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Incomplete-markets economies:
The seminal work of Diamond, Drèze and Radner
by
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Abstract

The present note highlights the seminal contributions of Diamond, Drèze and Radner towards the integration of financial markets into general equilibrium modeling.

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Introduction

Edward Elgar Publishing has published in 2008 a collection of papers on incomplete markets, under the editorship of Magill and Quinzii, covering the contribution of general equilibrium theory to the analysis of financial markets. It turned out to be a timely publication in view of the recent financial crisis which has dramatically highlighted the shortcomings of current macroeconomic modeling in ignoring the crucial interdependence between the real and the financial spheres.

"Every now and then new concepts and techniques emerge which lead to a fresh way of looking at old problems: hitherto disjoint or disconnected subfields come together to form parts of a coherent whole. Economists have long been interested in the relationship between the real, financial and monetary sectors of an economy. Traditionally, the analysis of the real sector was the purview of equilibrium theory and microeconomics (price theory), financial markets were the subjects of finance, while monetary theory formed part of macroeconomics. The object of this book is to show that the newly emerging theory of incomplete markets provides a useful framework for unifying these subfields and for clarifying the mutual dependence between real, financial and monetary phenomena."

(Opening sentence in the book of Magill and Quinzii, *Theory of Incomplete Markets*, MIT Press, 1996.)

Much remains to be done, no doubt, to accomplish the full integration envisioned by the authors of this quotation. But they are undoubtedly right in pointing to the emergence of the theory of incomplete-markets economies in the late sixties and early seventies as a major development, which has already enriched significantly our understanding of the functioning of our economies and which carries the promise of the contemplated integration.

The purpose of the present note is to highlight the seminal contributions of Diamond, Drèze and Radner to these developments, following Arrow's paper on *"The Role of Securities in the Optimal Allocation of Risk-bearing"* (1964).¹

Market incompleteness is a characteristic feature of contemporary market economies. This feature is particularly notable for future and contingent markets. When facing substantial uncertainty, agents are indeed reluctant to make detailed commitments into the future, the associated transactions costs being prohibitive. Hence markets for future or contingent deliveries are missing: actual economies do not operate as postulated by the idealized theory of general equilibrium. A host of important questions arise. They start with the definition of a suitable equilibrium concept, its existence and properties, and of course the status of the two welfare theorems.

¹ The founding papers of Arrow, Diamond, Radner and Drèze are reproduced in Magill and Quinzii (2008).

Early work in the finance literature did not stress market incompleteness, because the questions addressed and the models used did not bring out the importance of that feature. The Modigliani-Miller theorem of 1958 relies on an arbitrage argument between existing assets (stocks and bonds). The CAPM model of Markowitz (1952), Tobin (1958) and their followers relies on mean-variance preferences, these parameters being priced on the markets, which are thus implicitly complete.

The contribution of Peter Diamond

A first model of an incomplete-markets production economy was introduced in 1967 in a seminal paper of Peter Diamond, "*The role of a stock market in a general equilibrium model with technological uncertainty*". That paper builds on Kenneth Arrow's representation of uncertainty through "states of the world" that led to the theory of general equilibrium known as the "Arrow-Debreu model". Diamond introduces (admittedly a special case of) the basic model of incomplete-markets economies – still in use today – where uncertainty is described by an event tree, markets exist only for trading in spot commodities and in a limited set of assets or contracts. The only assets considered by Diamond were shares of stock of business firms. In its more general formulation (see below), that model describes well the market organization prevailing in modern economies.² It lends itself both to formal generality and to specificity of market incompleteness. It is often referred to as the model of "stock market economies".

In comparison with the standard general equilibrium model of the time, namely that of Debreu's *Theory of Value* (1959), there are two important new features: (i) the missing markets result in limited insurance opportunities, hence multiple budget constraints and limited information about prices of contingent claims; (ii) business firms serve a dual role, as producers of commodities and as suppliers of assets. Combining these two features, one notes that investment decisions of firms are not guided by market prices for future production in alternative events, these very markets being incomplete! Yet investment decisions are relevant, not only to the welfare of future consumers, but also to the future profits, hence dividends of the firms. Dividends accrue to shareholders. Somehow, the decisions of the firms must arbitrate between the interests of future consumers and those of current shareholders, with the latter holding control. And shareholders must reach collective decisions.

In Diamond's 1967 model, the new difficulties are eschewed by special assumptions. First, there is a single commodity, which limits the interests of future consumers to bare essentials namely the distribution of production possibilities across future events – *state-distribution* for short. Second, the technology has fixed coefficients i.e. the state-distribution of future output by each firm is given and is not a decision variable. Thus, the decision problem of each firm

² Perhaps the most significant further development is the extensive securitisation of otherwise non-marketable assets or contracts. It was discovered early on that creation of new assets is not invariably conducive to Pareto improvements. More recently, an example in Drèze et al. (2008) illustrates how securitisation together with asymmetric information can be conducive to proliferation of "toxic" assets.

boils down to a choice of scale (of investment level). Third, there are two periods only, with a single commodity, no markets are needed in the future, and there is no price uncertainty.

Under these assumptions, existence of equilibria is not problematic and Diamond is mostly concerned with efficiency of equilibria. The relevant concept is not first-best Pareto efficiency, because incomplete markets entail incomplete, hence inefficient risk-sharing. Instead, a concept of "constrained efficiency" is introduced, efficiency *relative to the set of allocations that can be realized under the prevailing market structure*. Competitive equilibrium on the stock market³ brings about a constrained-efficient allocation of investment unanimously endorsed by the shareholders of each firm; and every constrained-efficient allocation can be sustained by competitive stock prices. Thus, the two welfare theorems hold, under a suitable (constrained) reformulation. Intuitively, Diamond's conclusions are not surprising. In his model, every feasible investment project is priced on the stock market and this by-passes market-incompleteness.⁴

Diamond's Model (as it is commonly referred to) was inspired to a considerable extent by the finance literature of his days, which also concentrated on two-period single-commodity models. What Diamond added is the investment decisions. A first degree of integration between price theory and finance had been realized.

There is more to come but the path was open, and the significance of Diamond's paper to the sequel stands out clearly. He introduced the stock-market economies model that still retains attention today and the concept of constrained efficiency that proved central in later work.

The contribution of Roy Radner

Whereas Diamond took advantage of his special assumptions to build a transparent model and reached important conclusions, a general formulation was investigated by Roy Radner. In two papers (1967 and 1968) Radner reconsiders the Arrow-Debreu model and introduces differences in information across agents, but does not come to grips with market incompleteness. The decisive contribution comes with "*Existence of equilibrium of plans, prices and price expectations in a sequence of markets*" which appeared in (1972), a result that he later generalized (1979).

The 1972 paper considers a model with several goods, several time periods and general technologies. Agents being endowed with single-valued, common correct expectations about future prices, the equilibrium concept embodies perfect foresight.⁵ The aspect of collective decisions in firms is eschewed, by endowing firms with preferences of their own.

³ Alternatively stated: investment following Tobin's q .

⁴ This property has been studied extensively under the name of "spanning", starting with Ekern and Wilson (1974).

⁵ That property is definitely more demanding than rational expectations, as defined in the meantime by Muth (1961).

Radner proves existence of allocations at which the trades and plans of all agents are optimal, and all markets clear, provided share prices on the stock market are uniformly positive, a gap that has been partly filled in the ensuing literature when more suitable decision criteria for firms were introduced.⁶

To this date, the name "*Radner Equilibrium*" is still associated with perfect foresight equilibria in incomplete-markets economies.⁷ The significance of his work lies on the consistency of an equilibrium concept defined for a general model embodying rationality of plans and expectations. It is a remarkable achievement, and a benchmark of lasting value.

The contribution of Jacques Drèze

The collective decision aspect remained to be tackled. This was accomplished by Jacques Drèze in two papers, which appeared in 1972 and 1974. Drèze starts from Diamond's model, but with general convex technologies instead of fixed coefficients. This simple extension leads however to a "collective decision" problem because shareholders will typically not be unanimous about the production decisions of firms.

In an attempt at assessing the scope for efficiency, Drèze studies the conditions for *constrained efficiency* of production (investment) decisions. The conditions sought for are clear: it is *necessary* that the decision of each individual firm be Pareto-efficient from the viewpoint of the *final* shareholders of that firm, holding constant the production decisions of the other firms and the portfolios of the shareholders. Note that the production plan of a firm entails the same state-distribution of dividends for all its shareholders. For them, it has the properties of a *public good*. So, the conditions for efficiency are analogous to the Lindahl-Samuelson conditions for efficient production of a public good. In the case at hand, they receive a transparent formulation: each firm should maximize profits at shadow prices obtained as weighted averages of the shadow prices of shareholders, the weights being given by shareholdings. This is known as the "*Drèze Criterion*" for business decisions under incomplete markets, a criterion which has received since a number of extensions and applications, and remains of current interest.

More importantly perhaps, Drèze noted that the set of feasible allocations in a stock-market economy is *not convex*. The budget constraints of the consumers (multiple constraints under incomplete markets) are indeed *bilinear* in production plans and portfolio shares. Simultaneous adjustments in these two sets of variables allow efficiency gains that are not in sight under separate adjustments. As a consequence, *competitive equilibria of the stock market coupled with Pareto-efficient decisions of all firms need not be constrained efficient*, a property that has been later shown to hold generically by Geneakoplos, Magill, Quinzii and Drèze (1990). A crucial implication of this property is that there almost always exists scope

⁶ "Partly", due to the model specification in Drèze (1974) or to special assumptions in Grossman and Hart (1979). More recently, Drèze et al. (2009) introduced a concept of "investment-constrained equilibrium" to cover the case of undersubscribed stock emissions, thereby opening a new link to finance theory.

⁷ The more commonplace term is "General Equilibrium with Incomplete markets" or GEI.

for Pareto improvements through departures from competitive outcomes on either commodity or asset markets (or both). A significant application to “second-best wage rigidities” is cited below.

With this important discovery of constrained sub optimality of stock-market equilibria, the theory of incomplete-markets economies was definitely under way. It has received substantial attention from researchers ever since. For evidence of global interest in the model, one can refer to the special issue of the *Journal of Mathematical Economics* (Vol. 19, 1990), to the survey by Magill and Shafer in the *Handbook of Mathematical Economics* (Vol. 4, North-Holland, 1991), to the book by Magill and Quinzii, *Theory of Incomplete Markets*, quoted at the outset, or to the collection of papers on incomplete markets edited by Magill and Quinzii (2008).⁸

Extensions of the standard model of a stock-market economy

The question arises whether firms would serve the interests of their shareholders by maximizing future stock market value. It was noted by Drèze (1974) that a change in the production plan of a firm might not be advantageous to the shareholders, even if it raised market value, unless “the change had positive value at the *new* stock prices”. This is a different condition altogether. It brings in the stock market *responses* to production *adjustments*. These responses are in the nature of price derivatives, not prices. They are not directly observable, hence not covered by the assumption of perfect foresight.

Grossman and Hart introduced in 1979 the special assumption that each shareholder expects future stock prices to validate his or her own shadow prices on future states. A more general formulation is developed in Drèze, Lachiri and Minelli (2009). A recent paper by Bonnisseau and Lachiri (2004) established that constrained efficiency in the multi-period setting requires firms to adopt at each node (period–state) the Drèze criterion with weights given by shareholdings *at that node*. Again, the relevant information is not observable, thus confirming the limitations of competitive outcomes when markets are incomplete.

Other forms of corporate governance have been proposed based on majority voting among shareholders, an approach that runs into the Condorcet paradox of voting which precludes existence of equilibria. Drèze (1985 and 1989) has explored the more realistic alternative of introducing an *endogenous* board of directors with veto powers, the decisions of which are subject to approval by a majority vote among shareholders. In his 1989 lecture, Drèze has introduced the possibility of *incomplete preferences* using the Shafer-Sonnenschein (1975) framework. Incompleteness of preferences is however limited to firms and ought to be extended to households.

A further extension consists in introducing labor contracts. It was noted early on by James Meade (1972, p. 426) that “*While property owners can spread their risks by putting small bits of their property into a large number of concerns, a worker cannot put small bits of his effort*

⁸ This collection of papers in two volumes covers the period 1964 – 2005.

into a large number of different jobs. This presumably is a main reason why we find risk-bearing capital hiring labor rather than risk-bearing labor hiring capital". That observation was already at the root of the theory of implicit labor contracts developed in the mid-seventies. There, firms are risk-neutral and workers are risk-averse. Stock-market economies provide a natural framework for a more sophisticated analysis of labor contracts. This is done in Drèze (1989), and in Drèze and Gollier (1993) where risk-sharing considerations are shown to validate (at the second-best level) some degree of wage rigidity. This establishes a link to macroeconomics and the concept of equilibrium with price rigidities earlier introduced by Drèze (1975).

Concluding remarks

The foregoing suggests that models of stock-market economies are providing a suitable framework to integrate price theory and finance, and that a beginning has been made to extend the integration in the direction of macroeconomics. The opening quotation refers specifically to money; that is natural for Magill and Quinzii, who devote a chapter of their book to monetary economies – but with a particular modeling of money. Other authors have linked monetary policy to stock prices along different lines, either by noting the influence of stock prices on aggregate demand⁹ or by suggesting that monetary policy could play a role towards stabilizing stock prices, hence economic activity. That literature is however not yet linked to the work surveyed above. But the distance between the two has narrowed, as in the work of Greenwald and Stiglitz on "Financial Market Imperfections and Business Cycles" (1993) where firms are modeled as maximizing an expected utility of future market value.

To conclude, there are several clear indications that *"the newly emerging theory of incomplete markets provides a useful framework for unifying these subfields and for clarifying the mutual dependence between real, financial and monetary phenomena"*. The contributions of Peter Diamond (1967), Roy Radner (1972) and Jacques Drèze (1974) were indeed seminal to the emergence of a theory that keeps evolving. The long awaited second volume of Magill and Quinzii's book will be most welcome in summing up its recent developments.

⁹ A consumption channel inspired by the life-cycle theory of savings and an investment channel inspired by Tobin's q .

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