LMAT 2240 Knot theory and low-dimensional topology
2014/2015 - 1Q

6.0 Credits ECTS
Instructor: Pedro Vaz

The purpose of this course is to present the mathematical theory of knots and links as part of 3-dimensional topology. The techniques that will be used are connected to combinatorics, topology, group theory and algebraic topology. At the end of the course the student should be capable of working autonomously on an advanced subject in knot theory. All sessions consist of lectures and exercises.

Contents

1. Definitions and basic concepts
Definition of knot, projection and diagram, Reidemeister moves, operations on knots, 3-coloring, linking number.

2. The Jones polynomial
The Kauffman bracket, the Jones polynomial, relation between the number of crossings of an alternating knot and the Jones polynomial.

3. Topology of surfaces applied to knot theory
Seifert surface, classification of oriented surfaces with boundary, linking number as an intersection number with a Seifert surface, knot genus, additivity of the genus and decomposition in terms of prime knots.

4. The Alexander polynomial
Presentation matrices of modules, the Seifert matrix and the homology of the knot complement, the infinite cyclic cover of a knot, the Alexander module, the Alexander polynomial and its properties, the Alexander polynomial and the genus of a knot.

5. The knot group and the topology of the knot complement
The fundamental group of the knot complement, the Wirtinger presentation, the p-coloring of knot diagrams via the knot group.

6. Special topics (examples)
- The Witten invariant version of the Jones polynomial,
- knots and 3-manifolds,
- invariants of 3-manifolds via link invariants (Kirby calculus),
- braids, Alexander theorem, Markov theorem,
- tangles, operator invariants of tangles,
- Khovanov homology,
- Vassiliev invariants,
- applied knot theory (e.g. biology, chemistry, cryptography),
- knots and complexity theory.

Bibliography
- An Introduction to Knot Theory, W.B.R. Lickorish, GTM 175, Springer 1997.
- Knot knotes, par Justin Roberts (http://math.ucsd.edu/~justin/Papers/knotes.pdf)

Other resources
- K. Murasugi, *Knot theory & its applications*, Birkhäuser, Springer 1996.
- L. Kauffman, Knots and Physics, World Scientific 1991.
- C. Livingston, Knot Theory, Mathematical association of America, 1993.

Teaching method
Two sessions per week (2h+1h). Lectures of items (1)-(5), complemented with examples and exercises. The special topics proposed in (6) are object of oral presentations.

Prerequisite
A course on algebraic topology.

Evaluation
Written test (in class, with a weight of 65%) on subjects (1) to (5) and oral presentation (with a relative weight of 35%) on one of the special topics (6).

The students obtaining less than 10 in the evaluation are admitted to (written) exam on items (1) to (6).

Online resources
web page in iCampus.