# IDENTIFYING POSITIVE REAL MODELS IN SUBSPACE IDENTIFICATION BY USING REGULARIZATION

**Ivan Goethals** *,[1] **Tony Van Gestel** *,[2] **Johan Suykens** *,[3]
**Paul Van Dooren** **,[4] **Bart De Moor** *,[5]

* *Kasteelpark Arenberg 10, B3001 Heverlee-Leuven, Belgium*
** *Av. Georges Lemaître 4, B1348 Louvain-la-Neuve, Belgium*

Abstract: This paper deals with the lack of positive realness of identified models that may be encountered in many stochastic subspace identification procedures. Lack of positive realness is an often neglected, but important problem. Subspace identification algorithms fail to return a valid linear model if the so-called covariance model, which is obtained from an intermediate realization step in the subspace identification algorithm, is not positive real. The main contribution of this paper is to introduce a regularization approach to impose positive realness on the covariance model. It is shown that positive realness can be imposed by adding a regularization term to a least squares cost function appearing in the subspace identification procedure.

Keywords: stochastic systems, subspace methods, robustness, regularization

## 1. INTRODUCTION

In this paper, we will consider stable systems and models of the form:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k, \\ y_k &= Cx_k + v_k, \end{aligned} \tag{1}$$

with

$$\mathcal{E}\left\{ \begin{bmatrix} w_p \\ v_p \end{bmatrix} \begin{bmatrix} w_q^T & v_q^T \end{bmatrix} \right\} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{pq} \geq 0, \tag{2}$$

where $\mathcal{E}\{\cdot\}$ denotes the expected value operator and $\delta_{pq}$ the Kronecker delta. It is assumed that $\mathcal{E}\left\{ \begin{bmatrix} w_p \\ v_p \end{bmatrix} x_k^T \right\} = 0, \forall p \geq k$. The elements of the vector $y_k \in \mathbb{R}^l$ are given observations at the discrete time index $k$ of the $l$ outputs of the system. The vector $x_k \in \mathbb{R}^n$ is the unknown state vector at time $k$. The unobserved process and measurement noise $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^l$ are assumed to be white, zero mean, Gaussian with covariance matrices as given in (2). The system matrices $A, C$ and the covariance matrices $Q, S$, and $R$ have appropriate dimensions.

Stochastic subspace identification methods are ideally suited to identify models of the form (1). Typically, in a first step the measured output sequence $y_0, y_1, \ldots, y_{N-1}$ is stored in block Hankel matrices containing a user defined number of block rows $i$, and a certain number of columns $j$, so that $N = 2i + j - 1$, see (Van Overschee and De Moor, 1993)(Van Overschee and De Moor, 1996) for an extensive survey of this procedure. Kalman filter state sequences $\hat{X}_i \in \mathbb{R}^{n\times j}$ and $\hat{X}_{i+1} \in \mathbb{R}^{n\times j}$ of the system and an estimate of the system order $\hat{n}$ are then obtained by using geometric operations of subspaces spanned by the column and row vectors of these Hankel matrices.

In a second step, a so called covariance model $(\hat{A}, \hat{G}, \hat{C}, \hat{D})$, is estimated, where $\hat{G}$ is an estimate for the covariance matrix between states and observations $G = \mathcal{E}\left\{x_{k+1}y_k^T\right\}$ and $\hat{D} = \frac{\hat{\Lambda}_0}{2}$ is an estimate for $\frac{\Lambda_0}{2}$, with $\Lambda_m = \mathcal{E}\left\{y_{k+m}y_k^T\right\}$, $m \geq 0$ the output

---

[1] I. Goethals is a Research Assistant with the Fund for Scientific Research-Flanders (FWO-Vlaanderen).
[2] T. Van Gestel is a Postdoctoral Researcher with the FWO-Vlaanderen.
[3] J. Suykens is a Postdoctoral Researcher with the FWO-Vlaanderen.
[4] P. Van Dooren is Professor at the Catholic University of Louvain.
[5] B. De Moor is a full professor with the KULeuven.

covariance matrices. $\hat{A}$ and $\hat{C}$ are estimates for the matrices $A$ and $C$ in (1), which are obtained as the solution to a least squares problem:

$$(\hat{A}, \hat{C}) = \text{arg min}_{A,C} J_1(A, C), \qquad (3)$$

with

$$J_1(A, C) = \left\| \begin{bmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{bmatrix} - \begin{bmatrix} A \\ C \end{bmatrix} \cdot \hat{X}_i \right\|_F^2, \qquad (4)$$

where

$$Y_{i|i} = \begin{bmatrix} y_i \ y_{i+1} \ \dots \ y_{i+j-1} \end{bmatrix}. \qquad (5)$$

Using the definitions for $G$ and $\Lambda_m$ above, one can derive that

$$\Lambda_m = CA^{m-1}G, \ \Lambda_{-m} = \Lambda_m^T, \qquad m \geq 1. \qquad (6)$$

Hence, the output covariances can be considered as Markov parameters of a deterministic linear time invariant system with system matrices $(A, G, C, D)$.

From the estimated model $(\hat{A}, \hat{G}, \hat{C}, \hat{D})$, In a last step, a model is constructed in forward innovation form:

$$\begin{aligned} \hat{x}_{k+1} &= \hat{A}\hat{x}_k + \hat{K}e_k, \\ y_k &= \hat{C}\hat{x}_k + e_k, \end{aligned} \qquad (7)$$

from which estimates of the error covariances of the system can be derived. The forward innovation model is obtained by first calculating the forward state covariance matrix $\hat{P} = \mathcal{E}\left\{\hat{x}_k\hat{x}_k^T\right\}$ of the covariance model through the solution of the forward algebraic Riccati equation:

$$\begin{aligned} \hat{P} = \hat{A}\hat{P}\hat{A}^T + (\hat{G} - \hat{A}\hat{P}\hat{C}^T)\cdot \\ (\hat{\Lambda}_0 - \hat{C}\hat{P}\hat{C}^T)^{-1}(\hat{G} - \hat{A}\hat{P}\hat{C}^T)^T, \end{aligned} \qquad (8)$$

with the forward Kalman filter gain $\hat{K} = (\hat{G} - \hat{A}\hat{P}\hat{C}^T)(\hat{\Lambda}_0 - \hat{C}\hat{P}\hat{C}^T)^{-1}$. The resulting model matrices of the stochastic system are $(\hat{A}, \hat{K}, \hat{C}, I_l)$ and the covariance matrix $\mathcal{E}\left\{e_k e_k^T\right\}$ is given by $\hat{R} = \hat{\Lambda}_0 - \hat{C}\hat{P}\hat{C}^T$.

It is important to note here, that the forward innovation model can only be obtained if the forward algebraic Riccati equation (8) has a positive definite solution. It can be shown that this is the case if and only if the infinite sequence $\{\hat{\Lambda}_m\}_{m=0}^{\infty}$ with $\hat{\Lambda}_m = \hat{C}\hat{A}^{m-1}\hat{G}$, $m > 0$, and $\hat{\Lambda}_0 = \frac{1}{j}Y_{i|i}Y_{i|i}^T$, is a "valid" covariance sequence with positive definite Toeplitz matrix (Dahlén *et al.*, 1998; Faurre *et al.*, 1978). This is equivalent with the model $(\hat{A}, \hat{G}, \hat{C}, \hat{D})$, with $\hat{D} + \hat{D}^T = \hat{\Lambda}_0$, being positive real. Hence, if the positive realness property is not satisfied, no meaningful stochastic model will be obtained. This problem may appear in practical applications. The covariance model, for example, is built on a finite number of observed covariances. Even if these were exact ($j \to \infty$), the realization algorithm does not ensure that the infinite covariance sequence $\{\hat{\Lambda}_m\}_{m=0}^{\infty} = \hat{C}\hat{A}^{m-1}\hat{G}$, derived from the finite sequence $\{\hat{\Lambda}_m\}_{m=0}^{2i-1}$, is positive. Hence the choice of $i$ has a direct influence on the possible occurence of positivity problems (Oono, 1981; Lindquist and Picci, 1996). Secondly, for $j$ finite, the observed covariances are subject to statistical errors that may increase the probability for positive realness problems to occur. Finally the ability of $(\hat{A}, \hat{G}, \hat{C}, \hat{D})$ to model the observed covariance sequence is clearly dependent on the choice of the model order $\hat{n}$. The influence of the parameters $i$, $j$ and $\hat{n}$ will be illustrated in section 3. For a further theoretical description, the reader is referred to (Lindquist and Picci, 1996).

In this paper we propose a solution to impose positive realness on a formerly identifed stochastic model by adding a regularization term that involves the system matrices $\hat{A}$ and $\hat{C}$, and we analyse its performance and compare it with already existing techniques.

## 2. IMPOSING POSITIVE REALNESS BY USING REGULARIZATION

### 2.1 *Main idea*

The estimation problem that we consider is the following: given matrices $\hat{X}_{i+1}, Y_{i|i}$ and $\hat{X}_i$ and given the estimates $\hat{G}$ and $\hat{\Lambda}_0$, estimate the model matrices $\hat{A}, \hat{C}$ such that the resulting model $\hat{A}, \hat{G}, \hat{C}, \hat{\Lambda}_0$ is positive real. To impose positive realness, we will add a regularization term to the cost function $J_1(A, C)$ from (3):

$$(\tilde{A}_c, \tilde{C}_c) = \text{arg min}_{A,C} J_1(A, C) + cJ_2(A, C), \qquad (9)$$

with

$$J_2(A, C) = \text{Tr}\left(\begin{bmatrix} A \\ C \end{bmatrix} W \begin{bmatrix} A \\ C \end{bmatrix}^T\right), \qquad (10)$$

where $c \geq 0$ is a positive real scalar and $W$ a positive definite matrix of appropriate dimensions that satisfies $W - \hat{G}\hat{\Lambda}_0^{-1}\hat{G}^T > 0$. A similar regularization term $\text{Tr}\left(AWA^T\right)$, involving only the system matrix $A$ was described in (Van Gestel *et al.*, 2001), and was shown to impose stability on a model. We will show that by adding the output matrix $C$ to the regularization term, the model can not only be made stable, but also positive real, provided the regularization coefficient $c$ is chosen sufficiently large.

By the choice of the regularization term $J_2(A, C)$, the optimal solution of the minimization problem is found as

$$\begin{aligned} \begin{bmatrix} \tilde{A}_c \\ \tilde{C}_c \end{bmatrix} &= \begin{bmatrix} \hat{X}_{i+1} \\ Y_{i|i} \end{bmatrix} \cdot \hat{X}_i^T \cdot \left[\hat{X}_i\hat{X}_i^T + cW\right]^{-1} &(11) \\ &= \begin{bmatrix} \hat{A} \\ \hat{C} \end{bmatrix} \hat{X}_i\hat{X}_i^T \left[\hat{X}_i\hat{X}_i^T + cW\right]^{-1}. &(12) \end{aligned}$$

From the optimality of the least squares estimate (11), it follows that for $c_1, c_2 \geq 0$:

$$\begin{aligned} J_1(\tilde{A}_{c_2}, \tilde{C}_{c_2}) + c_1 J_2(\tilde{A}_{c_2}, \tilde{C}_{c_2}) \\ \geq J_1(\tilde{A}_{c_1}, \tilde{C}_{c_1}) + c_1 J_2(\tilde{A}_{c_1}, \tilde{C}_{c_1}), \end{aligned} \qquad (13)$$

$$J_1(\tilde{A}_{c_1},\tilde{C}_{c_1}) + c_2 J_2(\tilde{A}_{c_1},\tilde{C}_{c_1}) \geq J_1(\tilde{A}_{c_2},\tilde{C}_{c_2}) + c_2 J_2(\tilde{A}_{c_2},\tilde{C}_{c_2}), \quad (14)$$

where (14) can be rewritten as:

$$\begin{aligned} J_1(\tilde{A}_{c_1},\tilde{C}_{c_1}) &+ c_1 J_2(\tilde{A}_{c_1},\tilde{C}_{c_1}) \\ &+ (\Delta c) J_2(\tilde{A}_{c_1},\tilde{C}_{c_1}) \\ \geq J_1(\tilde{A}_{c_2},\tilde{C}_{c_2}) &+ c_1 J_2(\tilde{A}_{c_2},\tilde{C}_{c_2}) \\ &+ (\Delta c) J_2(\tilde{A}_{c_2},\tilde{C}_{c_2}), \quad (15) \end{aligned}$$

where $\Delta c = c_2 - c_1$. Combining (13) and (15) it is easily seen that the regularization term $J_2(\tilde{A}_c, \tilde{C}_c)$ is a non-increasing function of $c$.

## 2.2 *Choosing the regularization parameter*

### 2.2.1. *An upper-bound*

The following lemma, (Goethals *et al.*, 2002), states that positive realness can always be imposed, by using the regularization term introduced in (9), provided the regularization coefficient $c$ is chosen sufficiently large.

*Lemma 1.* Let $\hat{G}$, $\hat{\Lambda}_0$ be given. Let $W = Q_W Q_W^T > 0$, $W - \hat{G}\hat{\Lambda}_0^{-1}\hat{G}^T > 0$, and define $\hat{\Sigma} = X_i X_i^T$, $P_0 = \hat{\Sigma}W^{-1}\hat{\Sigma} - \hat{\Sigma}\begin{bmatrix}\hat{A}^T & \hat{C}^T\end{bmatrix}\begin{bmatrix}W & \hat{G}\\ \hat{G}^T & \hat{\Lambda}_0\end{bmatrix}\begin{bmatrix}\hat{A}\\ \hat{C}\end{bmatrix}\hat{\Sigma}$. Then there exists a $c^*$ such that the system $\tilde{A}_c$, $\hat{G}$, $\tilde{C}_c$, $\hat{\Lambda}_0$, with $\tilde{A}_c$ and $\tilde{C}_c$ as in (11), is positive real for $c \geq c^*$, with $c^* = \max_{i|\theta_i \in \mathbb{R}^+} \theta_i$, and $\theta$ the set of generalized eigenvalues of the following eigenvalue problem:

$$\theta = \lambda\left(\begin{bmatrix}0_{\hat{n}} & -I_{\hat{n}}\\ P_0 & 2\hat{\Sigma}\end{bmatrix}, -\begin{bmatrix}I_{\hat{n}} & 0_{\hat{n}}\\ 0_{\hat{n}} & W\end{bmatrix}\right). \quad (16)$$

Hence, provided the conditions of Lemma 1 are met, a positive real model is always obtained for $c \geq c^*$, and in particular $c = c^*$, with $c*$ as in Lemma 1. Furthermore, since any positive real model is necessarily stable, which follows immediately from the upper left part of the Schur complement of the Algebraic Riccati equation

$$\begin{bmatrix} P & G \\ G^T & D + D^T \end{bmatrix} - \begin{bmatrix} APA^T & APC^T \\ CPA^T & CPC^T \end{bmatrix} \geq 0, \quad (17)$$

stability is automatically guaranteed. However, $c^*$ can be a too conservative estimate. In general it seems reasonable to keep the amount of regularization as low as possible. Hence, one should search for the smallest possible $c \leq c^*$ for which a positive real model is found.

### 2.2.2. *A lower bound*

A lower bound $c_s$ for $c$ can be found from a theorem presented in (Van Gestel *et al.*, 2001) that states that all eigenvalues of $A_c$ can be made to lie within a closed disc with a given radius $\gamma$, provided $c \geq c_s = \max_{i|\vartheta_i \in \mathbb{R}^+} \vartheta_i$, where $\vartheta = \lambda\left(\begin{bmatrix}0 & -I\\ P_0 & P_1\end{bmatrix}, -\begin{bmatrix}I & 0\\ 0 & P_2\end{bmatrix}\right)$ is the set of eigenvalues of a Generalized Eigenvalue problem with $P_2 = -\gamma W \otimes \gamma W$, $P_1 = -\gamma W \otimes \hat{\Sigma} - \gamma\hat{\Sigma} \otimes W$, and $P_0 = \hat{A}\hat{\Sigma}\otimes\hat{A}\hat{\Sigma} - \gamma\hat{\Sigma}\otimes\gamma\hat{\Sigma}$. Furthermore $c_s$ is shown to be the smallest regularization coefficient with this property. Hence, as shown in figure 1, a minimal $c$ imposing positive realness will always satisfy $c_s \leq c \leq c^*$.

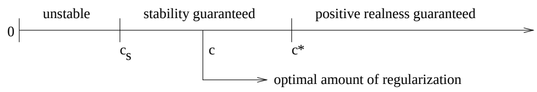


Fig. 1. Finding the optimal amount of regularization

When the realization $(\tilde{A}_{c_s}, \hat{G}, \tilde{C}_{c_s}, \hat{\Lambda}_0)$ is not yet positive real, i.e., $S_z(z) + S_z^T(z^{-1}) < 0$ for a certain $z = e^{j\theta}$, we can find a $c \geq c_s$ imposing positive realness, for instance by applying a bisection algorithm on the interval $c_s \leq c \leq c^*$.

Some alternative techniques have been reported in the literature in order to impose positive realness on a covariance model (Van Overschee and De Moor, 1996; Vaccaro and Vukina, 1993; Peternell, 1995; Marí *et al.*, 2000), many of whom are related to regularization principles. Apart from changing $\hat{A}$ and $\hat{C}$, regularization could also be applied to $\hat{G}$, $\hat{\Lambda}_0$, or a combination of both. A common problem for many of these alternatives is that they cannot be used if the covariance model is unstable. Apart from the technique proposed in this paper, which will be abbreviated as $\text{REG}_{\hat{A},\hat{C}}$ and of whom performance results will be given in the following sections, we will also discuss the performance of the following techniques:

- **SDP:** In (Marí *et al.*, 2000) a new identification scheme based was proposed, based on existing stochastic subspace methods and Semi Definite Programming (SDP). A stable $\tilde{A}$ is obtained by solving:

$$\begin{array}{ll} \min_{\tilde{A},\hat{P}} & \|(\hat{A} - \tilde{A})\hat{P}\|_2 \\ \text{s.t.} & \hat{P} > 0 \\ & \hat{P} - \tilde{A}\hat{P}\tilde{A}^T > 0. \end{array} \quad (18)$$

  Positive realness is thereafter imposed by solving a similar SDP-problem involving vectors of stacked covariance sequences. The performance of the SDP-technique was evaluated using software written by the authors and published on their website.

- **RES:** In (Van Overschee and De Moor, 1996) the residuals $\rho_w$ and $\rho_v$ of the least squares problem

$$\begin{bmatrix}\hat{X}_{i+1}\\ Y_{i|i}\end{bmatrix} = \begin{bmatrix}A\\ C\end{bmatrix}\hat{X}_i + \begin{bmatrix}\rho_w\\ \rho_v\end{bmatrix} \quad (19)$$

  are used to get estimates for $Q$, $S$ and $R$ that are guaranteed to be positive:

$$\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix} = \mathcal{E}\left\{\begin{bmatrix}\rho_w\\ \rho_v\end{bmatrix}\begin{bmatrix}\rho_w^T & \rho_v^T\end{bmatrix}\right\}. \quad (20)$$

The algorithm leads to biased estimates, unless $i \to \infty$, and is only applicable to stable models.

- **REG$_{\hat{\Lambda}_0}$:** Regularization on $\hat{\Lambda}_0$, as proposed in (Peternell, 1995). From $S_z(z) = \hat{D} + \hat{C}(zI_n - \hat{A})^{-1}\hat{G}$ and $\hat{\Lambda}_0 = \hat{D} + \hat{D}^T$ it is easily seen that $S_z(z) + S_z^T(z^{-1})$ can always be made positive provided $\hat{\Lambda}_0$ is chosen large enough. The method only works for stable models.

- **REG$_{\hat{G}}$:** In (Vaccaro and Vukina, 1993) one starts from REG$_{\hat{\Lambda}_0}$ to make the spectrum positive and solve the Riccati equation (8) for $\hat{P}$. The new $\hat{\Lambda}_0$, which will be denoted as $\tilde{\Lambda}_0$ and $\hat{P}$ are used to obtain an adjusted $\hat{G}$, denoted as $\tilde{G}$, after which $\hat{\Lambda}_0$ is again set to its initial value.

$$\begin{aligned}\hat{P} &= \hat{A}\hat{P}\hat{A}^T + (\hat{G} - \hat{A}\hat{P}\hat{C}^T)\cdot \\ &\quad (\tilde{\Lambda}_0 - \hat{C}\hat{P}\hat{C}^T)^{-1}(\hat{G} - \hat{A}\hat{P}\hat{C}^T)^T \\ &= \hat{A}\hat{P}\hat{A}^T + (\tilde{G} - \hat{A}\hat{P}\hat{C}^T)\cdot \\ &\quad (\hat{\Lambda}_0 - \hat{C}\hat{P}\hat{C}^T)^{-1}(\tilde{G} - \hat{A}\hat{P}\hat{C}^T)^T \qquad (21)\end{aligned}$$

The model $(\hat{A}, \tilde{G}, \hat{C}, \hat{D})$ can be shown to be positive real. The technique works on stable models only.

## 3. EMPIRICAL EVALUATION AND SIMULATION RESULTS

A known system was used to create output samples from Gaussian, zero mean, unit variance, white noise sequences. For each output sequence the stochastic subspace approach described in section 1 was used in combination with techniques to impose positive realness where necessary. The following system was used for the simulation:

$$H(z) = \frac{(z - 0.99e^{\pm 2j})(z - 0.98e^{\pm 1.4j})}{(z - 0.8e^{\pm 2.1j})(z - 0.8e^{\pm j})} \qquad (22)$$

$$\cdot \frac{(z - 0.99e^{\pm 0.6j})(z \pm 0.9)}{(z - 0.8e^{\pm 1.7j})(z - 0.8e^{0.8j})} \qquad (23)$$

The results of the simulations are reported in Table 1. The table contains the results of 4 different experiments, each with a different choice of the parameters $\hat{n}$ (order of the model), $i$ (number of block-rows), and $N$ (number of observations). For each experiment, 1000 noise-sequences were generated with the desired length $N$, and an equal number of covariance models were produced. The number of covariance models that needed corrections for stability and/or positive realness are reported. As unstable models are always non positive real, the latter number will always be greater than the former. Below this information, the performance of each technique on these non positive real models is given. The performance on all non-positive real, but stable models is given at the left hand side. The results for the unstable models are given at the right hand side. The performance measures $d_\infty, d_2, d_1$ used in the tables are norms of the differences between the transfer functions of the simulated and the identified stochastic models:

$$d_p = ||H(z) - \hat{H}(z)||_p \qquad (24)$$

with $p = 1, 2, \infty$ and $\hat{H}(x)$ is the identified model. Note that results for the techniques REG$_{\hat{G}}$, REG$_{\hat{\Lambda}_0}$, and RES on unstable models are sometimes available in the tables, even though it was stated earlier in this paper that these techniques do not work for unstable models. The reason is that for these techniques the regularization procedure described in (Van Gestel *et al.*, 2001) was used to impose stability on the covariance model prior to imposing positive realness. This to avoid ending up in a hard-failure mode during the experiments, and to maintain the possibility to compare the performance of all the techniques, even on unstable models. In some cases however, the experiments did return invalid results for some of the entries in the table (denoted by '-'), for instance if the total number of unstable models is zero. In this case, averaging over these models was impossible.

**Performance of the techniques**

Two techniques, RES and REG$_{\hat{A},\hat{C}}$ clearly outperform the others. For some experiments the former results in slightly better estimates, however problems with this method might occur as the system order is increased. To visualize this, in Figure 2 the estimated spectral densities for the fourth experiment ($\hat{n} = 10$, $i = 16$, $N = 500$) averaged over all 1000 runs (including the ones which did not need correction) are given, together with the spectral density of the original model and a 95% error region. Note the spikes in the average spectral density and its confidence bounds in many techniques, indicating bad performance on at least some of the 1000 sequences used for the experiment. Note also that in principle, without adaptation, the RES technique only works for stable models, a condition which is seldomly satisfied for non positive real models.

**Influence of $i$, $\hat{n}$ and $N$**

It is interesting to have a look into the influence of the parameters $\hat{n}$, $i$ and $N$ on the occurrence of positive realness problems. In Table 1, decreasing $i$ from 16 to 12 clearly resulted in a much higher number of non positive real models. It is well known that when the modeling order $\hat{n}$ increases, the probability to obtain unstable models increases considerably (see also (Van Gestel *et al.*, 2001)). This can also be observed in the table. Finally, it is observed that for the example described in this paper the influence of $N$ on the occurrence of positivity problems is relatively low compared to that of $\hat{n}$ and $i$.

## 4. CONCLUSIONS

Stochastic subspace methods for the identification of linear time-invariant systems are known to be asymptotically unbiased. However, for a finite amount of data, and depending on the choice of some used defined variables as the modeling order and the number of covariance lags used in the identification procedure, the procedure might break down due to positive realness problems. In this paper a regularization approach was proposed to impose positive realness on a formerly identified covariance model. It was shown that, if an adequate amount of regularization is used, a positive real model can always be obtained. The simulation results clearly indicate that this new approach yields better models than other existing techniques.

## 5. ACKNOWLEDGEMENTS

Our research is supported by grants from several funding agencies and sources:

**Research Council KUL:** Concerted Research Action GOA-Mefi sto 666 (Mathematical Engineering), IDO (IOTA Oncology, Genetic networks), several PhD/postdoc & fellow grants;

**Flemish Government:** Fund for Scientifi c Research Flanders (several PhD/postdoc grants, projects G.0256.97 (subspace), G.0115.01 (bio-i and microarrays), G.0240.99 (multilinear algebra), G.0197.02 (power islands), G.0407.02 (support vector machines), research communities ICCoS, ANMMM), AWI (Bil. Int. Collaboration Hungary/ Poland), IWT (Soft4s (softsensors), STWW-Genprom (gene promotor prediction), GBOU-McKnow (Knowledge management algorithms), Eureka-Impact (MPC-control), Eureka-FLiTE (flutter modeling), several PhD grants);

**Belgian Federal Government:** DWTC (IUAP IV-02 (1996-2001) and IUAP V-10-29 (2002-2006): Dynamical Systems and Control: Computation, Identifi cation & Modelling), Program Sustainable Development PODO-II (CP-TR-18: Sustainibility effects of Traffi c Management Systems);

**Direct contract research:** Verhaert, Electrabel, Elia, Data4s, IPCOS;

## 6. REFERENCES

Dahlén, A., A. Lindquist and J. Mari (1998). Experimental evidence showing that stochastic subspace identification methods may fail. *Systems and Control Letters* **34**, 302–312.

Faurre, P., M. Clerget and F. German (1978). Opérateurs rationnels positifs, application à l'hyperstabilité et aux processus aléatoires. *Méthodes Mathématiques de l'Informatique*.

Goethals, I., T. Van Gestel, J. Suykens, P. Van Dooren and B. De Moor (2002). Identification of positive real models in subspace identification by using regularization, submitted for publication.

Lindquist, A. and G. Picci (1996). Canonical correlation analysis, approximate covariance extension, and identification of stationary time series. *Automatica* **32 no 5**, 209–233.

Marí, J., P. Stoica and T. McKelvey (2000). Vector arma estimation: A reliable subspace approach. *IEEE Transactions on Signal Processing* **SP-48**, 2092–2104.

Oono, Y. (1981). Introduction to pseudo-positive-real functions. *Proc. 1981 Int. Symp. Circuits and Systems* **Chicago**, 469–472.

Peternell, K. (1995). *Identification of linear dynamic systems by subspace and realization-based algorithms, PhD thesis*. T.U. Wien. Vienna.

Vaccaro, R.J. and T. Vukina (1993). A solution to the positivity problem in the state-space approach to modeling vector-valued time series. *Journal of Economic Dynamics and Control* **17**, 401–421.

Van Gestel, T., J. Suykens, P. Van Dooren and B. De Moor (2001). Identification of stable models in subspace identification by using regularization. *IEEE Transactions on Automatic Control* **46(9)**, 1416–1420.

Van Overschee, P. and B. De Moor (1993). Subspace algorithms for the stochastic identification problem. *Automatica* **29, no 3**, 649–660.

Van Overschee, P. and B. De Moor (1996). *Subspace Identification for Linear Systems: Theory - Implementation - Applications*. Kluwer Academic Publishers. Boston/London/Dordrecht.

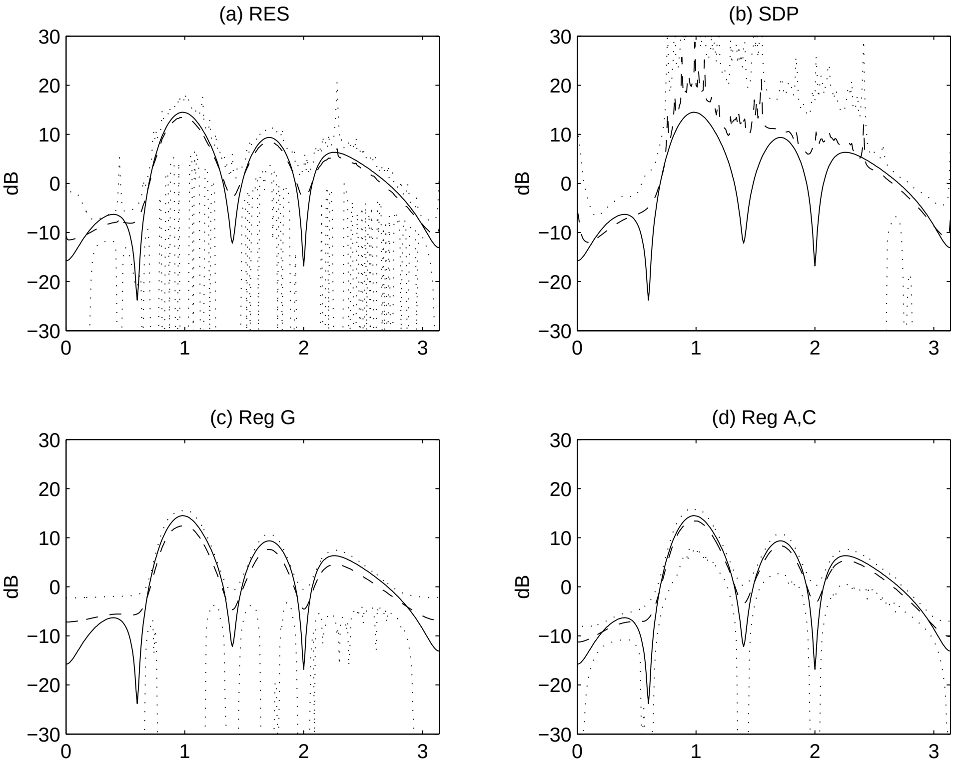


Fig. 2. Averaged Spectral density over 1000 runs for the simulation example ($H(z)$) with $\hat{n} = 10$, $i = 16$, $N = 500$ (dashed line) with $95\%$ error region (dotted line). The solid line is the spectral density of the original model used for simulation. The numerical results are summarized in Table 1

.

Table 1. Performance for various techniques on the simulation example ($H(z)$). Results for REG$_{\hat{G}}$, REG$_{\hat{\Lambda}_0}$, RES on unstable models are emphasized to stress the fact that they cannot be obtained without making the model stable first

| $n = 8, i = 16, N = 500$ | | | Not positive real 528/1000 | | | | Unstable 0/ 1000 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Stable models | | | | | Unstable models | | | | |
| | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP |
| Mean($d_\infty$) | 1.6 | 2.05 | 2.24 | 1.46 | 9.54 | - | - | - | - | - |
| Var($d_\infty$) | 0.324 | 0.666 | 0.624 | 0.239 | 504 | - | - | - | - | - |
| Mean($d_2$) | 0.571 | 0.695 | 0.771 | 0.549 | 1.93 | - | - | - | - | - |
| Var($d_2$) | 0.0181 | 0.0573 | 0.0566 | 0.0146 | 3.41 | - | - | - | - | - |
| Mean($d_1$) | 1.35 | 1.76 | 1.82 | 1.32 | 3.47 | - | - | - | - | - |
| Var($d_1$) | 0.0813 | 0.459 | 0.31 | 0.0673 | 3.38 | - | - | - | - | - |
| $n = 8, i = 12, N = 500$ | | | Not positive real 794/1000 | | | | Unstable 4/ 1000 | | | |
| | Stable models | | | | | Unstable models | | | | |
| | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP |
| Mean($d_\infty$) | 1.55 | 2.19 | 2.42 | 1.48 | 3.59 | 2.48 | *3.82* | *4.67* | *10.1* | 8.01 |
| Var($d_\infty$) | 0.253 | 0.684 | 0.665 | 0.518 | 37.2 | 0.0523 | *0.388* | *0.00468* | *15.1* | 68.4 |
| Mean($d_2$) | 0.577 | 0.75 | 0.846 | 0.549 | 1.12 | 1 | - | - | *1.19* | 1.62 |
| Var($d_2$) | 0.0171 | 0.0662 | 0.0716 | 0.0159 | 0.274 | 0.00826 | - | - | *0.035* | 0.8 |
| Mean($d_1$) | 1.37 | 1.87 | 2.02 | 1.29 | 2.54 | 2.47 | *2.93* | *4.56* | *2.59* | 3.16 |
| Var($d_1$) | 0.0881 | 0.475 | 0.426 | 0.0662 | 0.843 | 0.0872 | *0.222* | *0.000303* | *0.115* | 1 |
| $n = 8, i = 16, N = 1000$ | | | Not positive real 544/1000 | | | | Unstable 1/ 1000 | | | |
| | Stable models | | | | | Unstable models | | | | |
| | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP |
| Mean($d_\infty$) | 1.15 | 1.58 | 1.75 | 1.05 | 8 | 1.46 | *1.63* | *4.68* | *1.48* | 41.9 |
| Var($d_\infty$) | 0.157 | 0.495 | 0.445 | 0.0977 | 1.84e+03 | 253 | - | - | - | - |
| Mean($d_2$) | 0.413 | 0.533 | 0.602 | 0.418 | 1.48 | 0.55 | - | - | 0.607 | 8.05 |
| Var($d_2$) | 0.00939 | 0.0456 | 0.0407 | 0.00594 | 4.41 | - | - | - | - | - |
| Mean($d_1$) | 0.972 | 1.5 | 1.41 | 1.03 | 2.74 | 1.44 | *1.54* | *4.54* | *1.63* | 10.3 |
| Var($d_1$) | 0.0431 | 0.716 | 0.208 | 0.0275 | 2.43 | 11.6 | - | - | - | - |
| $n = 10, i = 16, N = 500$ | | | Not positive real 727/1000 | | | | Unstable 182/ 1000 | | | |
| | Stable models | | | | | Unstable models | | | | |
| | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP | REG$_{\hat{A},\hat{C}}$ | REG$_{\hat{G}}$ | REG$_{\hat{\Lambda}_0}$ | RES | SDP |
| Mean($d_\infty$) | 1.7 | 2.45 | 2.63 | 2.19 | 18.3 | 2.48 | *4.34* | *4.93* | *4.46* | 15.1 |
| Var($d_\infty$) | 0.488 | 1.57 | 1.04 | 6.02 | 4e+03 | 2.31 | *5.69* | *0.293* | *16* | 3.76e+03 |
| Mean($d_2$) | 0.579 | 0.784 | 0.889 | 0.591 | 2.46 | 0.696 | - | - | *0.723* | 2.06 |
| Var($d_2$) | 0.0172 | 0.0907 | 0.117 | 0.0294 | 8.59 | 0.0298 | - | - | *0.0623* | 9.65 |
| Mean($d_1$) | 1.36 | 1.98 | 2.12 | 1.35 | 4.08 | 1.65 | *2.9* | *4.53* | *1.6* | 3.47 |
| Var($d_1$) | 0.0709 | 0.643 | 0.76 | 0.0655 | 6.7 | 0.174 | *0.698* | *0.00353* | *0.168* | 6.82 |