Vintage capital growth theory: Three breakthroughs*

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Abstract

Vintage capital growth models have been at the heart of growth theory in the 60s. This research line collapsed in the late 60s with the so-called embodiment controversy and the technical sophisitication of the vintage models. This paper analyzes the astonishing revival of this literature in the 90s. In particular, it outlines three methodological breakthroughs explaining this resurgence: a growth accounting revolution, taking advantage of the availability of new time series, an optimal control revolution allowing to safely study vintage capital optimal growth models, and a vintage human capital revolution, along with the rise of economic demography, accounting for the vintage structure of human capital similarly to physical capital age structuring. The related literature is surveyed.

Keywords: Vintage capital, embodied technical progress, growth accounting, optimal control, endogenous growth, vintage human capital, demography.

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1 Introduction

Traditional aggregate productions functions are built on the assumption of homogenous capital in the sense that all capital goods constituting the operating stock of capital have the same marginal contribution to output. In particular, new and old capital goods contribute equally in conveying technical progress within the neoclassical paradigm (see Solow, 1956 and 1957). Such a view of capital denies *de facto* any connection between the pace of investment and the rate of technological progress in the long run as it can be readily inferred from the Solow decomposition apparatus. However, the assumed *disembodied* nature of technical progress looks barely unrealistic, as acknowledged by Solow himself (1960) in a posterior contribution:

"*...This conflicts with the casual observation that many if not most innovations need to be embodied in new kinds of durable equipment before they can be made effective. Improvements in technology affect output only to the extent that they are carried into practice either by net capital formation or by the replacement of old-fashioned equipment by the latest models...*" (page 91)

Accounting for the age distribution of investment goods sounds as the natural way to cope with the latter criticism, and this actually suggested a central stream of the growth theory literature of the 50's and 60's, giving birth to what we shall refer to as vintage capital growth theory. Surprisingly, this stream almost collapsed in the late 60s for different reasons. One is certainly due to the fact that the embodiment debate, involving such pre-eminent economists as Denison, Jorgenson, Phelps or Solow, got stuck at that time. As clearly outlined by Hercowitz (1998), this blockage was partly due to limited statistical resources. Indeed, the construction of time series on the relative price of durable goods by Gordon (1990) plays a decisive role in th spectacular revival of the vintage capital literature in the 90s. In the same vein, the recent conception of innovative tools for the mathematical treatment of delay-differential equations (both in terms of optimal control and numerical solution) to which a large class of vintage capital models lead, has definitely helped in the development of a new vintage capital literature based on intertemporal optimization and departing from the constant saving rate assumption so widely adopted in the 60s.

In this paper, we analyze the evolution of the vintage capital growth literature over the last 50 years. We start by highlighting the salient characteristics and implications of the seminal vintage capital models built up in the late 50s and 60s (see also Boucekkine, de la Croix and Licandro, 2008, for a summary). Then, we focus on the analysis of the recent impressive resurgence in the vintage capital literature. In particular, we identify three methodological breakthroughs:

1. *The growth accounting breakthrough*: The availability of times series on the relative price of equipment settles the old embodiment debate in the 90s and opens the door to a new growth accounting methodology based on two-sector modelling and stressing the importance of investment-specific or embodied technological progress.

2. *The optimal control breakthrough*: Vintage capital frameworks allow to address the issue of replacement of obsolete capital goods and technologies. Such a mechanism was thought to generate original short and long-run dynamics compared to the traditional neoclassical growth model. All the analytical attempts to uncover such an original outcome in the 60s did fail though. Recent advances in computational mathematics and optimal control of infinite dimensional dynamic systems allow to depart from the traditional fixed saving rate assumption and to move to vintage capital settings with intertemporal optimization and an explicit handling of transitional dynamics. Such a departure ultimately allows to identify new properties concerning transitional dynamics in vintage capital optimal growth models related to the replacement problem mentioned just above. An endogenous growth vintage capital theory also emerges with either technological and/or environemental concerns.

3. *The vintage human capital breakthrough*: The early vintage capital models only deal with age-structured physical capital though, as mentioned in the introduction of Section 5, extension to human capital seemed natural to many authors. Together with the rise of economic demography and to so-called unified growth theory (see Galor and Weil, 1999, for example), modeling vintage structures of human capital and analyzing their impact on the development process, technology diffusion and income distribution becomes an important line of research from the 90s.

The structure of the paper follows the presentation above. Section 2 is devoted to the description of the salient characteristics of the seminal vintage capital models constructed in the 60s. Section 3 presents the embodiment debate and ends with a short exposition of the new two-sector accounting methodology. The new vintage capital optimal growth theory is presented in the next section with a somehow detailed (though non-tecnical) analysis of the mathematical peculiarity of these models. Section 5 is a detailed description of models putting forward vintage human capital to tackle several key development issues.

2 Vintage capital models: Seminal theory

2.1 The Johansen vintage capital model

Johansen's 1959 model is the first historical vintage capital model. It has two main features, a putty-clay assumption and a vintage capital structure. Capital is non-malleable: while substitution between labor and capital is permitted *ex-ante*, it is not allowed once capital is installed. Capital goods embody the best available technology at the date of their construction and the number of workers operating them is, as formulated by Sheshinski (1967), "fixed by design". The output produced by capital goods of vintage v at date $t \geq v$, say $Y(v,t)$, is given by:

$$Y(v,t) = F\left(K(v,t), e^{\gamma v}\ L(v,t)\right),$$

where $K(v,t)$ is the amount of capital of vintage v still operated at date t and $L(v,t)$ the amount of labor assigned, while $\gamma \geq 0$ designates the rate of embodied technical progress[1]. The production function $F(.)$ is assumed *neoclassical*: it has constant returns to scale and diminishing marginal rates of substitution. $K(v,t)$ is typically related to the amount of capital of vintage v constructed, say $I(v) = K(v,v)$, by a depreciation-based law of motion of the type:

$$K(v,t) = e^{-\delta(t-v)}\ I(v),$$

where $\delta \geq 0$, is the rate of instantaneous capital depreciation. Because proportions of production factors are fixed *ex-post*, that is the amount of labor (measured in efficiency terms) associated with capital of vintage v is fixed for every $t \geq v$, the output per vintage can be written in the much simpler form:

$$Y(v,t) = F\left(1, \lambda(v)\right)\ K(v,t) = g\left(\lambda(v)\right)\ K(v,t),$$

where $\lambda(v) = e^{\gamma v}\ \frac{L(v,t)}{K(v,t)}$. Notice that, by construction, function $g(.)$ is strictly increasing and strictly concave.

A fundamental mechanism at work in the Johansen model is the **obsolescence** scheme, determining the range of active vintages at any date. The quasi-rent of a vintage v at date t, $t \geq v$, is given by

$$\mu(v,t) = g\left(\lambda(v)\right) - \lambda(v)\ e^{-\gamma(t-v)}\ w(t),$$

where $w(t) = g'\left(\lambda(t)\right)$, is the wage rate in terms of labor efficiency. An equipment of vintage v is operated as long as its quasi-rent remains positive. At any date t,

[1] Of course, disembodied technical progress could be trivially introduced.

the installation of a new vintage is always profitable, the associated quasi-rent being strictly positive, $\mu(t,t) = g(\lambda(t)) - \lambda(t)\, g'(\lambda(t)) > 0$, because function $g(.)$ is strictly increasing and concave. For t fixed, and for the associated wage rate, $w(t)$, the operation of the previously installed and less efficient vintages may not be profitable, and some are eventually scrapped. Therefore Johansen's framework naturally leads to optimally finite-lived capital goods.

If we denote $\Omega(t,w)$ the set of vintages still utilized at date t for a given wage $w(t)$,[2] total demand for labor is equal to $\int_{\Omega(t,w)} L(v,t)\, dv$. In a Solow growth set-up, namely under a constant investment rate s such that $K(t) = s \int_{\Omega(t,w)} Y(v,t)\, dv$, and the labor market clearing (with exogenously given labor supply), Sheshinski (1967) proved that the Johansen model converges to a unique stable balanced growth path with finite capital lifetime. We shall come back to this important asymptotic result when dealing with the Leontief vintage capital model studied by Solow *et al.* (1966).

2.2 The Solow vintage capital model

The vintage capital model proposed by Solow (1960) builds on the seminal work of Johansen but differs in a fundamental aspect: factor proportions are not fixed *ex-post*, they are freely variable over the lifetime of capital goods (putty-putty). The output per vintage follows a Cobb-Douglas technology

$$Y(v,t) = e^{\gamma v}\, K(v,t)^{1-\alpha}\, L(v,t)^{\alpha},$$

while total output is given by $Y(t) = \int_{-\infty}^{t} Y(v,t)\, dv$. In sharp contrast to Johansen's model, optimal capital lifetime need not be finite: due to the Cobb-Douglas production function, the marginal productivity of labor at $L(v,t) = 0$ is infinite, and the wage cost could be *ceteris paribus* covered by assigning arbitrary small amounts of labor to the oldest equipment, which is always possible in a putty-putty setting. Therefore, obsolescence does not show up through finite time scrapping but through a decreasing labor allocation to vintages over time, which in turn reflects a declining pattern for the value of vintages, as we shall see.

A striking outcome of Solow's 1960 model is its aggregation properties. Denote by $L(t)$ the total labor supply, that is $L(t) = \int_{-\infty}^{t} L(v,t)\, dv$, and define $K(t)$ as

$$K(t) = \int_{-\infty}^{t} e^{\sigma v}\, I(v) e^{-\delta t}\, dv,$$

where $I(v)$ stands for, as in the Johansen's model, the amount of capital of vintage v installed in the economy, and $\sigma = \delta + \frac{\gamma}{1-\alpha}$. If labor is homogenous, implying the

[2] As pointed out by Sheshinski (1967), this set consists in one or more time intervals, provided $\lambda(v)$ is continuous (for $v < t$).

marginal productivity of labor to be equal to the same wage rate regardless of the vintage operated, then the aggregate production function, $Y(t) = \int_{-\infty}^{t} Y(v,t)\, dv$, can be exactly written as

$$Y(t) = K(t)^{1-\alpha}\ L(t)^{\alpha}.$$

As mentioned by Solow (1960), page 93, the putty-putty vintage capital model degenerates into the neoclassical model with the same aggregate production function and an exponential life-table assumption (at the rate δ), when $\gamma = 0$. Of course, the Cobb-Douglas specification is crucial to get such a result. As we will show in Section 3, this aggregate model has been intensively used from the 90s to analyze the viability of a growth regime driven by embodied (or investment-specific) technological progress, which is a characteristic of the technological regime conveyed by the information technologies. It is important to notice here that this aggregate model can be interpreted as a two-sector model. A first sector is the one described by the aggregate production above, which features a consumption good sector. In the formulation above, technological progress is fully embodied in capital. Of course we can introduce neutral technological progress as well, this will be done in Section 3. A second sector, more implicit, is the capital sector: production in this sector follows the law of motion

$$\dot{K}(t) = e^{\gamma_q t} I(t) - \delta K(t),$$

where $\gamma_q = \frac{\gamma}{1-\alpha}$ is the rate of embodied technical progress. The term $e^{\gamma_q t}$ can be interpreted as productivity in the capital good sector, and as such, it also represents investment-specific technological progress because it only affects contemporaneous investment. This two-sector model, which is equivalent to the 1960 Solow vintage capital model, has become a key framework in the late 90s, with important implications for growth accounting as we will see later.

A major contribution of Solow's work, clearly apparent in the 1960 model, is his thorough study of the implications of technical progress and changes in the quality of capital goods for the valuation of (durable) assets, which has ultimately led him to bring out a very sound theory of net output, depreciation and obsolescence. Without getting into too many details, one can get a flavor of this theory through the following simple point. Suppose output can be used only for consumption and investment, that is:

$$Y(t) = C(t) + I(t).$$

As we will see in the next section, the accounting identity above is not innocuous since investment is reported without adjusting for productivity (or quality). Suppose now that one unit of consumption is forgone to invest in the capital sector at date t. Because the productivity in this sector is $e^{\gamma_q t}$, one can produce $e^{\gamma_q t}$ units of capital good. This implies a relative price of capital equal to $e^{-\gamma_q t}$. Therefore, while technological progress operates as a steady improvement in the quality of machines at a rate $\frac{\gamma}{1-\alpha}$, it also induces an instantaneous obsolescence process (at the same rate) of the previously installed equipment. If the rate of embodied technical progress is significantly different from zero, such a process is likely to distort the typical growth aggregates, like net output, the growth rate of (net) output and productivity growth figures.

This important point has been at the heart of a recent rich literature around the productivity slowdown puzzle (see among many others, Greenwood and Yorokoglu, 1997, Whelan, 2002, and Greenwood and Jovanovic, 2003). Actually, the discussion on the economic growth implications of embodied technical progress was tremendously controversial in the 60s. It has witnessed an astonishing resurgence in the 90s with the rise of the so-called *New Economy* and the availability of new economic statistical series (a move led by Robert Gordon, 1990), and has stimulated a quite important debate on growth accounting. We shall come back to this question in Section 3.

2.3 The vintage capital model with fixed factor proportions

In a famous joint contribution with Tobin, Yaari and Von Weizsacker, Solow examined the polar case of a Leontief vintage capital production function. In this case, factor substitution is not allowed neither *ex-ante* nor *ex-post*. Not surprisingly, this model shares some key characteristics with the Johansen putty-clay model, notably concerning the obsolescence mechanism at work and the (qualitative) asymptotic behavior. A roughly general formulation of the Leontief production function per vintage is

$$Y(v,t) = a(v)K(v,t) = b(v)e^{\gamma v}\, L(v,t).$$

One unit of vintage capital v produces $a(v)$ units of output once combined with $\frac{a(v)}{b(v)}\, e^{-\gamma v}$ units of labor. In the simple case where $a(v) = b(v) = 1$, for all v, and no capital depreciation, so that $K(v,t) = I(v)$ for all v and t, the production function per vintage takes the elementary form

$$Y(v,t) = Y(v) = I(v) = e^{\gamma v}\, L(v),$$

for all $t \geq v$. We shall use this simple specification hereafter.

As in the Johansen model, and basically for the same reasons, capital goods should be scrapped at a finite time. Because of fixed factor proportions, capital goods become obsolete some time after their installation when their associated quasi-rents can no longer cover the labor cost. In the Leontief vintage capital model, the obsolescence conditions are straightforward. Under the usual competitive conditions, the equilibrium (real) wage is equal to the marginal productivity of labor, which equals the inverse of the labor requirement of the oldest operating vintage in the Leontief case. If we denote by $T(t)$ the age of the oldest machines still in use (or scrapping time) at date t, the equilibrium wage is simply $e^{\gamma(t-T(t))}$. The quasi-rent associated with an equipment of vintage v at date $t \geq v$ is therefore $1 - e^{-\gamma v}\ e^{\gamma(t-T(t))}$, and it is exhaustible at finite time.

The main result established by Solow *et al.* (1966) concerns the asymptotic stability of the model under a constant saving rate and a clearing labor market (with an exogenously given labor supply). Under the condition that the saving rate is larger than the rate of embodied technological progress, and some more technical assumptions, Solow *et al.* show that the economy converges to a unique balanced growth path. As previously pointed out, Sheshinski came out with the same result on the Johansen model one year later. Therefore, it turned out that the vintage capital growth models, whatever their assumed technological structures, do deliver the same qualitative asymptotic behavior as the much simpler neoclassical growth model (with homogenous capital). This was a quite disappointing result because one would have expected that the obsolescence mechanisms at work in these models, specially when the equilibrium lifetime of equipment is finite, would distort the short term dynamics and long run outcomes. Indeed, when the equilibrium lifetime of equipment is finite, one would expect that replacement investment, here variable $I(t)$, would burst from time to time, giving rise to the so-called *replacement echoes.* Solow *et al.* and Sheshinski's results rule out such an occurrence in the long run. A recent stream of the vintage literature inspired by Malcomson (1975) has revisited this finding: relaxing the assumption of constant saving rate, common to the related literature of the 60's, it is shown that whence intertemporal optimization is introduced (which involves the determination of an optimal scrapping time), investment cycles may show up as a a result of optimal replacement of obsolete machines (see notably Boucekkine et al., 1997b). This recent "optimal growth" vintage capital literature is reviewed in Section 4.

3 The embodiment debate and implications for empirical growth: The accounting breakthrough

3.1 The embodiment controversy: Solow (with the help of Gordon) strikes back

In a famous statement, Denison (1964) claimed that "the embodied question" is unimportant. His argument was merely quantitative and starts with the assumption that embodiment should exclusively show up through the age distribution of the stock of capital. Using his own estimates of US growth in the period 1929-57, he argued that if the average age can be changed by one year from 1960 to 1970, this cannot alter the annual growth rate (in the extreme cases) by more than 0.06 percentage points in the period 1960-70.

Of course, Denison's reasoning is specific to a period of time and it uses a quite conventional one-sector-based growth accounting exercise with a restrictive identification of the embodiment channels (exclusively, the average age of capital). In particular, it omits *de facto* the relative price of capital (in terms of the consumption good) channel: the latter variable must go down under an acceleration in the rate of embodied (or investment-specific) technical progress as explained in Section 2.2, and such an effect can be neatly represented and its growth implications studied in two-sector models, as recently emphasized, among others, by Greenwood, Hercowitz and Krusell (1997). This idea was also strongly stressed by Solow (1960) but the lack of a compelling computation of the relative price of capital series was a clear limit to his claims. In this sense, Robert Gordon's relative price of equipment series for the US have been very good news for the Solowian view, as brilliantly pointed out by Hercowitz (1998) in a recent essay on the embodiment controversy.

We shall give here a brief account of the central point of the discussion as started by Solow (1960) and Jorgenson (1966), and as it has evolved over time, especially after Gordon (1990).[3] The major difference between the accounting approach prescribed by Solow and the one defended by Jorgenson is in the resource constraint:

$$Y(t) = C(t) + I(t).$$

As already mentioned in Section 2.2, Solow does not adjust investment for quality while Jorgenson does. Precisely, Jorgenson would write the previous identity as:

$$Y(t) = C(t) + I^*(t) = C(t) + e^{\gamma_q t} I(t),$$

with the notations of Section 2 (γ_q denoting the rate of investment-specific technical progress). Let us also introduce neutral (or disembodied) technological progress in the production function of the consumption good as it is traditional in the neoclassical model: $Y(t) = A(t)\ K(t)^{1-\alpha}\ L(t)^{\alpha}$ where $A(t)$ designates neutral technological progress (in the sense that it affects all the stock of capital and not only contemporaneous investment). So the unique difference between the two frameworks is in the adjustement for quality of investment in the resource constraint. Incidentally, this difference is quite substantial: it means that investment-specific technological progress is costly in the Jorgenson setting: the larger γ_q, the larger the amount of final good diverted from consumption (or the larger the labor input required if one has in mind the aggregate production function in the final good sector). This does not occur in the Solow set-up. This said, the main point raised by Jorgenson to dismiss Solow's proposal is that it is impossible from the available data (at that time) to distinguish between investment-specific and neutral technological progress, that is between $A(t)$

[3] A more theortical point is made by Phelps (1962) against what he called the "new investment" view inspired by Solow (1960). We shall restrict our discussion to the growth accounting debate.

and $e^{\gamma_q t}$. The point is rather straightforward: if the available data consist of time series on $C(t)$, $I(t)$ and $L(t)$, then the system of equations composed of the resource constraint (in both frameworks) and the law of motion of capital, together with the production function postulated, is trivially undetermined. Unless an ad-hoc identifying assumption is added, one cannot identify separately $A(t)$ and $e^{\gamma_q t}$. Traditional one-sector growth accounting builds on the restriction $\gamma_q = 0$. However such an assumption which may have looked reasonable in the 60s sounds as roughly counterfactual since Gordon (1990), who clearly uncovered a downward trend in the series of the relative price of capital, which amounts to finding $\gamma_q > 0$. Ultimately it turns that Gordon's work has not only broken down the "indeterminacy" argument put forward by Jorgenson, it has also paved the way for a new accounting framework based on the two-sector model already described in Section 2.2. We shall say some words on this matter in the next section.

3.2 Growth accounting under embodiment

Growth accouting under embodiment has been a fertile research line in the 90s with several important contirbutions. Key contributors are, among others, Hulten (1992), Greenwood, Hercowitz and Krusell (1997) and Whelan (2002). Hulten's findings are discussed in Greenwood et al. (1997). To make the presentation as simple as possible, we rely on the latter (see also Greenwood and Jovanovic, 2003, for a broader perspective on this question). Let us consider the two-sector model in Section 2.2 enriched with a neutral technological progress component (as just above) and let us introduce a distinction between equipement and structures, as the former are the predominant channel of embodied technical progress. More specifically, we shall assume that investment-specific technological progress is exclusively conveyed through equipement.[4] The equations of the "accounting" two-sector model are:

$$Y(t) = A(t) K_e(t)^{\alpha_e} \; K_s(t)^{\alpha_s} \; L(t)^{1-\alpha_e-\alpha_s},$$

$$\dot{K}_e(t) = e^{\gamma_q t} I_e(t) - \delta_e K_e(t),$$

$$\dot{K}_s(t) = I_s(t) - \delta_s K_s(t),$$

$$Y(t) = C(t) + I_e(t) + I_s(t).$$

Standard computations yield the following decomposition of the growth rate of output (with the notation γ_x for the growth rate of x):

[4] This is clearly an assumption. It is not obvious at all that the fraction of embodied technical progress in structures is negligible. Preliminary evidence tend to prove the contrary, see for example, Gort, Greenwood and Rupert (1999).

$$\gamma_Y = \frac{1}{1-\alpha_e-\alpha_s}\,\gamma_A + \frac{\alpha_e}{1-\alpha_e-\alpha_s}\,\gamma_q.$$

The formula gives the growth rate of the economy resulting from the two components of technological progress, neutral and investment-specific. One can exploit it to measure the contribution of each component to economic growth. This exercise is done by Greenwood et al. (1997) on US postwar data. Using Gordon's work, one is able to directly "observe" γ_q (around 4%). As in standard growth accounting, the rate of growth of neutral technical change, γ_A, is then computed residually, once the model conveniently calibrated.[5] Finally, one can evaluate the contribution to growth of each form of technological progress. Greenwood et al. found that more 60% of output growth can be attributed to embodied technical progress, which is indeed a huge figure. Even though some aspects of the methodology can be discussed, this exercise is clean enough to suggest that embodied technical progress is an important source of US growth. This calls for a deep revisiting of growth accounting procedures (see Whelan, 2002, for a careful discussion). This finding also makes clear that not only vintage capital models are realistic technological representations, they are crucially important to understand how the growth process set in and how to control it. The next section provides a state of art in vintage modelling in the economic literature.

4 Optimal vintage capital growth models: The optimal control breakthrough

As outlined earlier, the seminal vintage capital models built up in the 60s very often entail the assumption of a constant saving rate, and almost systematically concentrate on balanced growth paths characterization. The main reason behind is the complexity of vintage capital models, which involve a particular class of optimization problems and dynamic systems. This will be made clear along this section.

4.1 The mathematical peculiarity of vintage capital models

To fix ideas, let us shed light on the peculiar dynamic structure of the famous vintage capital model with fixed proportions (Solow et al., 1966) already described in Section 2.3. A key feature in the latter model is the determination of the scrapping time, $T(t)$. Suppose without loss in generality that labor supply is normalized to unity. Then, the

[5]Greenwood et al. (1997) take the following numbers: $\alpha_e = 0.17$, $\alpha_s = 0.13$.

labor market clears at date t if and only if

$$\int_{t-T(t)}^{t} I(v)\, e^{-\gamma v}\, dv = 1.$$

At $t = 0$, and for a given initial investment profile, $i(t)$ for $t < 0$, the equilibrium condition just above determines the initial scrapping time, $T(0)$. The latter statement already gives an idea of the peculiar dynamic systems one has to deal with when studying vintage capital models. In contrast to the more traditional growth models, there is no way to have a definite solution to the model if past investment profile is not given at least on a time interval $[-t_0\ 0]$, $t_0 > 0$, while a single initial condition $K(0)$ is enough in traditional growth models. This features the infinite dimension nature of the dynamic systems induced by vintage capital models, a feature highlighted in Fabbri and Gozzi (2008). Differentiating the clearing-market condition above one gets the following peculiar law of motion for the scrapping time:

$$\dot{T}(t) = 1 - \frac{I(t)e^{-\gamma T(t)}}{I\left(t - T(t)\right)}.$$

The law of motion of scrapping is no longer an ordinary differential equation, it is a delay-differential equation: a delayed term $I\left(t - T(t)\right)$ shows up. The latter reflects the replacement activity at finite scrapping time, $T(t)$, taking place in the economy. Moreover, the delay, $T(t)$, is an endogenous variable, which is a further complication.[6] So even abstracting away from dynamic optimization, the mathematical challenge one faces when dealing with transition dynamics in vintage capital models is at first glance a daunting task. Fortunately enough, recent advances in computational mathematics do allow to handle the class of dynamic systems discussed here. Indeed using up-to-date numerical methods, Boucekkine, Licandro and Paul (1997a) solved for the transition dynamics of the 1966 Solow et al.'s model. More precisely, they considered the 3 dimensional differential system obtained by adding to the delay-differential identified above the Leontief vintage capital production function and the constant saving rule differentiated with respect to time. Because the system does include a delayed term representing replacement at finite time, as explained above, one would expect that some kind of replacement-induced oscillations will show up in the transition to an otherwise standard balanced growth path (previously characterized by Solow et al., 1966). Boucekkine et al. (1997a) found that such short-term replacement echoes do not show up for a large class of past investment profiles.Transition dynamics do set in in the sense that all variables (including the scrapping time) vary over time and converge to their corresponding balanced growth values but the dynamics are mostly monotonic. Therefore, the work of Boucekkine et al. (1997a) essentially extends the no-replacement echoes finding of Solow et al. (1966) to the short-term dynamics.

[6] Delay-differential equations with endogenous delays are called state-dependent, see Boucekkine et al. (1997b).

4.2 Vintage capital optimal growth models

The latter surprising (and disappointing) result has suggested further research. Among other lines of research, one direction taken builds on the following observation. Suppose that the scrapping time is constant, equal to T^o. Time differentiation of the labor market equilibrium condition above yields

$$\hat{I}(t) = \hat{I}\left(t - T^o\right),$$

where $\hat{I}(t) = i(t)\, e^{-\gamma t}$ is detrended investment. Therefore, if the scrapping time were to be constant, replacement echoes would govern investment dynamics, and thus, output dynamics too since technology is Leontief. Incidentally, the constancy of optimal scrapping rules is a salient property in a related operation research literature: such a rule is usually referred to as the Terborgh-Smith result as developed in Malcomson (1975).[7] Relying on this literature, Boucekkine, Germain and Licandro (1997b) suggested to move to optimal growth set-ups for replacement echoes to show up. Accordingly, they replaced the constant saving rule by a Ramsey intertemporal optimal control problem with linear utility function while keeping all the other technological assumptions in the Solow *et al.* 1966 model. In such a case, the optimal scrapping time turns out to be effectively constant after a finite time adjustment period, which generates everlasting fluctuations in investment, output and consumption, following the simple replacement echoes mechanism outlined just above. When utility of consumers is strictly concave, these fluctuations do arise in the short run but they get dampened in the long run by the induced consumption smoothing mechanism (see Boucekkine *et al.*, 1998). These results are in sharp contrast to the neoclassical growth model which typically gives rise to monotonic transition dynamics[8]

Therefore, vintage capital optimal growth models do strikingly differ from the neoclassical growth model, at least in terms of short run dynamics, provided capital and labor are to some extent complementary. Admittedly, compared to the standard neoclassical model, the generated non-monotonic investment paths are much more consistent with the observed dynamics either at the plant level (Doms and Dunne, 1998) or at the aggregate level (Cooper, Haltiwanger and Power, 1999).

To end this section, some comments on the optimization techniques needed to handle vintage capital optimal growth models are in order. As explained in Section 3.1, the state equations involved in these models are infinite dimensional because they belong

[7]See also the excellent book of Hritonenko and Yatsenko (1996) which brings out an extensive material on vintage capital modelling in this literature.

[8]A flavor of these results could be found in an early seminal contribution due to Benhabib and Rustichini (1991) who were the first to show that a departure from the typical exponential decay depreciation rule for physical capital in the neoclassical model is enough to give birth to non-monotonic adjustment paths.

to the class of delay-differential or delay-integral equations. There is no obvious way to deal with the optimal control of such dynamic motions, which probably explains why the vintage capital literature has resumed so late. The natural preliminary question turns out to be whether it is safe to apply the typical optimal control apparatus (or slight variations of it), which is conceived to be applied for the optimal control of finite dimension ordinary differential equations, or to develop alternative techniques more adapted to the finite dimensional models under scrutiny. The inherent methodological debate is addressed in Boucekkine, de la Croix and Licandro (2004). These two approaches have been both taken in the recent vintage capital literature. On one hand, variational methods to derive the maxiumum principle have been successfully applied by Malcomson (1975), Boucekkine et al. (1997b), and more systematically by Hritonenko and Yatsenko (1996, 2005, 2006). The techniques applied extend quite naturally and straightforwardly the classical variational method. On the other hand, Fabbri and Gozzi (2008) and Faggian and Gozzi (2010) have applied a novel dynamic programming method to solve vintage capital optimal growth models explicitly dealing with the infinite dimension of the problems tackled. A comparison between the two approaches is beyond the scope of this paper. Nonetheless, and by construction, the dynamic programming method allows to obtain a finer characterization of optimal solutions, especially when the value function is explicitly obtainable as it is the case in some linear models (see again Fabbri and Gozzi, 2008). The variational methods requires more additional work, for example to figure out the sufficient optimality conditions or to study asymptotic stability issues.[9] In the absence of explicit solutions for the value function, the gap between the two groups of methods is certainly smaller: combining both seems a preferable strategy in such a case (as argued by Fabbri and Gozzi, 2008). To conclude, it is worth pointing out that the area of optimal control of age-structured human and non-human populations (to which vintage capital optimal growth models belong) has become quite active in the recent years.[10] It is highly important for economists aiming to incorporate age structures into the analysis to track these methodological developments; many questions are still open in this respect.

4.3 Vintage capital with endogenous growth

The models surveyed so far build on exogenous (embodied) technical progress. Indeed, until the late 90s, there has been no work combining wintage capital and endogenous growth. The rise of the information technologies in the 90s and the debate on the

[9]It is easy to understand why: When the (linearized) dynamic systems are ordinary (linear) differential equations, stability involves the location of a finite number of roots of the corresponding characteristic polynomial. When the dynamic systems are delay-differential equations, the counterpart is the location of the infinite number of roots of a transcendental function.

[10]See a compilation of applications and methods in Boucekkine, Hritonenko and Yatsenko, Eds. (2010).

viability of what appeared as a new growth regime have stimulated a new stream of literature aiming at endogenizing embodied technical progress (see the excellent survey of Greenwood and Jovanovic, 2003). Before this new trend, vintage capital models have been mostly used to study the impact of obsolescence (that is the so-called replacement problem) and exogenous embodied technical progress. It is interesting to notice that even when technological progress is exogenous, purposive modernization strategies (namely, strategies increasing the productivity of the operated capital stock) can be conducted using two different tools: investment (because technological progress is embodied) and scrapping. Scrapping may involve downsizing but in such a case downsizing entails modernization: old and obsolete capital goods are replaced by newer and more productive vintages. In the traditional neoclassical mode with homogenous capital, investment is not the vehicle of (exogenous) technological progress and such modernization strategies are not well-founded. As a consequence, vintage capital models are much more natural to frame these strategies and to evaluate the macroeconomic impact of modernization policies. Concrete modernization policies include investment subsidies, scrapping subsidies and tax treatment of capital income, which happen to be popular policy tools in several advanced economies, including the US (See Cooley, Greenwood and Yorokoglu, 1997). Endogenizing technological progress allows to reach an even more comprehensive set of modernization strategies and policies. Of course, this further refinement comes at a non-negligible analytical marginal cost. Combining a vintage capital structure (with or without endogenous scrapping) and an R&D sector is a difficult task. Hereafter we shall briefly present a sample of contributions to this line of research.

Most of the papers endogenizing embodied technical progress have not encompassed the replacement problem, that is the possibility for the firms to shut down obsolete plants and to scrap obsolete capital goods. As correctly outlined by Greenwood and Jovanovic (2003), these contributions typically build on (or are formally equivalent to) the Solow putty-putty vintage capital model described in our Section 2.2. An important contribution to this line of research is due to Krusell (1998). To our knowledge, the latter paper is the first endogenizing embodied technical change through R&D activities. More precisely, research is conducted by the producers of capital goods who act as monopolists: the exclusive aim of these producers is to increase the productivity of capital goods, that is the level of investment-specific (or embodied) technical progress. Beside some weaknesses (like the presence of a scale effect), the model constructed by Krusell illustrates very well how purposive research activity in the capital sector can drive the growth of an economy, with some remarkable features like the decline of the relative price of equipment, which is admittedly a key stylized fact of modern economies after the 60s. Boucekkine, del Rio and Licandro (2003, 2005) have taken a similar approach: in the 2003 paper, embodied technical progress is modeled using Arrowian learning-by-doing; the 2005 paper is closer to Krusell's contribution in that it incorporates purposive R&D activities aiming to boost the productivity of capital goods. In both, the authors highlight the fact that a larger part of embodied technical progress in the overall growth rate of technological progress has a cost (obsolescence, which shows up in the decline of the relative price of capital) and an advantage (modernization, as new capital goods are more efficient). As a result, an R&D subsidy has an a priori ambiguous effect on growth. Nonetheless Boucekkine et al. (2005) establish that the modernization effect generally dominates the obsolecence costs inherent to embodied technical progress.

A few papers have attempted to introduce an explicit vintage structure in an endogenous growth setting. One is due to Cooley et al. (1997) mentioned above. A large proportion of these papers are motivated by environmental and sustainability considerations. It is easy to guess how the vintage capital structure could be exploited in environmental frameworks. One could differentiate between successive vintages in terms of productivity, but differentiation can be operated on an environmental basis. Suppose that part of the R&D activities of an economy are devoted to decrease energy consumption in the production process (energy-saving technical progress) by conceiving cleaner machines over time. In such a case, successive vintages will be differentiated by their energy consumption, that is by their emissions. In this context, modernization would in first place aim to decrease energy consumption and emissions over time to cope with environmental sustainability criteria. Hart (2004) and Feichtinger et al. (2005) have taken this avenue. In the former, two types of R&D activities are possible: a first activity tends to develop more environment-friendly capital goods, and the other is the traditional productivity-enhancing research. Under some conditions, optimal combinations of the two types of R&D activities are shown to give rise to environmentally sustainable growth regimes.

In both papers,[11] the number of vintages is fixed, so that no endogenous scrapping is allowed. Yet, it seems obvious that scrapping of the least productive and the most polluting capital goods is a valid tool, and should be accounted for in any broad modernization strategy. Of course, adding endogenous scrapping to a general equilibrium model with vintage capital and R&D is analytically intractable in general, which explains why this task has not been undertaken so far. A shortcut has been recently taken by Boucekkine, Hritonenko and Yatsenko (2011) who consider a firm problem with a vintage capital production function in line with Solow et al. (1966), endogenous scrapping and R&D activities to increase the rate of energy-saving technical progress. With this broad set of modernization action in hands, the firm has to design a strategy assuring long-run growth given scarcity of resources (through given exogenously rising resource price) and pollution quota constraints. General equilibrium extensions (with non-linear utility functions for consumers) seem definitely not manageable analytically.

[11]And also in the Cooley et al. paper, which has the additional feature that growth is endogenized via a Lucas human capital accumulation mechanism in contrast to all the papers cited in this section.

5 Vintage human capital: The third breakthrough

Traditional vintage capital growth models typically consider an homogenous labor force. It is clear that just like physical capital is heterogenous, so is the labor force. This observation is commented in this way by Solow (1960), page 91, footnote 2: "...Of course, I could completely reverse the roles of L and K, and then it would be legitimate to speak of successive generations of workers as more efficient than their forbears...". It also goes without saying that introducing heterogenous labor force into one of the seminal vintage (physical) capital models seen before would dramatically complicate the analysis and surely break down some of the nicest equilibrium relationships established (as the Cobb-Douglas aggregate production function in the 1960 Solow model).

Yet the concept of vintage human capital has been more explicitly used in the literature since the early 90s to treat some specific issues related to technology diffusion, inequality and economic demography. A few models try to put in the same model physical and human vintage capital, among them Jovanovic (1998). We shall briefly outline here three lines of research taken with the human vintage capital approach.

5.1 Vintage human capital and technology diffusion

The basic idea conveyed by the recent contributions exploring this line of research is already in Zeckhauser's short note published in 1968. In a world with a continuous pace of innovations, a representative individual faces the typical question of whether to stick to an established technology or to move to a new and better one. The trade-off is the following: switching to the new technique would allow him to employ a more advanced technology but he would loss the expertise, the *specific human capital*, accumulated on the old technique. Since it is costly to learn a new technique, there is an optimal switching timing problem here, which is solved in a very simple partial equilibrium set-up by Zeckhauser 40 years ago. More importantly, notice that in such frameworks the generated vintage human capital distributions pretty much mimic the vintage distribution of technologies, the time sequence of innovations being generally exogenously given. This is clearly the case in the recent papers by Chari and Hopenhayn (1991) and Parente (1994).

In the former (highly elegant) contribution, not only different technology vintages (and therefore vintage human capital) coexist, but in contrast to the early vintage capital models studied above, individuals keep on investing in old technologies for a while even though superior technologies are available. Here, the Zeckhauser mechanism plays a crucial role: human capital is technology-specific and immediate switching to a superior technology is not necessarily optimal: one may be better off skilled in an inferior technology than unskilled in an advanced one [12]. In Chari and Hopenhayn, the story is a bit more specific, and therefore more accurate. Each individual leaves 2 periods; every period, she can only work in one vintage. If she decides to work in a particular vintage, she becomes expert (or skilled) *in that vintage* at the end of the period, and then she can decide either to continue working with this technology or to switch to another vintage as unskilled. Therefore, at every period, any (surviving) vintage may be operated by skilled and unskilled individuals. In the spirit of the paper, the production function of any (operated) vintage may be written as

$$Y(v,t) = \gamma^v \, F\left(N(v,t), Z(v,t)\right),$$

where γ is the rate of technological progress, $\gamma > 1$, $N(v,t)$ the number of unskilled people operating vintage v at date t (part of them being old individuals who decide to switch technologies), and $Z(v,t)$ the number of skilled (and therefore old) individuals choosing this technology.

The fundamental contribution of Chari and Hopenhayn is to highlight the role of complementarity between skilled and unskilled, between old and new vintage human capital, a crucial aspect which was barely discarded, often by construction, in the early vintage literature. Chari and Hopenhayn show that this complementarity is key in shaping technology diffusion. In their framework, technology diffusion may be captured by the distribution of skilled (and old) individuals across vintages. The authors establish two main results for stationary distributions: first of all, the distributions are single-peaked, and second, the more old and new vintage human capital are complementary, the slower technological diffusion (or equivalently, the less people choosing to operate the newest technologies). Thus, beside replicating some of the crucial observed features of technology diffusion, the way different vintages interact is shown to be a decisive determinant of such a diffusion. The intuition behind this outcome is straightforward: for a young individual, the marginal return to choose an old technology is high if his (human capital) investment in such a technology is complementary enough with the pre-existing specific human capital. In such a case, few young workers will invest in the new technology, and technology diffusion is therefore likely to slowdown as the complementarity between old and new vintage human capital rises.

5.2 Vintage human capital and inequality

An obvious implication of relaxing the hypothesis of labor force homogeneity is to generate labor income discrepancy (under the typically competitive conditions): *ceteris paribus* more skilled (and therefore productive) workers would receive higher wages. Of

[12]A more recent exploitation of this mechanism in a vintage capital partial equilibrium set-up is due to Feichtinger et al. (2006).

course, one has to be more specific about how capital and labor are actually combined in the workplace: a highly skilled worker on an out-dated machine is not likely to be so productive. Yet one could legitimately think that considering vintage human capital is a good approach to income inequality. In a recent contribution, Jovanovic (1998) takes the argument a bit further and argues that vintage capital models are particularly well suited to explain income disparity across individuals and across countries. They are specially appropriate if one is primarily interested in getting beyond the typical exogenously-generated inequalities: the latter would not arise as a consequence of given policy or initial endowments discrepancies but are the result of different investment strategies.

The complete argument relies on a natural nonconvexity in vintage capital models deriving from the indivisibility of machines. Though new machines are more productive, it is generally infeasible to replace all the old vintages (because of the resource constraint of the economy for example). Therefore, different vintages will coexist, and under the assumption that new technologies and skills are complementary, the best machines will be operated by the best skilled individuals, which in turn generates inequality. Moreover, "...small differences in skills will translate into larger differences in productivities. This is due to nonconvexity. Had the economy been convex...we would have improved all of the existing machines by a small increment..." (page 498). Notice that the argument is easily extensible to income inequality across countries[13].

On the theoretical ground, Jovanovic's work is an important contribution to the vintage capital literature to the extent that it settles in a quite appealing way the hard problem of combining vintage physical capital and vintage human capital in a framework where the vintage distributions of both assets are endogenous. As argued above, this is a highly sensitive issue. Jovanovic proposes to resort to the assignment model *à la* Sattinger (1975) to go through it. In such a framework, firms combine machines and workers in fixed proportions, say one machine for one worker, at every instant. The typical assignment problem by a firm having acquired capital of a given vintage is to find the optimal vintage human capital or skill of the associated worker. This could be trivially formulated in standard vintage theory notations but we shall follow Jovanovic's elementary formalism here, and speak about quality of machines (indexed by k) and quality of skills (indexed by s). The assignment problem faced by a firm owning a machine of quality k takes the form of the maximization with respect to the index s of the profit function:

$$\pi(k) = F(k, s) - w(s),$$

at every date t, with obvious notations. In particular, $w(.)$ is a given price function for labor skill. The production function $F(.)$ is linearly homogenous with $F_{12} > 0$.

[13]Hereafter, we shall omit the economic development reading of this contribution and focus on the vintage theory value-added.

The first-order condition of the problem determines an optimal relationship $k = \Phi(s)$, which in turn fixes the profit of the firm which owns vintage capital of quality k as: $\pi(k) = F(k, \phi(k)) - w(\phi(k))$. If function $\phi(.)$ is increasing, then the best machines will be paired with the best skills, as needed. It is very important to note that the assignment problem is settled at any instant, and that reassignment is consequently assumed to be frictionless (think of workers free to move among firms or plants).

An interesting characteristic of Jovanovic's model is the obsolescence mechanism involved. Actually, it is very close to the one at work in the Johansen and Leontief vintage capital models, which is by no way surprising since firms combine workers and machines in fixed proportions in the underlying assignment model. Because labor resources are fixed and due to fixed factor proportions, old machines become unprofitable at a finite time and are eventually scrapped. By the envelope theorem, one actually gets $\pi'(k) = F_1(k, \phi(k)) > 0$, which simply reflects that the best machines are the most profitable. Under free entry, the condition $\pi(k) = 0$ will determine the minimal quality of machines still operated, and by the relationship, $k = \phi(s)$, one can also deduce the (minimal) skill paired with the worse machine still in use.

More importantly, and in contrast to Chari and Hopenhayn (1991), growth is endogenous in the Jovanovic paper: It comes from human capital accumulation à la Lucas (1988) and this has some very important implications compared to the set-ups based on the Zeckhauser mechanism. Growth of the stock of human capital determines the maximal quality of human capital available: if the worker has human capital, h, and works a fraction of time u (in production), then her skill is given by $s = u\ h$. If the best worker at date 0 has $h = 1$, then if all individuals choose the same u, the maximal skill is u, paired with best machine of quality $\phi(u)$ at the same date. Finally, the model is closed by an equipment sector: at any instant, a machine producer can produce one machine of any quality at a cost increasing in the quality of the machine, and including cost reducing external effects of the knowledge spillovers type. Not surprisingly, the shape of the involved cost function is a decisive determinant of the inequality obtained: the flatter is the marginal cost of improving machines, the larger the inequality. In case of large knowledge spillovers making the latter marginal cost steeper, the resulting inequality will be smaller.

In the balanced growth path, human capital will grow at a constant rate, and so will be shifted the distributions of vintage physical and human capital. When new vintages of physical capital come out, the best skilled workers will be immediately assigned to the latter vintages, the second best will go to the machines just abandoned by the best skilled workers...etc... This goes at odds with Chari and Hopenhayn's set-up where human capital vintage specificity induce a much slower switching of technologies. As correctly pointed out by Jovanovic, frictionless reassignment has its virtue: it implies persistent inequality in contrast to many competing vintage technology models like Parente's (1994) which bear leapfrogging and cannot serve to explain the persistent income discrepancies across countries. Nonetheless it is clear that a theory of economic development relying on technological decisions, and thus on technology diffusion, should certainly admit a certain degree of frictions in the reassignment process. This and other open issues are to be put to the credit of the highly stimulating 1998 vintage capital contribution of Jovanovic.

5.3 Demographic vintage human capital models

The relationship between demographic trends and economics is an area of research that is now expanding quickly. The importance of the economic growth process in fostering improvements in longevity has been stressed by the literature, but the feed-back effect of past demographic trends on growth, and in particular on the take-off of the Western World, remains largely unexplored. One likely channel through which demographics affect growth is the size and quality of the work force. In this view, generations of workers can be understood as being vintages of human capital, and studied with the same tools as vintages of physical capital.

An interesting point stressed by the empirical literature is that the relation between demographic variables, such as mortality, fertility and cohort sizes, and growth is anything but linear. Kelley and Schmidt (1995) highlight the ambiguous effect of crude death rates. Indeed, growth is slowed by the deaths of the workers but can be enhanced by the deaths of dependents. They provide several elements showing the importance of age-specific mortality rates. These non-linear relationships stress the need to model the vintage structure of population. A key element is that different generations have different learning experiences and that the aggregate stock of human capital is built from the human capital of the different generations. This view is taken by de la Croix and Licandro (1999) and Boucekkine, de la Croix and Licandro (2002) within an endogenous growth set-up through human capital accumulation *à la* Lucas. In such frameworks, the vintage specificity of human capital does not rely on technological vintages as in Chari and Hopenhayn (1991) but on generation (or cohort) specific demographic characteristics.

Of course, the relevance of age-specific characteristics for economic analysis, including for human capital accumulation, is by no way new in the literature- see for example the life-cycle model with human capital investment of Ben-Porath (1967). The Ben Porath mechanism has been recently subject to criticisms by Hazan and Zoabi (2006) and Hazan (2009). The latter paper shows that the lifetime labor input of American men born in 1840-1970 declined despite the dramatic gains in life expectancy. Hazan further argues that a rise in the lifetime labor supply is a necessary implication of the Ben-Porath type model, which casts doubts on the possibility of such a model to explain the rise in schooling. Hazan's critique however only applies when survival curves are rectangular, see Cervellati and Sunde (2009).[14]

The novelty in the recent contributions is the general equilibrium and endogenous growth focus allowing to capture the nonlinear relationships of the type mentioned just above. Another new trend in the recent literature is the attempt to incorporate more realistic demographic ingredients, at least more realistic than the traditional Blanchard-Yaari-like models (see for example, Blanchard, 1985). A typical demographic vintage human capital model runs as follows (Boucekkine, de la Croix and Licandro, 2002). The general structure is an overlapping generations in continuous time. The vintage structure primarily relies on the demographic ingredients. The set of individuals born in t, forming the cohort of vintage t, has a size say $\zeta\ e^{nt}$ where ζ is a scale parameter and n is the growth rate of population. The probability at birth of surviving at least until age a for any member of cohort t is given by :

$$m(a,t) = \frac{e^{-\beta_t(a)} - \alpha_t}{1-\alpha_t}.$$

The survival law depends on two cohort-specific parameters, $\alpha_t > 1, \beta_t < 0$, and is a concave function of age. There is thus an upper bound on longevity obtained by solving $m(\mathrm{A}_t, t) = 0$: $\mathrm{A}_t = -\log(\alpha_t)/\beta_t$. Accordingly, α_t and β_t also determine life expectancy of the individuals of cohort t.

The vintage specificity of human capital comes from the schooling decision taken by the individuals of the same cohort. Precisely, the individuals have to decide how many years to spend in schooling, knowing that a better education will increase their labor income later. Optimal decisions are taken by maximization of expected intertemporal utility under the intertemporal budgetary constraint

$$\int_t^{t+\mathrm{A}} c(t,z)R(t,z)\mathrm{d}z = \int_{t+\mathrm{T}(t)}^{t+\mathrm{P}(t)} h(t)w(z)R(t,z)\mathrm{d}z,$$

and to the rule of accumulation of human capital:

$$h(t) = \frac{\mu}{\eta}\bar{H}(t)\mathrm{T}(t)^{\eta}.$$

The choice variables are consumption $c(t,z)$, schooling length $\mathrm{T}(t)$, and retirement age $\mathrm{P}(t)$. The retirement decision would makes sense in such a framework if for example the objective function includes a disutility of work. $R(t,z)$ is the contingent price of the consumption good, i.e. the price at time t for buying one unit of good at time z conditional on being alive at time z. The parameter μ measures the efficiency

[14]More precisely, the analysis in Hazan (2010) is developed under the assumption of a perfectly rectangular survival probability: Individuals are assumed to survive with probability one during all their life, and die with probability one when reaching their life expectancy.

in the production of human capital, and the parameter η is the elasticity of income with respect to an additional year of schooling. θ represents the time discounting rate. Finally, one can postulate a simple aggregate production function to close the model, $Y(t) = H(t)$, where $H(t)$ is aggregate human capital (across cohorts). Such a technology implies that the real wage par unit of human capital should be equal to 1 to ensure equilibrium in the labor market.

As it stands, the model has two important (and pro-factual) properties. First of all, schooling time is positively correlated with life expectancy. In the simplest case, i.e. $\alpha = 0$, $\eta = 1$, and no disutility of work (as in de la Croix and Licandro, 1999), we can solve the model explicitly for optimal schooling, and obtain

$$\mathrm{T}(t) = \frac{1}{\beta_t + \theta}.$$

We clearly see here that improvements in longevity of individuals in cohort t (drop in β_t) raise the optimal length of schooling, and thus they increase the level of human capital of this generation. Second, the model is able to generate nonlinear relationships between growth and demographic variables, consistent with Kelley and Schmidt findings. For example, Boucekkine, de la Croix and Licandro (2002) establish that the relationship between the growth rate per capita of the economy is a nonlinear function of the population growth rate n. In particular, there exists a value of demographic growth which maximizes economic growth par capita. The same nonlinearity arises in the relationship between the growth rate of the economy and life expectancy. This is not surprising in such models: a longer life has several conflicting effects. On one hand, it raises the incentives to educate and it reduces the depreciation rate of aggregate human capital. But on the other, it implies that the economy will be populated with more old people who did their schooling a long time ago, and such an effect is clearly harmful for economic growth.

The model of Boucekkine, de la Croix and Licandro (2002), although involved, paves the way for quantitative analyzes of the role of mortality in the industrial revolution. To exploit the full possibilities of the model, cohort life tables for the pre-industrial era are needed. Boucekkine, de la Croix and Licandro (2003) use Perrenoud's data who constructed life tables from 1625 to 1825 on the basis of a wide nominative study in Geneva (Switzerland) and Beltrami's work based on parish registers to reconstitute age-group dynamics of the Venetian population over the period 1600-1790. The main finding of Boucekkine et al. (2003) is that the observed changes in adult mortality from the last quarter of the seventeenth century to the first quarter of the eighteenth century played a fundamental role in launching modern growth. This study thus promotes the view that the early decline in adult mortality is responsible for a large part of the acceleration of growth at the dawn of the modern age.

Various authors stressed that the rising density of population may have played a role in fostering the rise in literacy and education. Higher density can lower the cost of education through facilitating the creation of schools. Externalities can also be generated by denser population. High population density spurs technological change. Unified growth theory argues for population-induced" technological progress. Population needs to reach a threshold for productivity to accelerate. In de la Croix, Lindh and Malmberg (2008), we extend the model to allow for an ad-hoc effect of density on total factor productivity:

$$\mu(t) = f(\text{density})$$

This allows to evaluate the respective importance of different mechanisms relating income growth to demographic change. The exercise is conducted by calibrating the model on Swedish long-term time series of mortality, education, age structure, and per capita income. The conclusion is that changes in longevity may account for as much as 20% of the observed rise in education over the period $1800 - 2000$. Thus, longevity plays an important role, but by itself cannot explain more than a part of the rise in the education level in a model with no credit restrictions. The remaining 80% should be sought elsewhere, probably in the development of public subsidies to education and/or to the acceleration of skill-biased technical progress. The total effect of the demographic variables on growth is higher. Most income growth over this period would not have materialized if demographic variables had stayed constant since 1800.

To go beyond ad-hoc effects of density, Boucekkine, de la Croix, and Peeters (2007) build a theory connecting the creation of schools to population density thereby providing microfoundations to the relationship between density and productivity. We choose a simple geographical setting: a circle of unit circumference. We assume that, at every point of time, the cohort of the newborn generation is uniformly spread along the circle and has the same distribution of abilities at every location. We suppose that every point on the circle can accommodate a school and that schools are identical in their characteristics (same services, same quality, same reputation, etc.). The length of schooling is still chosen by individuals who maximize lifetime income, and depends on future wages, longevity, and the distance to the nearest school. The demand for schooling arising from each point on the circle as a function of the distance to the nearest school. Given the hypothesis on the dispersion of the population, it is obvious that schools will be optimally located if they are evenly spaced. Hence, for a given number of facilities, we can determine the literacy rate of the population, the total amount of fees paid by the pupils, and the total transportation costs. Accordingly, the school location problem is reduced to the single question: how many schools (or classrooms) will be founded at every date t to educate the newborn cohort? The number and location of educational facilities are determined, either chosen by the optimizing state or following a free entry process (market solution). Higher population density makes it optimal to increase school density, opening the possibility to attain higher educational levels.

Population size is a major determinant of school creation because the main source of a schools revenues, tuition fees, depends on this demographic variable. This is true for both institutional arrangements (central planner and market). No school is viable below a certain threshold of population size. When the newborn population is low, the school creation or set-up costs are unlikely to be covered, hence no schools are created. Once the population reaches a threshold value, many schools may be created at once. The process by which illiteracy is eliminated is thus initiated by a jump. After this initial jump, the process takes place much more smoothly over time depending on the evolution of population density and of the attendance rate at schools of the successive cohorts, which in turn depends on the demographic, technological, and geographic factors outlined above.

Finally, let us briefly explain the dynamics in these models. The transition to the constant growth solutions usually follows second-order differential-difference equations, as time is continuous but the agents take discrete timing decisions (as schooling an retirement time). No theorem is available to assess directly the asymptotic behavior of the solutions of this kind of dynamic system. Such a theorem, called Hayes theorem, is only available for scalar and autonomous delay differential equations with a single delay. No direct stability theorem is available for delay differential systems with more than one delay since in this case the stability outcomes depend on the particular values of the delays. We can however study the stability of the dynamic system for different parameterizations of the model. Simulations can be ran usually with the assumption that the economy was initially on a balanced growth path. Dynamics turn out to be oscillatory. Indeed, generations of people are somehow like generations of machines, and we observe replacement echoes which are typical of models with delays. A simplified version of these echoes is found in discrete time overlapping generations models, where the eigenvalues are most of the times complex numbers which generate non-monotonic dynamics (see Azariadis, Bullard and Ohanian, 2004).

Such a line of research is quite recent and of course, many tasks remain to be addressed within the same framework. For example, endogenizing at least partially some of the demographic parameters like life expectancy (for example by including private and/or public health expenditures) is an interesting extension. More issues are still open, and the field of demographic vintage human capital models is by now quite promising.

6 Concluding

The vintage capital literature has experienced a quite interesting evolution, with expansions, collapses and revivals, some spectacular in nature and magnitude. It is probably the case for many areas in many scientific disciplines. Yet we believe that this literature illustrates perfectly how technical constraints can limit the development of a research line, and how technical advances, some borrowed from other disciplines, can allow to relax these constraints and to spectacularly resuscitate a dying theory or paradigm. We have paid a special attention to clarify to which extent disciplines like computational mathematics and demography have helped resuming the vintage capital research program. Of course, many issues and extensions are still open. In particular, we believe that vintage capital models are a natural framework to study the environmental viability and, more broadly, the sustainability of a growth regime, as advocated in Section 4.3. Efforts in vintage capital research should be directed to study the latter questions. Last but not last, the dissemination of the vintage capital framework requires a much more intensive quantitative validation work in general equilibrium. So far only few researchers have undertaken this daunting task (notably, Cooley et al., 1997, and Gilchrist and Williams, 2000).

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