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The problem

Bézier spline!

Find a C^1 curve $\mathbf{B}$, s.t.

$$\operatorname*{argmin}_{\mathbf{B}\in\Gamma} E_\lambda(\mathbf{B}) := \int_{t_0}^{t_n} \left\| \frac{D^2\mathbf{B}(t)}{dt^2} \right\|^2_{\mathbf{B}(t)} dt + \lambda \sum_{i=0}^{n} d^2(\mathbf{B}(t_i), d_i),$$

regularizer — data attachment

Motivated by:
Denoising or resampling
Medical applications
Computer graphics

A Riemannian manifold $\mathcal{M}$.
A set of m+1 manifold-valued data points $d_0, \dots, d_m \in \mathcal{M}$.
A set of m+1 time-parameters $t_0, \dots, t_m \in [0, n]$.
The Riemannian exponential "Exp"...
... and its inverse "Log".

And that's all!

What it needs

What it returns

A composite C^1 curve made out of n pieces

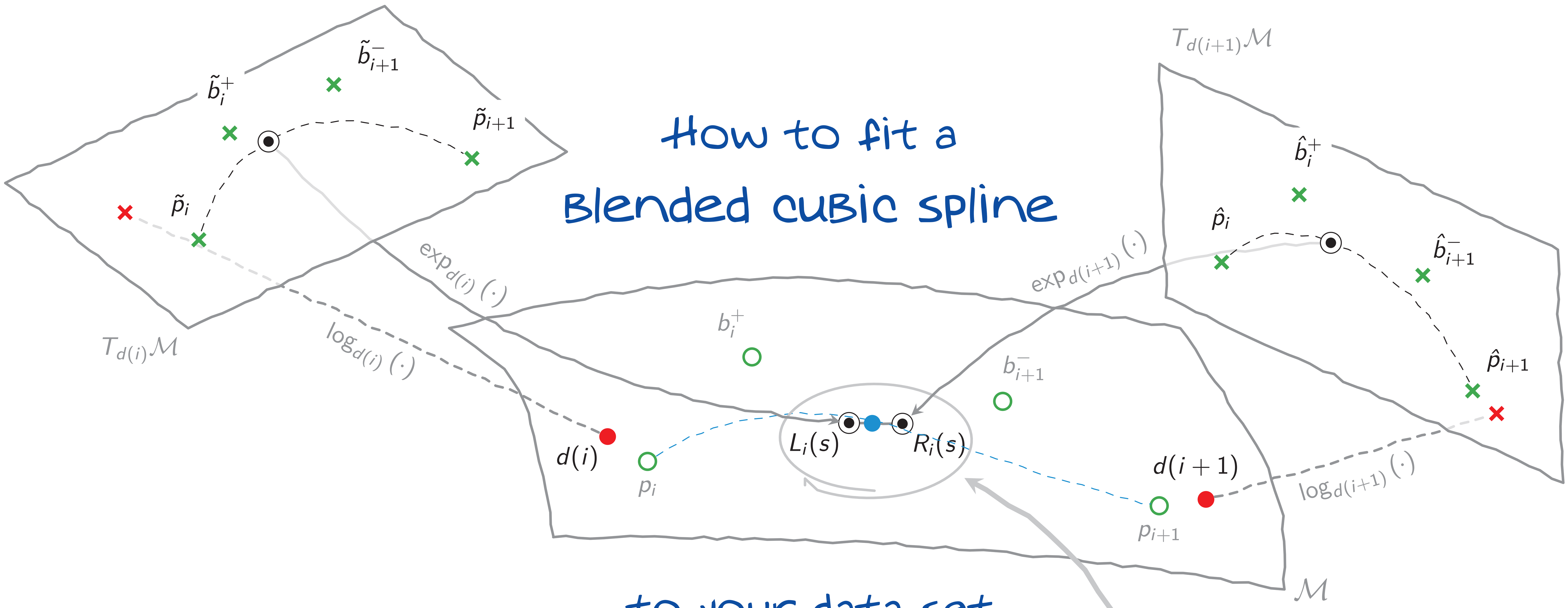
$$\mathbf{B} : [0, n] \to \mathcal{M} : t \mapsto \mathbf{B}(t) = \beta_i(t - i), \text{ with } i = \lfloor t \rfloor \dots$$

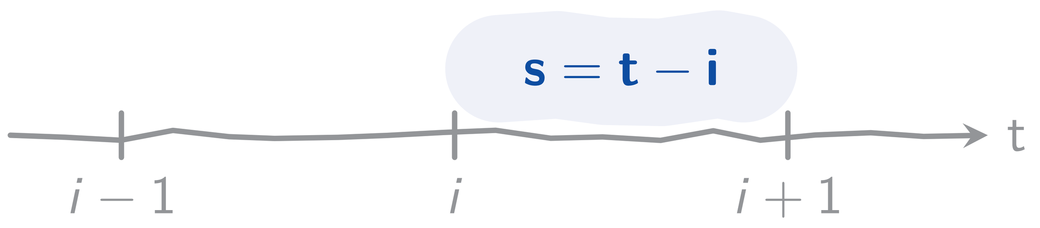
(i) ... differentiable on $[0, n]$,
(ii) ... that interpolates the data points if $m = n$ when $\lambda \to \infty$,
(iii) ... that is the natural cubic smoothing spline when $\mathcal{M} = \mathbb{R}^r$.

An efficient and easy-to-use method

(iv) ... that only uses Exp's and Log's on $\mathcal{M}$;
(v) ... that only stores $\mathcal{O}(n)$ tangent vectors;
(vi) ... that needs $\mathcal{O}(1)$ operations to compute $\mathbf{B}(t)$ at a given t.

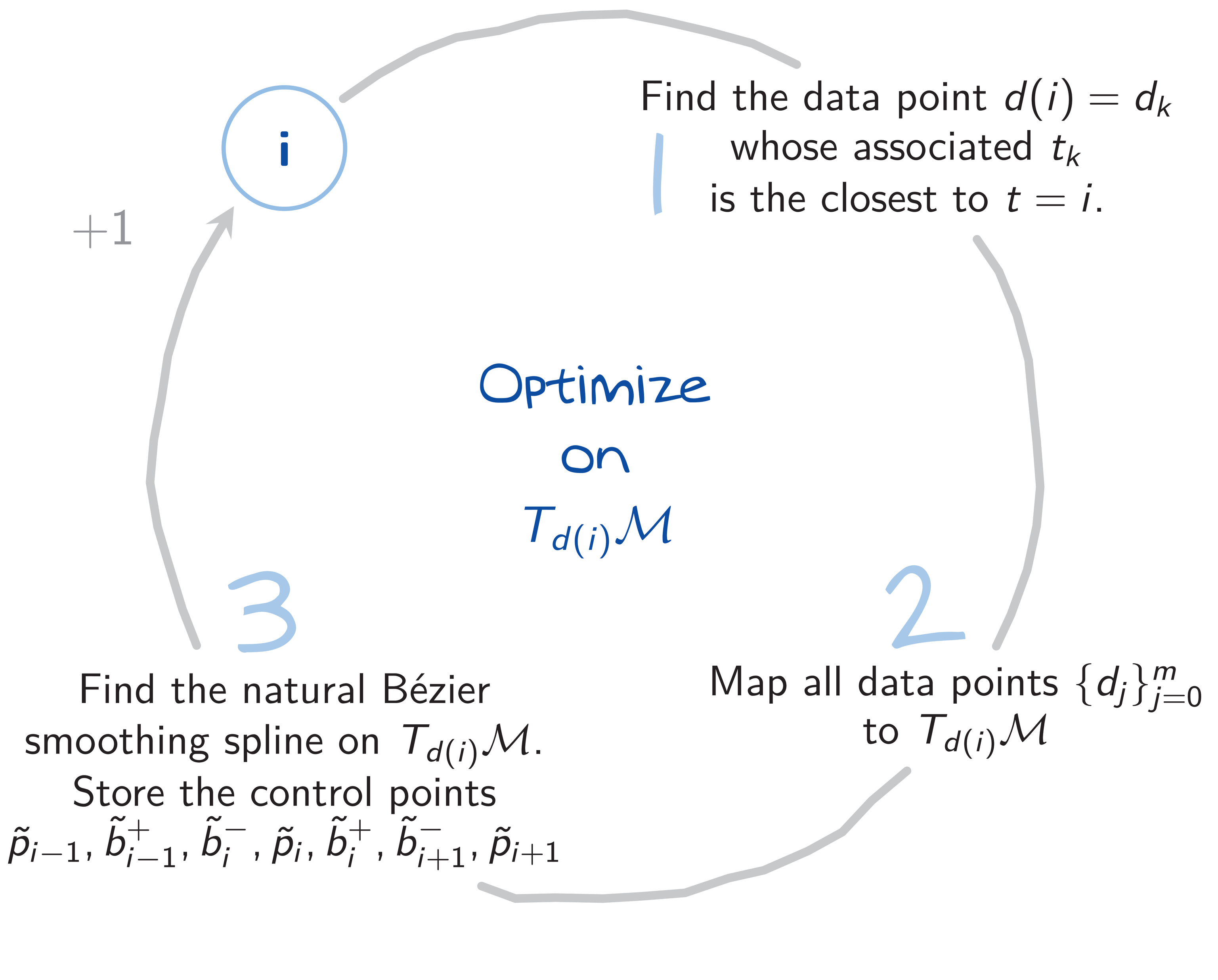
How to fit a Blended cubic spline to your data set on a manifold?





$\mathbf{B}(t) = \beta_i(s)$ is a weighted geodesic averaging of $L_i(s)$ and $R_i(s)$ with a weight $w(s) = 3s^2 - 2s^3$

How it gets to it



What it looks like

$\lambda = 100,\ m = 100,\ n = 4$ — $\lambda = 10^8,\ m = n = 10$