Interpolation on manifolds with B-splines

Pierre-Yves Gousenbourger
pierre-yves.gousenbourger@uclouvain.be

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From a set of $n+1$ points p_i on a manifold $\mathcal{M}$ associated to nodes $i \in \mathbb{Z}$, we seek a $\mathcal{C}^1$ function $\mathfrak{B} : \mathbb{R} \to \mathcal{M}$ such that $\mathfrak{B}(i) = p_i$.

To this end, we restrict $\mathfrak{B}$ to a family of manifold-valued piecewise-Bézier curves where the first and last segments are quadratic while the others are cubic (as in [AGSW16]). We then compute the *control points* of $\mathfrak{B}$ by generalizing the Euclidean concept of natural $\mathcal{C}^2$-splines.

One of the benefits of this application arise in problems whose solutions $(p_i)_{i=0}^n$ depend on only one parameter and are hard to compute, but are evaluated on a manifold $\mathcal{M}$. Hence, for a new value of the parameter, instead of solving the complicated problem, one can estimate the solution $p^\star$ by interpolating $(p_i)_{i=0}^n$ on $\mathcal{M}$.

The advantages of this technique are (i) a lower space complexity as the solution curve is represented by a few B ézier control points on the manifold, and (ii) a considerably simpler method that only requires two objects on the manifold: the Riemannian exponential and the Riemannian logarithm.

References

[AGSW16] P.-A. Absil, Pierre-Yves Gousenbourger, Paul Striewski, and Benedikt Wirth. Differentiable piecewise-bzier surfaces on riemannian manifolds. *SIAM Journal on Imaging Sciences*, 9(4):1788–1828, 2016.