Comments on “Minimum-gain minimum-time deadbeat controllers”

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Abstract: We point out that the problem formulated in the above paper by Elabdalla and Amin [1] was solved in an earlier paper [3] using a direct approach.

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Introduction

Consider the following state-space system:
\[ x_{t+1} = \begin{bmatrix} A & B \\ 0 & A \end{bmatrix} x_t, \quad t = 0, 1, 2, \ldots, \quad x_0 \text{ given}, \] (1)
where \( A \) is an \( n \times n \) matrix and \( B \) is an \( n \times m \) matrix. Assume the system is reachable and let the reachability indices be equal to \( \mu_1, \mu_2, \ldots, \mu_m \), i.e.:
\[ \text{rank}[B \ A \ B^2 \ \ldots \ A^{k-1}B] = \sum_{i=1}^{k} \mu_i. \] (2)

Substituting a control law \( u_t = Fx_t \) in (1) we obtain the modified system
\[ x_{t+1} = \left( A + BF \right) x_t, \quad t \geq 0 \text{ and } x_0 \text{ given}. \] (3)

The problem of minimum-gain minimum-time deadbeat control as formulated in [1] is to find a feedback law \( F \) that is of minimum (Frobenius) norm, and that will drive an arbitrary initial state \( x_0 \) to \( x_k = 0 \) after a minimum number of steps \( k \).

Since a unitary transformation \( U \) does not affect the Frobenius norm of a matrix, one can as well look for the minimum norm solution \( F_u = FU \) in the transformed coordinate system (6). In [3] it is shown that in that coordinate system one merely has to solve \( k \) linear systems of equations – corresponding to the block columns of zeros in (6) – in a minimum norm sense in order to obtain the minimum norm solution for \( F_u \). The whole process of constructing \( U \), solving for \( F_u \) and computing \( F = F_uU^T \) requires less than \( 8n^2(n + m) \) floating point operations and is shown to be numerically stable in a mixed sense in [3]. The algorithm not only constructs \( U \) and \( F \) but also passes via the staircase form of the system \( (A, B) \) in order to compute its reachability indices via the numbers \( r_i \) (see [2]). This procedure should in general be much faster than an algorithm based on an iterative scheme as presented in [1].
We now show the results for the examples of [1] using this method. The tests were run in double precision on a VAX/VMS machine with relative precision $\varepsilon = 2^{-56} \approx 1.4 \times 10^{-17}$. The program is written in FORTRAN 77 and is available in the subroutine library SLICOT [4]. We only show the first 5 digits of the results. For the matrices $F$, the remaining digits happen to be zero, and our routine also computed these numbers correctly up to the first 16 digits.

For Example 1 the resulting feedback matrix is:

$$F = \begin{bmatrix}
1.5520 & -3.2240 & 0.0000 \\
-0.6640 & 1.1680 & -1.0000 \\
\end{bmatrix}$$

with Frobenius norm 3.9507 as in [1]. For Example 2 the resulting feedback matrix is:

$$F = \begin{bmatrix}
0.2500 & -0.7500 & 0.0000 & 0.0000 \\
-1.2500 & 0.7500 & 0.0000 & -1.0000 \\
0.0000 & 0.0000 & 0.0000 & -1.0000 \\
\end{bmatrix}$$

with Frobenius norm 2.1794 which is incorrectly evaluated in [1].

References