Systems & Control Letters 12 (1989) 93–94
North-Holland

Comments on "Minimum-gain minimum-time deadbeat controllers"

P. Van DOOREN
Philips Research Laboratory, Brussels, Belgium

Received 17 October 1988

Abstract: We point out that the problem formulated in the above paper by Elabdalla and Amin [1] was solved in an earlier paper [3] using a direct approach.

Keywords: Deadbeat control; numerical methods.

Introduction

Consider the following state-space system:

$$x_{t+1} = Ax_t + Bu_t, \quad t = 0, 1, 2, \ldots, \quad x_0 \text{ given}, \tag{1}$$

where A is an $n \times n$ matrix and B is an $n \times m$ matrix. Assume the system is reachable and let the *reachability indices* be equal to $\mu_1, \mu_2, \ldots, \mu_m$, i.e.:

$$\operatorname{rank}[B \quad AB \quad A^2B \quad \cdots \quad A^{k-1}B] = \sum_{i=1}^{k} \mu_i. \tag{2}$$

Substituting a control law $u_t = Fx_t$ in (1) we obtain the modified system

$$x_{t+1} = (A + BF)x_t, \quad t \geq 0 \text{ and } x_0 \text{ given}. \tag{3}$$

The problem of *minimum-gain minimum-time deadbeat control* as formulated in [1] is to find a feedback law F that is of minimum (Frobenius) norm, and that will drive an arbitrary initial state x_0 to $x_k = 0$ after a *minimum* number of steps k. The latter is equivalent to requiring all eigenvalues of $A + BF$ to lie at $\lambda = 0$ with Jordan chains of *minimal* length μ_i. In [3] it is shown how to solve this problem via a direct method, hence involving no iteration. Moreover, the approach does not require B to have rank m as in [1].

It suffices to construct a nested orthonormal basis for the *controllability subspaces*:

$$S_i \doteq \operatorname{Im}[B \quad A^{-1}B \quad \cdots \quad A^{-i+1}B], \tag{4}$$

where the inverse A^{-1} stands for the functional inverse (see [3]). Indeed, let U be a unitary matrix such that

$$S_i \doteq \operatorname{Im}[U_1 \quad U_2 \quad \cdots \quad U_i], \tag{5}$$

i.e. whose first i blocks span the space S_i. Then *any* minimum-time feedback must satisfy

$$U^{\mathrm{T}}AU + U^{\mathrm{T}}BFU = \begin{array}{c} \left[\begin{array}{ccccc} 0 & \bar{A}_{1,2} & \cdots & \cdots & \bar{A}_{1,k} \\ & 0 & & & \vdots \\ & & \ddots & & \vdots \\ & & & 0 & \bar{A}_{k-1,k} \\ & & & & 0 \end{array}\right] \\ \begin{array}{ccccc} \underbrace{\quad}_{r_1} & \underbrace{\quad}_{r_2} & & & \underbrace{\quad}_{r_k} \end{array} \end{array} \begin{array}{l} \}r_1 \\ \\ \\ \}r_{k-1} \\ \}r_k \\ \\ \end{array}, \tag{6}$$

where the block sizes r_i are connected to the reachability indices μ_i via the rule [3]:

$$\text{there are } r_i \text{ indices } \mu_j \text{ equal to } i \text{ for } i = 1, \ldots, k. \tag{7}$$

Since a unitary transformation U does not affect the Frobenius norm of a matrix, one can as well look for the minimum norm solution $F_u \doteq FU$ in the transformed coordinate system (6). In [3] it is shown that in that coordinate system one merely has to solve k linear systems of equations – corresponding to the block columns of zeros in (6) – in a minimum norm sense in order to obtain the minimum norm solution for F_u. The whole process of constructing U, solving for F_u and computing $F = F_uU^{\mathrm{T}}$ requires less than $8n^2(n+m)$ floating point operations and is shown to be numerically stable in a mixed sense in [3]. The algorithm not only constructs U and F but also passes via the staircase form of the system (A, B) in order to compute its reachability indices via the numbers r_i (see [2]). This procedure should in general be much faster than an algorithm based on an iterative scheme as presented in [1].

We now show the results for the examples of [1] using this method. The tests were run in double precision on a VAX/VMS machine with relative precision $\varepsilon = 2^{-56} \approx 1.4\text{D} -17$. The program is written in FORTRAN 77 and is available in the subroutine library SLICOT [4]. We only show the first 5 digits of the results. For the matrices F, the remaining digits happen to be zero, and our routine also computed these numbers correctly up to the first 16 digits.

For Example 1 the resulting feedback matrix is:

$$F = \begin{bmatrix} 1.5520 & -3.2240 & 0.0000 \\ -0.6640 & 1.1680 & -1.0000 \end{bmatrix}$$

with Frobenius norm 3.9507 as in [1]. For Example 2 the resulting feedback matrix is:

$$F = \begin{bmatrix} 0.2500 & -0.7500 & 0.0000 & 0.0000 \\ -1.2500 & 0.7500 & 0.0000 & -1.0000 \\ 0.0000 & 0.0000 & 0.0000 & -1.0000 \end{bmatrix}$$

with Frobenius norm 2.1794 which is incorrectly evaluated in [1].

References

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