Many products and services are not sold on open platforms but on competing for-profit platforms, which charge buyers and sellers for access. What is the effect of for-profit intermediation on seller investment incentives? Since for-profit intermediaries reduce the available rents in the market, one might naively suspect that sellers have weaker investment incentives with competing for-profit platforms. However, we show that for-profit intermediation may lead to overinvestment when free access would lead to underinvestment because investment decisions affect the strength of indirect network effects and, thus, access prices. We characterize the effect of for-profit intermediation on investment incentives depending on the nature of the investment and on which side of the market singlehomes.

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1. Introduction

How does the market environment affect manufacturers’ investment incentives? It is well-known that, in general, manufacturers may underinvest in technology or marketing because they cannot fully appropriate the surplus that is generated when selling a product. However, little is known about the influence of market microstructure or trading environment on investment incentives. Addressing this issue is important, as we observe that most consumer products are not sold directly but via intermediaries. These intermediaries come in various forms. For example, retailers rent shelf space to producers; shopping mall developers rent stores to retail chains (or franchisees); trade fairs rent booths to exhibitors. In all these market environments, the prices for the goods to be traded are set by the “producers” and not by the intermediary. Similarly, Internet shopping sites list sellers on their platform. Intermediaries obtain revenues by charging for access to and usage of the platform. This is true not only for trading platforms, but also, for instance, for software platforms, which grant licenses to application software developers and charge users for access (by selling the respective operating systems).

In this paper, we analyze seller investment decision in such market environments. More precisely, we analyze the following question: How does for-profit intermediation affect manufacturers’ investment incentives? The central message

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of the paper is that a manufacturer's investment decision is substantially affected if intermediaries strategically set access prices to their platforms. We elaborate on this message in a particular setting that is motivated below.

With the rise of B2B and B2C commerce, the above question has become even more relevant. Intermediaries may become active in different ways. They may set bid and ask prices and, therefore, alleviate search inefficiencies, which arise, for example, under random matching. The presence of a dealer-intermediary can be seen as an implicit screening device between seller and buyer types (see, e.g., Gehrig, 1993; Spulber, 2003). In many markets, however, search inefficiencies may be so pronounced that buyers and sellers always trade via a platform. This is clearly the case if the platform provides part of a system that complements the product provided by the seller. A good example of this is the video-game industry (and other software industries), in which game developers write their applications for game platforms. In this case, a video-game platform aggregates demand and balances the two sides of the market through the use of price instruments (as in the literature on two-sided markets: see, e.g., Rochet and Tirole, 2003; Armstrong, 2006). In such a platform industry, we can abstract from any search efficiencies and, instead, focus on indirect network effects that arise due to group size.

We analyze how seller investment incentives are affected by the presence of competing for-profit platforms. To this end, we present a stylized model with two-sided indirect network effects on two competing platforms. Participants on both sides of the market choose which platform to visit; we contrast different scenarios according to whether buyers and/or sellers are allowed to trade via both platforms (i.e., to multihome) or are restricted to use a single platform (i.e., to singlehome). We capture size effects in the form of variety-seeking buyers who have a downward-sloping demand function for each available product. We may call trade taking place through for-profit intermediaries intermediated trade. These intermediaries set access or membership fees on both sides of the market. Conversely, in the absence of for-profit platforms that can restrict access and use of the platform, trade is non-intermediated or takes place via open trading platforms, which can be accessed without charge. As our benchmark, we choose a market in which buyers and sellers interact through two open platforms to which access is free of charge. While there are a number of real-world examples of open platforms (e.g., Linux as a software platform or PCs for PC-based video games), the main reason for doing so is conceptual: it uncovers the effect of strategic price setting by platforms on seller investment incentives. In a nutshell, to address the role of (imperfectly) competing intermediaries, we compare the seller investment incentives of two competing for-profit platforms with those of two open platforms.

Seller investments may, for example, take the form of cost reduction, quality improvement or marketing measures that facilitate price discrimination or expand demand. We model such investments as long-term variables that give commitment to the sellers—i.e., sellers make their investment decision before they know the opportunity cost of visiting each platform and before platforms set their prices. Take the video-game industry as an illustration. Software publishers reportedly invest in order to reduce their development costs and/or improve the quality of their games. For instance, in 2007, Ubisoft (one of the world's largest video game publishers) opened a new video-game development studio in Chengdu, China. The company chose Chengdu because it offers “long-term growth opportunities based on a talented and highly educated local population (with over 35,000 software programming graduates per year).” It seems reasonable to assume that investments of this kind are long-term decisions that software publishers make before knowing the exact “membership fee” that they will have to pay to the console manufacturer. This view is reinforced by the fact that console manufacturers regularly modify the price of their development kits.6

Why should the type of platform matter for seller investment incentives? Clearly, the presence of for-profit intermediaries reduces the rents that are available in the market. Therefore, one might naively suspect that sellers have unambiguously weaker investment incentives with intermediated trade. However, this ignores margin effects. Investments affect the distribution of gains from trade for buyers and sellers (i.e., the division of economic surplus within a buyer–seller pair) and, thus, the size of the network effects. This drives competition between for-profit intermediaries, which is reflected by the access fees. In particular, when innovations increase buyer surplus, intermediaries react to the corresponding investments by lowering access fees on the seller side. As a consequence, sellers internalize changes in buyer surplus if products are traded on for-profit platforms, whereas they do not in the context of open platforms. Thus, investment incentives can be stronger with competing for-profit platforms than with open platforms. The exact relationship between investment incentives and for-profit intermediation depends on which side of the market singlehomes and on the nature of the investment effort. In our linear specification with a finite number of sellers, we obtain the following results: (i) When both sides singlehome, trade via for-profit platforms raises seller incentives to invest in cost reduction and in quality, but lowers incentives to invest in price discrimination (and the effect depends on parameter values for investments in demand expansion); furthermore, in such a market, a social underinvestment problem with open platforms translates into a social overinvestment problem with proprietary for-profit platforms; (ii) when sellers can multihome and buyers singlehome, trade via for-profit platforms leads to weaker investment incentives.


6 For instance, in November 2007, Sony slashed the price of the PlayStation 3 development kit by almost half—from $20,500 to $10,250 in the U.S. (see “Sony halves cost of PS3 development kit,” by Matt Martin, www.gamesindustry.biz, November 19, 2007). In March 2009, Sony further lowered the cost of its PS3 development kit to $2,000 (see “Sony tries to boost PS3 development with dev kit price cut,” by Blake Snow, arstechnica.com, March 23, 2009).
whatever the nature of the innovation; and (iii) when sellers singlehome and buyers can multihome, the opposite tends to prevail—i.e., if trade takes place via for-profit platforms, investment incentives are strengthened. In the last case, we focus on a market with a single seller.

Our results can be read as follows: as the intensity of competition for sellers increases, proprietary platforms are more likely than open platforms to provide better seller investment incentives. Indeed, this happens when the nature of platform competition moves from multihoming sellers and singlehoming buyers, to singlehoming sellers and buyers, and then to singlehoming sellers and multihoming buyers.

In an empirical paper, Boudreau (2006) investigates the effect of the degree of openness of the platform on seller incentives in the computer industry. He finds that restricted access and some control over the platform led to more investments in innovation than did highly open strategies by platforms. A potential reason for these findings, as Boudreau (2006, p. 2) points out, are strategic effects: “In that the opening of a system will also surely affect the “within-system” competition (and perhaps even between system strategic interactions), suppliers’ strategic incentives to make investments in innovation might also be affected.” We present a formal framework to address the issue of competition between platforms.

Our paper connects to the burgeoning recent literature on two-sided markets. Compatible with this literature is the view that intermediaries possess property rights on a platform and, thus, can make profits from charging access or usage fees on both sides of the market. Seminal contributions to this literature include Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), and Armstrong (2006). Our setup of the two-sided market borrows from Armstrong’s models with singlehoming and with competitive bottlenecks where two competing intermediaries set access prices on both sides of the market.7 We provide a micro foundation of seller profits and consumer utilities that allows us to give a specific meaning to seller investment incentives. We compare seller investment incentives in a market with competing for-profit platforms to a market in which all sellers and buyers trade through open platforms. While most work has focused exclusively on for-profit platforms, Hagiu (2006a) and Nocke et al. (2007), among others, have compared for-profit to open platforms. However, the two-sided market literature has been silent about investment incentives.

More generally, our paper contributes to the micro-market structure and intermediation literature. Here, an alternative line of research has taken the view that intermediated trade (by dealers who set bid and ask prices) may avoid inefficiencies that arise in random matching environments. In this setting, the coexistence of matching and dealer markets leads to a self-selection of types (see Gehrig, 1993).8 In such an environment, Spulber (2003) analyzes sellers’ (and buyers’) investment incentives. He shows that the introduction of a dealer market in a decentralized matching market leads to stronger investment incentives. Our analysis complements Spulber’s (2003) work because we abstract from search inefficiencies and, instead, focus on market-size externalities.

Also, our paper borrows from and contributes to the literature on R&D incentives. The pioneering paper studying the effect of market structure on the incentives for R&D is Arrow (1962). He compares the incentive to innovate in monopolistic and competitive markets, concluding that perfect competition fosters more innovation than monopoly does.9 The contribution of this paper, then, is to show that investment incentives depend on the underlying intermediation structure. In particular, the incentives may be affected by the need to obtain access to one of the competing platforms.

The rest of the paper is organized as follows. In Section 2, we set up the model. We then analyze three particular versions of the model: In Section 3, we assume that both buyers and sellers singlehome; in Section 4, only buyers singlehome, while sellers are allowed to multihome; in Section 5, the opposite prevails, as sellers singlehome, while buyers are allowed to multihome. In Section 6, we provide a microfoundation for the generic surplus functions used in the previous sections and examine the seller investment incentives in cost-reduction, in quality improvement, in price discrimination, and in demand expansion. We conclude and discuss possible extensions in Section 7. An appendix complements the analysis in the main text.

2. The model

We provide an abstract model of trade on a platform that closely follows the literature on two-sided markets and, in particular, Armstrong (2006) and Armstrong and Wright (2007). There are two sides of the market, the buyer side and the seller side. Suppose that there is a unit mass of buyers and a finite number K of sellers, each seller selling mass 1/K of products (so that the total mass of products is equal to 1). Buyers and sellers can interact on two platforms, 1 and 2, which are assumed to be located at the extreme points of the unit interval. Sellers are ex ante identical and learn their location only after their investment decision. Given investment decisions, platforms simultaneously set prices for accessing the
platform. Before choosing which platform to visit, buyers and sellers are assumed to independently draw their location from a uniform distribution on the unit interval. It is further assumed that each seller and each buyer has private information about his or her location. Buyers and sellers incur an opportunity cost of visiting a platform that increases linearly in distance at rates $\tau_b$ and $\tau_s$, respectively.

In order to analyze how the market microstructure affects sellers’ ex ante investment incentives, we compare two different organizations of the trading platforms: intermediated trade (in which platforms are run by strategic profit-maximizing intermediaries) and non-intermediated trade (in which platforms are open). The latter case can be seen as a natural benchmark because we do not want to introduce other differences with respect to the for-profit duopoly model. The comparison between intermediated and non-intermediated trade gives us an answer to the following thought experiment: What would happen, in terms of seller investment incentives, if platforms were open?

Under both trade patterns, we characterize the subgame-perfect equilibrium of the multi-stage game in which, for given investment levels, agents first choose which platform to join and then interact on the platform(s) of their choice. We then examine how the sellers’ equilibrium payoffs would change as a result of some investment that modifies sellers’ and buyers’ payoff functions in the multi-stage game. This change in the sellers’ equilibrium payoffs measures the sellers’ ex ante incentives to achieve the given investment. The investment choice is analyzed at a stage 0, where sellers choose simultaneously and non-cooperatively whether or not to invest in some innovation. At this point, sellers do not yet know their location.

In the case of intermediated trade, we analyze the following four-stage game. At stage 1, intermediaries simultaneously set membership fees $M_i^L, M_i^R$ on the two sides of the market. The membership fee on the seller side is expressed per product so that a seller who sells mass $1/K$ products has to pay $M_i^L/K$ if he sells his products at platform $i$. Sellers and buyers learn their location—this is private information for each of them. At stage 2, sellers and buyers decide which platform to visit. At stage 3, sellers set the price of their goods simultaneously. Finally, at stage 4, buyers make purchasing decisions. In the case of non-intermediated trade, buyers and sellers interact through open platforms to which access is assumed to be free-of-charge. As the for-profit intermediaries disappear, so does stage 1. Thus, we solve for subgame-perfect equilibria of the multi-stage game consisting of stages 2 through 4. According to our timing, the intermediaries’ pricing decision is more flexible than the sellers’ investment decision. In particular, product and process innovations are lumpy decisions, whereas intermediaries can more flexibly change prices.

Some of the timing assumptions are worthwhile discussing. Regarding stage 1, sellers are ex ante identical. We view the investment decision by a seller as a long-term decision which can be seen as independent of the realized preferences with respect to via which intermediary to trade. Regarding stage 2, buyers and sellers make their decision as to which intermediary to visit after learning their location. Note that with for-profit intermediaries, it is immaterial whether sellers and buyers learn their location before or after platforms have set their access fees. Regarding stage 3, we assume that the buyers’ pricing decisions are independent, so that we do not need to make particular assumptions on the timing decision at this pricing stage (we can think of sellers producing perfectly differentiated varieties). This assumption simplifies the analysis and allows us to focus on comparing investment incentives under intermediated and non-intermediated trade. Since the pricing decision of sellers is assumed to be very flexible, sellers do not have to be concerned with any impact of their pricing decision on platform demand, despite the fact that sellers are non-atomless.

Regarding stage 4, we assume that a buyer at platform $i$ has a downward-sloping demand function for each product that is traded on this platform. In Section 6, we provide a number of micro models of buyer–seller relationships and analyze particular investment decisions in such settings. In particular, we consider sellers with independent and downward-sloping demand who engage in cost-reducing R&D, who invest in quality improvements, or who invest in technologies or marketing efforts that allow them to price discriminate between consumers or to expand demand. For now, we use a reduced-form representation of buyer–seller interaction. Absent any investment, the net gains from trade for a seller and for a buyer buying one of the seller’s products are given by $\pi_0$ and $u_0$, respectively. The investment changes these values to $\pi_1 = \pi_0 + \Delta_2$ and $u_1 = u_0 + \Delta_3$, respectively. In most applications, $\Delta_2 > 0$, but the sign of $\Delta_3$ depends on the nature of the investment (for instance, an investment in cost reduction tends to benefit consumers, $\Delta_3 > 0$, while an investment in enhanced price discrimination tends to harm consumers, $\Delta_3 < 0$).

In the next three sections, we contrast investment incentives in intermediated and non-intermediated trade under three different scenarios regarding the agents’ ability to interact simultaneously on more than one platform. We first consider situations where both sides of the market singlehome (Section 3). We turn next to situations where one side of the market singlehomes, while multihoming is feasible on the other side of the market (Sections 4 and 5). As we will illustrate in Section 6, the investment level affects the demand curve or the cost function that firms face and, thus, has an impact on $u$ and $\pi$ (it may also impact price-discrimination possibilities).

---

10 In an earlier version of this paper, we considered the coordinated investment decision of a continuum of ex ante heterogeneous sellers. In the discussion section of this paper, we return to this alternative setup. Arguably, the present setup applies to a larger number of industries.

11 In line with most work on two-sided markets, our analysis covers only independent sellers. An earlier discussion paper included an analysis of an imperfectly competitive seller side. Under imperfect competition between sellers, buyers benefit from more competition between sellers through lower prices (see Gehrig, 1998; Hagiu, 2006a, 2009, and Nocke et al., 2007).

12 To analyze the case of multihoming on both sides of the market, additional restrictions would need to be imposed, which would largely complicate the model. Because of this difficulty and of the limited interest of this case (competition between platforms is largely reduced when both sides can multihome), we do not consider this case further.
3. Two-sided singlehoming

In this section, both sides of the market are assumed to singlehome. Singlehoming environments in the real world can be motivated by indivisibilities and limited resources, or by contractual restrictions. The former applies to certain real-world marketplaces where buyers and sellers can physically locate in only one of them (flea and farmers’ markets come to mind). For the latter, we find more examples. For instance, taxi companies in Germany sign exclusive contracts with taxi call centers. There also appears to be little multihoming on the consumer side. Similarly, some employment agencies for temporary work can be characterized by singlehoming on both sides of the market. It is less clear to which extent video-game platforms can be approximated by two-sided singlehoming. Some gamers have more than one platform, and today’s leading game developers develop the same game for different platforms. Also, it has been claimed that the market for innovation. Accordingly, seller surplus per product and buyer surplus gross of any opportunity cost of visiting a platform are respectively given by

\[
\nu_i^* = \begin{cases} 
  n_i^b \pi_1 - M_i^b & \text{if seller has invested,} \\
  n_i^b \pi_0 - M_i^b & \text{otherwise,}
\end{cases}
\]

\[
\nu_i^b = n_i^b (ku_1 + (1-k)u_0) - M_i^b,
\]

where \(n_i^b\) is the mass of buyers joining platform \(i\), and \(n_i^s\) is the mass of products available at platform \(i\).

We start by deriving the number of buyers and the number of sellers (and, thus, products) going to each platform. The indifferent buyer type is identified by \(b_{12}\) such that \(\nu_{12}^b - \tau_b n_{12} = \nu_{12}^s - \tau_b(1-b_{12})\), which is equivalent to

\[
b_{12} = \frac{1}{2} + \frac{(n_i^b - n_i^s)(ku_1 + (1-k)u_0) + M_i^2 - M_i^1}{2\tau_b}.
\]  

(1)

The indifferent seller type who has invested is identified by \(s_{12}\) such that \(n_{12}^b \pi_1 - M_{12}^b - \tau_s n_{12} = n_{12}^s \pi_1 - M_{12}^s - \tau_s(1-s_{12})\). The indifferent seller type who has not invested is identified by \(s_{12}\) such that \(n_{12}^b \pi_0 - M_{12}^b - \tau_s n_{12} = n_{12}^s \pi_0 - M_{12}^s - \tau_s(1-s_{12})\). That is,

\[
s_{12} = \frac{1}{2} + \frac{(n_i^b - n_i^s) \pi_1 + M_i^2 - M_i^1}{2\tau_s},
\]  

(2)

\[
s_{12} = \frac{1}{2} + \frac{(n_i^b - n_i^s) \pi_0 + M_i^2 - M_i^1}{2\tau_s}.
\]  

(3)

Two-sided singlehoming and full participation imply the following:

\[
\begin{cases} 
  n_i^b = b_{12} \text{ and } n_i^s = 1 - b_{12}, \\
  n_i^s = \kappa s_{12} + (1-\kappa) n_{12} \text{ and } n_i^s = 1 - n_i^s.
\end{cases}
\]  

(4)

We introduce the following notation: \(\bar{u}(\kappa) \equiv ku_1 + (1-k)u_0\) and \(\bar{u}(\kappa) \equiv k\pi_1 + (1-k)\pi_0\) (we drop the reference to \(\kappa\) for the moment and return to it when considering investment decisions). Using expressions (1)-(4), we obtain the following
expressions for the numbers of buyers and sellers at the two platforms:

\[ n_b = \frac{1}{2} + \left( \frac{2n_b - 1}{2} \right) \frac{\bar{u} - (M^*_{b} - M_{b}^i)}{2\tau_b} \]

\[ n_s = \frac{1}{2} + \left( \frac{2n_s - 1}{2} \right) \frac{\hat{p} - (M^*_{s} - M_{s}^i)}{2\tau_s} \]

This shows that, for given membership fees of buyers, an additional product attracts \( \bar{u}/\tau_b \) additional buyers. Similarly, an additional buyer attracts \( \hat{p}/\tau_s \) additional products in expectation. Combining these two findings, we see that the indirect network effects on each side of the market are measured by the ratio \( \bar{u} \hat{p}/\tau_B \tau_s \). If the indirect network effects are too strong, two intermediaries cannot be active. To exclude this possibility, we require that \( \tau_B \tau_s > \bar{u} \hat{p} \); i.e., that the opportunity costs \( \tau_B \) and \( \tau_s \) (which measure the perceived horizontal differentiation between the two platforms) be sufficiently large with respect to the gains from trade \( \bar{u} \hat{p} \). To make sure that the second-order conditions are satisfied in the maximization programs of stage 1, we impose a slightly more restrictive condition, namely \( 4\tau_B \tau_s > (\bar{u} + \hat{p})^2 \). We can then solve the above implicit expressions for the number of buyers and sellers to obtain the following formulas:

\[ n_b = \frac{1}{2} + \frac{\bar{u} (M^*_{b} - M_{b}^i) + \tau_s (M^*_{b} - M_{b}^i)}{2(\tau_B \tau_s - \bar{u} \hat{p})} \]

(5)

\[ n_s = \frac{1}{2} + \frac{\hat{p} (M^*_{s} - M_{s}^i) + \tau_B (M^*_{s} - M_{s}^i)}{2(\tau_B \tau_s - \bar{u} \hat{p})} \]

(6)

The number of buyers at one platform is decreasing not only in the membership fee for buyers on this platform, but also, due to indirect network effects, in the membership fee for sellers. If the fees set by the intermediaries are such that \( 0 < n_b, n_s < 1 \) (which will indeed be the case), expressions (5) and (6) define a unique and stable equilibrium for stage 2 (see the appendix for a proof).

Let us next turn to the first stage of the game at which platforms set prices (for given sellers’ investment levels). Assuming that the intermediary’s cost per buyer is \( C_b \) and per seller is \( C_s \), we can write platform \( i \)'s expected profit as

\[ \Pi^i = (M^i_{b} - C_b) \left( \frac{1}{2} + \frac{\bar{u} (M^*_{b} - M_{b}^i) + \tau_s (M^*_{b} - M_{b}^i)}{2(\tau_B \tau_s - \bar{u} \hat{p})} \right) + (M^*_{s} - C_s) \left( \frac{1}{2} + \frac{\hat{p} (M^*_{s} - M_{s}^i) + \tau_B (M^*_{s} - M_{s}^i)}{2(\tau_B \tau_s - \bar{u} \hat{p})} \right). \]

The two intermediaries simultaneously choose membership fees on both sides of the market. First-order conditions of profit maximization in a symmetric equilibrium—i.e., \( M^*_{b} = M^*_{b} \equiv M_b \) and \( M^*_{s} = M^*_{s} \equiv M_s \)—can be written as

\[ M_b = C_b + \tau_B - \frac{\hat{p}}{\tau_s} (\bar{u} + M_s - C_b) \]

\[ M_s = C_s + \tau_s - \frac{\bar{u}}{\tau_B} (\bar{u} + M_b - C_b) \]

Equilibrium prices on the seller side are equal to marginal costs plus the product differentiation term as in the standard Hotelling model, adjusted downward by the term \( (\bar{u}/\tau_B)(\bar{u} + M_b - C_b) \). Recall that each additional seller attracts \( \bar{u}/\tau_b \) additional buyers. These additional buyers allow the intermediary to extract \( \bar{u} \) per product without affecting the sellers’ surplus. In addition, each of the additional \( \bar{u}/\tau_b \) buyers gives a profit per product of \( M_b - C_b \) to the intermediary. Thus, \( (\bar{u}/\tau_{B})(\bar{u} + M_b - C_b) \) represents the value of an additional buyer to the intermediary. The higher this value, the more aggressive the price setting among intermediaries on the seller side.

Solving for the Nash equilibrium membership fees, one finds

\[ M^*_b = C_b + \tau_B - \hat{p} \]

\[ M^*_s = C_s + \tau_s - \bar{u} \]

Sellers’ membership or access fees are lower if there are larger gains from trade on the buyer side. It follows that, at equilibrium, \( n_b^* = n_s^* = 1/2 \) and \( n_s^* = n_s^* = 1/2 \), so that, gross of transportation cost, the equilibrium net surplus of sellers (per product) and the one of buyers are equal to\(^{16}\):

\[ v^*_b = \frac{1}{2} \bar{u} + \hat{p} - (C_b + \tau_B) \]

\[ v^*_s = \frac{1}{2} \bar{u} + \hat{p} - (C_b + \tau_B) \]

---

\(^{16}\) We still need to check that all buyers and sellers participate. This is so, provided that \( v^*_b - (c_b/2) > 0 \) and \( v^*_s - (c_s/2) > 0 \), which are respectively equivalent to \( \tau_s < \delta (\bar{u} + \hat{p} - C_b) \) and \( \tau_s < \delta (\bar{u} + \hat{p} - C_s) \).
\begin{align*}
v_s^* &= \begin{cases} 
\frac{1}{2} \pi_1 + \bar{u} - (C_s + \tau_s) & \text{if the seller has invested}, \\
\frac{1}{2} \pi_0 + \bar{u} - (C_s + \tau_s) & \text{otherwise}.
\end{cases}
\end{align*}

We observe that $v_s^*$ and $v_s^*$ are increasing in the gains from trade that accrue on the other side of the market and, to a lesser extent, also in the gains from trade on the buyer's (resp. seller's) own side. The intermediaries' expected equilibrium profits are

\[ \Pi^i = \frac{1}{2} (\tau_s - \bar{u}) + \frac{1}{2} (\tau_b - \bar{u}) = \frac{1}{2} (\tau_s + \tau_b) - \frac{1}{2} (\bar{u} + \bar{u}). \]

Note that only the joint net gain from trade by buyers and sellers determines each intermediary's profit. Thus, the distribution of net gains among sellers and buyers does not affect its profit. Furthermore, this profit is decreasing in $(\bar{u} + \bar{u})$; i.e., in markets in which gains from trade are high, the intermediary's profits are low. This result may seem counterintuitive but can be explained as follows. Net gains $\bar{u}$ and $\bar{u}$ determine the strength of network effects in the industry. If $\bar{u} + \bar{u}$ is large, this means that additional buyers and sellers are very valuable for intermediaries. Therefore, they compete more aggressively in the market place. Recall that we restrict attention to situations in which two intermediaries are viable so that $\Pi^i > 0$.\footnote{This is the case under the assumption we made above, $4\tau_s \tau_b > (\bar{u} + \bar{u})^2$, which implies that platforms are sufficiently differentiated.}

3.2. Seller investment incentives

We now investigate a seller's investment incentives. Since each seller can coordinate his investment decision for all his product of mass $1/K$ under his control, and since all products are symmetric, we can restrict attention to the case in which the seller invests in either all or none of its products. Supposing that $0 \leq k < K$ sellers invest, a seller who does not invest achieves a net surplus per product given by

\[ V_s(k) = \frac{1}{2} \pi_0 + \bar{u}(k) - (C_s + \tau_s). \]

If the seller invests, his net surplus per product becomes

\[ V_s^i \left(\frac{k+1}{K}\right) = \frac{1}{2} \pi_1 + \bar{u} \left(\frac{k+1}{K}\right) - (C_s + \tau_s). \]

Hence, the seller's incentives to innovate under \textit{intermediated trade} are given by

\[ I^m = V_s^i \left(\frac{k+1}{K}\right) - V_s(k) = \frac{1}{2} \pi_1 - \frac{1}{2} \pi_0 + \frac{1}{K} (u_1 - u_0). \]

In the case of \textit{non-intermediated trade}, each seller interacts with half of the buyers and, therefore, gets a surplus of $\frac{1}{2} \pi_1$ or $\frac{1}{2} \pi_0$ whether he has invested or not. It follows that a seller's incentives to innovate under \textit{non-intermediated trade} are equal to

\[ I^n = \frac{1}{2} \pi_1 - \pi_0. \]

Comparing the latter two expressions, we obtain

\[ I^m - I^n = \frac{1}{K} (u_1 - u_0) = \frac{1}{K} \Delta_u, \]

which implies that

\[ I^m > I^n \iff \Delta_u > 0. \]  

(7)

This observation allows us to state the following simple result.

\textbf{Proposition 1.} In the two-sided singlehoming model, for-profit trading platforms give stronger investment incentives for sellers if and only if the investment increases the buyer's surplus.

To understand this condition, recall that in intermediated trade, the net surplus per product for a seller who has not invested is equal to $V_s^* = \pi_0 - M_s^* = \frac{1}{2} \pi_0 - (C_s + \tau_s - \bar{u})$. Recall that $\bar{u}(k) = ku_1 + (1-k)u_0 = u_0 + k \Delta_u$. Hence, if the investment increases the buyers' surplus, we have that $\bar{u}(k)$ is an increasing function of $k$ as $\Delta_u > 0$. It follows that if an additional seller invests, for-profit platforms will charge a lower fee to sellers. This provides extra investment incentives compared to open platforms (where this price effect is absent): clearly, the opposite prevails if the investment decreases the buyers' surplus (i.e., for $\Delta_u < 0$). Note also, that the difference in incentives (between intermediated and non-intermediated trade) is a decreasing function of $K$; that is, the fewer sellers there are, the more the incentives to innovate differ in the two market microstructures. If we take the limit $K \to \infty$ and maintain the assumption that sellers cannot coordinate their investment decisions themselves, investment incentives are the same under the two different intermediation structures.
In Section 6, we will have a closer look at the microstructure of the buyer–seller relationship and the nature of seller investments. Note that since \( n^1_2 = 1/2 \) under both types of platform organization, the ranking of per-seller incentives implies the same order of total investment (i.e., the sum of sellers’ willingness to pay for the innovation). While the precise form of condition (7) is an artifact of the linear structure, a general feature is that a higher buyer surplus is desirable for sellers on a for-profit platform but irrelevant on an open platform. Hence, under for-profit platforms, sellers partly internalize improvements of buyer surplus when making their investment decision.\(^{18}\)

**Remark 1.** In a second-best world (where the social planner decides only about the investment level, and trade takes place on both platforms), total surplus is maximized with the innovation if

\[
\frac{1}{2} [\pi_1 - \pi_0] + \frac{1}{2} [u_1 - u_0] > C,
\]

where \( C \) is the social cost to carry out the innovation. More generally, let us denote by \( y \) the size of the innovation and define accordingly \( \pi(y) = \pi_1 - \pi_0, \ u(y) = u_1 - u_0 \) and \( C(y) \). If these functions are differentiable, the planner solves \( \pi(y) + u(y)/2 = C(y) \). Sellers in a market with two open platforms (which simply charge zero access prices) choose an investment level that solves \( \pi(y)/2 = C(y) \); and sellers in a market with two proprietary platforms choose an investment level that solves \( \pi(y)/2 + u(y) = C(y) \). If \( u \) is strictly increasing in \( y \), there is social underinvestment on open platforms and social overinvestment on for-profit platforms. Conversely, if \( u \) is strictly decreasing in \( y \), there is social overinvestment on open platforms and social underinvestment on for-profit platforms.

The remark implies that public policy steered towards private investments should depend critically on the prevailing intermediation structure. For instance, while the analysis under open platforms suggests that an R&D subsidy may be an appropriate remedy to the social underinvestment problem that prevails in the context of R&D investments (see Section 6), such a subsidy may worsen the overinvestment in a market with for-profit platforms.

4. Competitive bottlenecks when sellers multihome

In this section, we analyze investment incentives in market environments in which sellers have the possibility to multihome. As noted by Evans (2003), personal computers constitute a typical example of this situation: End-users (i.e., buyers) singlehome (they almost always use a single operating system), while application developers (i.e., sellers) multihome.\(^{19}\) Other examples include retail chains that can locate in competing shopping malls, firms that list in competing yellow pages, and shops that accept competing credit cards (provided that consumers hold only one card).

4.1. Equilibrium for given investment levels

At stage 2, because sellers now have the possibility to multihome, there are three subintervals of the unit line to consider: If a seller draws a location “on the left,” he will visit platform 1 only; if the location is “around the middle,” the seller visits both platforms; and if the location is “on the right,” the seller visits platform 2 only. To identify these three subintervals, we define the locations where a seller would be indifferent between visiting platform 1 (resp. 2) and not visiting any platform (thereby getting a utility of zero). For a seller who has not invested, these locations are given by

\[
s_{10} = \frac{n_1^1 \pi_0 - M^1}{\tau_s} \quad \text{and} \quad s_{20} = 1 - \frac{n_2^2 \pi_0 - M^2}{\tau_s}.
\]

Similarly, for a seller who has invested, we define

\[
s_i' = \frac{n_i^1 \pi_1 - M^i}{\tau_s} \quad \text{and} \quad s_{20}' = 1 - \frac{n_2^2 \pi_1 - M^2}{\tau_s}.
\]

We assume for now that \( 0 < s_{20} < s_{10} < 1 \) and \( 0 < s_{20}' < s_{10}' < 1 \) (we express necessary and sufficient conditions below). The number of products available on platforms 1 and 2 are, thus, given by \( n_1^1 = \kappa s_{10} + (1 - \kappa)s_{10} \) and \( n_2^2 = \kappa (1 - s_{20}) + (1 - \kappa)(1 - s_{20}) \), respectively, or, equivalently, by\(^{20}\)

\[
n_i^1 = \frac{n_i^1 \pi_1 - M^i}{\tau_s}.
\]

\(^{18}\) We could then even observe types of investment that decrease the seller surplus per buyer \( \pi \), provided that this decrease is more than compensated by an increase in the buyer surplus per seller \( u \). By contrast, sellers would never choose to make such investments if platforms were open.

\(^{19}\) See Lerner (2002) for data about the developers that develop for various operating systems.

\(^{20}\) We could have followed an alternative route to derive the number of products on the two platforms (we thank an anonymous referee for this suggestion). We could have assumed that each seller is not identified by one draw on the Hotelling line but instead, by two independent draws \( (x_1^1, x_2^1) \) from \([0,1]^2\), which measure the “distance” between the seller’s location and each of the two platforms. Then, assuming that \( \tau_s \) is large enough so that the seller market is never covered by either platform, we would have had that sellers for whom both \( x_1^1 \) and \( x_2^1 \) are low multihome, sellers for whom \( x_1^1 \) is low and \( x_2^1 \) is high singlehome on platform 1, etc. It is easy to see that all our results would be preserved in this alternative model; however, the comparison with the previous two-sided singlehoming model would not be as natural.
The buyers are in the same situation as in the previous section: As they singlehome, they divide into two groups, one that goes to platform 1 (i.e., buyers located between 0 and \( b_{12} \)) and one that goes to platform 2 (i.e., buyers located between \( b_{12} \) and 1). We recall from the previous section that

\[
b_{12} = \frac{1}{2} + \frac{(n_1^1 - n_1^2)\bar{u} + M_1^2 - M_1^1}{2\tau_b}, \quad n_1^1 = b_{12}, \quad n_2^1 = 1 - b_{12}.
\]

Solving the system of four equations in four unknowns, we obtain

\[
\begin{align*}
n_b^1 &= \frac{1}{2} + \frac{\bar{v} (M_b^1 - M_b^2) + \tau_b (M_b^1 - M_b^2)}{2(\tau_b\tau_s - \bar{u}\bar{\pi})}, \\
n_s^1 &= \frac{\bar{v}}{\tau_s} \left( \frac{1}{2} + \frac{\bar{v} (M_b^1 - M_b^2) + \tau_b (M_b^1 - M_b^2)}{2(\tau_b\tau_s - \bar{u}\bar{\pi})} \right) - M_b^1.
\end{align*}
\]

As in the previous section, our reference point is the situation of two open platforms located at the extremes of the unit interval (that is, they have the same locations as for-profit platforms). Suppose that access is free. Setting \( M_b^1 = M_b^2 = M_s^1 = M_s^2 = 0 \) in expressions (8) and (9), we compute the numbers of buyers and sellers on each platform as

\[
n_b^1 = n_s^1 = \frac{1}{2} \quad \text{and} \quad n_b^2 = n_s^2 = \frac{1}{2\tau_s}.
\]

It follows that the (per platform) net surplus of buyers and the one of sellers are equal to

\[
\begin{align*}
v_b^0 &= \frac{1}{2\tau_s} \bar{\pi} \bar{u} \quad \text{and} \quad v_s^0 = \frac{1}{2} \bar{\pi},
\end{align*}
\]

where \( \pi = \pi_1 \) or \( \pi_0 \) according to whether or not the seller has invested.

Assuming that \( \bar{\pi} < 2\tau_s \), we have that sellers visit both open platforms if they are located between \( 1 - \bar{\pi}/(2\tau_s) \) and \( \bar{\pi}/(2\tau_s) \), whereas they visit only the platform closer to their location if they are located outside this interval. As long as the equilibrium fee set by for-profit platforms, \( M_b^2 \), is positive, open platforms attract more sellers than for-profit platforms do (which also means that more sellers multihome if trade is organized through open platforms).

Let us now turn to the pricing stage of the game in which platforms are for-profit. Each platform \( i \) solves the problem \( \max_{M_b^i, M_s^i} \Pi^i \) where

\[
\Pi^i = (M_b^i - C_b) n_b^i (M_b^i, M_s^i, M_s^i) + (M_s^i - C_s) n_s^i (M_b^i, M_s^i, M_s^i).
\]

Equilibrium prices are\(^{21} \)

\[
M_b^2 = M_b^{1*} = M_b^{2*} = C_b + \tau_b - \frac{\bar{\pi}}{4\tau_s} (3\bar{u} + \bar{\pi} - 2C_s),
\]

\[
M_s^2 = M_s^{1*} = M_s^{2*} = \frac{1}{2} C_s + \frac{1}{4} (3\bar{\pi} - \bar{u}).
\]

On the buyer side, platforms have monopoly power. If the intermediary focused only on sellers and ignored effects on the buyer side, it would charge a monopoly price equal to \( C_b/2 + \bar{\pi}/4 \) (assuming that each seller would have access to half of the buyers and, therefore, would have, on average, a gross willingness to pay equal to \( \bar{\pi}/2 \)). We observe that this price is adjusted downward by \( \bar{u}/4 \) when the indirect network effect that sellers exert on the buyer side is taken into account (and remains positive as long as \( \bar{\pi} + 2C_s > \bar{u} \)).\(^{22} \) On the buyer side, platforms charge the Hotelling price, \( C_b + \tau_b \), less a term that depends on the size of the indirect network effects.

It is useful to compare price changes in the competitive bottleneck model to those in the two-sided singlehoming model. In equilibrium, we observe that the membership fee for sellers is increasing in the strength of the indirect network effect (\( \partial M_b^i / \partial \pi > 0 \)), whereas it is constant in the two-sided singlehoming model. This is due to the monopoly pricing feature on the multihoming side. Everything else equal, if sellers are multihoming, the platform operators directly appropriate part of the rent generated on the multihoming side by setting higher membership fees. This is not the case in the singlehoming world, where the membership fee does not react to the strength of the network effect on the same side since platforms compete for sellers (and buyers). This observation is relevant for the analysis of investment incentives below.

---

\(^{21}\) Firms’ best responses are implicitly defined by the first-order conditions, which can be expressed as \( M_b^1 = [-\bar{u} + \bar{\pi} - M_b^1 + \bar{u}M_b^1 + \bar{\tau}_b M_b^1 - \bar{\tau}_b (\bar{u} - C_s) + \bar{u}(\bar{\tau}_b + \bar{\tau}_s)] / [2\bar{u} + \bar{\tau}_b] \) and \( M_s^1 = [-\bar{u} + \bar{\pi} - M_s^1 + \bar{u}M_s^1 + \bar{\tau}_s M_s^1 - \bar{\tau}_s (\bar{u} + C_s) + \bar{u}(\bar{\tau}_b + \bar{\tau}_s)] / [2\bar{u} + \bar{\tau}_b] \). Second-order conditions require that \( 8\bar{u} \bar{\tau}_b > \bar{\tau}_b^2 + 4\bar{u}^2 + 6\bar{u} \). This condition is also sufficient to have a unique and stable interior equilibrium at stage 2. Note that in the special case where parameters are symmetric on both sides of the market—i.e., \( \bar{\tau}_b = \bar{\tau}_s = \bar{\tau} \) and \( \bar{u} = \bar{\pi} = \bar{\tau} \)—this inequality simplifies to \( \bar{u} > \bar{\tau} \).

\(^{22}\) In contrast with the two-sided singlehoming case, \( M_i \) does not depend here on the sellers’ transportation cost (\( \bar{\tau}_s \)). This is due to the monopoly power platforms have on the seller side and on the linearity of our model (\( \bar{u} \) affects the slope of the sellers’ demand but not the optimal price). When sellers singlehome, platforms compete for them and \( \bar{\tau}_s \) (which measures how sellers perceive the differentiation between the platforms) positively affects the equilibrium prices.
It follows that, at equilibrium,

\[ n^*_b = n^*_s = \frac{1}{2}. \]

\[ n^*_s = n^*_s = \frac{1}{4\tau_s} (\bar{\rho} - \bar{\pi} - 2C_s). \]

Thus, we must have \( 0 < n^*_s < 1 \) if \( 2C_s < \bar{u} + \bar{\pi} < 2C_s + 4\tau_s \) for obtaining an interior solution.\(^{23}\) Under these conditions, the equilibrium net surplus of sellers and the one of buyers (gross of transportation costs and for one platform) are equal to:

\[ v^*_s = \begin{cases} \frac{1}{2} \pi_1 - \frac{1}{4} (\bar{\pi} - \bar{\rho}) - \frac{1}{2} C_s & \text{if seller has invested,} \\ \frac{1}{2} \pi_0 - \frac{1}{4} (\bar{\pi} - \bar{\rho}) - \frac{1}{2} C_s & \text{otherwise.} \end{cases} \]

\[ v^*_b = \frac{1}{4\tau_s} (\bar{u} - 4\pi\bar{\pi} + \bar{\pi}^2 - 2(\bar{u} + \bar{\pi})C_s) - \tau_b - C_b. \]

Note that \( v^*_s \) is the seller's per-product surplus on one platform. If \( n^*_s > 1/2 \), the sellers located between \( 1 - (v^*_s / \tau_s) \) and \( v^*_s / \tau_s \) multihome and, therefore, earn a surplus per product of \( 2v^*_s \). Concerning those who decide to access only one platform, \( v^*_s \) is the surplus earned by the sellers located between \( 0 \) and \( 1 - (v^*_s / \tau_s) \), who choose to visit platform 1 only, and by the sellers located between \( v^*_s / \tau_s \) and \( 1 \), who choose to visit platform 2 only. We observe that \( v^*_s \) and \( v^*_s \) are increasing in the net gain of the other side and in the net gain of the buyer's (resp. seller's) own side. The intermediaries' expected equilibrium profits are

\[ \Pi^* = \frac{1}{16\tau_s} (8\tau_0 \tau_s - (\bar{\pi}^2 - \bar{\pi} + \bar{\pi}^2 + 6\pi\bar{\pi}) + 4C_s^2) > 0. \]

### 4.2. Seller investment incentives

As before, we compare the seller investment incentives under two different organizations of the trading platforms, a situation in which there are two for-profit platforms and a situation in which there are two open platforms. We start from a situation where \( 0 \leq k < K \) sellers invest, and we measure the incentive for an additional seller to invest. Under intermediated trade, a seller that does not invest achieves (per platform) a total surplus given by

\[ V_s(k) = \frac{1}{2} \pi_0 - \frac{1}{4} \bar{\pi} (k + 1/k) - \frac{1}{2} C_s. \]

If the seller invests, his total surplus (per platform) becomes

\[ V_s(k + 1/k) = \frac{1}{2} \pi_1 - \frac{1}{4} \bar{\pi} \left( k + 1/k \right) - \bar{\rho} \left( k + 1/k \right) - \frac{1}{2} C_s. \]

Using the same notation as in the previous section, with two strategic intermediaries, sellers are willing to pay up to the increase in their (equilibrium) net surpluses \( f^m \equiv V_s(k + 1/k) - V_s(k) \). Here, we need to distinguish between the sellers who multihome and those who singlehome at equilibrium. Recalling that \( \bar{u}(k) = u_0 + \kappa A_u \), we have that \( \bar{u}(k + 1/k) - \bar{\rho}(k + 1/k) = (1/k) A_u \); similarly, \( \bar{\rho}(k + 1/k) - \bar{\pi}(k + 1/k) = (1/k) A_\pi \). Then, the increase in net surplus is given by

\[ f^m = \begin{cases} A_\pi + \frac{1}{2K} (A_u - A_\pi) & \text{for multihoming sellers,} \\ \frac{1}{2} A_\pi + \frac{1}{4K} (A_u - A_\pi) & \text{for singlehoming sellers.} \end{cases} \]

If trade takes place on two open platforms, the increase in the sellers' net surpluses is

\[ f^o = \begin{cases} A_\pi & \text{for multihoming sellers,} \\ \frac{1}{2} A_\pi & \text{for singlehoming sellers.} \end{cases} \]

We can now compare the expressions for \( f^m \) and \( f^o \) under the assumption that sellers multihome or singlehome under both organizations. We obtain\(^{24}\):

\[ f^m > f^o \iff A_u > A_\pi, \]

which allows us to state the following result.

\(^{23}\) More precisely, the conditions for an interior solution can be written as: \( Z/(8\tau_s) < \tau_s < Z/(6\tau_s) \) with \( Z = \bar{u}^2 + 4\pi\bar{\pi} + \bar{\pi}^2 - 2(\bar{u} + \bar{\pi})C_s - 4\tau_s C_\pi \), and \( \frac{1}{2} (\bar{u} + \bar{\pi} - 2C_s) < \tau_s < \frac{1}{2} (\bar{u} + \bar{\pi} - 2C_s) \). Under these conditions, the equilibrium we describe is unique. In this respect, our model differs from Armstrong and Wright (2007), where multiple equilibria may coexist in the competitive bottleneck case. This difference is due to the fact that Armstrong and Wright assume that sellers view the two platforms as homogeneous (i.e., \( \tau_s \) is set to zero), whereas we consider that they see them as differentiated (i.e., \( \tau_s > 0 \)).

\(^{24}\) The sellers located in \( [1 - \pi/(2\tau_s), 1 - v^*_s / \tau_s] \) and in \( [\pi/(2\tau_s), v^*_s / \tau_s] \) multihome if platforms are open, but singlehome if platforms are strategic. For them, the condition for incentives to innovate to be higher under intermediated trade is more stringent: \( f^m > f^o \iff A_u > (2K + 1)A_\pi \).
Proposition 2. In the competitive bottleneck model in which sellers are on the multihoming side, for-profit trading platforms give stronger investment incentives for sellers if and only if the change of the buyer’s surplus is larger than the change of the seller’s surplus.

Note that the latter condition is more demanding than the corresponding condition in the singlehoming environment, condition (7), provided that profits are increasing in the investment level ($\alpha > 0$). This is due to the fact that the sellers’ surplus can be better extracted by for-profit platforms when the sellers can multihome, but that they are less critical for the overall success of the platform. The intuition is that each platform does not directly react to a change in membership fee charged by the competitor on the multihoming side (i.e., for given $n_i$) so that there are no direct multiplier effects between $M_1^s$ and $M_2^s$. As an equilibrium outcome in our linear model, a larger $\pi$ does not affect sellers’ membership fees if sellers singlehome, but leads to higher membership fees if they multihome. This implies that an increase in investment leads to an increase in the equilibrium membership fee on the seller side, which makes the above inequality more demanding than the corresponding inequality under two-sided singlehoming.

5. Competitive bottlenecks when buyers multihome

We analyze the same model as in the previous section, with the difference that the roles of buyers and sellers are reversed—i.e., sellers singlehome and buyers can multihome. For example, every Sunday morning, there are two flea markets in Brussels. Their locations are sufficiently close for consumers to be able to visit both on the same morning; however, sellers are not mobile and stay put on a single platform. Similarly, owner-managed shops may set up in only one of the shopping areas, but consumers may be able to make it to both areas for their shopping. Such a situation also arises in cases in which sellers sign exclusivity contracts with platforms but where buyers multihome.

We can combine the results of the two previous sections to analyze this situation. Referring to the two-sided singlehoming case, we find the number of products available on platform $i$ to be

$$n_i^s = \frac{1}{2} + \frac{\left( n_i^b - n_i^s \right) \bar{\pi} - \left( M_i^b - M_i^s \right)}{2\tau_s}. \quad (10)$$

On the buyers’ side, let $b_{10}$ (resp. $b_{20}$) denote the buyer who is indifferent between visiting platform 1 (resp. 2) and not visiting any platform:

$$v_b^1 - \tau_b b_{10} = 0 \iff b_{10} = \frac{v_b^1}{\tau_b},$$

$$v_b^2 - \tau_b (1 - b_{20}) = 0 \iff b_{20} = 1 - \frac{v_b^2}{\tau_b}.$$

Assuming that $0 < b_{20} < b_{10} < 1$, we have that $n_i^b = b_{10}$ and $n_i^b = 1 - b_{20}$. It follows that

$$n_i^b - n_i^s = \frac{\left( n_i^b - n_i^s \right) \bar{u} - \left( M_i^b - M_i^s \right)}{\tau_b}.$$

Plugging this expression into (10), using the fact that $n_i^1 = 1 - n_i^s$ and solving for $n_i^s$, we find

$$n_i^s = \frac{1}{2} + \frac{\bar{\pi} \left( M_i^b - M_i^s \right) + \tau_b \left( M_i^b - M_i^s \right)}{2(\tau_b \bar{\pi} + \bar{u} \bar{\pi})}.$$

It follows that

$$n_i^b = \frac{1}{2} + \frac{\bar{u} \left( M_i^b - M_i^s \right) + \tau_b \left( M_i^b - M_i^s \right)}{2(\tau_b \bar{\pi} + \bar{u} \bar{\pi})} - \frac{M_i^b}{\tau_b}.$$  

It is easily seen that the latter two expressions coincide with expressions (8) and (9) if the buyer and seller indices (as well as $\bar{u}$ and $\bar{\pi}$) are reversed. Following the analysis in the previous section, we find

$$M_i^s = M_i^{s+} = C_s + \tau_s - \frac{\bar{u}}{4\tau_b} (3\bar{\pi} + \bar{u} - 2C_b),$$

$$n_i^{s+} = \frac{1}{4\tau_b} (\bar{u} + \bar{\pi} - 2C_b).$$

The equilibrium net surplus of sellers (gross of transportation costs) is equal to:

$$v_i^s = \begin{cases} \frac{1}{4\tau_b} (\bar{u}^2 + \bar{u} (\pi_1 + 3\bar{\pi}) + \bar{u} \bar{\pi} - 2(\bar{u} + \pi_1) C_b) - \tau_s - C_s & \text{if the seller has invested,} \\
\frac{1}{4\tau_b} (\bar{u}^2 + \bar{u} (\pi_0 + 3\bar{\pi}) + \pi_0 \bar{\pi} - 2(\bar{u} + \pi_0) C_b) - \tau_s - C_s & \text{otherwise.} \end{cases}$$
If trade takes place via two open platforms, \( n_b^k = n_b^o = \bar{u}/(2\tau_b) \) and the surplus of sellers is equal to 
\[
\nu^o = \frac{1}{2\tau_b} \pi \bar{u},
\]
where \( \pi = \pi_o \) or \( \pi_0 \) according to whether the seller has invested or not.

Again, we compare seller investment incentives under the two organizations of the trading platforms. Computations are more complicated here as the equilibrium number of buyers visiting each platform depends on the investment level. As a result, the condition for larger investment incentives under intermediated trade depends on the specific values of \( K \) and \( k \) and does not, therefore, simplify as nicely as in the previous two cases. A few lines of computations establish that the difference \( m^o - m^i \) has the same sign as the following expression:
\[
L(K,k,A_n,A_u) = A_n \left( \pi_0 - u_0 - 2C_b \right) K^2 + A_u \left( A_u + 3A_n \right) (2k + 1) + (kA_n(A_n - A_u) - 2A_u C_b) K
+ \left( 3u_0 A_n + \pi_0 A_n + 2u_0 A_n + 2\pi_0 A_n - A_n A_u + A_n^2 \right) K.
\]

Despite the complexity of this expression, we are able to relate its sign to an intuitive condition, which is necessary and sufficient in a particular case, and sufficient in all other cases.

Consider, first, the special case where there is only one seller—i.e., \( K = 1 \). Note that this is equivalent to a market with a finite or infinite number of sellers who all coordinate their investment decision among themselves. With a single seller, \( k = 0 \) before the investment and \( k = 1 \) after. Hence, setting \( k = 0 \) and \( K = 1 \) in the above expression, we find
\[
L(1,0,A_n,A_u) = (A_n + A_u)(2u_0 + A_n + 2\pi_0 + A_n - 2C_b),
\]
where the second bracketed term is positive (otherwise, for-profit platforms would not be able to make a profit). It follows that \( m^o > m^i \Rightarrow A_n + A_u > 0 \). That is, in the single-seller case, trade via for-profit platforms leads to stronger investment incentives than via open platforms if and only if the joint buyer’s and seller’s surplus \( A_n + A_u \) increases, which has to be the case under any potentially welfare-improving increase in the investment level. Note that this condition is less demanding than condition (7). Here, the membership fee on the singlehoming side is substantially lower since platforms compete more fiercely for the singlehoming side. However, the reaction of the membership fee to the level of investment is relevant for investment incentive. Since sellers are critical to the success of a platform, intermediaries are more likely to refrain from increasing the membership fee.

Let us now examine the extent to which the latter condition can be generalized to the cases with \( K \geq 2 \). First, we compute \( L(K,k,A_n,A_u) \) while imposing \( A_u + A_n = 0 \):

\[
L(K,k,A_n,−A_u) = A_n \left( K(1) \pi_0 - u_0 - 2C_b \right) + 2K \left( K(1) - 1 \right) A_n.
\]

For \( A_n > 0 \), a sufficient condition for the latter expression to be positive is \( \pi_0 > u_0 + 2C_b \), which we assume to be the case.\(^{25}\)

Second, we take the derivative of \( L(K,k,A_n,A_u) \) with respect to \( A_u \); we find
\[
\frac{\partial L(K,k,A_n,A_u)}{\partial A_u} = 2K \left( u_0 + \pi_0 - C_b \right) + 2(A_u + A_n)(2k + 1) - (K(1) - 1)(K(2) - K) A_n.
\]

We want to show that, for \( A_u + A_n \geq 0 \), this expression is positive. This holds, if, evaluated at \( A_u + A_n = 0 \), this derivative is positive. We can write
\[
\frac{\partial L(K,k,A_n,A_u)}{\partial A_u} \bigg|_{A_u = −A_n} = 2K(\pi_0 - u_0 - 2C_b)(K(1) - 1)(K(2) - K) A_n,
\]
which is positive if \( A_n \) is sufficiently small.

Third, we take the derivative of \( L(K,k,A_n,A_u) \) with respect to \( A_n \); we find
\[
\frac{\partial L(K,k,A_n,A_u)}{\partial A_n} = K^2 \left( \pi_0 - u_0 - 2C_b \right) + (3u_0 + \pi_0)K + 2K(k+1)(A_u + A_n) - 3(K(1) - 1)(K(2) - K) A_n.
\]

Evaluated at \( A_u + A_n = 0 \), we can write
\[
\frac{\partial L(K,k,A_n,A_u)}{\partial A_n} \bigg|_{A_u = −A_n} = K^2(\pi_0 - u_0 - 2C_b) + (3u_0 + \pi_0)K - 3(K(1) - 1)(K(2) - K) A_n.
\]

With a similar argument as before, we obtain a restriction on \( A_u \) for the derivative to be positive for \( A_u + A_n \geq 0 \).

Thus, we have found conditions such that, at \( A_u + A_n = 0 \), for-profit trading platforms give stronger investment incentives. This also holds for all \( A_u + A_n > 0 \) since \( L \) is increasing in \( A_n \) and \( A_u \) for any \( A_n,A_u \) with \( A_u + A_n \geq 0 \). We can thus write that
\[
A_u + A_n > 0 \Rightarrow m^o > m^i.
\]

We summarize our result in the following proposition.

\(^{25}\) In the example that we develop in the next section, we have that \( u = \pi/(\pi + 1) \), with \( \pi > 0 \). Hence, \( u < \pi \) and the condition is satisfied for \( C_b \) sufficiently small.
Proposition 3. Suppose that $\Delta_x > 0$ and $\Delta_u$ are sufficiently small such that expressions (11) and (12) are positive. In the competitive bottleneck model, in which sellers are on the singlehoming side, for-profit trading platforms give stronger incentives for sellers to innovate if the joint buyer’s and seller’s surplus increases.

6. Applications

In line with most of the literature on two-sided markets, we have not provided a microfoundation of $u$ and $\pi$. To fill the gap, we present a simple parametric model according to which each seller offers a set of independent products—i.e., products that are neither a substitute nor a complement for the other products. Each consumer has independent variable demand for each of the products. Suppose that inverse demand for each product is given by $P(q) = \max(1-q^\alpha,0)$ for $\alpha > 0$. If $\alpha = 1$, demand is linear; if $0 < \alpha < 1$, demand is convex; and if $\alpha > 1$, demand is concave where positive. Suppose that the marginal cost of production is constant and equal to $0 \leq c < 1$.

We consider four specific types of investments: Sellers can invest in order to (1) reduce their marginal cost of production, (2) improve the quality of their product, (3) enhance their ability to price discriminate, or (4) expand demand. For each type of investment, we examine whether incentives to invest are higher under intermediated or non-intermediated trade. At the end of the section, we collect our findings with respect to these investment examples.

To facilitate the analysis, it is useful to summarize the results of Propositions 1 through 3: Investment incentives are stronger under intermediated trade if and only if

- $\Delta_u > 0$ in a market in which both sides singlehome;
- $\Delta_u > \Delta_x$ in a market in which buyers singlehome and sellers can multihome;
- $\Delta_u + \Delta_x > 0$ in a market in which sellers singlehome and buyers can multihome (under the restriction that there is a single seller).

The most simple specification of demand is the linear case. In this case, monopoly pricing implies that $u = \pi/2$ for all investment which may affect costs or demand. Hence, in a market in which both sides singlehome or in which sellers singlehome and buyers can multihome, investment incentives are stronger under intermediated trade if and only if the investment increases the gross profit per buyer, $\Delta_x > 0$. The reverse holds in a market in which buyers singlehome and sellers can multihome. Below, we look at a more flexible specification of demand.

6.1. Cost-reducing R&D

In this example, the investment level determines $c$. Each seller’s decision problem at the last stage (for each of his independent products) reduces to the simple monopoly maximization problem $\max_q \pi(c) = (1-c-q^\alpha)q$. The profit-maximizing quantity is computed as:

$$q = \left( \frac{1-c}{\alpha+1} \right)^{1/\alpha}.$$

We obtain the (per product) firm’s profit and the consumer’s surplus at the profit-maximizing quantity as a function of $c$:

$$\pi(c) = \frac{\alpha(1-c)}{\alpha+1} \left( \frac{1-c}{\alpha+1} \right)^{1/\alpha},$$

$$u(c) = \frac{\alpha(1-c)}{(\alpha+1)^2} \left( \frac{1-c}{\alpha+1} \right)^{1/\alpha} = \frac{1}{\alpha+1} \pi(c).$$

Suppose that there exists a process innovation that allows sellers to decrease the marginal cost of production from $c$ to $c'$, with $0 < c' \leq c$. Both the firm’s profit and the consumer’s surplus increase as a result of the cost reduction; that is, $u(c') - u(c) > 0$ and $\pi(c') - \pi(c) > 0$. It follows that intermediated trade provides higher incentives to invest in cost reduction when both sides singlehome (from Proposition 1), and when sellers singlehome while buyers can multihome (from Proposition 3). As for the situation where buyers singlehome and sellers can multihome, incentives are higher under intermediated trade if and only if (from Proposition 2):

$$u(c') - u(c) > \pi(c') - \pi(c) \iff \frac{1}{\alpha+1} (\pi(c') - \pi(c)) > \pi(c') - \pi(c),$$

which is never true as, by assumption, $\alpha > 0$. It follows that, in the case where buyers singlehome and sellers can multihome, incentives to invest in cost reduction are lower under intermediated trade.
6.2. Quality-improving R&D

By simply relabeling variables, we can replicate the analysis for a type of quality-improving R&D. Suppose that there exists a product innovation that shifts the inverse demand curve outward; namely, consider quality \( s \geq 0 \) with \( P(q) = 1 + s - q^2 \). Marginal costs here are set equal to zero. Then, the profit maximization problem of each monopoly seller becomes: \( \max_q \pi = (1 + s - q^2)q \), which is made equivalent to the above analysis by simply substituting \(-c\) for \( s \). The results of the previous subsection, therefore, carry over.

6.3. Investment in price discrimination

The third type of investment we consider consists of data-collection activities about consumers’ willingness to pay. This investment allows sellers to practice some form of price discrimination and, thereby, to capture a share \( 0 < \beta < 1 \) of the consumer surplus at a given price \( p \). We can think of each seller setting a two-part tariff of the form: \( T(p,q) = \beta CS(p) + pq \), where \( CS(p) \) is the consumer surplus at price \( p \). A few lines of computation establish that

\[
CS(p) = \frac{p^\alpha}{\alpha+1}(1-p)^{\alpha+1/2}.
\]

The investment allows sellers to capture a larger share of the consumer surplus (i.e., to increase \( \beta \)). For a given value of \( \beta \), the seller chooses \( p \) so as to maximize (to ease the computations, we set the marginal cost to zero):

\[
\max_{p} \pi(p,\beta) = p(1-p)^{1/2} + \beta p \frac{\alpha}{\alpha+1}(1-p)^{\alpha+1/2}.
\]

The profit-maximizing price is easily found as

\[
p^* (\beta) = \frac{\alpha(1-\beta)}{1+\alpha(1-\beta)}.
\]

We compute the (per product) net gain from trade for each seller and for each buyer respectively as:

\[
\pi(\beta) = \beta CS(p^* (\beta)) + p^*(\beta)(1-p^*(\beta))^{1/\alpha} = \frac{\alpha}{\alpha+1} \left( \frac{1}{1+\alpha(1-\beta)} \right)^{1/\alpha},
\]

\[
u(\beta) = (1-\beta)CS(p^* (\beta)) = (1-\beta) \frac{\alpha}{\alpha+1} \left( \frac{1}{1+\alpha(1-\beta)} \right)^{(\alpha+1)/\alpha}.
\]

Observe that \( \pi(\beta) \) increases and \( \nu(\beta) \) decreases with \( \beta \):

\[
\frac{d}{d\beta} \nu(\beta) = -\frac{\beta \alpha^2}{\alpha+1} \left( \frac{1}{1+\alpha(1-\beta)} \right)^{(2\alpha+1)/\alpha} < 0.
\]

Therefore, \( \nu(\beta) - \nu(\beta') < 0 < \pi(\beta') - \pi(\beta) \), which implies, using Propositions 1 and 2, that intermediated trade provides lower incentives to invest in price discrimination when both sides singlehome and when buyers singlehome while sellers can multihome. As for the third case (sellers singlehome, buyers can multihome), we observe that total surplus increases with \( \beta \):

\[
\pi(\beta) + \nu(\beta) = \frac{\alpha}{\alpha+1} \left( \frac{1}{1+\alpha(1-\beta)} \right)^{1/\alpha} \left( \frac{2-\beta+\alpha(1-\beta)}{1+\alpha(1-\beta)} \right),
\]

\[
\frac{d}{d\beta} (\pi(\beta) + \nu(\beta)) = \frac{\alpha(1-\beta)}{(1+\alpha(1-\beta))^2} \left( \frac{1}{1+\alpha(1-\beta)} \right)^{1/\alpha} > 0.
\]

Applying Proposition 3, we have that, here, intermediated trade provides higher seller investment incentives.

6.4. Investment in demand expansion

In this simple model, we model demand expansion as an increase in \( \alpha \). Note that the parameter \( \alpha \) also determines the curvature of the demand function given by \( q = (1-p)^{1/\alpha} \). Here, a higher \( \alpha \) not only increases the average willingness-to-pay per unit but, as we will see, also affects the rent distribution between consumers and sellers in the profit-maximizing solution. We obtain that

\[
\frac{d\alpha}{d\alpha} = -\frac{1}{2\alpha}(1-p)^{1/\alpha} \ln(1-p) > 0,
\]

which means that at a given price \( p \), the quantity demanded increases as \( \alpha \) increases.

\[\text{We check that } p^*(0) = \alpha/(1+\alpha) \text{ (profit-maximizing uniform price) and } p^*(1) = 0 \text{ (the variable part of the tariff is equal to the marginal cost in the case of perfect price discrimination).}\]
Suppose that the sellers’ investment has the effect of increasing $\alpha$ from $\alpha_0$ to $\alpha_1 = \alpha_0 + A_2$. If $A_2$ is small, the impact of the investment on the firm’s profit and the consumer surplus can be approximated by $\pi'(\alpha)A_2$ and $u'(\alpha)A_2$, respectively. To simplify the exposition, let us fix again $c=0$. We compute:

\[
\pi'(\alpha) = \frac{1}{\alpha(\alpha+1)} \left( \frac{1}{\alpha+1} \right)^{1/2} \ln(\alpha+1) > 0,
\]

\[
\frac{1}{\alpha(\alpha+1)^2} \left( \frac{1}{\alpha+1} \right)^{1/2} [(\alpha+1)\ln(\alpha+1) - \alpha^2] > 0 \text{ if and only if } \alpha < 1.537
\]

\[
\pi'(\alpha) - u'(\alpha) = \frac{1}{\alpha(\alpha+1)^2} \left( \frac{1}{\alpha+1} \right)^{1/2} [(\alpha+1)\ln(\alpha+1)+\alpha] > 0,
\]

\[
\pi'(\alpha) + u'(\alpha) = \frac{1}{\alpha(\alpha+1)^2} \left( \frac{1}{\alpha+1} \right)^{1/2} [(\alpha+2)(\alpha+1)\ln(\alpha+1) - \alpha^2] > 0.
\]

We observe that the consumer surplus has a U-inverted shape with respect to $\alpha$. Hence, for small values of $\alpha$ (namely, for $\alpha < 1.537$), the consumer surplus increases in $\alpha$, meaning that investment incentives are higher under intermediated trade when both sides singlehome; the opposite result prevails for larger values of $\alpha$ (namely, for $\alpha > 1.537$). Hence, at a large $\alpha_0$, buyers obtain consumer surplus $u(\alpha_0)$ with $u'(\alpha_0) < 0$. Then, a larger $\alpha_1$ reduces $u$ and makes platforms increase the sellers membership fees $M_s^p$ in response.

We also observe that $\pi'(\alpha) - u'(\alpha) > 0$ and $\pi'(\alpha) + u'(\alpha) > 0$ for all values of $\alpha$. The former inequality implies that investment incentives are lower under intermediated trade when sellers can multihome; the latter implies that investment incentives are higher under intermediated trade when buyers can multihome.

6.5. Summary

We collect the results of the four applications in the following remark.

**Remark 2.** Whether investment incentives are stronger under intermediated or non-intermediated trade depends on which side of the market singlehomes and on the nature of the investment. In our parametric specification, results are summarized by the following table (where “+” stands for stronger seller investment incentives if platforms are for-profit).

<table>
<thead>
<tr>
<th></th>
<th>Cost reduction</th>
<th>Quality improvement</th>
<th>Price discrimination</th>
<th>Demand expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sellers multihome buyers singlehome</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Both sides singlehome</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+/–*</td>
</tr>
<tr>
<td>Sellers singlehome buyers multihome</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

* “+” if demand sufficiently convex (“–” otherwise)

7. Conclusion

In this paper, we have analyzed whether and how the fact that products are not sold on open platforms, but on competing for-profit platforms, affects sellers’ investment incentives. Investments may take the form of cost reductions, quality improvements, or marketing measures aimed at capturing a larger share of consumer surplus or at expanding demand. We show that, in general, the trading environment is not neutral to such investment incentives because the trading environment affects the distribution (and level) of total surplus that is attained by buyers and sellers. In the trading environment with for-profit platforms, sellers can strategically use their investment strategy to affect buyers’ and sellers’ gains from trade, which will affect platform pricing.

We build a model with many manufacturers and consumers and two competing intermediaries who charge membership or access fees on both sides of the market. We compare this situation to an environment in which manufacturers and consumers have free access to platforms. Clearly, the presence of for-profit intermediaries reduces the available rents in the market. Therefore, one might suspect that sellers have weaker investment incentives with competing for-profit platforms. However, this is not necessarily the case. The reason is that investment incentives affect the size of the network effects and, thus, competition between intermediaries.

In particular, we show that the relative strength of investment incentives depends on which side of the market singlehomes and on the nature of the investment. For instance, if both sides singlehome, incentives to invest in cost reduction are stronger with competing for-profit platforms, whereas incentives to invest in consumer targeting (that improve the possibility of price discrimination, for example) are weaker. Our results are relevant for the debate on...
innovation policy. For instance, a large part of the discussion around the protection of intellectual property focuses exclusively on seller surplus. However, as we have shown, innovators may also benefit from higher consumer surplus, as they internalize some of its improvements in the form of lower access fees.

In an earlier version of this paper, we showed that similar results obtain in a setting with a continuum of ex ante heterogeneous firms and coordinated investment incentives. Future research may want to consider a number of extensions. First, our analysis allows only for access fees—platforms do not charge for usage. In many real-world examples, platforms also charge for usage on at least one side of the market (e.g., video-game platforms receive royalties from game developers for each game sold). Unfortunately, a meaningful analysis of platforms that charge for both usage and membership would be required—in line with one strand of the two-sided market literature, we decided to concentrate on access fees.\(^{27}\)

Second, we have assumed that whether one side of the market single- or multihomes is exogenously given. In some markets, this may be technologically given. However, in other markets, some buyers and sellers singlehome, whereas others multihome. As Section 2 clarified, we have looked at three extreme situations. As pointed out above, one may want to generalize our analysis to cases where only a share of buyers or of sellers has the possibility to multihome. In addition, single- versus multihoming may be endogenously determined. Future work may want to study this issue.\(^{28}\)

Third, the scope of our analysis is limited by our assumption that sellers take their pricing decisions independently of one another. In an earlier version of this paper, we showed that our results carry over to a situation of imperfect competition among sellers captured by a negative direct network effect among sellers. This can be interpreted as a congestion effect on the platform. In our specification, sellers’ profits decrease linearly with the number of sellers present on a platform, but, as we can show, pricing decisions remain independent. A more general approach would be to consider strategic interaction in pricing decisions (e.g., that sellers produce imperfectly differentiated products). However, such interaction makes the platform choice game much more complex to solve than in the present setting and cannot be incorporated into our linear model. We leave this for future research.

While the precise comparison of investment incentives under the two governance structures (for-profit versus open) of platforms is due partly to our linear specification, the general insight of the paper is that profit-maximizing intermediaries adjust their access fees strategically to seller investments, with the effect that for-profit intermediation may provide stronger investment incentives to sellers. This insight is not restricted to a setup with competing intermediaries. With respect to the number of intermediaries, we note that, also in the monopoly model, the intermediary adjusts prices to the strength of indirect network effects. Further work might focus on the role of competition between intermediaries for seller investment incentives.

This paper is a first step in analyzing seller investment in platform markets. Future work may want to focus, in particular, on market asymmetries. For instance, it would be interesting to extend our analysis to the case in which one of the two platforms is for-profit. Furthermore, platforms may be asymmetric in the sense that they offer different platform qualities. How do these asymmetries between platforms affect investment incentives? Also, sellers in our setting are ex ante symmetric. It is conceptually straightforward to include heterogeneous sellers who take different investment decisions. However, with asymmetric platforms, it would be interesting to analyze the sorting of sellers and the implications for investment decisions.

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Appendix. Unique and stable equilibrium at stage 2

We show here that the equilibrium we derived at stage 2 of the two-sided singlehoming game is unique and stable. From the identification of the buyer and the seller who are indifferent between the two platforms, we defined the number of buyers visiting platform \( i \) as a function of the number of sellers visiting (more precisely, of products being sold on) this platform, and vice versa:

\[
\begin{align*}
\tilde n_i^b (n_i^s) &= \frac{1}{2} + \frac{1}{2\tau_b} \left[ (2n_i^s - 1)h - M_i^b - M_i^b \right], \\
\tilde n_i^s (n_i^b) &= \frac{1}{2} + \frac{1}{2\tau_s} \left[ (2n_i^b - 1)\hat s - M_i^s - M_i^s \right].
\end{align*}
\]

\(^{27}\) For a discussion of the use of different price instruments, see Rochet and Tirole (2006). Also, timing issues are likely to affect equilibrium prices.

\(^{28}\) Armstrong and Wright (2007) provide such an analysis with exogenous investment levels.
The solution to this system of equation was given by expressions (5) and (6):

\[
\begin{align*}
\nu_b^i &= \frac{1}{2} + \frac{\tilde{u} (M_b^i - M_0^i) + \tau_b (M_b^i - M_b^0)}{2(\tau_b \tau_s - \bar{u} \pi)}, \\
\nu_s^i &= \frac{1}{2} + \frac{\pi (M_b^i - M_b^0) + \tau_s (M_s^i - M_b^0)}{2(\tau_b \tau_s - \bar{u} \pi)}.
\end{align*}
\]

Assuming that \(\tau_b \tau_s > \bar{u} \pi\), we have that \(n_b^i, n_s^i > 0\). To have an interior solution, we further need \(n_b^i, n_s^i < 1\). This is so provided that

\[
\begin{align*}
\tilde{u} (M_b^i - M_0^i) + \tau_b (M_b^i - M_b^0) < \tau_b \tau_s - \bar{u} \pi, \\
\pi (M_b^i - M_b^0) + \tau_s (M_s^i - M_b^0) < \tau_b \tau_s - \bar{u} \pi.
\end{align*}
\]

Uniqueness. Let us examine if there could be another equilibrium under these conditions. The only candidates are the situations where all agents on one and/or on the other side of the market concentrate on the same platform. Consider the situation where all buyers and all sellers visit platform 1. For this to be an equilibrium, the buyer and the seller located at 1 must prefer platform 1 over platform 2, expecting that all agents will be on platform 1. That is, the following two conditions must be met: (B) \(v_b^i - \tau_b \geq \bar{u} \geq M_b^1 - M_b^0 + \tau_b\), and (S) \(v_s^i - \tau_s \geq \bar{u}\) with and without investment, which is equivalent to \(\min(n_0, n_1) \geq M_b^1 - M_b^0 + \tau_b\). Multiplying both sides of condition (S) by \(\bar{u}\), we get \(\bar{u} \min(n, n_1) \geq \bar{u} (M_b^1 - M_b^0) + \tau_s\). Using condition (B), we further have that \(\bar{u} \pi \geq \bar{u} (M_b^1 - M_b^0) + \tau_s (M_s^0 - M_s^0) + \tau_b \tau_s\), which can be rewritten as \(\tau_b \tau_s - \bar{u} \pi \leq \bar{u} (M_b^1 - M_b^0) + \tau_s (M_s^0 - M_s^0)\). It is easily seen that the latter condition contradicts the first condition in (13). This proves that we cannot have an equilibrium where all agents concentrate on the same platform when the conditions in (13) are satisfied. Similar arguments can be used to show that conditions (13) also exclude situations where only one side of the market concentrates on the same platform.

Stability. We show now that expressions (5) and (6) also define a stable market configuration. Graphically, in a \((n_b^i, n_s^i)\) plane, the line \(n_b^i(n_s^i)\) must cross the line \(n_b^i(n_s^i)\) from below. Analytically, following Hagiu (2006b), we justify this by postulating a dynamic adjustment process for fixed \((M_b, M_s, M_b^0, M_s^0)\) of the following type: starting from any \((n_b^0, n_b^0))\), the market configuration \((n_b^i(t), n_s^i(t))\) evolves according to

\[
\begin{align*}
\left( \begin{array}{c}
\nu_b^i(t) \\
\nu_s^i(t)
\end{array} \right) &= \left( \begin{array}{c}
E \left( \frac{1}{2} + \frac{1}{2 \tau_s} \right) \left(2n_b^i(t) - 1\right) \pi - (M_b^i - M_0^i) \\
F \left( \frac{1}{2} + \frac{1}{2 \tau_b} \right) \left(2n_s^i(t) - 1\right) \bar{u} - (M_s^i - M_b^0)
\end{array} \right),
\end{align*}
\]

where \(E\) and \(F\) are two positive constants. Fig. 1 depicts the phase diagram of this process (in the case where \(M_b^0 = M_s^0\) and \(M_b^0 = M_b^0\)). It is easily seen that the assumption \(\tau_b \tau_s > \bar{u} \pi\) guarantees that the process converges to \((\tilde{n}_b^i, \tilde{n}_b^i)\), defined by expressions (5) and (6).

References