1990) argued that it is illusory to think that it would be possible to detect this kind of firm or to produce a complete ideal model. Other international studies concluded that a "typical" high growth firm does not exist (OECD, 2000; Delmar et. al., 2003).

From a theoretical perspective, scholars have shown that research has largely failed to generate cumulative results regarding new firm growth because there is a great variability in researchers' use of growth conceptualization and operationalization (Davidsson and Wiklund, 2000; Delmar 1997; Delmar et al, 2003; Murphy et al, 1996 Chandler and Hanks, 1993; Weinzimmer et al, 1998; Wiklund and Shepherd, 2005). Indeed from a theoretical perspective the phenomenon of entrepreneurial growth has been mostly studied within individual academic disciplines, which does not encourage an integrated and systemic analysis (Garnsey and Heffernan, 2005). Research was done from different theoretical imperatives such as industrial organization, resource-based view, strategic adaptation and evolutionary economic perspective. Research from each discipline tends therefore to ignore important findings from competing schools. For example, since the original "theory of the growth of the firm" from Penrose (1959), where managerial resources played a pivotal role, diverse factors have been suggested as affecting growth. Some of them, such as environmental carrying capacity or market forces, are external to the organization (Aldrich, 1990; Singh and Lumsden, 1990). Others are internal, like capabilities, culture or strategy and have been mainly addressed from the resource-based view of the firm (Wernerfelt, 1984; Teece et al., 1997; Boeker, 1997; Zahra et al., 2000; Canals, 2000).

From an empirical perspective, growth is a multi-faceted phenomenon, but this heterogeneous nature is often neglected by scholars. Despite the diversity of approaches in terms of indicators, formulae and time spans used to measure growth (Delmar, 1997), empirical research has also largely failed to generate cumulative results (Davidsson and Wiklund, 2000; Delmar, 1997; Delmar et al, 2003; Weinzimmer et al, 1998). The common explanation is the absence of consensus regarding which firm growth indicators should be used (Weinzimmer et al, 1998; Delmar, 1997; Murphy et al, 1996; Chandler and Hanks, 1993). Another explanation is that researchers often measure growth along a single dimension (Weinzimmer et. al., 1998) although this approach has been widely criticized as firm growth is heterogeneous in nature (Birley and Westhed, 1990; Delmar and Davidsson, 1998; Delmar et al, 2003). As a consequence, using a single measure of growth defined by a single criterion actually investigates only one particular kind of growth and the results are unlikely to be applicable to other forms of growth (Delmar and Davidsson, 1998).

Finally, most studies about growth tend to focus on specific sectors (for example high-tech) although the economic contribution of new firm growth appears to be spread across various sectors (Delmar et al, 2003). Indeed most papers looking at "promising firms" have focused on samples limited to new technology based firms, from sectors such as software products, telecommunications or biotechnology (Baldwin, Chandler

and Papailiadis, 1994; Vyakarnam, Jacobs and Handelberg, 1997; Woywod and Lessat, 2001; Calvo and Lorenzo, 2001; Julien, 2001).

In order to address those theoretical and empirical issues, recent entrepreneurship research argues that there is a strong need for a conceptual scheme and for longitudinal growth studies (Busenitz et al, 2003; Chandler and Lyon, 2001; Davidsson and Wiklund, 2000; Delmar et al, 2003; Garnsey et al., 2006; Pettigrew et al., 2001). The underlying assumption is that growth is a heterogeneous phenomenon that naturally happens over time; it should therefore be analyzed in a dynamic process perspective and across multiple organizational contexts. Indeed, while most new and small firm growth studies have focused on the explanation of the performance using cross-sectional data and/or have assumed that growth is an uninterrupted process, longitudinal approaches have shown that regular growth is the exception rather than the rule (Delmar et al., 2003; Garnsey et al., 2006; Garnsey & Heffernan, 2005; McMachon, 2001; OECD, 2002; Stam, 2003).

However longitudinal approaches generate methodological challenges which require new research methods (Huber and Van de Ven, 1995; Poole et al, 2000; Van de Ven, 1992), which in particular involve more than static comparisons between initial and end states (Davidsson, 2004; Davidsson and Wiklund, 2000). Based on their respective dissertations and works in the field, Dadvidsson, Delmar and Wiklund (2006, p.5) argue that "firm growth is a complex phenomenon. It is not uni-dimensional. It is hard to predict and assess. Further, it can manifest itself in various ways, and consequently it can have differential effects on several different levels".

In this context, the purpose of this research is to present an original method that can accommodate, in a systematic way, the longitudinal analysis of new firm growth trajectories based on a multidimensional construct of growth. More specifically, our objective is to answer the following research question : is the early growth of a firm a process essentially idiosyncratic i.e. related to the individual characteristics of each firm or do typical growth trajectories that are adopted by a majority of firms exist? We analyzed the initial growth trajectories of 741 Belgian firms created between 1992 and 2002 which have grown above micro-firm size during that period. We developed and tested an original methodology allowing an empirical taxonomy of early growth.

In the following sections we detail our research design and introduce the experimental setting. We then document the empirical results of this research and discuss major findings and limitations.

Research design

To deal with the challenges generated by the analysis of the complex nature of new firm growth, we have elaborated a research design at the crossroad of entrepreneurship research and applied mathematics (Lévesque, 2004). More specifically, at a conceptual level, our research design integrates multidimensional and dynamic approaches of growth across sectors in order to conceptualize the early growth processes of firms. The key aspects of this research design are addressed hereafter.

Multidimensional approach and ubiquity

When considering the measurement of growth, there is as discussed in the introduction no consensus regarding which and how many indicators should be used. Moreover, the majority of researchers do not justify theoretically their choice of variables, although those choices can have consequences on the results (Delmar, 1997; Janssen, 2005). We therefore choose for this research design to jointly use multiple indicators, based upon commonly used measures of firm economics and financials, and let the empirical data show which indicators are the most meaningful. Let us stress that those "tangible" measures of growth do not allow to distinguish organic from acquisition-driven growth and tend to only indirectly reflect the "intangible" aspects of growth, such as the intellectual capital, the culture or the strategy of a firm, which might play an important role but cannot be taken directly into consideration in the context of our research design.

Moreover we adopt the assumption that firm growth is an ubiquitous phenomenon. In other words, we will not limit ourselves a priori to specific sectors, on the basis of the assumption that firms develop in various manners whatever their sector. We believe that the relevance of the sector dimension should be checked empirically a posteriori and not preconceived a priori.

Growth trajectory

As discussed above, growth is a process of change that needs to be studied over time (Davidsson, Delmar and Wiklund, 2006 : 40). Indeed, Penrose (1995) saw growth as a

cumulative process in which firm members build knowledge and competence. According to this author, firms are "[...] a result of development process [...] in which interacting series of internal changes lead to increase in size accompanied by changes in the characteristics of the growing object" (Penrose, 1995:1).

In the entrepreneurship context, recent authors emphasized the underlying assumption of the growth process, suggesting that firm growth is driven by a "productive opportunity" (Penrose, 1995) in a cumulative process of interaction between the firm's productive base and its market opportunities (Garnsey, 1995; Garnsey et al, 2006). Thus, process studies of firms should examine interconnected causes, outcomes and further feedback effects (Van de Ven, 1992). We need longitudinal research because it mainly allows direct observation of such change, causal statements, temporal context and feedback effects (Davidsson and Wiklund, 2000; Chandler and Lyon, 2001; Pettigrew et al., 2001). As a consequence, our research design will focus on taking into account successive measures of the growth process as the firm evolves over time, rather than only considering the initial and end states.

Empirical Taxonomy

As emphasized by most scholars in organizational studies (Archibugi, 2001; de Jong and Marsili, 2006; Ketcher and Shook, 1996; Pavitt, 1998; Rich, 1992), a useful empirical taxonomy can reduce the complexity of empirical phenomena to a few constructs. Thus, a widely accepted and usable taxonomy is a fundamental element in the development of a scientific body of knowledge (Sabherwal and Robey, 1995) and can serve as an empirically based framework to theory development. Indeed, previous researches suggest that, contrary to a typology considered as an individual creativity invention, taxonomy is an empirical classification tool for building the complex filing systems that allow both the ordering and retrieval of large amounts of data (Mckelvey, 1975; Pugh, Hickson and Hinings, 1969). Moreover, according to Rich (1992), a taxonomy is more than a simple classification of items into separate groups. It is a specific classification scheme that expresses the overall similarity between organisms in a hierarchical fashion. In addition, in their innovative small firm research, de Jong and Marsili (2006) emphasized that taxonomy classifies and labels many different items into groups or clusters that share common traits.

We therefore adopted in this research a taxonomy approach with the objective to attempt to reduce the complexity and therefore to improve our understanding of early firm growth. Hence our research design consists in mobilizing advanced applied mathematics tools in order to develop an empirical taxonomy of firm trajectories based on the initial evolution across sectors and over time of multiple economic and financial indicators.

Experimental setting

Having specified our research design, we will present briefly in this section the methodology adopted to identify the typical growth trajectories, with regard to the choice of the sample and variables and to the methods of analysis adopted.

Choice of sample and variables

A valuable opportunity to address the key study issue in this chapter has been provided by the availability of the BEL-FIRST database developed by the Bureau van Dijk Electronic Publishing (BvD), one of Europe's leading electronic publishers of business information. Our research collected longitudinal financial information and demographic indicators about all Belgian firms. This study focusing on the initial growth of the firm, the population considered here includes all the firms created after 1992 and still in existence in 2002 (N = 152.064).

Among these young firms, we selected all those which, since their creation, exceeded the stage of micro-firm level (as defined by the European Commission, 2003). This enables us to identify firms whose growth can be regarded as "promising" in the broadest sense, i.e. which can be considered as having contributed somewhat to economic development. This allows us to build a sample that goes beyond exceptional cases of very high and regular growth, often publicized but not at all representative of a "typical" growth firm. Moreover, in order to eliminate most "false creation" cases (such as a firm created through the incorporation of an existing subsidiary), we eliminated firms that had already exceeded the size of a micro-firm at the time of their creation.

Regarding the choice of variables, we selected as our main measures of growth three economic indicators: Sales, Employment and Total Assets, which have all been considered as suitable indicators of growth (see Davidsson and Wiklund, 2000). Those indicators are combined with seven financial variables traditionally linked to firm performance (Value added, Operating income, Current income, Net income, Cash-flow, Working capital and Shareholders' equity). In line with previous researchers (Davidsson

and Wiklund, 2000; Birley and Westhead, 1990; Weinzimmer et al., 1998; Wiklund, 1999), we think that the combination of multiple size and financial indicators provides richer information and therefore allows better investigating the growth process. Finally our data also included information regarding each firm's main sector of activity, type of ownership and legal form. On the basis of this choice of variables, the firms for which available data were complete and coherent or could be reconstituted by simple interpolation were selected.

Methods of analysis

The method used to analyze the existence of typical growth trajectories consists in considering a firm growth trajectory as a sequence of states (corresponding to the successive years of existence) in a space with 10 dimensions (corresponding to the three economic and seven financial indicators). In this space, mathematical tools for classification and discrimination such as a principal components analysis¹ (PCA) and empirical clustering based on density estimation (Cuevas, 2001) can be mobilized (see François et al, 2004 for other examples of tools for classification). These tools make it possible to identify and validate through density distribution potential clusters, each cluster corresponding to firms in similar stages of development. Once these clusters are identified and tagged ("stage A", "stage B", etc...), the trajectory of a firm can be described as a sequence or Markov chains corresponding to the various stages it experienced successively. For example AAABABB represents seven years of the trajectory of a firm evolving between states close to the clusters "stage A" and "stage B".

The firms having adopted similar growth trajectories will be characterized by similar sequences. Those sequences can be compared through graphical interpretation and Markov chain analysis (Bakerman and Gottman, 1986; Howard, 1971) and then analyzed through a systematic sequence analysis (Poole et. al., 2003), in order to evaluate the heterogeneity of growth trajectories and test the existence, validity and characteristics of typical trajectories.

¹ PCA is basically a projection method (Lawley and Maxwell, 1997) which can be used to develop cluster-based taxonomies (Evrard et al., 2003; de Jong and Marsili, 2006).

Empirical results

We will present our results in four sections. The first section relates to the sampling results and to the validation of the use of multiple indicators and sectors. The second section presents the result of the clustering of the successive growth states into four "stages". The following two sections explore the heterogeneity of growth trajectories and the existence and characteristics of typical trajectories through first a graphical comparison and then a systematic analysis.

Sampling results

As discussed above, our research is based on a sample including all Belgian firms created since 1992 and still in existence in 2002 (n = 152.064). From theses firms, we selected those which grew above micro-firm size anytime during that period. There were 17,168 such firms identified in our sample. Those "promising" firms represented 6% of all the existing Belgian firms in 2002 and 11% of the firms created since 1992 and still in existence in 2002. However, they generated (in 2002) respectively 19% and 80% of overall gross job creation.

Among the firms selected, 33% had missing values regarding Employment, 53% regarding Sales and 17% regarding Cash flow. We excluded the firms which did not publish complete data for more than two consecutive years, or firms which published less than 40% of the data available for two consecutive years. Using those filters, the final data set included 741 firms.

Most sectors were represented in our sample (13 out of the 17 principal sectors included in the standard NACE industry classification). The majority related to service industries (71%), while only 11% were related to Manufacturing and another 17% to the Construction industry. Only 19% of the firms in our sample were high-tech firms². This confirms the relevance of our cross-sector approach.

Finally, the 741 firms in our sample were sorted according to the measure of size (employment, sales or total assets) along which they had grown above the micro-firm size threshold. 50% of the firm in our sample had reached only the micro-firm

² To identify the firms related to "high tech" sectors, we defined a conversion table between the Code NACE-BEL and the Code US SIC for the classification of industry sectors according to technological intensity (from the Bureau of the Census and Walcott (2001)). We then validated our approach by matching this with other existing classification approaches (OECD, 1998) of industrial sector classification and of some new innovative firm classification indicators in the literature.

employment threshold (more than 9 employees) while respectively 33% and 8% had reached only the sales and the asset micro-firm thresholds. Less than 10% had reached more than one threshold and 2% had reached all three thresholds (they simultaneously had more than 9 employees, more than one million euro in sales and more than two million euros in assets). This confirms the relevance of our use of multiple indicators to measure growth (Delmar et al, 2003; Janssen 2005; St-Pierre et al, 2005), as using different indicators leads to different selections of "promising" firms.

Clustering

In order to explore the growth trajectories of the firm in our sample, we first tested the existence of clusters among the various states a firm experiences as it initially grows. We used for this purpose a Principal Component Analysis based on the successive absolute value of our three economic and seven financial indicators for each firm. This analysis produced three principal axes, with a cumulated variance of 82% (the two first axes accounted for 72%). Other choices of variables were tested, in particular using relative rather than absolute financial values (ratios). Nevertheless, all these alternative choices proved less relevant in terms of restitution of information, i.e. generating a weaker cumulated variance.

The three principal axes identified enabled us to represent (through a linear projection) all the successive states of the firms in a space with three dimensions. These axes represent composite variables which can be regarded as "latent dimensions" of the problem (Evrard et al., 2003), making it possible to apprehend fundamental dimensions of the studied phenomenon. The three axes are detailed in Table 1 below, according to their correlation³ with the 10 starting variables after a Varimax⁴ rotation.

³ Our initial variables being standardized, the coefficient of correlation is a good indicator of the relation between a variable and a principal axis (Evrard and Al, 2003).

⁴ This algorithm of rotation is based on the maximization of the coefficients of correlation of the most correlated variables (Hendrickson and White, 1964; Kaiser 1958).

Axis	Empl.	Sales	Value added	Oper. inc.	Current income	Net income	Work. capital	Cash flow	Shar. equity	Total assets
1. Perf.	0.12	0.03	-0.06	-0.47	-0.49	-0.51	-0.12	-0.44	-0.23	0.05
2. Size	0.42	0.47	0.45	0.03	-0.01	-0.11	0.26	0.03	0.027	0.50
3. Res.	0.33	0.12	0.29	0.14	0.07	0.05	-0.71	0.20	-0.47	-0.04

Table. 1: Results of PCA analysis

Those results indicate that the first axis is more correlated with four financial variables (Operating income, Current income, Net income and Cash flow) which can be linked to the "Performance" of a firm. The second axis is more correlated with four indicators (Employment, Sales, Value added and Total assets) which can be linked to the "Size" of a firm. Finally, the third axis is more correlated to the variables Working capital and Shareholder equity, which can be linked to the "Resources" of a firm.

Hence the results of the clustering allowed us to identify three principal axes which can be used to extract significant information relative to the evolution of each firm over time, across our sample. Let us stress that this reduction from the ten initial variables to three axis does not remove the multidimensional aspect of our approach, as those axis have been identified empirically (rather than defined a priori) and reflect the fundamental dimensions of our initial data set, i.e. the most relevant measures of how individual firm trajectories differ.

Furthermore, by the design of our sample all the firms considered will move from relatively low initial values to relatively high values of sales, employment and/or assets, as they grow above micro-firm size. The informational value of the "Size" axis is therefore limited from a clustering point of view. From a modeling point of view, this "Size" axis could actually be considered more as the dependent variable of this study.

We therefore focused our clustering analysis on the "Performance" and "Resources" axes. We can represent the successive states any firm in our sample goes through as it grows along those two axes, projecting their initial value along the 10 indicators on the two principal axis identified and taking as a reference point the average value of the sample (Figure 1).

However those axes are only numerical constructs produced by the PCA, which can only approximately be related to actual dimensions of the firms and have no direct managerial interpretation for a given firm. In order to test whether the two selected principal axes could be used in order to define meaningful clusters (i.e. whether they relate to actual differences between actual firms from a managerial point of view), we measured the density distribution of all 10 economic and financial variables and of four commonly used financial ratios (return on equity, return on asset, capital productivity and labor productivity) along the two axis.



Figure. 1: Two-dimensional representation of firm successive states

As an illustration, the density distributions of the variable Operating income within the four quadrants are presented in Figure 2, where the vertical scale represents the probability, the horizontal scale represents the value of the firm's operating income and each of the four lines represent the distribution within one of the four quadrants, as numbered in Figure 1. This figure indicates that there appears to be a cutting point (at around 30.000 Eur of annual operating income) between the distribution curves related to the quadrants 1 and 2 (corresponding to firms with below average "performance") and the distribution curves related to the quadrants 3 and 4 (corresponding to firms with above average "performance").



Figure 2: Density distribution of operating income along the principal axes

The analysis presented in Figure 2 indicates that the firms which achieve a higher value along the "performance" principal axis are indeed different from a managerial point of view, as their operating income will be significantly higher.

The application of this process to the 10 economic and financial variables and to the four financial ratios indicate that the "performance" principal axis is significantly related to operating income, current income, net income, cash flow and labor productivity while the "resources" principal axis is significantly related only to shareholder equity. Combining those two axes and their managerial interpretation allows us therefore to identify four different stages a new firm can reach as it grows:

- "Questions" are firms located at the bottom-left of Figure 1. They tend to combine lower than average operating income, current income, net income, cash flow and labor productivity (low performance) with lower than average shareholder equity (low resource). Their future development might at first sight seem at risk.
- 2. "Seeds" are firms located at the top-left of Figure 1. They tend to have lower than average operating income, current income, net income, cash flow and labor productivity (low performance) but higher than average shareholder equity (high resource). A firm associated to this profile could for example have been able to raise relatively important funds to ensure its initial development but needs time to improve its performance.
- 3. "Boutiques" are firms located at the bottom-right of Figure 1. They tend to have higher than average operating income, current income, net income, cash flow and

labor productivity (high performance) but lower than average shareholder equity (low resource).

4. "Stars" are firms located at the top-right of Figure 1. They tend to combine higher than average operating income, current income, net income, cash flow and labor productivity (high performance) with higher than average shareholder equity (high resource). Their future development seems a priori promising.

Hence the PCA and the density analysis have allowed us to identify two axes along which four stages of growth can be identified which are both empirically valid (as indicated by the PCA) and correspond to actual managerial dimensions (as indicated by the density analysis); We will discuss in the next two sections how those two axes and four stages can be exploited from a graphical and systematic point of view in order to test the heterogeneity of the growth trajectories of young firms, and ultimately build a taxonomy.

Graphical analysis

Several representations of firm trajectories using the two principal axes we identified above are presented in Figure 3, where each box represents the successive states adopted by a given firm along the two axes using a normalized scale. In this figure, a selection of trajectories of similar shape have been gathered together. The first group (the top six boxes) represents rather linear trajectories, from high resources/low performance to low resources/high performance states. The second group gathers sigma-shaped trajectories. They illustrate that the growth of those firms has not been smooth over the years, with some periods that may even correspond to decay. The third group (bottom line) presents angular trajectories going up first (increase in resources) then bifurcating to the left (increase in performance).

Going further, the trajectory of a given firm can be characterized by the sequence of successive stages it goes through. Following a Markov chain approach, the way firms in our sample move from one stage to another (dependencies) can be represented through a digraph (Figure 4), which details the probability that a firm starts in a given stage (boxes) and move from one stage to another (arrows). Such a digraph renders visible how stages are sequenced over time (Bakerman and Gottman,1986).



Figure 3: Examples of two dimensional projections of 18 firm growth trajectories.



Figure 4: Growth trajectories digraph.

The results presented in Figure 4 indicate that the probability to remain within a given stage is quite high (p > 0.61 for all for stages). This behavior can be interpreted as firm inertia. Moreover, "magic recoveries" and "catastrophes", i.e. directly moving from low

levels of performance and resource ("questions" stage, n°1) to high levels ("stars" stage, n°4) and reciprocally is very rare (respectively p < 0.02 and p < 0.03).

Those graphical analyses both indicate that growth trajectories can be differentiated along a "performance" and a "resources" axis and that this type of representation gives evidence of some homogeneity between the trajectories of some of the firms. There appear to be at the same time a diversity of trajectories but also groups of trajectories that share similar shapes and that are worth investigating further, in a more systematic way. Indeed whether the trajectories such as the ones presented in Figure 3 amount to a taxonomy, i.e. whether we can identify a small number of trajectories that are adopted by a majority of firm cannot be tested through graphical analysis. Testing this hypothesis in a systematic way is the subject of the following section.

Systematic taxonomy

In order to explore the existence of typical growth trajectories based on the characterization of firm trajectories as sequences of successive stages, we first considered only the firms for which at least four years of data was available, and looked at their trajectories during those four initial years of existence. There were 602 firms in our sample (out of 741) for which such data was available. The initial trajectory of each of those 602 firms can be characterized by a four-long sequence of stages (four years) among the four possible stages we identified through our clustering ("seeds," "stars", "boutiques" and "questions" stages). This leads to 256 (4⁴) theoretical possibilities. The distribution of the sequences observed among the 602 firms is presented in Figure 5, where the observed combinations are listed on the horizontal axis and where the vertical axis represent the occurrence of each of those combinations. The shape of this distribution curve indicates that they are not at all uniformly distributed.



Figure 5: Distribution of growth trajectories observed.

A closer analysis of this curve shows that only 115 different sequences (not visible on the figure) have actually been observed, and that only 22 sequences have been adopted by more than 1% of the firms. Those 22 sequences collectively cover 71% of the firms and are presented in Table 2 hereafter.

	Sequences	Freq.	Cumul. Freq.
		(n = 602)	(n = 602)
1	S2222	22%	22%
2	S2111	5%	27%
3	S2221	5%	32%
4	S1111	5%	37%
5	S4444	5%	42%
6	S2211	3%	45%
7	S2224	3%	48%
8	S3333	3%	51%
9	S2223	2%	54%
10	S2244	2%	56%
11	S2444	2%	58%
12	S1311	2%	59%
13	S2113	1%	61%
14	S1333	1%	62%
15	S2242	1%	63%
16	S4222	1%	65%
17	S4333	1%	66%
18	S1131	1%	67%
19	S1133	1%	68%
20	S2122	1%	69%
21	S2233	1%	70%
22	S3111	1%	71%

Table. 2: Most frequent sequences

If one follows a Markov chain approach and focuses on transitions between stages rather than on the time spent within each stages (the sequences 4333, 4433 and 4443 are therefore regarded as equivalent), seven typical trajectories emerge from those 22 sequences. These trajectories are presented in Figure 6 below.



"Resource"

Figure 6: Typical trajectories

Those seven typical trajectories can be split between four "stable" ones (a firm remains in a given stage over the time period considered) and three "unstable" ones. The four stable and three unstable trajectories are described hereafter (Table 3), highlighting the corresponding sequences and presented by decreasing frequency.

Description		Corresp.	Freq.	Example
		sequ.	(n = 602)	
Stable trajectories				
В	Grow as a "seed" (high	S2222	22%	Firm having raised
	resources but low performance)			capital to reach break-
	-			even
D	Grow as a "star" (high resources	S4444	5%	"Gazelle"; potential
	and high performance)			high-growth firm
Α	Grow as a "question" (low	S1111	5%	Firm experiencing
	resources and low performance)			troublesome growth
С	Grow as a "boutique" (low	S3333	3%	Profitable service firm
	resources but high performance)			with limited assets

Table. 3	3: '	Taxonomy	of	growth	traject	ories
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Unstable trajectories						
E	Grow from a "seed" to "question"	S2111, S2211	13%	"Seed" who burns capital before break-		
	question	S2221		even		
G	Grow from a "seed" to "star"	S2444, S2244, S2224	7%	"Seed" having developed towards profitability		
F	Grow from a "seed" to "boutique"	S2333, S2233, S2223	4%	"Seed" having lowered ambitions		
То	tal		59%			

Those seven typical trajectories collectively include the first 11 and the 21st sequences of Table 2 and therefore correspond to 59% of the firms in our sample for which at least four years of data is available (602 firms).

While those seven trajectories, considered individually, might appear as quite natural development paths for firms to follow (in particular the three "unstable" ones), it is clearly a non-obvious result to have identified empirically that it is those seven trajectories (and not any other possible subset of the 24 theoretical combinations) which a majority of firms follow. Moreover, it provides an empirical validation of the prevalence of non obvious (atypical) trajectories, such as trajectories "A" and "C".

Hence this taxonomy shows a relatively great heterogeneity of growth trajectories both in terms of frequency and development paths. The detailed characterization of each one of these trajectories would require individual case studies which exceed the framework of this research. Indeed, no significant correlations were observed (through χ^2 - test) between the seven typical growth trajectories identified and individual measures of growth (sales, employment or assets) or between the growth trajectories and particular demographic data (sector, type of ownership or legal form).

The results presented above indicate that, based on empirical evidence, new firm growth trajectories are neither "linear" nor a random or idiosyncratic phenomenon, and that a taxonomy of seven typical trajectories corresponding to a majority of firms can be identified.

Discussion and conclusions

Growth has been extensively studied in the fields of entrepreneurship, strategic management and industrial organization. However most studies have concentrated on

the explanation of growth using cross-sectional data or case studies and have explicitly or implicitly assumed that growth is essentially an uninterrupted process. However, the longitudinal studies of growth suggest that regular (or linear) growth is the exception rather than the rule.

With the aim to contribute to a better understanding of the growth process of new firms, we have presented an original methodological approach allowing the systematic analysis of early growth trajectories based on a multidimensional construct of growth across sectors. This method made it possible to track systematically typical growth trajectories of firm having grown beyond micro-firm size and to select through a PCA two key independent dimensions of the problem leading to four clusters which were used to identify seven typical growth trajectories. Those seven trajectories were adopted by 59% of the new firms considered in our sample.

Hence our findings indicate that this original systematic approach is useful for taxonomy development and therefore contributes to reduce the gap between the complexity of new firm growth process and the standard approaches often used to deal with it. They also have several implications and limitations, that will be discussed hereafter.

Implications

Our results first confirm that organizational growth constitutes a multiform and crosssector phenomenon by nature, which should not therefore be reduced to a single dimension or studied within a single sector. Thus, the results of our analysis support a multidimensional conceptualization of growth, contrary to what many researches used, particularly in the studies on the determinants of the growth process (Birley and Westhead, 1990). Moreover, they reinforce the recent work emphasizing the heterogeneity of new firm growth (Delmar et al., 2003; François et al. 2004; Janssen, 2005; and Weinzimmer, 1998). They also raise the question of the relevance of the uniform quantitative approaches, focused only on the criterion of a high relative growth rate in sales and /or employment adopted by policymakers and venture capitalists to evaluate the potential of a new firm.

Moreover, our empirical results suggest that new firm growth appears to be neither a continuous (or life-cycle based) nor idiosyncratic (or completely random) process. It can be adequately described through a limited number of typical growth trajectories that can

be identified in a systematic way. This original contribution of our research has important theoretical and practical implications.

Our results also confirm that a cross-sectional approach can fail to capture the complex reality of the evolving new firm. They support the recent studies based on longitudinal approaches such as those of Delmar et al. (2003), Garnsey et al. (2006), Garnsey and Heffernan (2005), Mustar (2002), McMachon (2001) and Stam and Garnsey (2006).

Our findings are important from a theoretical perspective because their bring insight regarding how new firms evolve over time. They contribute to our understanding and appreciation of the heterogeneity of the growth trajectory phenomenon. Thus, researchers should develop more nuanced explanations of new firm growth process than the simple uninterrupted or "linear" dynamic growth process. Our findings indicate that nonlinearities exist in new firm growth trajectories and emphasize the rarity of very high growth trajectories.

Our findings are also important from a practical prospective as they confirm that there is no such thing as "a growth firm". Whether a given firm will be qualified as a "growth firm" is strongly related to the criteria used to measure growth and to the corresponding thresholds adopted. Researchers should align their definition of "growth firm" with the specific context and/or objective of their research, be it managerial performance, economic development or job creation. In particular, there is no universal criteria to determine whether firm A has grown more over time than firm B, for any pair of firms. Indeed what constitutes a meaningful measure of the size of a firm should be a function of the nature of the firm's activity (for example manufacturing versus trading) and of its governance structure (for example hierarchical and closed structure versus flat and open organizations).

In addition, this research provides a useful original taxonomy of new firm growth trajectories calling for explanations from case studies. Our taxonomy also extends previously developed taxonomies by taking into account the firm's financial characteristics as complementary information to its economics.

This study also provides a useful methodology contribution by showing the value added of the use of advanced applied mathematic methods to deal with the complex and dynamic nature of firm growth and therefore to contribute to theory development. We provided an original methodological approach based on systematic clustering and sequence analysis to derive an empirical taxonomy. In sum, our approach supports the recent work (Lévesque, 2004) emphasizing how mathematics can provide important contributions to current theories of management and organizations.

Limitations

This study has several limitations that should be noted. Firstly, in order to examine growth trajectories, we needed a significant number of longitudinal data which justified the use of retrospective secondary data limited to only one country. Moreover, the nature of these data does not enable us to measure the effect on the evolution of new firms of some important qualitative factors such as strategies, entrepreneurial motivation, partnerships and human capital.

Our research is also limited by the natural selection bias excluding failed firms in the sample. Therefore, we have no way of knowing what type of distinct growth trajectories such firms would exhibit, and how these growth paths might differ from the seven typical growth trajectories identified in this research.

Although these limitations are important and must be taken into account, we are nevertheless convinced that this study should contribute to a better understanding of new firm growth process.

Future research directions

This research opens many future research directions. Firstly, the replication in various contexts of the developed empirical taxonomy and specific data analysis methodology should be considered.

Secondly, while this research can be considered as a first step towards a better understanding of the start-up growth trajectories, further research is needed to improve our understanding of the dynamic growth process of new ventures. It should explore which endogenous and exogenous factors might explain why a majority of start-ups follow the seven identified typical growth trajectories. To this end, our retrospective approach and data analysis methodology should be complemented by other methods such as surveys and multiple case studies.

In this context, among other orientations, it could be highly relevant to refine our taxonomy by examining the relationship between innovative and technological sources and growth trajectories, both in high and low technological industries.

Finally, while our findings provide empirical and methodological support in new firm growth research, one of the most important future research directions is to test the accuracy of the proposed taxonomy in the stability of the firm dynamic growth process beyond the limited period of four years retained in this research.

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References

- Aldrich, H. (1990), 'Using an ecological perspective to study organizational founding dates', *Entrepreneurship Theory and Practice* Spring: 7-24.
- Archibugi, D. (2001), 'Pavitt's taxonomy sixteen years on: a review article', *Economic Innovation and New Technology*, **10**, 415–425.
- Audrestch, D.B. and A.R. Thurik (2000), 'Capitalism and democracy in the 21st Century: from the managed to the entrepreneurial economy', *Journal of Evolutionary Econmics* **10**, pp. 17-34.
- Bakerman, R., and Gottman, J. M. (1986), *Observing interaction: An introduction to sequential analysis*, Cambridge: Cambridge University Press.
- Baldwin, J.R., C. Chandler et T. Papailiadis (1994), 'Stratégies pour le succès. Le profil des PME en croissance au Canada', Ottawa, *Statistique Canada*, **61**-523ER.
- Barney, J., (1991). Firm resources and sustained competitive advantage. *Journal of Management*, 17, 99–119.
- Birch, D. (1981), 'Who Creates Jobs', The Public Interest 65 (Fall),3-14.
- Birch, D., A. Haggetty, and W. Parsons (1997), *Corporate Almanac*, Cambridge (Mass.), Cognetic Inc.
- Birley, S., and P.Westhead (1990), 'Growth and Performance Contrasts Between 'Types' of Small Firms', *Strategic Management Journal*, **2**: 535–557.

Boeker, W. (1997), 'Strategic change: The influence of managerial characteristics and organizational growth', *Academy of Management Journal*, **40** (1): 152-170.

- Busenitz, L.W., West, P., Shepherd, D., Nelson, T., Zacharakis, A., and G. Chandler, (2003), 'Entrepreneurship in emergence: Past trends and future directions', *Journal of Management*, **29** (3): 285-308.
- Calvo, J. L., and M. J. Lorenzo (2001), 'Une caractérisation des entreprises manufacturières espagnoles à forte croissance: 1994-1998', *Revue Internationale P.M.E.*, 14, n°s 3-4.
- Canals, J. (2000), Managing Corporate Growth, New York: Oxford University Press.
- Chandler, G. N., and D. W. Lyon (2001), 'Issues of Research Design and Construct Measurement in Entrepreneurship Research: the Past Decade', *Entrepreneurship Theory and Practice*, **25** (4), pp. 101-113.
- Chandler, G. N., and S. H. Hanks (1993), 'Measuring Performance of Emerging Businesses', *Journal of Business Venturing*, **8**: 32-40.
- Cuevas A., M. Febbero, R. Fraiman (2001), 'Cluster analysis: a further approach based on density estimation', *Computational statistics and data analysis*, **36** (4), 441-459.
- Davidsson, P. (1995), 'SMEs and job creation in Sweden', Communication présentée au séminaire OCDE sur "les PME: l'empoi, l'innovation et la croissance." Washington, D.C.
- Davidsson, P. (2004), 'The Domain of Entrepreneurship Research: Some Suggestions'.In Katz, J. and S. Shepherd, (eds) *Advances in Entrepreneurship, Firm Emergence and Growth* Elsevier: JAI Press, 6, 315-372.
- Davidsson, P., and J. Wiklund, (2000), 'Conceptual and empirical challenges in the study of firm growth', In Sexton, D. and H. Landström, (Eds.), *The Blackwell Handbook of Entrepreneurship*. Oxford, MA: Blackwell Business.
- Davidsson, P., F. Delmar and J. Wiklund (2006), *Entrepreneurship and the growth of firms*, Edward Elgar Publishing.
- Davidsson, P., L. Lindmark and C. Olofsson (1998). 'The extent of overestimation of small firm job creation - an empirical examination of the regression bias'. *Small Business Economics*, **11**, 87-100.
- De Jong, J.P.J. and O. Marsili (2006), 'The Fruit Flies of Innovation: A Taxonomy of Innovative Small Firms', *Research Policy*, **35**, 213-229.

- Delmar, F. (1997), 'Measuring growth: methodological considerations and empirical results', In: Donckels, R., A. Miettinen, (Eds.), *Entrepreneurship and SME Research: On its Way to the Next Millennium*, Aldershot, UK:Ashgate, pp. 199–216,
- Delmar, F., Davidsson, P. (1998), 'A taxonomy of high growth firms', in Reynolds,
 P.D., W.D. Bygrave, M., Carter, S., Manigart, C.M. Mason, G.D. Mayer, K.G. Shaver,
 (Eds), *Frontiers of Entrepreneurship Research*, Center for Entrepreneurial Studies,
 Babson College, Wellesley, MA, pp. 399-413.
- Delmar, F., P. Davidsson (1998). A Taxonomy of High-Growth Firms. *Frontiers of Entrepreneurship Research*. http://www.babson.edu/entrep/fer/papers98/XIV/XIV_A

Delmar, F., P. Davidsson, W. Gartner (2003), 'Arriving at the high-growth firm'. *Journal of Business Venturing*, **18**, 189–216.

- Evrard, Y., B. Pras et E. Roux (2003), Market: *Etudes et Recherches en Marketing*. (3^{ème} Eds), Duno, Paris.
- François, D., M. Biga Diambeidou, B. Gailly, V. Wertz, M. Verleysen (2004), 'Observer des Trajectoires de 'Start-up' sur des Cartes de Kohonen', ACSEG2004 proceedings – Connectionist Approaches in Economics and Management Sciences Lille (France), 18-19 November 2004, 302-307.
- Garnsey, E. (1995), 'High technology renewal and the UK investment problem', Journal of General Management, **20** (4), 1-22.

Garnsey, E. and P. Heffernan (2005) 'Growth setbacks in new firms' *Futures*, **37**(7): 675-697.

- Garnsey, E., E. Stam, and P. Heffernan (2006), 'New Firm Growth: Exploring processes and paths', *Industry and Innovation* 13.1, 1-20.
- Geroski, P A, (2001), 'Exploring the Niche Overlaps between Organizational Ecology and Industrial Economics,' Industrial and Corporate Change, Oxford University Press, 10(2), 507-40.
- Gibb, A.A. and L.G. Davies, (1990), 'In pursuit of frameworks for the development of growth models of the small business', *International Small Business Journal* 9, 15-31.
- Hendrickson, A.E., P.O. White (1964), 'Promax: a quick method for rotation to oblique simple structure', *British Journal of Statistical Psychology*, **17**, 65-70.
- Howard, R. (1971), Dynamic probabilistic systems, vol. 2. New York: Wiley
- Huber G. and A. H. Van de Ven (1995), *Longitudinal field research methods: Studying processes of organisational change*, Organization Sciences, Sage publications.

- Hugo, O. and E. Garnsey (2005), 'Problem-solving and competence creation in the early development of new firms', *Managerial and Decision Economics* **26**, 139-148.
- Janssen F. (2005), 'La conceptualisation de la croissance: l'emploi et le chiffre d'affaires sont-ils des représentations interchangeables d'un même phénomène?', *Revue Gestion* 2000, 6, pp 267-291.
- Julien, P.-A. (2001), 'Les PME à forte croissance et la métaphore du jazz. Comment gérer l'improvisation de façon cohérente', *Revue Internationale P.M.E.* **14**, n°s 3-4.
- Julien, P.-A., P. Mustar, M.-F. Estimé (2001), 'Les PME à forte croissance: une comparaison internationale', *Revue Internationale P.M.E.*, vol. **14**, n°s 3-4.
- Kaiser H.F. (1958), 'The varimax criterion for analytic rotation in factor analysis', *Psychometrika* **23**, 187-200.
- Ketchen D. J., C. L. Shook (1996), 'The application of cluster analysis in strategic management research: An analysis and critique', *Strategic Management Journal*, 17 (6).
- Lawley P.A., A.E. Maxwell (1971), *Factor Analysis as a Statistical Method*, (2nd Ed.), American Elsevier Publishing and Co. New York.
- Lévesque M. (2004), 'Mathematics, theory, and entrepreneurship', *Journal of Business Venturing*, **19** (5), 743-765.
- Levie, J. (1997), 'Patterns of Growth and Performance: An Empirical Study of Young, Growing Ventures in France, Ireland and Scotland', In *Frontiers of Entrepreneurship Research 1997*, P. D. Reynolds, W. D. Bygrave, N. M. Carter, P. Davidsson, W. B. Gartner, C. M. Mason and P. P. McDougall (Eds), Babson College, Wellesley, MA, pp. 375–389.
- MCKelvey, B. (1975), 'Guidelines for the Empirical Classification of Organization', *Administrative Science Quarterly*, **20**, 509-525.
- McMahon, R. G. P. (2001), 'Deriving an Empirical Development Taxonomy for Manufacturing SMEs using Data from Australia's Business longitudinal survey', *Small Business Economics*, **17**, 197-212.
- Murphy, G.B., J.W. Trailer, R.C. Hill (1996), 'Measuring performance in entrepreneurship', *Journal of Business Research*, **36**, 15–23.
- Mustar, P. (2002), 'Diversité et unité des entreprises à forte croissance du secteur manufacturier en France', *Gestion 2000*, Dossier la PME à forte croissance **5** (2002), 77-98.
- OECD (1994), The OECD Jobs Study, Paris

OECD (1998), Technology, Productivity and Job Creation: Best Policy Practices, Paris. OECD (2000), High-Growth SMEs and Employment, Paris

- OECD (2002), High-Growth SMEs: Their Contribution to Employment and Their Characteristics, Paris
- Pavitt, K. (1998), 'Technologies, Products and Organization in the Innovating Firm: What Adam Smith Tells Us and Joseph Schumpeter Doesn't', *Industrial and Corporate Change*, **7**, 433-452.
- Penrose, E. (1959), *The Theory of the Growth of the Firm*, Oxford : Oxford University, Press.
- Penrose, E. (1995), *The Theory of the Growth of the Firm*, (3rd eds), Oxford: Oxford University Press.
- Pettigrew A.M., R.W. Woodman, K.S. Cameron (2001), 'Studying Organizational Change and Development: Challenges for Future Research', Academy of Management Journal, 44 (4), 697-713.
- Poole M. S., A. H. Van de Ven, K. Dooley, M. E. Holmes (2000), Organisational Change and Innovation Process: Theory and Methods for Research. (eds) Oxford University Press.
- Pugh, D. S., D. J. Hickson, C. R. Hinings (1969), 'An empirical taxonomy of structures of work organizations', *Administrative Science Quarterly*, 14, 115-126.
- Rich, P. (1992), 'The organization Taxonomy: Definition and Design', Academy of Management Review 17, 758-781.
- Roberts, E. B. and C. A. Berry, (1985), 'Entering New Businesses: Selecting Strategies for Success', *Sloan Management Review*, **26**, 3–17.
- Sabherwal, R. and D. Robey (1993), 'An empirical taxonomy of implementation processes based on sequences of events in information system development', *Organization Science*, **4**(4), 548-576.
- Singh, J., C. Lumsden (1990), 'Theory and research in organizational ecology', *Annual Review of Sociology*, **16**, 161-195.
- Stam, E. (2003), *Why Butterflies don't Leave. Locational Evolution of Evolving Enterprises. Utrecht:* Utrecht University (PhD thesis).
- Stam, E. and E. Garnsey (2006), 'New Firms Evolving in the Knowledge Economy; Problems and solutions around turning points', In: Dolfsma, W. and Soete, L. (eds.), Understanding the Dynamics of a Knowledge Economy, Cheltenham, Edward Elgar, pp. 102-128.

Storey, D. (1994), Understanding the small business sector. Londres, Routledge.

- Storey, D.J. (1997), *Understanding the Small Business* Sector, London: International Thomson Business Press.
- ST-PIERRE J., F. Janssen, P. A. Julien, C. Therrien (2005), 'Les facteurs de croissance des PME manufacturières sur les marchés locaux et internationaux', IXes Journées scientifiques de l'Agence universitaire de la Francophonie, Cluj Napoca, Roumanie, 2005.
- Teece, D. J., G. Pisano, A. Shuen (1997), 'Dynamic capabilities and strategic management', *Strategic Management Journal*, **18**(7), 509-533.
- Van de Ven A.H., D., Polley (1992), 'Learning while innovating', *Organisation Science*, 3(1), 92-116.
- Van de Ven, A. H., R. Engleman (2004), 'Event and outcome driven explanations of entrepreneurship', *Journal of Business Venturing*, 19, 343-358.
- Vyakarnam, S., R. C. Jacobs, J. Handelberg (1997). Formation and development of entrepreneurial teams in rapid growth businesses. *Frontiers of Entrepreneurship Research*. http://www.babson.edu/entrep/fer/papers97/sum97/vya.htm
- Weinzimmer, L.G., P.C. Nystron, S.J., Freeman (1998), 'Measuring organizational growth: issues, consequences and guidelines', *Journal of Management*, **24** (2), 235– 262.
- Welbourne, T.M. (1997), 'Valuing employees: a success strategy for fast growth firms and fast paced individuals'. In: Reynolds, P.D., W.D. Bygrave, P. Davidsson, W.B.Gartner, C.M. Mason, P.P. McDougall, (Eds.), *Frontiers of Entrepreneurship Research*. Center for Entrepreneurship Research, Babson Park, MA, pp. 17–31.
- Wernerfelt, B. (1984), 'A resource-based view of the firm', *Strategic Management Journal*, **5**, 171-181.
- Wiklund, J. and D. Shepherd (2005), 'Knowledge accumulation in growth studies : the consequences of methodological choice'. Paper prepared for ERIM workshop "Perspectives on the Longitudinal Analysis of New Firm Growth", Erasmus University Rotterdam, the Netherlands, 18-19 May.
- Wiklund, J., (1999), 'The sustainability of the entrepreneurial orientation-performance relationship', *Entrepreneurship Theory and Practice*, **24**, (1), 37-48.
- Woywod, M., V. Lessat (2001), 'Les facteurs de succès des entreprises à croissance rapide en Allemagne', *Revue Internationale P.M.E.*, **14**, n° s 3-4.

Zahra, S. A., R. D. Ireland and M. A. Hitt (2000), 'International expansion by new venture firms: International diversity, mode of market entry, technological learning, and performance', *Academy of Management Journal*, **43**(5), 925-950.