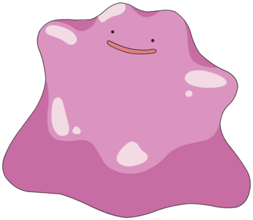
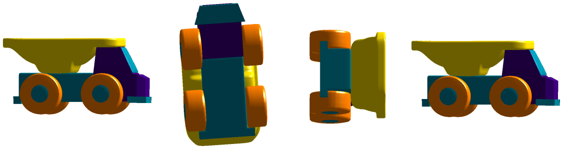
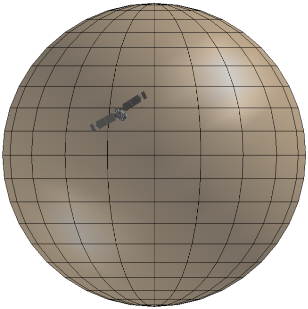


Poké-Collab: Kanto / 151 Pokemon by 151 Artists
July 22 - August 10 2013

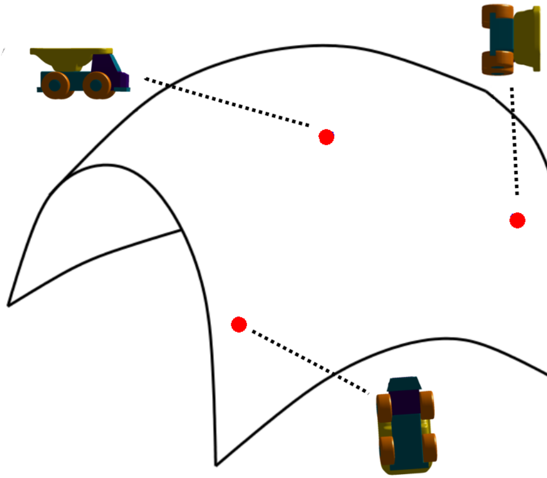


SO(3)

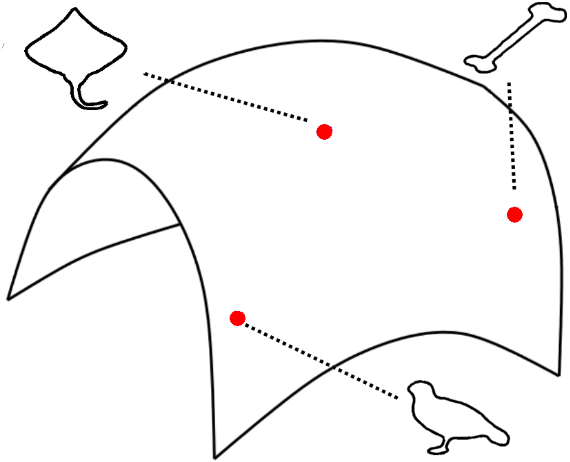


$SO(3) \times \text{Sphere}$

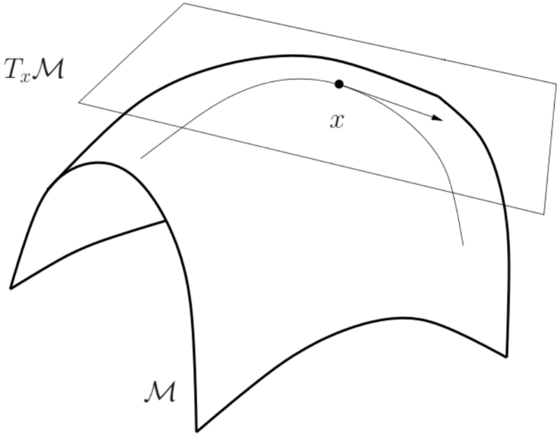
What's a manifold?



What's a manifold?

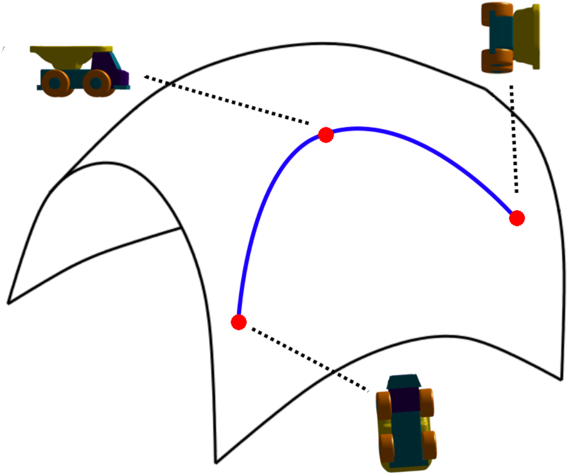


Hopefully, the tangent space at x is Euclidean

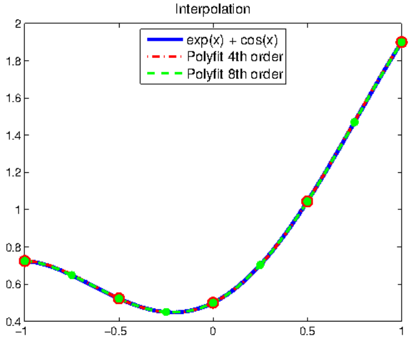


Manifolds. ✔

Interpolation.

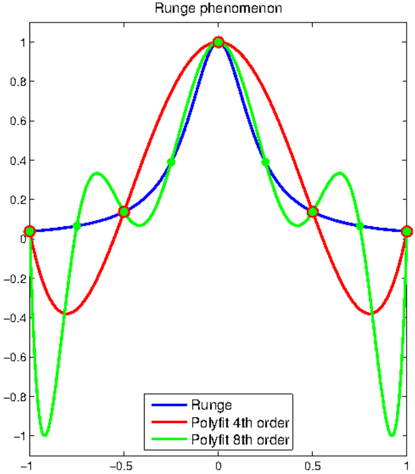


Interpolation on $\mathcal{M} = \mathbb{R}^n$



- Lagrange polynomials
- Cubic splines
- Bernstein
- curve fitting
- ... and many more (ask V.Legat).

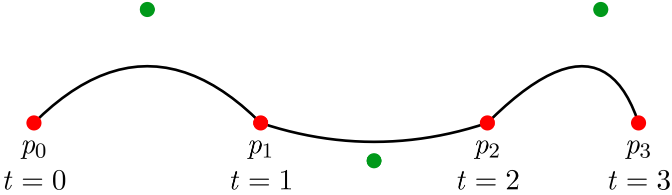
Interpolation on $\mathcal{M} = \mathbb{R}^n$



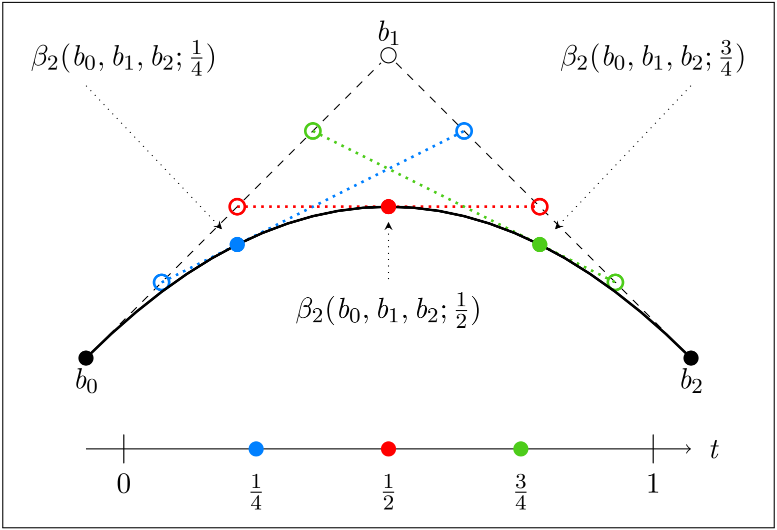
- Runge phenomenon
- Extrapolation error
- How to solve?
  Piecewise curves!

How to interpolate?

Each segment between two consecutive points is a **Bézier function**.



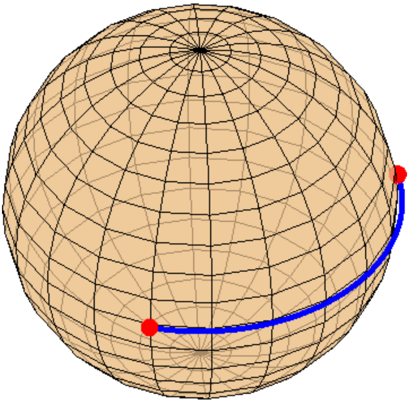
Reconstruction: the De Casteljau algorithm



How to generalize to manifolds?

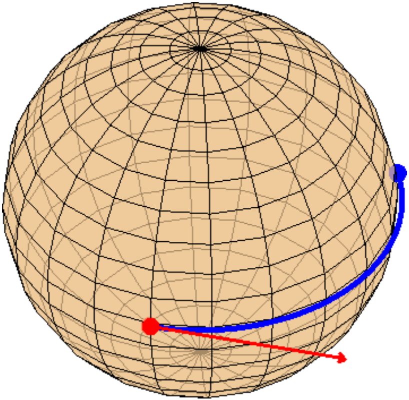
(

Geodesics are straight lines



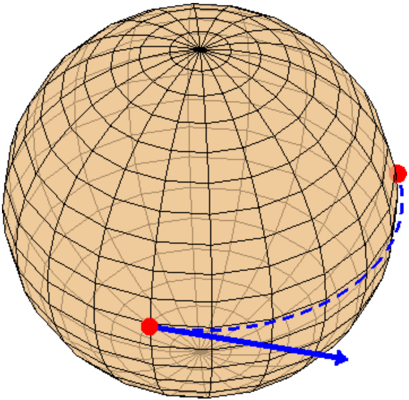
I'm a straight line!

Exponential maps computes geodesics



I compute the straight line!

Logarithmic maps are in the tangent space

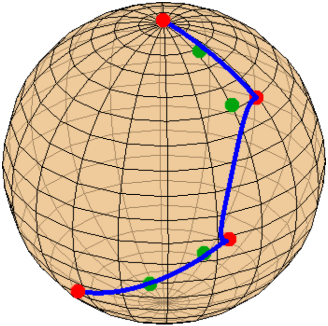


I'm in the tangent space!

(And I'm the velocity needed to compute the straight line!)

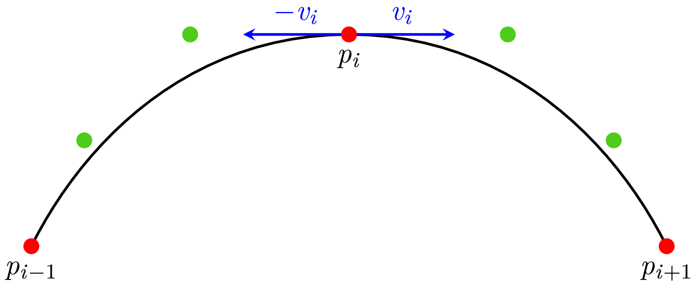
$$\bigg)$$

Example on the sphere



It’s ugly. Make it **smooth**!

$\mathcal{C}^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)



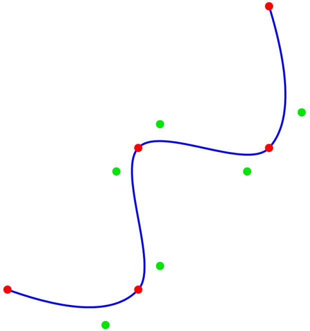
$$b_i^- = p_i - v_i \qquad \text{and} \qquad b_i^+ = p_i + v_i$$
$$b_i^- = \mathrm{Exp}_{p_i}(-v_i) \qquad \text{and} \qquad b_i^+ = \mathrm{Exp}_{p_i}(+v_i)$$

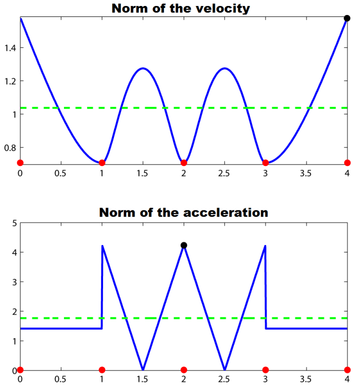
Manifolds. ✔

Interpolation. ✔

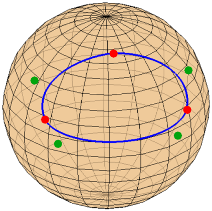
Results?

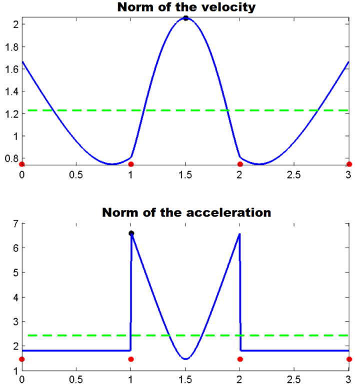
A result on $\mathbb{R}^2$



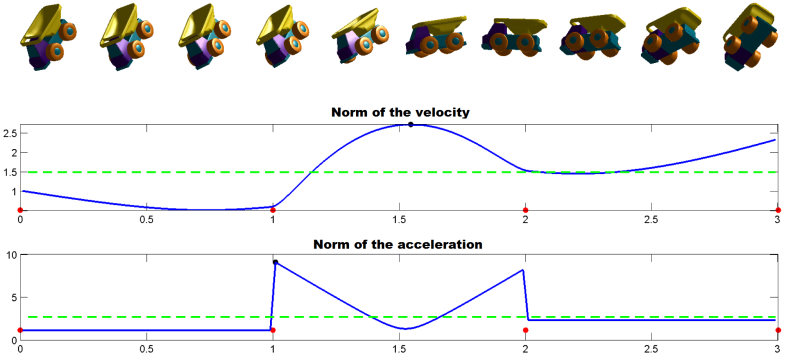


A result on the sphere

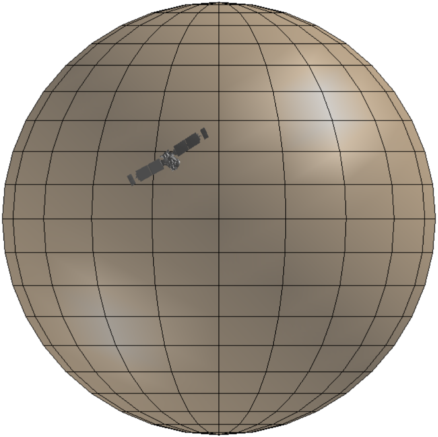




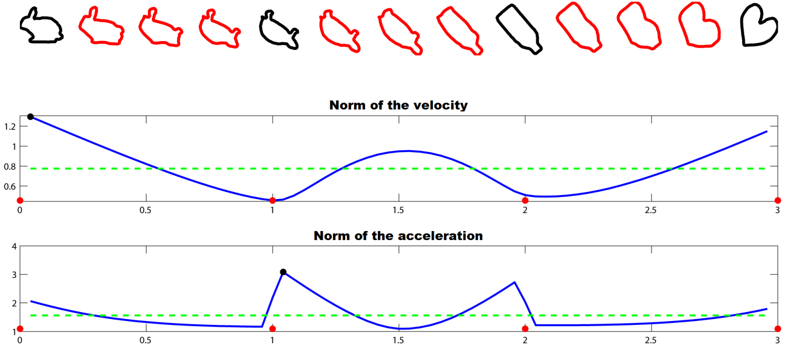
A result on SO(3)



Satellite moving



Morphing...

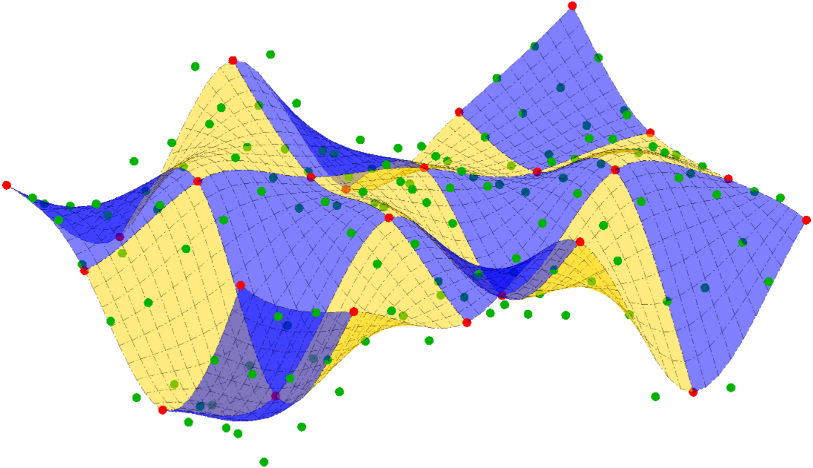


Smooth Bézier path

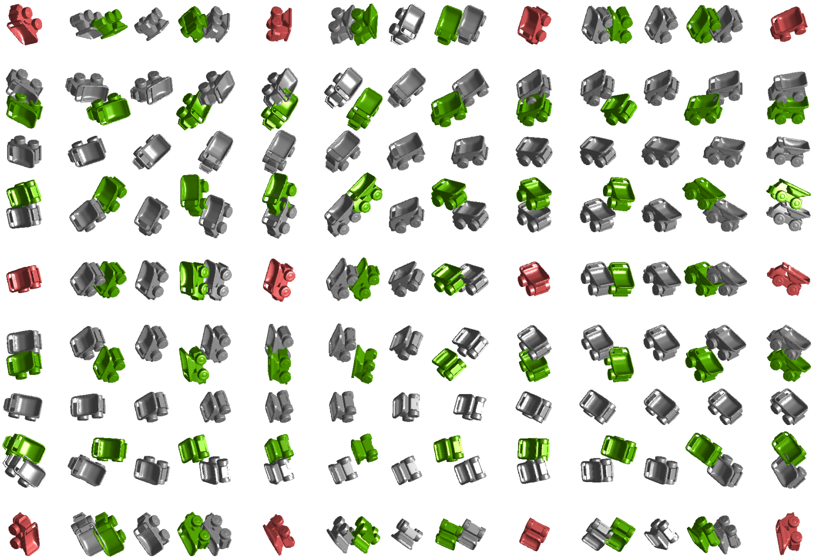
Piecewise geodesic (ugly) path

2D

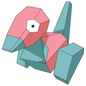
A result on $\mathbb{R}^2$



A result on SO(3)



A result on the space of triangulated shells (just because the result is cool)

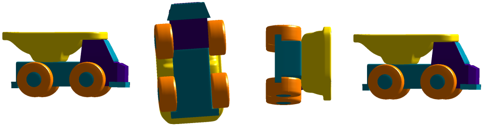


Any questions?

Bézier interpolation on Riemannian manifolds

ASCII's tutorial seminar

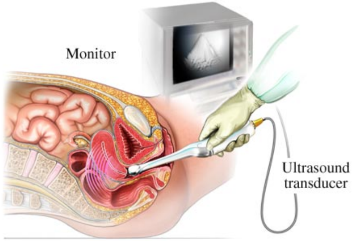
P.-Y. Gousenbourger

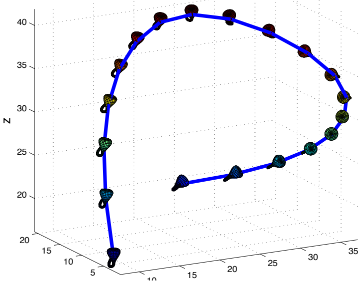


pierre-yves.gousenbourger@uclouvain.be

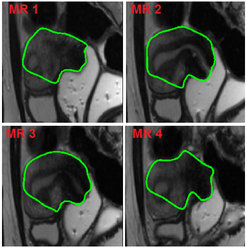
27.11.2015

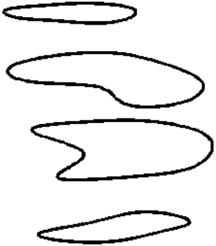
Application 1: MRI navigation

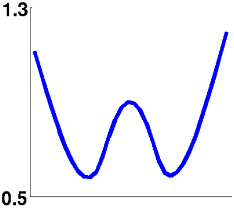


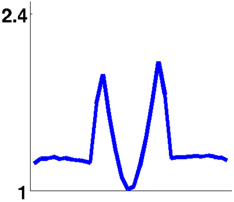


Application 2: Endometrial volume reconstruction









Optimal $\mathcal{C}^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

Minimization of the mean square acceleration of the path

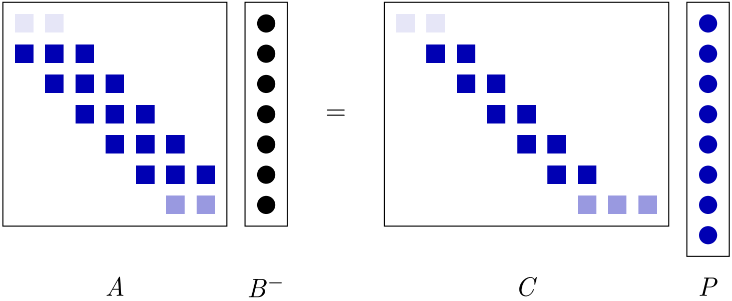
$$\underbrace{\min_{b_i^-} \int_0^1 \|\ddot{\beta}_2^0(b_1^-;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(b_i^-;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(b_{n-1}^-;t)\|^2 \mathrm{d}t}_{\text{Second order polynomial } P(b_i^-)}$$

$$\nabla P(b_i^-) \ !$$

Optimal $\mathcal{C}^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

Minimization of the mean square acceleration of the path

$$\underbrace{\min_{b_i^-} \int_0^1 \|\ddot{\beta}_2^0(b_1^-;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(b_i^-;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(b_{n-1}^-;t)\|^2 \mathrm{d}t}_{\text{Second order polynomial } P(b_i^-)}$$



Optimal $\mathcal{C}^1$-piecewise Bézier interpolation (on $\mathcal{M}$)

- The control points are given by:

$$b_i^- = \sum_{j=0}^{n} D_{i,j} p_j$$

- These points are invariant under translation, *i.e.*:

$$b_i^- - p^{ref} = \sum_{j=0}^{n} D_{i,j}(p_j - p^{ref})$$

- Transfer to the manifolds setting using the Log as $a - b \Leftrightarrow \mathrm{Log}_b(a)$

$$\mathrm{Log}_{p^{ref}}(b_i^-) = \sum_{j=0}^{n} D_{i,j} \mathrm{Log}_{p^{ref}}(p_j)$$