Endometriosis: MRI navigation and surface reconstruction on manifolds

GSI2015

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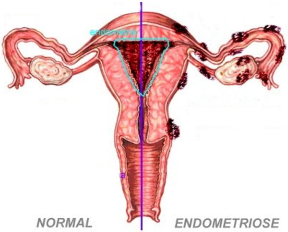
What is endometriosis ?

10 %

of women



Ovaries • Uterosacral ligaments • Colon • vagina • bladder

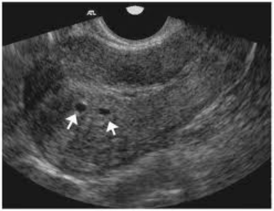
How to diagnose and cure ?

Before surgery

location • size • depth

MRI

TVUS



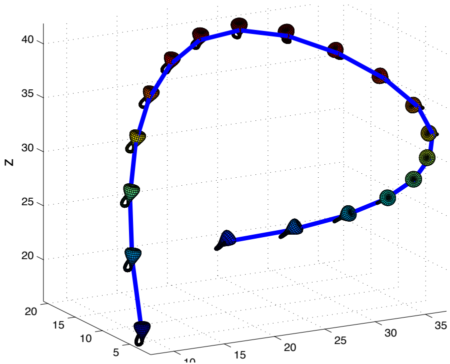
Question 1 : How to merge both techniques ?

Question 2 : How to evaluate the size of the cyst ?

When endometriosis meets manifolds

Answer 1 :

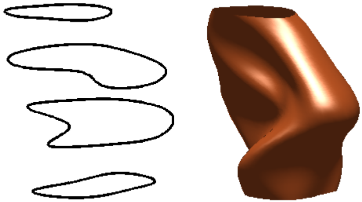
MRI navigation as a path on SE(3)



When endometriosis meets manifolds

Answer 2 :

Endometrial volume reconstruction as path on shape manifold

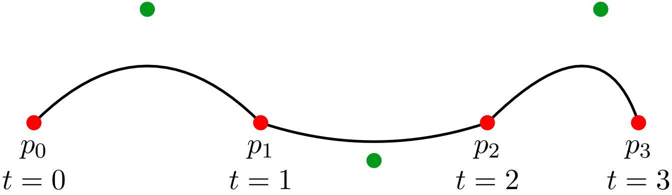


?

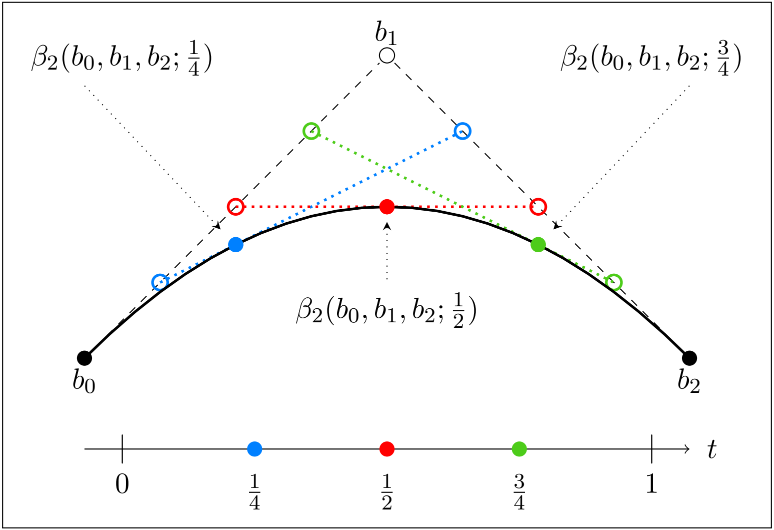
How to interpolate points on manifolds ?

How to interpolate ?

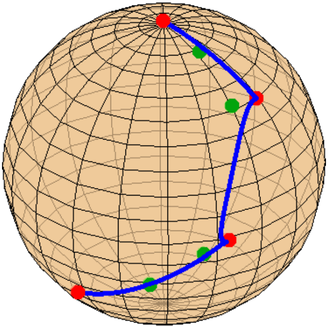
Each segment between two consecutive points is a **Bézier function**.



Reconstruction : the De Casteljau algorithm

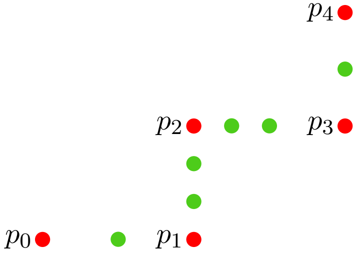


Example on the sphere



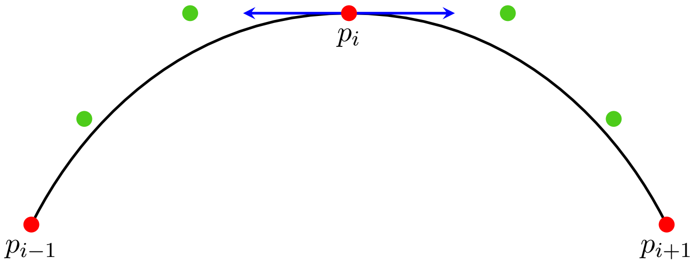
It's ugly. Make it **smooth**!

Smooth interpolation with Bézier (in $\mathbb{R}^n$)



Find the optimal position of control points

$\mathcal{C}^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)



$$b_i^+ = 2p_i - b_i^-$$

Optimal $\mathcal{C}^1$-piecewise Bézier interpolation (in $\mathbb{R}^n$)

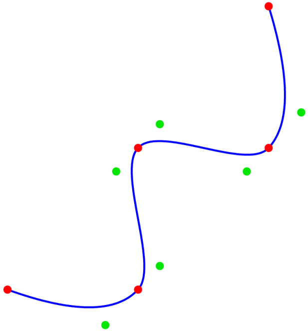
Minimization of the mean square acceleration of the path

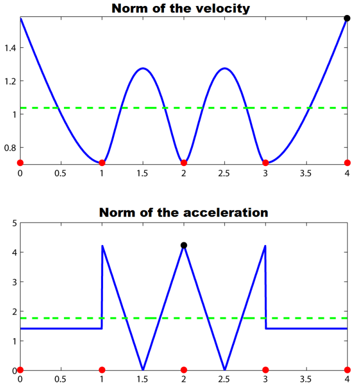
$$\min_{\alpha_i} \int_0^1 \|\ddot{\beta}_2^0(b_1^-;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(b_i^-;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(b_{n-1}^-;t)\|^2 \mathrm{d}t$$

$$\underbrace{\min_{\alpha_i} \int_0^1 \|\ddot{\beta}_2^0(b_1^-;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(b_i^-;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(b_{n-1}^-;t)\|^2 \mathrm{d}t}_{\text{Second order polynomial } P(b_i^-)}$$

$$\underbrace{\min_{\alpha_i} \int_0^1 \|\ddot{\beta}_2^0(b_1^-;t)\|^2 \mathrm{d}t + \sum_{i=1}^{n-1} \int_0^1 \|\ddot{\beta}_3^i(b_i^-;t)\|^2 \mathrm{d}t + \int_0^1 \|\ddot{\beta}_2^n(b_{n-1}^-;t)\|^2 \mathrm{d}t}_{\text{Second order polynomial } P(b_i^-)}$$

A result on $\mathbb{R}^2$





Optimal $\mathcal{C}^1$-piecewise Bézier interpolation (on $\mathcal{M}$)

- The control points are given by :

$$b_i^- = \sum_{j=0}^{n} D_{i,j} p_j$$

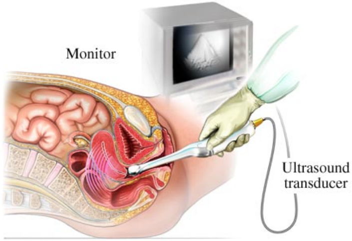
- These points are invariant under translation, *i.e.* :

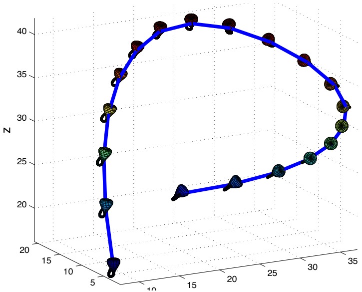
$$b_i^- - p^{ref} = \sum_{j=0}^{n} D_{i,j} (p_j - p^{ref})$$

- Transfer to the manifolds setting using the Log as $a - b \Leftrightarrow \mathrm{Log}_b(a)$

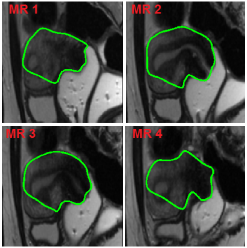
$$\mathrm{Log}_{p^{ref}}(b_i^-) = \sum_{j=0}^{n} D_{i,j} \mathrm{Log}_{p^{ref}}(p_j)$$

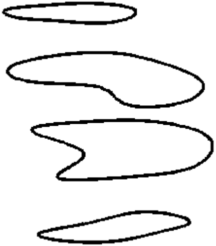
Application 1 : MRI navigation

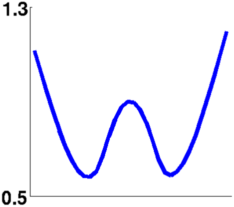


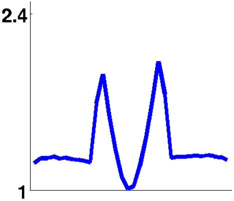


Application 2 : Endometrial volume reconstruction









Conclusions

General C^1-**interpolative** method on **manifolds**...
applied in medical imaging.

- It's light ;
- It's fast ;
- It's general ;

- Bézier interpolation can be extended to multidimentional interpolation (surfaces) ;

Any questions ?

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