Identification of Port-Hamiltonian Systems from Frequency Response Data

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Abstract

In this paper, we study the identification problem of a passive system from tangential interpolation data. We present a simple construction approach based on the Mayo-Antoulas generalized realization theory that automatically yields a port-Hamiltonian realization for every strictly passive system with simple spectral zeros. Furthermore, we discuss the construction of a frequency-limited port-Hamiltonian realization. We illustrate the proposed method by means of several examples.

Keywords: Passive systems, port-Hamiltonian system, identification, tangential interpolation

1. Introduction

We study linear and finite-dimensional dynamical systems that are *passive*. We look at port-Hamiltonian realizations of such transfer functions which play an important role in the robustness of passive systems. We consider continuous-time systems that can be represented in the standard state-space form with real coefficients and real inputs, outputs and states:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \qquad x(0) = 0, \\ y(t) &= Cx(t) + Du(t). \end{aligned} \tag{1}$$

Denoting real and complex n-vectors ($n \times m$ matrices) by $\mathbb{R}^n$, $\mathbb{C}^n$ ($\mathbb{R}^{n\times m}$, $\mathbb{C}^{n\times m}$), respectively, then $u : \mathbb{R} \to \mathbb{R}^m$, $x : \mathbb{R} \to \mathbb{R}^n$, and $y : \mathbb{R} \to \mathbb{R}^m$ are vector-valued functions denoting the *input*, *state*, and *output* of the system, and the coefficient matrices satisfy $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{m\times m}$. The Hermitian (or conjugate) transpose of a vector or matrix V is denoted by V^{H} $\left(V^{\mathsf{T}}\right)$ and the identity matrix is denoted by I_n or I if the dimension is clear. We furthermore require that input and output dimensions are equal to m since we aim to interpolate with (square) passive transfer functions. We denote the set of symmetric matrices in $\mathbb{R}^{n\times n}$ by $\mathbb{S}_n$. Positive definiteness (semi-definiteness) of $M \in \mathbb{S}_n$ is denoted by $M \succ 0$ ($M \succeq 0$).

Model-order reduction for passive systems has been an active research area and has been investigated by several researchers in e. g., [1, 2, 3, 4, 5, 6, 7]. However, this requires the availability of system matrices, which may not be easily available, especially when the necessary parameters to model a dynamical process are not known. Hence, we aim at identifying system realizations using frequency response. The structure of the paper is as follows. In Section 2, we briefly recall some important properties of passive systems. Subsequently, in Section 3, we discuss state-space representations and properties of port-Hamiltonian realizations. This is followed by a discussion of degrees of freedom of a port-Hamiltonian system in the subsequent section in order to have an understanding how many parameters are needed to describe a minimal port-Hamiltonian system. In Section 5, we propose a variant of the Loewner-based approach, realizing the system in port-Hamiltonian form when data are available at spectral zeros along with zero directions. Furthermore, we discuss the estimation of the dominant spectral zeros and zero directions using the data given on the imaginary axis in Section 6. In Section 7, we illustrate the proposed identification approach by means of a couple of numerical examples, which is followed by a short summary.

2. Passive Systems and Port-Hamiltonian Realizations

Passive systems and their relationships with *positive-realness and stability conditions* are well studied. We briefly recall some important properties by following [8], and refer to the literature for a more detailed survey. We consider continuous-time systems with a real rational transfer matrix $Z(s)$ and define the associated spectral density function:

$$\Phi(s) := Z^{\mathsf{T}}(-s) + Z(s), \tag{2}$$

which coincides with the Hermitian part of $Z(s)$ on the $\imath\omega$ axis:

$$\Phi(\imath\omega) = [Z(\imath\omega)]^{\mathsf{H}} + Z(\imath\omega).$$

Definition 2.1. *The rational transfer function $Z(s)$ is called* strictly positive-real *if $\Phi(\imath\omega) \succ 0$ for all $\omega \in \mathbb{R}$ and it is called* positive-real *if $\Phi(\imath\omega) \succeq 0$ for all $\omega \in \mathbb{R}$.*

The transfer function $Z(s)$ is called asymptotically stable *if the poles of the transfer function are in the open left half-plane, and it is called* stable *if all the poles are in the closed left half-plane, with any pole occurring on the imaginary axis being first-order.*

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The transfer function $Z(s)$ is called strictly passive *if it is strictly positive-real and asymptotically stable and it is called* passive *if it is positive-real and stable.*

We will assume throughout the paper that this realization is minimal (i. e. controllable and observable) and will restrict ourselves in this paper to *strictly* passive systems, which implies that the matrix A is invertible and that the transfer matrix is proper since poles cannot be on the imaginary axis or at infinity. Moreover, $\Phi(\imath\omega) \succ 0$ at $\omega = \infty$, implies that we must have $D^{\mathsf{T}} + D \succ 0$ as well. We will see that this restriction simplifies our discussion significantly. It is also a reasonable restriction because passive systems can be viewed as limiting cases of strictly passive systems.

Since the transfer function is proper, we can represent it in standard state-space form $Z(s) = C(sI_n - A)^{-1}B + D$. For proper transfer functions $Z(s)$ with minimal realization $\mathcal{M} := \{A, B, C, D\}$, there is a necessary and sufficient condition for passivity, known as the Kalman-Yakubovich-Popov linear matrix inequality. An elegant proof of this can be found in [8].

Theorem 2.1. *Let $\mathcal{M} := \{A, B, C, D\}$ be a minimal realization of a proper rational transfer function $Z(s)$ and let*

$$W(X, \mathcal{M}) = \begin{bmatrix} -A^{\mathsf{T}}X - XA & C^{\mathsf{T}} - XB \\ C - B^{\mathsf{T}}X & D + D^{\mathsf{T}} \end{bmatrix}. \tag{3}$$

Then, $Z(s)$ is passive if and only if there exists a real symmetric matrix $X \in \mathbb{S}_n$ such that

$$W(X, \mathcal{M}) \succeq 0, \quad X \succ 0, \tag{4}$$

and is strictly passive if and only if there exists a real symmetric matrix $X \in \mathbb{S}_n$ such that

$$W(X, \mathcal{M}) \succ 0, \quad X \succ 0. \tag{5}$$

The solutions X of these inequalities are known as *certificates* for the passivity or strict passivity of the system $\mathcal{M}$.

Definition 2.2. *Every solution X of the LMI*

$$\mathbb{X}^{\succeq} := \{X \in \mathcal{S} \,|\, W(X, \mathcal{M}) \succeq 0,\ X \succ 0\} \tag{6}$$

is called a certificate *for the passivity of the model $\mathcal{M}$ and every solution of the LMI*

$$\mathbb{X}^{\succ} := \{X \in \mathcal{S} \,|\, W(X, \mathcal{M}) \succ 0,\ X \succ 0\} \tag{7}$$

is called a certificate *for the strict passivity of the model $\mathcal{M}$.*

3. Port-Hamiltonian Models

In this section, we provide a brief introduction to special realizations of passive systems, known as port-Hamiltonian system models.

Definition 3.1. *A linear time-invariant* port-Hamiltonian system model *of a proper transfer function, has the standard state-space form*

$$\begin{aligned} \dot{x}(t) &= (J - R)Qx(t) + (G - P)u(t), \quad x(0) = 0, \\ y(t) &= (G + P)^{\mathsf{T}}Qx(t) + (N + S)u(t), \end{aligned} \tag{8}$$

where the system matrices

$$\mathcal{V} := \begin{bmatrix} J & G \\ -G^{\mathsf{T}} & N \end{bmatrix}, \quad \mathcal{W} := \begin{bmatrix} R & P \\ P^{\mathsf{T}} & S \end{bmatrix}, \tag{9}$$

satisfy the conditions

$$\mathcal{V} = -\mathcal{V}^{\mathsf{T}}, \quad \mathcal{W} = \mathcal{W}^{\mathsf{T}} \succeq 0, \quad Q = Q^{\mathsf{T}} \succeq 0.$$

It readily follows from the properties of port-Hamiltonian models that when Q and $\mathcal{W}$ are invertible, we can choose $X = Q$ as certificate for the model

$$\mathcal{M} := \left\{(J - R)Q, G - P, (G + P)^{\mathsf{T}}Q, N + S\right\}$$

to show that it satisfies the strict passivity condition (5).

Remark 3.1. *The condition that Q is non-singular is automatically satisfied when the state transition matrix A is non-singular, which is the case for strictly passive systems. We can then also represent the system in generalized state-space form, using $\widehat{x} := Qx$, yielding:*

$$\begin{aligned} Q^{-1}\dot{\widehat{x}} &= (J - R)\widehat{x} + (G - P)u, \\ y &= (G + P)^{\mathsf{T}}\widehat{x} + (N + S)u. \end{aligned} \tag{10}$$

We use such models for representing intermediate results later on.

Conversely, let $\mathcal{M} := \{A, B, C, D\}$ be a state-space model, satisfying the strict passivity condition (5) with a given certificate $X \succ 0$. Then, it can always be transformed in the port-Hamiltonian form, as shown in [9]. We can use a symmetric factorization $X = T^{\mathsf{T}}T$, which implies the invertibility of T, and define a new realization

$$\{A_T, B_T, C_T, D\} := \{TAT^{-1}, TB, CT^{-1}, D\}$$

so that

$$\begin{aligned} &\begin{bmatrix} T^{-\mathsf{T}} & 0 \\ 0 & I_m \end{bmatrix} \begin{bmatrix} -A^{\mathsf{T}}X - XA & C^{\mathsf{T}} - XB \\ C - B^{\mathsf{T}}X & D^{\mathsf{T}} + D \end{bmatrix} \begin{bmatrix} T^{-1} & 0 \\ 0 & I_m \end{bmatrix} \\ &= \begin{bmatrix} -A_T & -B_T \\ C_T & D \end{bmatrix} + \begin{bmatrix} -A_T^{\mathsf{T}} & C_T^{\mathsf{T}} \\ -B_T^{\mathsf{T}} & D^{\mathsf{T}} \end{bmatrix} \succ 0. \end{aligned} \tag{11}$$

We can then use the symmetric and skew-symmetric part of the matrix

$$\mathcal{S} := \begin{bmatrix} -A_T & -B_T \\ C_T & D \end{bmatrix}$$

to define the coefficients of a port-Hamiltonian representation via

$$\mathcal{V} := \begin{bmatrix} J & G \\ -G^{\mathsf{T}} & N \end{bmatrix} := \frac{\mathcal{S} - \mathcal{S}^{\mathsf{T}}}{2}, \; \mathcal{W} := \begin{bmatrix} R & P \\ P^{\mathsf{T}} & S \end{bmatrix} := \frac{\mathcal{S} + \mathcal{S}^{\mathsf{T}}}{2}.$$

This construction yields $\mathcal{W} \succ 0$ and $Q = I_n$ because of the chosen factorization $X = T^{\mathsf{T}}T$. This is called a *normalized* port-Hamiltonian representation. This shows that port-Hamiltonian models with strict inequalities $Q \succ 0$ and $\mathcal{W} \succ 0$ are nothing but strictly passive systems described in an appropriate coordinate system. On the other hand, it was shown in [10] that normalized port-Hamiltonian systems have good robustness properties in terms of their so-called passivity radius.

4. Degrees of Freedom of a Transfer Function

In the literature, one can find a discussion of the degrees of freedom of a given strictly proper rational transfer function $Z(s)$ of a given MacMillan degree n [11]. This corresponds to the minimum number of parameters to describe such a function. Since this literature is quite opaque, we briefly re-derive the basic results using a generic $m \times m$ strictly proper transfer matrix of MacMillan degree n without repeated poles. Such a transfer function can be written in its partial fraction expansion as follows:

$$Z(s) = \sum_{k=1}^{n_r} u_k (s-\lambda_k)^{-1} v_k^\mathsf{T} + \sum_{k=1}^{(n-n_r)/2} U_k \left(sI_2 - \begin{bmatrix} \alpha_k & \beta_k \\ -\beta_k & \alpha_k \end{bmatrix} \right)^{-1} V_k^\mathsf{T},$$

which requires a total of $2(m+1)n$ real parameters. This can be seen as a state-space model in the real Jordan form with 1×1 diagonal elements for the n_r real poles and 2×2 diagonal blocks for the $n_c := n - n_r$ complex conjugate complex poles. But this representation is only unique up to a block diagonal state-space transformation with exactly m degrees of freedom: a scalar t_k for each real pole and a 2×2 block $t_k \begin{bmatrix} c_k & s_k \\ -s_k & c_k \end{bmatrix}$ for each complex conjugate pair, where the real rotation matrix depends on one real parameter. When taking the quotient of the manifold of block-diagonal models with respect to this state space transformation, we are left with the exact number of real degrees of freedom, which is $2mn$ for a strictly proper $m \times m$ transfer function of degree n with real coefficients.

When considering the larger class of real $m \times m$ *proper* rational transfer functions, one has to add the real parameters to *realize* the constant matrix D. If D is constrained to have a particular rank, then we again need to take that into account. A rank r matrix D can be represented by a rank factorization $D = UV^\mathsf{T}$ where we can again quotient out the degrees of freedom of an $r \times r$ factor T in an equivalent factorization $D = (UT)(T^{-1}V^\mathsf{T})$. Such a factor can thus be represented by $r(2m-r)$ degrees of freedom, which has to be added to those of the strictly proper part of $Z(s)$.

To summarize, a real rational $m \times m$ transfer function $Z(s)$ of MacMillan degree n has

- $2mn$ real degrees of freedom when $Z(s)$ is strictly proper,
- $2m(n+r)-r^2$ real degrees of freedom when $Z(s)$ is proper and $Z(\infty)$ has rank r.

This count of the number of degrees of freedom will determine the number of parameters we can assign using tangential interpolation conditions. For a rigorous discussion on these aspects, we refer to [11].

5. Loewner Approach for Identification of a port-Hamiltonian Realization

In this section, we discuss the identification problem of a strictly passive transfer function $Z(s)$ of degree n, which is defined via a set of left and right interpolation conditions. Since $Z(s)$ is strictly passive, it is proper and has a standard state-space realization $\{A, B, C, D\}$ with D of full rank and positive-real (i. e. $D + D^\mathsf{T} \succ 0$). We can then define the transfer function $Z(s)$ via a set of left and right tangential interpolation conditions

$$v_j := \ell_j Z(\mu_j), \quad w_j := Z(\lambda_j) r_j, \quad j = 1, \ldots, n, \quad Z(\infty) = D, \tag{12}$$

where (μ_j, ℓ_j, v_j), and (λ_j, r_j, w_j), $j = 1, \ldots, n$, are sets of self-conjugate left and right interpolation conditions with $\{\ell_j, v_j\} \in \mathbb{C}^{1\times m}$, $\{r_j, w_j\} \in \mathbb{C}^{m \times 1}$, $\{\lambda_j, \mu_j\} \in \mathbb{C}$. Then, the so-called *Loewner* and *shifted Loewner* matrices defined in [12] have dimensions $n \times n$ and look like

$$\mathbb{L} := \begin{bmatrix} \frac{\ell_1 w_1 - v_1 r_1}{\lambda_1 - \mu_1} & \cdots & \frac{\ell_1 w_n - v_1 r_n}{\lambda_n - \mu_1} \\ \vdots & \ddots & \vdots \\ \frac{\ell_n w_1 - v_n r_1}{\lambda_1 - \mu_n} & \cdots & \frac{\ell_n w_n - v_n r_n}{\lambda_n - \mu_n} \end{bmatrix}, \tag{13a}$$

$$\mathbb{L}_\sigma := \begin{bmatrix} \frac{\lambda_1 \ell_1 w_1 - \mu_1 v_1 r_1}{\lambda_1 - \mu_1} & \cdots & \frac{\lambda_n \ell_1 w_n - \mu_1 v_1 r_n}{\lambda_n - \mu_1} \\ \vdots & \ddots & \vdots \\ \frac{\lambda_1 \ell_n w_1 - \mu_n v_n r_1}{\lambda_1 - \mu_n} & \cdots & \frac{\lambda_n \ell_n w_n - \mu_n v_n r_n}{\lambda_n - \mu_n} \end{bmatrix}. \tag{13b}$$

They satisfy the following Sylvester equations

$$\mathbb{L}\Lambda - M\mathbb{L} = LW - VR, \quad \mathbb{L}_\sigma \Lambda - M \mathbb{L}_\sigma = LW\Lambda - MVR, \tag{14}$$

where we used the definitions

$$L := \begin{bmatrix} \ell_1 \\ \vdots \\ \ell_n \end{bmatrix}, \quad V := \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \quad M := \operatorname{diag}(\mu_1, \ldots, \mu_n), \tag{15}$$

and

$$R := \begin{bmatrix} r_1, \ldots, r_n \end{bmatrix}, \quad W := \begin{bmatrix} w_1, \ldots, w_n \end{bmatrix}, \quad \Lambda := \operatorname{diag}(\lambda_1, \ldots, \lambda_n). \tag{16}$$

The following interpolation result follows from the theory developed in [13] in the special case that the Loewner matrix $\mathbb{L}$ is invertible.

Theorem 5.1. *Let $Z(s)$ be a proper transfer function of MacMillan degree n, then the interpolation conditions* (12) *uniquely define $Z(s)$ if the Loewner matrix $\mathbb{L}$ is invertible. Moreover, a minimal generalized state-space realization is then given by*

$$Z(s) = (W - DR)(\mathbb{L}_\sigma - LDR - s\mathbb{L})^{-1}(V - LD) + D$$

and the corresponding system matrix is given by

$$\left[\begin{array}{c|c} A - sE & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c} \mathbb{L}_\sigma - s\mathbb{L} & V \\ \hline -W & 0 \end{array}\right] + \left[\begin{array}{c} -L \\ \hline I_m \end{array}\right] D \left[\begin{array}{c|c} R & I_m \end{array}\right].$$

Let us now apply this to the special case where the interpolation points are the so-called spectral zeros of $Z(s)$.

Definition 5.1. *Let $Z(s)$ be a real and strictly passive transfer function of MacMillan degree n with associated spectral density function $\Phi(s) := Z^\mathsf{T}(-s) + Z(s)$. Then the spectral zeros and zero directions of $Z(s)$ are the pairs (s_j, r_j) such that $\Phi(s_j) r_j = 0$.*

When the zeros are distinct (which is generic), there are exactly n zeros in the open right half-plane and n zeros in the open left half-plane because the spectral density function $\Phi(s)$ has degree $2n$ and has no zeros on the imaginary axis. The definition of the spectral zeros implies

$$\Phi(s_j)r_j = Z^{\mathsf{T}}(-s_j)r_j + Z(s_j)r_j = 0,$$

and hence

$$w_j := Z(s_j)r_j \iff Z^{\mathsf{T}}(-s_j)r_j = -w_j.$$

Since the spectral zeros and zero directions (s_j, r_j) form a self-conjugate set, we can distinguish two cases for these equations, depending on the condition that s_j is a real zero or not. In the real case, we have

$$s_j \in \mathbb{R}: \quad Z(s_j)r_j = w_j \iff r_j^{\mathsf{T}} Z(-s_j) = -w_j^{\mathsf{T}},$$

and in the complex case, we have

$$s_j \notin \mathbb{R}: \quad \begin{cases} Z(s_j)r_j = w_j & \iff \quad r_j^{\mathsf{H}} Z(-\overline{s}_j) = -w_j^{\mathsf{H}}, \\ Z(\overline{s}_j)\overline{r}_j = \overline{w}_j & \iff \quad r_j^{\mathsf{T}} Z(-s_j) = -w_j^{\mathsf{T}}. \end{cases}$$

Therefore, if we define $\lambda_j,\ j = 1, \ldots, n$, to be the spectral zeros of $Z(s)$ in the open right half-plane,

$$\Re\lambda_j \geq 0, \quad Z(\lambda_j)r_j = w_j,\ j = 1, \ldots, n,$$

then the set of right tangential conditions (λ_j, r_j, w_j) is self-conjugate. Moreover, to every right tangential condition $Z(\lambda_j)r_j = w_j$ (and its complex conjugate when λ_j is complex), there is a corresponding left tangential condition

$$r_j^{\mathsf{H}} Z(-\overline{\lambda}_j) = -w_j^{\mathsf{H}},\ j = 1, \ldots, n.$$

Therefore, we can define left tangential interpolation conditions $\ell_j Z(\mu_j) = v_j,\ j = 1, \ldots, n$ in such a way that

$$M = -\overline{\Lambda} = -\Lambda^{\mathsf{H}}, \qquad L = R^{\mathsf{H}}, \qquad V = -W^{\mathsf{H}}.$$

Using these definitions, the Loewner and shifted Loewner matrices now become

$$\mathbb{L} := \begin{bmatrix} \frac{r_1^{\mathsf{H}} w_1 + w_1^{\mathsf{H}} r_1}{\lambda_1 + \overline{\lambda}_1} & \cdots & \frac{r_1^{\mathsf{H}} w_n + w_1^{\mathsf{H}} r_n}{\lambda_n + \overline{\lambda}_1} \\ \vdots & \ddots & \vdots \\ \frac{r_n^{\mathsf{H}} w_1 + w_n^{\mathsf{H}} r_1}{\lambda_1 + \overline{\lambda}_n} & \cdots & \frac{r_n^{\mathsf{H}} w_n + w_n^{\mathsf{H}} r_n}{\lambda_n + \overline{\lambda}_n} \end{bmatrix}, \tag{17a}$$

$$\mathbb{L}_\sigma := \begin{bmatrix} \frac{\lambda_1 r_1^{\mathsf{H}} w_1 - \overline{\lambda}_1 w_1^{\mathsf{H}} r_1}{\lambda_1 + \overline{\lambda}_1} & \cdots & \frac{\lambda_n r_1^{\mathsf{H}} w_n - \overline{\lambda}_1 w_1^{\mathsf{H}} r_n}{\lambda_n + \overline{\lambda}_1} \\ \vdots & \ddots & \vdots \\ \frac{\lambda_1 r_n^{\mathsf{H}} w_1 - \overline{\lambda}_n w_n^{\mathsf{H}} r_1}{\lambda_1 + \overline{\lambda}_n} & \cdots & \frac{\lambda_n r_n^{\mathsf{H}} w_n - \overline{\lambda}_n w_n^{\mathsf{H}} r_n}{\lambda_n + \overline{\lambda}_n} \end{bmatrix} \tag{17b}$$

and they satisfy the equations

$$\mathbb{L}\Lambda + \Lambda^{\mathsf{H}}\mathbb{L} = R^{\mathsf{H}}W + W^{\mathsf{H}}R, \quad \mathbb{L}_\sigma\Lambda + \Lambda^{\mathsf{H}}\mathbb{L}_\sigma = R^{\mathsf{H}}W\Lambda - \Lambda^{\mathsf{H}}W^{\mathsf{H}}R. \tag{18}$$

We point out that the matrix $\mathbb{L}$ is Hermitian by the construction itself, while $\mathbb{L}_\sigma$ is skew-Hermitian by construction. For such symmetric conditions, the matrix $\mathbb{L}$ is also called the Pick matrix (see [14, 15]). It follows from Theorem 5.1 that a generalized state-space realization $\{A, B, C, D, E\}$ is given by

$$\left[\begin{array}{c|c} A - sE & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c} \mathbb{L}_\sigma - s\mathbb{L} & -W^{\mathsf{H}} \\ \hline -W & 0 \end{array}\right] + \left[\begin{array}{c} -R^{\mathsf{H}} \\ \hline I_m \end{array}\right] D \left[\begin{array}{c|c} R & I_m \end{array}\right]. \tag{19}$$

Notice that the introduction of complex matrices and vectors in this section is in fact artificial. Since the interpolation conditions are self-conjugate, we can transform the construction as follows. Let $v := v_r + \imath v_\imath$ be a complex vector associated with a complex interpolation point $\lambda := \alpha + \imath\beta$, then the unitary transformation $Q := \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -\imath \\ 1 & \imath \end{bmatrix}$ transforms pairs of complex conjugate data to real data, as can be seen below

$$\begin{bmatrix} v & \overline{v} \end{bmatrix} \cdot Q = \sqrt{2}\begin{bmatrix} v_r & v_\imath \end{bmatrix}, \quad Q^{\mathsf{H}} \cdot \begin{bmatrix} \lambda & 0 \\ 0 & \overline{\lambda} \end{bmatrix} \cdot Q = \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}.$$

If the pairs of complex conjugate vectors and interpolation points have been permuted to be adjacent, then it suffices to apply a block diagonal unitary similarity transformation U with diagonal blocks Q corresponding to each complex conjugate pair $(\lambda, \overline{\lambda})$, to transform the equations eqs. (16) to (18) to real equations:

$$\widehat{\mathbb{L}}\Omega + \Omega^{\mathsf{T}}\widehat{\mathbb{L}} = \widehat{R}^{\mathsf{T}}\widehat{W} + \widehat{W}^{\mathsf{T}}\widehat{R}, \text{ and} \tag{20a}$$

$$\widehat{\mathbb{L}}_\sigma\Omega + \Omega^{\mathsf{T}}\widehat{\mathbb{L}}_\sigma = \widehat{R}^{\mathsf{T}}\widehat{W}\Omega - \Omega^{\mathsf{T}}\widehat{W}^{\mathsf{T}}\widehat{R}, \tag{20b}$$

where

$$\widehat{\mathbb{L}} = U^{\mathsf{H}}\mathbb{L}U,\ \widehat{\mathbb{L}}_\sigma = U^{\mathsf{H}}\mathbb{L}_\sigma U,\ \Omega = U^{\mathsf{H}}\Lambda U,\ \widehat{W} = WU,\ \widehat{R} = RU,$$

and Ω is now block diagonal with 2×2 blocks corresponding to each pair of complex conjugate interpolation points. It then also follows from (19) that a *real* generalized state-space realization $\{\widehat{A}, \widehat{B}, \widehat{C}, D, \widehat{E}\}$ is then given by

$$\left[\begin{array}{c|c} \widehat{A} - s\widehat{E} & \widehat{B} \\ \hline \widehat{C} & D \end{array}\right] = \left[\begin{array}{c|c} \widehat{\mathbb{L}}_\sigma - s\widehat{\mathbb{L}} & -\widehat{W}^{\mathsf{T}} \\ \hline -\widehat{W} & 0 \end{array}\right] + \left[\begin{array}{c} -\widehat{R}^{\mathsf{T}} \\ \hline I_m \end{array}\right] D \left[\begin{array}{c|c} \widehat{R} & I_m \end{array}\right]. \tag{21}$$

Let us now look at the passivity condition we imposed on the transfer function $Z(s)$. The Loewner matrix $\mathbb{L}$ given in (17) has the structure of a Pick matrix (see e. g., [14]) since the spectral zeros used for the interpolation are assumed to be distinct. The strict passivity of $Z(s)$ implies that this matrix is positive definite. It follows that $Z(\infty) = D$, and hence that D must be strictly positive-real as well. Since $\mathbb{L}$ is positive definite, so is $\widehat{\mathbb{L}}$ and we can factorize it as $\widehat{\mathbb{L}} = \Gamma^{\mathsf{T}}\Gamma$, where Γ is invertible, by using, for instance, the upper triangular Cholesky factor. Defining

$$\widehat{W}_\Gamma := \widehat{W}\Gamma^{-1}, \quad \widehat{R}_\Gamma := \widehat{R}\Gamma^{-1}, \quad \widehat{\mathbb{L}}_{\sigma\Gamma} := \Gamma^{-\mathsf{T}}\widehat{\mathbb{L}}_\sigma\Gamma^{-1},$$

we obtain an equivalent quadruple for the state-space realization $\{\widehat{A}_\Gamma, \widehat{B}_\Gamma, \widehat{C}_\Gamma, D\} = \{\Gamma^{-\mathsf{T}}\widehat{A}\Gamma^{-1}, \Gamma^{-\mathsf{T}}\widehat{B}, \widehat{C}\Gamma^{-1}, D\}$ of $Z(s)$ as

$$\left[\begin{array}{c|c} \widehat{A}_\Gamma & \widehat{B}_\Gamma \\ \hline \widehat{C}_\Gamma & D \end{array}\right] = \left[\begin{array}{c|c} \widehat{\mathbb{L}}_{\sigma\Gamma} & -\widehat{W}_\Gamma^{\mathsf{T}} \\ \hline -\widehat{W}_\Gamma & 0 \end{array}\right] + \left[\begin{array}{c} -\widehat{R}_\Gamma^{\mathsf{T}} \\ \hline I_m \end{array}\right] D \left[\begin{array}{c|c} \widehat{R}_\Gamma & I_m \end{array}\right]. \tag{22}$$

We then show that this realization is in port-Hamiltonian form.

Theorem 5.2. *Let us construct an $m \times m$ real transfer function $Z(s)$ of MacMillan degree n using the self-conjugate interpolation conditions as follows:*

$$Z(\infty) = D, \quad Z(\lambda_j)r_j = w_j, \quad r_j^{\mathsf{H}} Z(-\overline{\lambda}_j) = -w_j^{\mathsf{H}}, \quad j = 1, \dots, n,$$

where $\Re\left(\lambda_j\right) > 0$, $D + D^{\mathsf{T}} > 0$ and $\widehat{\mathbb{L}} > 0$ in which $\Re(\cdot)$ denotes the real part. Then, $Z(s)$ is strictly passive and the quadruple $\{\widehat{A}_\Gamma, \widehat{B}_\Gamma, \widehat{C}_\Gamma, D\}$ is in normalized port-Hamiltonian form and its spectral zeros and zero directions are given by (λ_j, r_j), $j = 1, \dots, n$.

Proof. A necessary condition for strict passivity is that the Hermitian part of $Z(s)$ is positive definite on the whole imaginary axis, including infinity, and since $D = Z(\infty)$ and is a real matrix, we must have $D + D^{\mathsf{T}} > 0$. A necessary and sufficient condition for the passivity of $Z(s)$ with given interpolation data, is that the Loewner matrix $\widehat{\mathbb{L}}$ is positive semi-definite, but since we assume $\widehat{\mathbb{L}} > 0$, the transfer function is passive. Let us now decompose the real matrix D as $D = N + S$, where S is the symmetric part of D and N is its skew-symmetric part. Then, following the discussion of Section 2, we obtain

$$\mathcal{W} = \mathcal{W}^{\mathsf{T}} = \left[\begin{array}{c} \widehat{R}_\Gamma^{\mathsf{T}} \\ \hline I_m \end{array}\right] S \left[\begin{array}{c|c} \widehat{R}_\Gamma & I_m \end{array}\right] \geq 0,$$

$$\mathcal{V} = -\mathcal{V}^{\mathsf{T}} = \left[\begin{array}{c|c} -\widehat{\mathbb{L}}_{\sigma\Gamma} & \widehat{W}_\Gamma^{\mathsf{T}} \\ \hline -\widehat{W}_\Gamma & 0 \end{array}\right] + \left[\begin{array}{c} \widehat{R}_\Gamma^{\mathsf{T}} \\ \hline I_m \end{array}\right] N \left[\begin{array}{c|c} \widehat{R}_\Gamma & I_m \end{array}\right]$$

which are the conditions for the passivity of a normalized port-Hamiltonian system. The standard state-space realization (22) is therefore normalized port-Hamiltonian. It follows from the self-conjugate conditions that

$$\Phi^{\mathsf{T}}(-\lambda_j)r_j = \Phi(\lambda_j)r_j = Z(-\lambda_j)r_j + Z(\lambda_j)r_j = -w_j + w_j = 0,$$

for $j = 1, \dots, n$, and since $\Phi(s)$ has MacMillan degree bounded by $2n$, these are the only zeros of $\Phi(s)$, which also implies that $Z(s)$ is then strictly passive. ☐

Remark 5.1. *The conditions that the spectral zeros should be simple can be removed. The construction of the Loewner matrix $\mathbb{L}$ and of the shifted Loewner matrix $\mathbb{L}_\sigma$ then have to be adapted, as explained in [15, 13], but the properties of these matrices are preserved. The tangential interpolation conditions then also involve tangential conditions on the derivatives of $Z(s)$ at the spectral zeros λ_i, but the conclusions remain the same.*

Remark 5.2. *The conditions that we should know the zero directions of the corresponding spectral zeros of $Z(s)$ form a demanding constraint. But this is different in the scalar case since we only need to impose a scalar condition $Z(-\lambda_j) + Z(\lambda_j)$ in each spectral zero. We can then choose $R = \begin{bmatrix}1, \dots, 1\end{bmatrix}$ which implies that $W = \begin{bmatrix}Z(\lambda_1), \dots, Z(\lambda_n)\end{bmatrix}$.*

Finally, we summarize the construction of a port-Hamiltonian realization in the normalized form in Algorithm 1.

Algorithm 1 Construction of a port-Hamiltonian realization in a normalized form.

Input:
- Spectral zeros λ_j and zero directions r_j, $j = 1, \dots, n$,
- transfer function measurements, i.e. $w_j = Z(\lambda_j)r_j$, where $Z(s)$ denotes the transfer function,
- the feed-through term D.

Construct the Loewner and shifted Loewner matrices using w_j and r_j as shown in (17).
Define $W := \begin{bmatrix}w_1, \dots, w_n\end{bmatrix}$ and $R := \begin{bmatrix}r_1, \dots, r_n\end{bmatrix}$.
Construct the interpolating realization, ensuring the matching of the transfer function at infinity is:
$E = \mathbb{L}$, $A = \mathbb{L}_s - R^{\mathsf{H}}DR$, $B = -W^{\mathsf{H}} - R^{\mathsf{H}}D$, $C = -W + DR$.
Perform the unitary transformation to obtain a real realization $(\widehat{E}, \widehat{A}, \widehat{B}, \widehat{C})$.
Consider the Choleskey factorization of $\widehat{E} := \Gamma^{\mathsf{T}}\Gamma$.
Construct a port-Hamiltonian realization in the normalized form as follows:
$\tilde{A} = \Gamma^{-\mathsf{T}}\widehat{A}\Gamma^{-1}$, $\quad \tilde{B} = \Gamma^{-\mathsf{T}}\widehat{B}$, $\quad \tilde{C} = \widehat{C}\Gamma^{-1}$.

Output: A port-Hamiltonian realization: $\left(\tilde{A}, \tilde{B}, \tilde{C}, D\right)$.

Algorithm 2 Estimation of spectral zeros and directions using the data on the imaginary axis.

Collect enough samples on the imaginary axis.
Construct a realization using the Loewner approach, ensuring the matching of the transfer function at infinity.
Determine spectral zeros of the obtained realization.

6. Estimation of Spectral Zeros and Zero Directions using Data on the Imaginary Axis

So far, we have discussed how to construct a port-Hamiltonian realization from the transfer function measurements on spectral zeros along with zero directions. However, it may be restrictive as in practice, it is almost impossible to know the spectral zeros and zero directions a priori. Moreover, even if the zeros are known, taking measurements at those points and directions is not straightforward. On the other hand, there are methods allowing us to obtain measurements on the imaginary axis. Using these measurements, one can obtain a realization using the classical Loewner approach, proposed in [13], which interpolates the data. However, it is very likely that it will not yield a realization in normalized port-Hamiltonian form. But we are interested in a passive realization given there is an underlying passive system. To do so, we first propose a strategy in Algorithm 2 to estimate the spectral zeros and directions based on the data on the $j\omega$ axis. Once we have such a data set, we can obtain a passive realization directly using Algorithm 1.

The main motivation of proposing Algorithm 2 is as follows. As we know, if the transfer functions of two linear systems are the same, then there exists a state-space transformation, allowing us to go from one to another. Furthermore, it is also known that a minimal realization of a linear system can be obtained using the Loewner approach, assuming we have enough samples on the imaginary axis. Hence, if there exists a passive realization of the linear system, then such a passive realization can be determined using the realization obtained using the Loewner approach and a state-space transformation. However, a state-space transformation of a linear system does not change the spectral zeros and corresponding directions. Consequently, we can indeed directly use the realization obtained using the Loewner approach to estimate the spectral zeros and corresponding directions and further can evaluate the transfer function at spectral zeros and in the corresponding directions.

Remark 6.1. *One can also construct a reduced-order system as well by truncating singular values of the Loewner matrix at a desired tolerance. This can be followed by determining the spectral zeros and zero directions of the reduced-order system, which can be very different from the original ones; however, the spectral zeros and zero directions of the reduced-order system form a good representative, allowing us to compare the important dynamics of the original system.*

Remark 6.2. *If the transfer function measurements are given in a particular frequency band, then applying Algorithm 2 would yield spectral zeros and zero directions, corresponding to the considered frequency band. If a port-Hamiltonian realization in the normalized form is constructed using Algorithm 1, then we obtain the frequency-limited port-Hamiltonian realization. This is discussed and illustrated further in the subsequent section.*

7. Illustrative Examples and Application in Model-Order Reduction

In this section, we illustrate the proposed identification approach to construct a passive realization by means of several examples. All numerical simulations are carried out in MATLAB® version 7.11.0.584 (R2016b) 64-bit on an Intel®Core™i7-6700 CPU @ 3.40GHz, 6MB cache, 8GB RAM, Ubuntu 16.04.6 LTS (x86-64).

7.1. An analytical example

We first consider an analytical example, showing the necessary steps, precisely Algorithm 1, to identify an underlying passive realization whose transfer function is as follows:

$$Z(s) := dI_2 - (sI_2 - A)^{-1} \quad \text{with} \quad A := \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

and let us take $a = -1$, $b = 1$, $d = 2$ to make the system strictly passive since it is then port-Hamiltonian with positive definite matrix $\mathcal{W}$. The poles of $Z(s)$ are the eigenvalues of A and are equal to $-1 \pm \imath$ and hence asymptotically stable. The spectral zeros are the zeros of $\Phi(s) = Z^{\mathsf{T}}(-s) + Z(s)$ which can be obtained from

$$\begin{aligned} Q\Phi(s)Q^{\mathsf{H}} &= QZ^{\mathsf{T}}(-s)Q^{\mathsf{H}} + QZ(s)Q^{\mathsf{H}} \\ &= 2dI_2 - (-sI_2 - QA^{\mathsf{T}}Q^{\mathsf{H}})^{-1} - (sI_2 - QAQ^{\mathsf{H}})^{-1}, \end{aligned}$$

where

$$Q = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -\imath \\ 1 & \imath \end{bmatrix}, \qquad QAQ^{\mathsf{H}} = \begin{bmatrix} a + \imath b & 0 \\ 0 & a - \imath b \end{bmatrix}.$$

It then turns out that both $QZ(s)Q^{\mathsf{H}}$ and $Q\Phi(s)Q^{\mathsf{H}}$ are diagonal and equal to

$$\begin{aligned} QZ(s)Q^{\mathsf{H}} &= \operatorname{diag}\left(2 - \frac{1}{s+1-\imath}, 2 - \frac{1}{s+1+\imath}\right), \\ Q\Phi(s)Q^{\mathsf{H}} &= \operatorname{diag}\left(\frac{6 + 8\imath s - 4s^2}{2 + 2\imath s - s^2}, \frac{6 - 8\imath s - 4s^2}{2 - 2\imath s - s^2}\right). \end{aligned}$$

The spectral zeros in the right half-plane are $\lambda = \frac{\sqrt{2}}{2} + \imath$ and $\overline{\lambda} = \frac{\sqrt{2}}{2} - \imath$ and the corresponding zero directions are

$$Q\Phi(\lambda)Q^{\mathsf{H}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = 0 \quad \Longleftrightarrow \quad \Phi(\lambda)\begin{bmatrix} 1 \\ \imath \end{bmatrix} / \sqrt{2} = 0,$$

and

$$Q\Phi\left(\overline{\lambda}\right)Q^{\mathsf{H}}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \quad \Longleftrightarrow \quad \Phi\left(\overline{\lambda}\right)\begin{bmatrix} 1 \\ -\imath \end{bmatrix} / \sqrt{2} = 0.$$

The interpolation conditions then are

$$Z(\lambda)Q^{\mathsf{H}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = Z(\lambda)\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{\imath}{\sqrt{2}} \end{bmatrix} = W, \text{ and } Z(\overline{\lambda})\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{\imath}{\sqrt{2}} \end{bmatrix} = \overline{W},$$

where $QZ(\lambda)Q^{\mathsf{H}}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 - \frac{1}{\lambda-(a+\imath b)} \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}$ implies

$$QW = \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix}, \text{ and } W = \begin{bmatrix} 1 \\ \imath \end{bmatrix}.$$

The Loewner matrix then is obtained from $R = Q^{\mathsf{H}}$, $W = \sqrt{2}Q^{\mathsf{H}}$ and hence $W^{\mathsf{H}}R = \sqrt{2}I_2$, which finally yields

$$\begin{aligned} \mathbb{L} &= \frac{2\sqrt{2}}{\lambda + \overline{\lambda}} I_2 = 2I_2, \\ \mathbb{L}_\sigma &= \begin{bmatrix} \sqrt{2}\frac{\lambda-\overline{\lambda}}{\lambda+\overline{\lambda}} & 0 \\ 0 & -\sqrt{2}\frac{\lambda-\overline{\lambda}}{\lambda+\overline{\lambda}} \end{bmatrix} = 2\begin{bmatrix} \imath & 0 \\ 0 & -\imath \end{bmatrix}, \end{aligned}$$

and the generalized state space realization (19) becomes

$$\left[\begin{array}{c|c} A - sE & B \\ \hline C & D \end{array}\right] = \left[\begin{array}{c|c} \mathbb{L}_\sigma - s\mathbb{L} & -\sqrt{2}Q \\ \hline -\sqrt{2}Q^{\mathsf{H}} & 0 \end{array}\right] + 2\left[\begin{array}{c} -Q \\ \hline I_2 \end{array}\right]\left[\begin{array}{c|c} Q^{\mathsf{H}} & I_2 \end{array}\right].$$

Using the factorization $\mathbb{L} = \Gamma^{\mathsf{H}}\Gamma$ with $\Gamma := \sqrt{2}Q^{\mathsf{H}}$, we get $\mathbb{L}_{\sigma\Gamma} = \frac{1}{2}Q^{\mathsf{H}}\mathbb{L}_\sigma Q$ and

$$\begin{aligned} \left[\begin{array}{c|c} A_\Gamma & B_\Gamma \\ \hline C_\Gamma & D \end{array}\right] &= \left[\begin{array}{c|c} \mathbb{L}_{\sigma\Gamma} & -I_2 \\ \hline -I_2 & 0 \end{array}\right] + 2\left[\begin{array}{c} -I_2/\sqrt{2} \\ \hline I_2 \end{array}\right]\left[\begin{array}{c|c} I_2/\sqrt{2} & I_2 \end{array}\right] \\ &= \left[\begin{array}{cc|cc} -1 & 1 & -c & 0 \\ -1 & -1 & 0 & -c \\ \hline 1/c & 0 & 2 & 0 \\ 0 & 1/c & 0 & 2 \end{array}\right] \end{aligned}$$

with $c = 1 + \sqrt{2}$ and $1/c = \sqrt{2} - 1$. The smallest eigenvalue $\lambda_{\min}(\mathcal{W}_\Gamma)$ of the above model is 0 which is a poor estimate of its passivity radius. But we can apply to this model a similarity scaling with $T = cI_2$, which yields a model $\mathcal{M}_T$ where $A_T = A_\Gamma$ and $D_T = D$ are unchanged but $C_T = -B_T = I_2$. This corresponds to using [10, Lemma 3.2] with the certificate $X = c^{-2}I$, and transforming the model to a new port-Hamiltonian form which has a passivity radius equal to $\lambda_{\min}(\mathcal{W}_T) = \frac{1}{2}(3 - \sqrt{5}) \approx 0.382$.

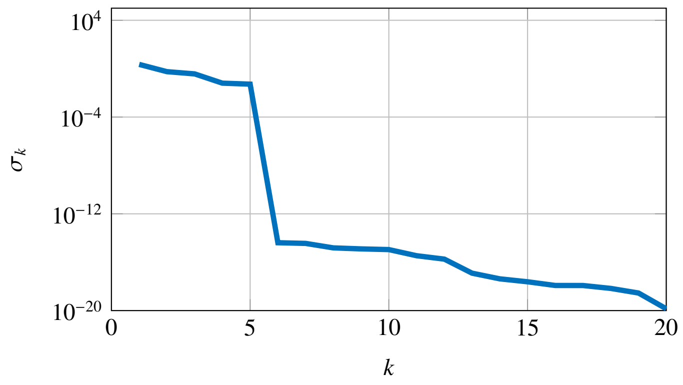

Figure 1: RLC example: The decay of the singular values of the Loewner matrix.

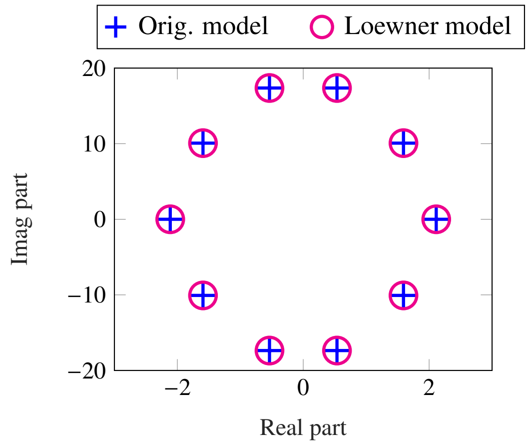

Figure 2: RLC example: Spectral zero of the original and Loewner model.

7.2. *Electric RLC circuit*

As second example, we discuss the electrical circuit example considered in [15]. The system dynamics in the state-space form is given by as follows:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t) + Du(t),\end{aligned}$$

where

$$A = \begin{bmatrix} -20 & -10 & 0 & 0 & 0 \\ 10 & 0 & -10 & 0 & 0 \\ 0 & 10 & 0 & -10 & 0 \\ 0 & 0 & 10 & 0 & -10 \\ 0 & 0 & 0 & 10 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^{\mathsf{T}},$$

and $D = 2$.

To identify the dynamics, we assume to have 20 points on the imaginary axis in a log-scale between $[10^{-1}, 10^{3}]$. We first employ the Loewner approach [13] to obtain a realization. In Figure 1, we plot the singular values of the Loewner matrix, which allows us to determine the order of a minimal realization. We observe that the singular values after the 5th are at the level of machine precision as one would expect. Hence, we determine a realization of order 5. Next, we show the spectral zeros of the original and Loewner model in Figure 2, indicating that the spectral zeros of both models are the same as expected.

It is not in the form of a passive port-Hamiltonian system. But we can use the spectral zeros and zero directions of the Loewner model, which in this case, are the same as the original system, and estimate the transfer function at the spectral zeros along with the respective zero directions. Consequently, we apply Algorithm 1 to obtain a realization in the generalized state-space form of a port-Hamiltonian system (10) where

$$Q^{-1} = \begin{bmatrix} 0.8795 & 0.0263 & -0.0304 & -0.0511 & 0.0938 \\ 0.0263 & 0.8515 & -0.0770 & -0.1574 & -0.0098 \\ -0.0304 & -0.0770 & 0.2545 & 0.0814 & 0.1136 \\ -0.0511 & -0.1574 & 0.0814 & 0.3560 & 0.0400 \\ 0.0938 & -0.0098 & 0.1136 & 0.0400 & 0.2891 \end{bmatrix},$$

$$J = \begin{bmatrix} 0 & -15.2595 & 0.5921 & 1.7823 & 0.5344 \\ 15.2595 & 0 & -0.4864 & -0.8033 & 1.6342 \\ -0.5921 & 0.4864 & 0 & 0.5204 & -0.5325 \\ -1.7823 & 0.8033 & -0.5204 & 0 & -3.3854 \\ -0.5344 & -1.6342 & 0.5325 & 3.3854 & 0 \end{bmatrix},$$

$$R = \begin{bmatrix} 4.0000 & 0.0000 & -2.8284 & -3.9606 & 0.5598 \\ 0.0000 & 0 & -0.0000 & -0.0000 & 0.0000 \\ -2.8284 & -0.0000 & 2.0000 & 2.8006 & -0.3959 \\ -3.9606 & -0.0000 & 2.8006 & 3.9216 & -0.5543 \\ 0.5598 & 0.0000 & -0.3959 & -0.5543 & 0.0784 \end{bmatrix},$$

$$G = \begin{bmatrix} -0.6563 & 0.3238 & 0.5378 & 0.6924 & -0.2925 \end{bmatrix}^{\mathsf{T}},$$

$$P = \begin{bmatrix} 2.8284 & 0.0000 & -2.0000 & -2.8006 & 0.3959 \end{bmatrix}^{\mathsf{T}},$$

$$N = 0, \quad S = 2.$$

Furthermore, we compare the Bode plots of the original, Loewner, and port-Hamiltonian model (3), illustrating that all three models have the same transfer functions, and also have the same spectral zeros and zero directions.

7.3. *A large scale electrical circuit*

Next, we consider a large scale RLC circuit, where 100 electrical capacitances, inductors, and resistances are interconnected. For more details on the circuit topology, we refer to [16]. The modeling of such a circuit leads to a model of order $n = 200$. Next, we assume that we obtain 200 points on the imaginary axis on a log-scale within the range $\left[10^{-1}, 10^{3}\right]$.

Towards constructing a port-Hamiltonian reduced-order system using the data, we first determine a realization using the classical Loewner method. We plot the decay of the singular values of the Loewner matrix in Figure 4, indicating a sharp decay. Having truncated singular values at 10^{-8} (relatively), we

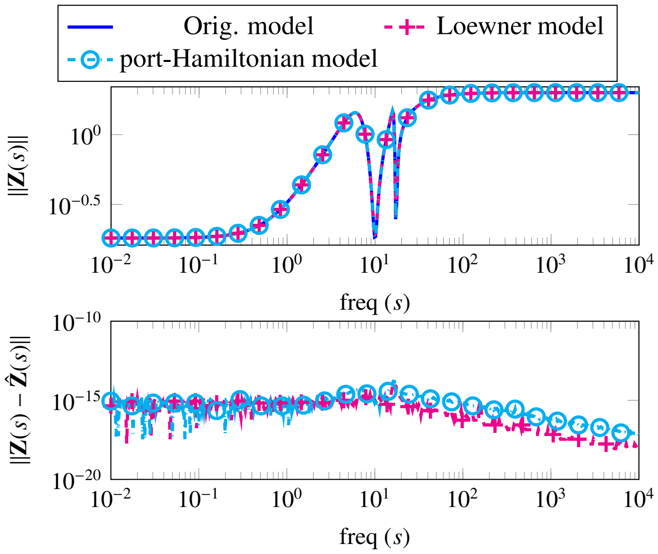

Figure 3: RLC example: Comparison of the Bode plots of the original, Loewner and port-Hamiltonian models.

construct a reduced-order (Loewner) model of order $r = 13$, which is expected to capture the dynamics very well. Next, we compare the spectral zeros of the original and Loewner models in Figure 5. It is interesting to see that spectral zeros of both models are very different. Somehow, one can think of representative spectral zeros of the original systems with a smaller number, yet capturing the dynamics of the original systems very accurately.

Subsequently, we determine the spectral zeros and zero directions using the Loewner model and evaluate the transfer function of the Loewner model at these zeros along with the respective directions. Then, we can determine a port-Hamiltonian realization using Algorithm 1. To compare the quality of models, we plot the Bode plots of the original, Loewner, and port-Hamiltonian models in Figure 6, showing the Loewner and port-Hamiltonian models approximate the original model very well.

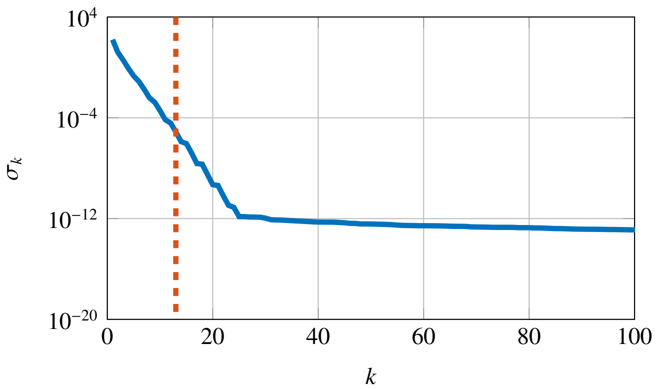

Figure 4: Large-scale RLC circuit: The decay of the singular values of the Loewner matrix.

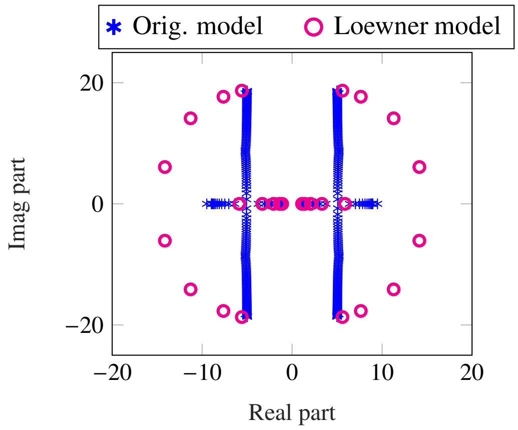

Figure 5: A large scale RLC example: The decay of the singular values of the Loewner matrix.

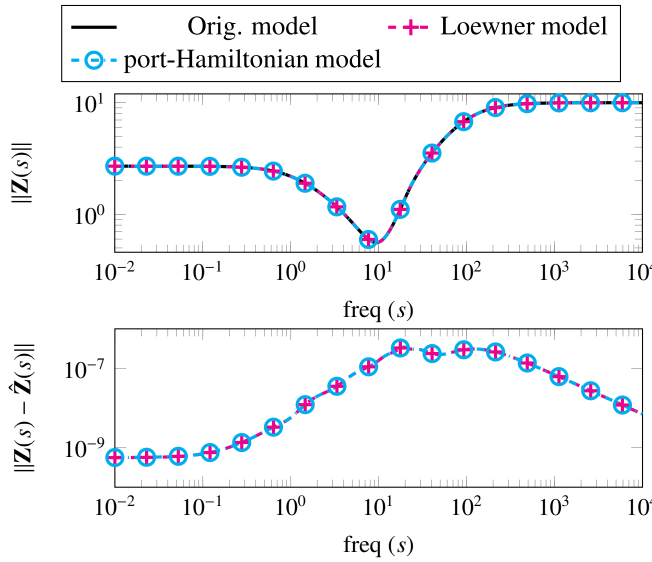

Figure 6: A large-scale RLC example: Comparison of the Bode plots of the original and Loewner model.

7.4. Frequency-limited port-Hamiltonian realization

Lastly, we discuss the construction of a frequency-limited port-Hamiltonian realization using the same example as in the previous subsection. This means that the transfer function of the port-Hamiltonian realization is required to very accurate in a given frequency band. Let us assume that we are given measurements in a frequency band $\left[5, 15\right]$. As the first, we construct a Loewner model. This is followed by determining a reduced-order model of order $r = 9$. Next, we compare the spectral zeros of original and Loewner models in Figure 7. It can be observed that the spectral zeros not only different from the original ones but also from those of the reduced-order model of order $r = 13$ in the previous example, see Figure 5.

Next, we plot the transfer functions of the original and the identified port-Hamiltonian realization in Figure 8. Comparing, in particular, the error plots in Figures 6 and 8, we observe that the identified port-Hamiltonian realization using the data

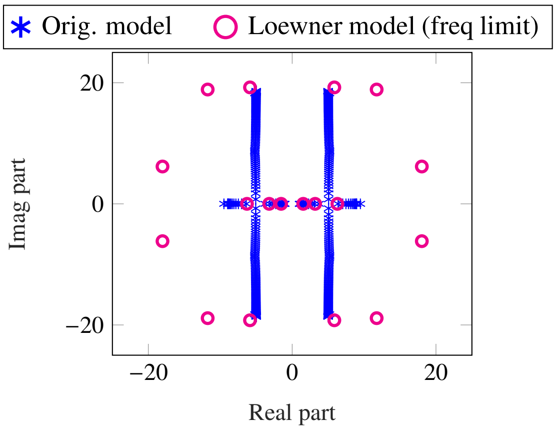

Figure 7: Frequency limited RLC circuit: Comparison of spectral zeros of the original and Loewner model.

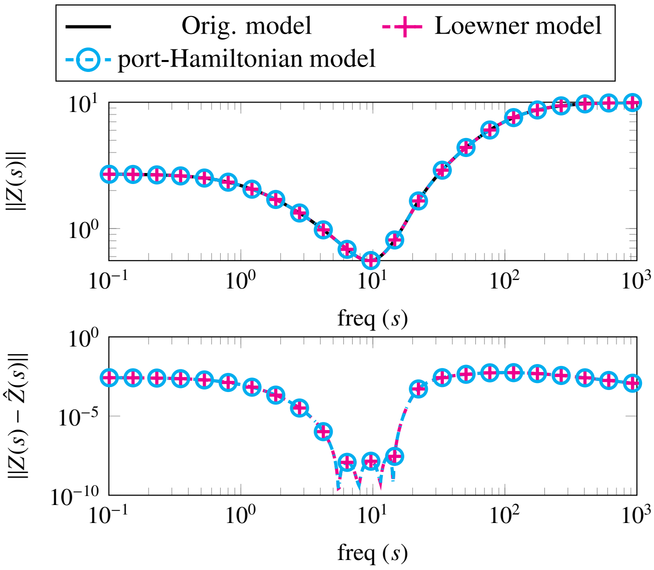

Figure 8: Frequency limited RLC circuit: Comparison of the Bode plots of the original and Loewner model.

in the frequency band is much more accurate in the considered frequency band (nearly by three orders of magnitude) than the model identified in the previous subsection, and moreover, it is of a lower dimension.

8. Conclusions

In this work, we have studied the identification problem for strictly passive systems. We have proposed a variant of the classical Loewner approach [13], which constructs a realization in port-Hamiltonian form. We have also discussed a two-step procedure which allows us to construct a port-Hamiltonian realization using data on the imaginary axis. Furthermore, we have investigated the construction of frequency-limited port-Hamiltonian realization, which can also be viewed as a frequency-limited model-order reduction scheme for passive systems. We have illustrated the proposed methods by means of a couple of variants of electrical circuits. As a future direction, it would be interesting to investigate an identification problem of second-order passive systems by extending the idea proposed in [17].

References

[1] P. Benner, E. S. Quintana-Ortí, G. Quintana-Ortí, Computing passive reduced-order models for circuit simulation, in: Proc. Intl. Conf. Parallel Comp. in Elec. Engrg. PARELEC 2004, IEEE Computer Society, Los Alamitos, CA, 2004, pp. 146–151.

[2] L. Daniel, J. Phillips, Model order reduction for strictly passive and causal distributed systems, in: Proc. 39rd ACM IEEE Design Automation Conference, 2002.

[3] R. W. Freund, P. Feldmann, Reduced-order modeling of large passive linear circuits by means of the SyPVL algorithm, in: Technical Digest of the 1996 IEEE/ACM International Conference on Computer-Aided Design, IEEE Computer Society Press, 1996, pp. 280–287.

[4] J. R. Phillips, L. Daniel, M. Silveira, Guaranteed passive balancing transformations for model order reduction, IEEE Trans. Comput.-Aided Design Integr. Circuits Syst. 22 (8) (2003) 1027–1041.

[5] S. Gugercin, R. V. Polyuga, C. Beattie, A. Van Der Schaft, Structure-preserving tangential interpolation for model reduction of port-Hamiltonian systems, Automatica 48 (9) (2012) 1963–1974.

[6] R. V. Polyuga, A. J. van der Schaft, Effort-and flow-constraint reduction methods for structure preserving model reduction of port-Hamiltonian systems, Systems Control Lett. 61 (3) (2012) 412–421.

[7] T. Wolf, B. Lohmann, R. Eid, P. Kotyczka, Passivity and structure preserving order reduction of linear port-Hamiltonian systems using krylov subspaces, Eur. J. Control 16 (4) (2010) 401–406.

[8] J. C. Willems, Dissipative dynamical systems part II: Linear systems with quadratic supply rates, Arch. Ration. Mech. Anal. 45 (5) (1972) 352–393.

[9] C. Beattie, V. Mehrmann, H. Xu, Port-Hamiltonian realizations of linear time invariant systems, Tech. rep., TU Berlin (2016).

[10] V. Mehrmann, P. Van Dooren, Optimal robustness of port-Hamiltonian systems, Tech. rep., TU Berlin (2019).

[11] M. Hazewinkel, R. E. Kalman, On invariants, canonical forms and moduli for linear, constant, finite dimensional, dynamical systems, in: Mathematical Systems Theory, Springer, 1976, pp. 48–60.

[12] A. C. Antoulas, Approximation of Large-Scale Dynamical Systems, SIAM Publications, Philadelphia, PA, 2005.

[13] A. J. Mayo, A. C. Antoulas, A framework for the solution of the generalized realization problem, Linear Algebra Appl. 425 (2-3) (2007) 634–662.

[14] D. C. Youla, M. Saito, Interpolation with positive real functions, Journal of the Franklin Institute 284 (2) (1967) 77–108.

[15] A. C. Antoulas, A new result on passivity preserving model reduction, Systems Control Lett. 54 (4) (2005) 361–374.

[16] S. Gugercin, A. C. Antoulas, A survey of balancing methods for model reduction, in: Proc. European Control Conf. ECC 2003, Cambridge, UK, 2003, CD Rom.

[17] I. P. Duff, P. Goyal, P. Benner, Data-driven identification of Rayleigh-damped second-order systems, arXiv:1910.00838 (2019).