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LINEAR ALGEBRA AND ITS APPLICATIONS

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Preface

Fourth Special Issue on Linear Systems and Control

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This is the fourth special issue of LAA devoted to Linear Systems and Control. The previous issues appeared in LAA 50 (1983), LAA 122–124 (1989), LAA 203–204 (1994) and were edited by, respectively, (i) *Brockett and Fuhrmann*, (ii) *Fuhrmann, Kimura and Willems* and (iii) *Antoulas, Fuhrmann, Hautus and Yamamoto*. The purpose of this series is to promote the exchange of ideas between the mathematical communities of systems theory and linear algebra. Since the relationship between these fields has been long lasting and is continuing, a brief glimpse into its history may be appropriate.

It all began in the second half of the 19th century when control theory had not yet emerged as a discipline and linear algebra was still in its early youth. At that time about 70 000 ‘governed’ steam engines worked in England and showed—under the influence of various technical improvements—an increasing tendency to ‘hunt’, i.e. a tendency to become unstable. This motivated Maxwell 1868 to pose the first mathematical problem of control engineering. He asked for algebraic conditions on the coefficients of a polynomial which guarantee that all its roots have negative real parts. The solution of this problem is linked with the names of Hermite, Sylvester, Routh and Hurwitz and is based on the use of quadratic forms for the location of polynomial roots. These linear algebraic stability criteria remained virtually the sole mathematical tools of the automatic control engineer until well into the 20th century.

In the period that followed, frequency–response techniques, originating from communication engineering, dominated the study of control systems (servomechanisms). Only single input single output systems described by real rational transfer functions

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were considered and their analysis and design did not require any help from linear algebra.

This changed abruptly when, motivated by the development of optimal control methods and the availability of stored program digital computers the *state-space approach* emerged in the second half of the 1950s. Now feedback control of multivariable linear systems became a possibility. At this early stage system theory became a disciple of linear algebra and Gantmacher's book on matrices turned into a bible for systems theorists. The basic concepts and constructions of state-space theory developed by Kalman and others in the early sixties were of linear algebraic nature and so were the methods employed in realization theory and the structure theory of linear systems. As a result the rapidly developing field of linear systems theory soon became a source of inspiration for linear algebra, providing a rich variety of new problems, concepts and ideas. A truly symbiotic connection developed between the two fields in the sixties and the seventies. The different pole placement problems for state and dynamic/static output feedback, the various canonical form problems for pairs and triplets of matrices, the problem of system equivalence, the partial realization problem for scalar and multivariable systems, and the detailed analysis of the algebraic Riccati equation represented new challenges for linear algebra. Kalman's module theoretic approach to the realization problem, Rosenbrock's polynomial system matrices, Fuhrmann's polynomial models and Wonham's geometric control theory can be viewed as different systematic approaches to incorporate central parts of linear systems and control theory into a linear algebraic framework. From this point of view the sixties and a good part of the seventies was the golden age of the partnership between linear algebra and systems theory. The enthusiasm and youthful spirit of discovery characteristic for that period still pervaded the preface of the first special issue devoted to linear systems and control (LAA 50 (1983)).

In the second half of the seventies, however, the control techniques of linear state-space theory, in particular the method of linear quadratic optimal control met with increasing criticism. The main point was that—in contrast with traditional frequency based methods—it failed to deal with the problems of modeling errors and the attenuation of unknown disturbances which are of fundamental importance in concrete applications. The principal raison d'être of feedback control in natural systems as well as in engineering is, in fact, to secure stability and satisfactory performance in the presence of uncertainty. The recognition of this basic task of feedback control turned the direction of systems theory away from linear algebraic structures and led to an influx of techniques from analysis, optimization theory, approximation theory, operator and perturbation theory. Inspired by classical frequency domain techniques a new method was developed, H^∞-control, which combined an optimization approach with worst case analysis. First solutions of the H^∞-control problem were based on techniques from complex analysis and operator theory (Hardy spaces, Nevanlinna–Pick interpolation, Nehari's theorem). It took nearly a decade before a linear algebraic approach to H^∞-theory was worked out utilizing algebraic Riccati equations and linear matrix inequalities.

This development casts some light on the relationship between linear algebra and systems theory, which is of a dialectical nature. Although it may have seemed so in the sixties and seventies, the two areas of mathematics have not fused into one inseparable field. This is fortunate, since the productivity of their relationship relies on their essential difference. The vitality of linear systems theory depends on its basic interdisciplinary links to communication and control engineering for which it creates the adequate mathematical tools, methods and languages. Therefore, driven by its basic areas of application, linear systems theory persistently escapes from the realm of linear algebra and employs tools from all branches of mathematics. On the other hand, because of the linear algebraic nature of its central object, there is an inherent tendency in linear systems theory to incorporate the new tools and results as much as possible into linear algebra. In this way linear systems theory has become an efficient catalyst for the interaction of linear algebra with other fields of mathematics. There is an important side effect of this interaction. By bringing objects from e.g. operator theory, function theory or approximation theory into linear algebraic form, linear systems theory prepares them for the algorithms of numerical linear algebra and makes them computable.

H^∞-theory and, more generally, the problems of uncertainty and robust control have dominated linear systems theory since the late seventies. In the same period problems of approximation and model reduction have moved to the center ground of linear systems theory. Geometrical and topological methods were applied to solve parameterization problems and study families and (orbit-) spaces of linear systems. Metric aspects of system properties gained importance and a variety of nearness problems were studied in order to determine quantitative indices of system properties like robust stability and controllability in the presence of structured perturbations. All of these current topics have in common that they combine linear algebraic constructions with methods from other fields of mathematics, and are related—to a lesser or larger degree—with problems of uncertainty and modeling errors. Another recent area whose methods show such a characteristic combination of linear algebra and analysis is the area of hybrid systems, i.e. systems that incorporate both continuous dynamics and discrete switching.

Along with the greater emphasis on ‘realistic’ aspects of control which motivated the development of H^∞-theory, numerical aspects have gained increasing importance in linear systems and control over the last two decades. This is visible not only in the number of research papers devoted to the numerics of our field (including the analysis of complexity issues) but also in the increasing availability of specialized software packages (see, for instance, the various MATLAB toolboxes).

Independent of these trends, there are continuing efforts to solve basic algebraic problems of linear systems theory in subject areas like pole placement, feedback invariants, system equivalence, system structure and partial realization. Another major enterprise which should be mentioned in this context is the behavioral approach to linear system proposed by J.C. Willems in the eighties. This approach takes up the efforts of the sixties and seventies of creating a unifying framework for linear systems and control. The behavioral approach puts the solution spaces into the center of the theory rather than the equations which determine them and replaces the input-state–output framework by a setting in which all signals connected with a system are treated on an equal footing. Polynomial matrices play a central role in behavioral equations as they do in Rosenbrock's state-space theory and Fuhrmann's polynomial models. One of the advantages of the behavioral approach is that its fundamental constructions can be extended to nD- and partial differential systems by replacing the ring of univariate polynomials with the ring of polynomials in n variables. However, this requires a substantial change in the mathematical tools, a switch from linear algebra to commutative algebra and algebraic geometry. The work of Oberst provides a framework for this extension.

All of the topics and trends mentioned above have already been represented to some extent in the previous two special issues (LAA 122–124 (1989), LAA 203–204 (1994)), but in the present issue they dominate the picture. This can be seen from the *Survey of topics and papers* which we attach to this preface for the convenience of the reader.

The contents of this special issue may reflect to some extent our own research interests, but we are confident that, altogether, the issue represents a reasonable cross section of current mathematical research in linear systems and control. A glance at the table of contents will convince the reader that the contributions to this issue are not only of interest for linear algebraists but also for specialists from other parts of mathematics. We hope that the scope of this issue makes it both a good reference to researchers active in the area as well as a convenient source to introduce the general mathematical reader to this field which brings together many branches of mathematics.

The number and high quality of the contributions demonstrates the vitality and creativity of linear systems theory and shows the continuing need for special issues of this kind. We would like to thank the authors for their contributions and Hans Schneider for his continuous support during the preparation of this issue.

Survey of topics and papers

1. *Rational matrix factorizations*
   - Minimal nonsquare spectral factors by M.A. Petersen and A.C.M. Ran
   - Nonsquare spectral factors via factorization of a unitary function by M.A. Petersen and A.C.M. Ran

     Both these papers are about spectral factorization of rational matrix functions into nonsquare factors. They present a full state-space parameterization of such factors, and show the relation to factorizations of a unitary rational function.

2. *System structure*
   - Silverman algorithm and the structure of discrete-time stochastic systems by A. Ferrante, G. Picci and S. Pinzoni
     This paper presents a method to obtain a reduced order Riccati equation for filtering non-regular stochastic processes based on Silverman's structure algorithm.
   - Rosenbrock models and their homotopy equivalence by V. Lomadze
     The paper introduces a homotopy equivalence of Rosenbrock systems and shows that this concept coincides with the classical equivalence of Rosenbrock and Fuhrmann.
   - Poles, zeros, and sheaf cohomology by B.F. Wyman
     The fundamental pole-zero exact sequence gives meaning to the statement that there are as many poles as there are zeros for a transfer function. Crucial for this sequence is the Wedderburn–Forney space, a notion introduced by the author. In this paper the Wedderburn–Forney space is identified with a cohomology group.
   - A simple state-space design of an interactor for a non-square system via system matrix pencil approach by X. Xin and T. Mita
     A state-space design of the interactor matrix is presented, based on the infinite eigenstructure of the system matrix pencil of the transfer function.

3. *Behavioral theory*
   - A study of behaviors by P.A. Fuhrmann
     This comprehensive paper presents an approach to behavior theory from the point of view of polynomial and rational models. Major themes of the paper are: characterizations of behavior homomorphisms and isomorphisms, equivalence results for different classes of behavior representations, controllability, and the minimality of representations.
   - On some special features which are peculiar of discrete-time behaviors with trajectories on $\mathbb{Z}_+$ by M.E. Valcher
     This paper points out some important differences between discrete time behaviors defined on the whole time axis and behaviors defined on the positive time axis.
   - Module theoretic approach to controllability of convolutional systems by P. Vettori and S. Zampieri
     The paper compares the behavioral point of view with the module theoretic point of view by Fliess. The main result shows that a certain topologization of the modules is needed in order to maintain the equivalence of the approaches.
   - Key problems in the extension of module-behaviour duality by J. Wood
     The paper studies two important questions in the behavioral literature over general signal spaces. (1) The Willems closure of an equation module and (2) the elimination problem.

4. *Feedback invariants*
   - Feedback invariants of restrictions and quotients: series connected systems by I. Baragaña and I. Zaballa
     This paper analyses the controllability indices and invariant factors of systems connected in series or parallel.
   - Output feedback invariants by M.S. Ravi, J. Rosenthal and U. Helmke
     In this paper the action of the group of static linear output feedback (plus coordinate transformations in the input, state and output spaces) is considered. Using tools from geometric invariant theory it is shown that there exists a quasi-projective variety whose points parameterize the output feedback orbits.
5. *Pole placement*
   - New counterexamples to pole placement by static output feedback by A. Eremenko and A. Gabrielov
     An important still unresolved question in the area of static output feedback pole placement is: when is $n = mp$ a sufficient condition for pole placement over the reals? The authors settle this question for many cases.
   - On minimal degree simultaneous pole assignment problems by B.K. Ghosh and X.A. Wang
     The paper provides new strong results for the problem of simultaneous static pole placement.
6. *Topology of observability and controllability subspaces*
   - A cellular decomposition of the manifold of observable conditioned invariant subspaces by F. Puerta, X. Puerta and I. Zaballa
     Given an observable system defined by a pair (C, A), a cellular decomposition of the manifold of (C, A)-invariant subspaces with fixed observability indices is specified. The decomposition is used to compute the homology groups of these manifolds.
   - On the geometry of the set of controllability subspaces of a pair (A, B) by F. Puerta and X. Puerta
     A new stratification of the set of d-dimensional controllability subspaces into smooth strata is introduced and a formula for the dimensions of the strata is derived.
7. *Interpolation, Padé approximation and partial realization*
   - Two-sided residue interpolation in matrix H_2 spaces with symmetries: conformal conjugate involutions by D. Alpay, V. Bolotnikov, and L. Rodman
     The paper studies two-sided and one-sided residue interpolation problems in classes of matrix-valued Hardy functions with various symmetries.
   - Gröbner basis solutions of constrained interpolation problems by H. O'Keeffe and P. Fitzpatrick
     The authors study a generalized Padé approximation problem for multi-dimensional systems. Algebraically the question asks for a solution of the congruence $a \equiv \sum_{i=1}^{s} b_i h_i \bmod I$ where $h_1, \ldots, h_s$ are given modulo a zero dimensional ideal I.

8. *Approximation, identification and model reduction*
   - A constrained approximation problem arising in parameter identification by B. Jacob, J. Leblond, J.-P. Marmorat and J. R. Partington
     The paper investigates the best approximation of a function on a subarc of the unit circle by an H^2 function, subject to a constraint on its imaginary part on the complementary arc and presents an algorithm for the computation of a best approximation. The problem is motivated by boundary parameter identification problems arising in non-destructive control.
   - The Sylvester equation and approximate balanced reduction by D.C. Sorensen and A.C. Antoulas
     The paper presents an iterative algorithm for computing approximately balanced reduced order systems. The approach is based on the computation of the cross gramian of the system. The cross gramian is the solution of a Sylvester equation and therefore some effort is dedicated to the study of this equation.
   - Canonical forms and parameter identification problems in perspective systems by S. Takahashi and B.K. Ghosh
     The paper derives a canonical form for perspective dynamical systems. This has applications in the area of parameter identification when the system is observed by a camera.
9. *Uncertain parameters*: *pseudospectra, stability radii, and other nearness problems*
   - Statistical learning methods in linear algebra and control problems: the example of finite-time control of uncertain linear systems by C.T. Abdallah, F. Amato, M. Ariola, P. Dorato and V. Koltchinskii
     Application of statistical learning methods to computationally difficult linear algebra and control problems.
   - Can spectral value sets of Toeplitz band matrices jump? by A. Böttcher and S.M. Grudsky
     Spectral values sets may jump in dependency on the uncertainty level. In this paper it is shown that this cannot happen for certain Toeplitz band matrices and compact structure operators.
   - More on pseudospectra for polynomial eigenvalue problems and applications in control theory by N.J. Higham and F. Tisseur
     Definitions and characterizations of pseudospectra are given for rectangular matrix polynomials expressed in homogeneous form. Pseudospectra are visualized on the Riemann sphere. Estimates and characterizations of the distance to the nearest non-regular polynomial and nearest uncontrollable dth order system are obtained in terms of pseudospectra.
   - Real and complex stability radii of polynomial matrices by Y. Genin, R. Ştefan and P. Van Dooren
     In this paper analytic expressions are derived for the complex and real stability radii of non-monic polynomial matrices with respect to an arbitrary stability

region of the complex plane. Numerical issues for computing these radii for different perturbation structures are also discussed.

- Detecting a definite Hermitian pair and a hyperbolic or elliptic quadratic eigenvalue problem, and associated nearness problems by N.J. Higham, F. Tisseur and P.M. Van Dooren
  This paper shows how to detect whether a pair of Hermitian matrices is positive definite (resp. a quadratic Hermitian matrix polynomial is hyperbolic), and how to determine the distance of such a pair (resp. quadratic Hermitian matrix polynomial) from the set of those which do not have the same property.

10. *Riccati equations, linear matrix inequalities, H^∞-control*
    - State-feedback H^∞-type control of linear systems with time-varying parameter uncertainty by T. Damm
      This paper is concerned with stabilization and disturbance attenuation problems for linear systems with stochastic and/or time-varying deterministic parameter uncertainties. The solvability of these problems is characterized by generalized Riccati equations. Newton's method is applied to the solution of these equations and a global convergence result is proved.
    - A survey of nonsymmetric Riccati equations by G. Freiling
      This paper gives a survey of recent and older results on nonsymmetric matrix Riccati differential equations and—in the time invariant case—on the corresponding algebraic Riccati equations.
    - H_∞-control of linear state-delay descriptor systems: an LMI approach by E. Fridman and U. Shaked
      This paper analyzes state-delay descriptor systems using linear matrix inequalities. It looks at the problems of delay-dependent and delay-independent stability for systems with uncertainties, and at the problems of filtering and output feedback.
11. *Stability and optimization*
    - Two numerical methods for optimizing matrix stability by J.V. Burke, A.S. Lewis, and M.L. Overton
      This paper presents two numerical methods for optimizing the stability of the dynamical system $\dot{z} = A(x)z$ where the affine matrix family $A(x) = A_0 + \sum x_k A_k$ maps a design vector x into the space of real matrices.
    - Root counting, phase unwrapping, stability and stabilization of discrete time systems by L.H. Keel and S.P. Bhattacharyya
      This paper determines the change of the argument of a real polynomial or real rational function along the unit circle via their Chebyshev representations. The result is applied to the problem of feedback stabilization of a discrete time control system by constant gain or by a two parameter controller.
    - Structured finite-dimensional controller design by convex optimization by C.W. Scherer

The paper solves a rather general structured controller design problem using convex optimization. The paper reveals the close interplay between structural controller design and so-called multi-objective control problems.

12. *Hybrid systems*
    - Existence and uniqueness of solutions for a class of piecewise linear dynamical systems by M.K. Çamlıbel and J.M. Schumacher
      This paper presents some sufficient conditions which guarantee the existence or/and the uniqueness of solutions for continuous time piecewise affine dynamical systems. A concept of solution is presented for these systems, and some sufficient conditions for existence and uniqueness of solutions are provided.
    - Hybrid static output feedback stabilization of second-order linear time-invariant systems by B. Hu, G. Zhai and A.N. Michel
      A SISO system of McMillan degree 2 can in general not be stabilized by static output feedback. This paper shows that a generic SISO system of McMillan degree 2 can be stabilized by hybrid static output feedback.
13. *Complexity*
    - The presence of a zero in an integer linear recurrent sequence is NP-hard to decide by V.D. Blondel and N. Portier
      This paper shows that several related problems (e.g. Pisot's problem and the problem to find the minimal dimension of a realization of a one-letter max-plus rational series) are NP-hard.