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Book Review

Optimal Control of Age-Structured Populations in Economy, Demography, and the Environment edited by R. Boucekkine, N. Hritonenko, and Y. Yatsenko. (2010). Oxon, UK, and New York, NY: Routledge, 295 pp., $150.

Mathematical population studies are pre-eminently the science of decay, renewal, and the resulting age structure in populations of living beings, at least since 1760 when its founder Leonard Euler established the relationship among age structure, mortality, and growth rate. This process also concerns machines and procedures, which become obsolescent with passing time. The introduction of aging into capital has often been avoided by economists, who routinely elude the difficulty in invoking “stylistic facts” or “toy models” to argue with homogenous capital, that is, a real unidimensional variable. Just like physical capital, as demographers know, considering a homogenous labor force may lead to far-fetched conclusions, because the age structure (among others) can change results entirely. The mathematics required to deal with heterogeneous models is demanding, requiring competence both in optimal control and in integro-partial differential equations and functional analysis. The three editors of *Optimal Control of Age-Structured Populations in Economy, Demography, and the Environment*, Natali Hritonenko and Yuri Yatsenko since 1996 and Raouf Boucekkine since 1997, have pioneered vintage capital growth models, publishing in leading mathematical or economic journals, including *Mathematical Population Studies*. In preparation for their volume, they gathered several contemporary experts in age-structured populations, in fields as various as economics, ecology, environment, management, demography, epidemiology, agro-diversity, and taxation policy.

Typically, vintage capital models consist of maximizing a discounted utility $w(c(\tau))$ depending on consumption $c(\tau)$ at time τ (Boucekkine et al., 2004; Hritonenko and Yatsenko, 2008a, 2008b, 2010):

$$\mathrm{Max}_{\mathrm{c}(\cdot),\mathrm{T}(\cdot),\mathrm{i}(\cdot)} \int_0^\infty w(c(\tau))e^{-\rho\tau}d\tau \tag{5.1}$$

subject to

$$\begin{cases} \text{(a)} & 1 = \int_{\mathrm{t-T(t)}}^{\mathrm{t}} \mathrm{i}(\tau)\mathrm{e}^{-\gamma\tau} d\tau \\ \text{(b)} & p(t) = \int_{\mathrm{t-T(t)}}^{\mathrm{t}} \mathrm{i}(\tau) d\tau \\ \text{(c)} & c(t) = p(t) - i(t) \\ \text{(d)} & p(t) \geq i(t) \geq 0 \\ \text{(e)} & T(t) \geq 0 \end{cases}, \tag{5.2}$$

where $p(t)$ denotes production, $T(t)$ scrapping age, $i(t)$ investment, ρ the constant discount rate, and $c(t)$ consumption at continuous time t. Firms scrap the oldest and presumably less efficient machines (beginning then with machines of age $T(t)$). The utility function w is increasing and concave; γ is the rate of embodied technological progress. Constraint (5.2)(a) reflects the equilibrium condition in the labor market equilibrium, where labor supply is normalized to 1. In constraint (5.2)(b), each machine of vintage τ produces one unit of output, but because technological progress is labor-saving, the labor required to operate a machine decreases exponentially at the same speed as the vintage index. Equation (5.2)(c) expresses consumption $c(t)$ as what is left over from production after investment. Constraints (5.2)(d) and (e) are feasibility conditions (people invest less than they produce).

In fact, this integro-differential system yields the controlled differential system, after replacing $p(t)$ by its expression (5.2)(a) in (5.2)(c) and differentiating with respect to time:

$$\begin{cases} \text{(a)} & c'(t) = i(t)(1 - e^{-\gamma T(t)}) - u(t) \\ \text{(b)} & T'(t) = 1 - \frac{i(t)}{i(t-T(t))} e^{-\gamma T(t)} \\ \text{(c)} & i'(t) = u(t) \end{cases}. \tag{5.3}$$

The additional constraint:

$$1 - T'(t) \geq 0$$

used to guarantee that a scrapped machine will not be reused in the future (Hritonenko and Yatsenko, 2006) is contained in the combination of constraint $i(t) \geq 0$ of (5.4) and dynamics (b) of (5.3): indeed the constraint $1 - T'(t) \geq 0$ amounts to $\frac{i(t)}{i(t-T(t))} e^{-\gamma T(t)} \geq 0$, or $\frac{i(t)}{i(t-T(t))} \geq 0$ or $\forall t$, $i(t) \geq 0$, which is the constraint $i(t) \geq 0$ of (5.4).

The key question in vintage models is how (control on $i(t)$ or $i'(t)$) and when (deciding on the scrapping time $T(t)$) to switch to a new technique, which normally should yield a higher return, but at the cost of losing the expertise accumulated with the old technique and at the additional cost of learning the new technique. This trade-off results in an optimal switching time problem. Moreover, by distinguishing between skilled and outdated workers, those who have learned the new technique should be paid more, engendering income inequality, be it at the firm level or between countries.

In engineering economics and operation research, taking account of age-structure is often mandatory. However, real applications cannot afford dealing with continuous-time models because engineers count machine by machine, worker by worker, piece by piece. The continuous-time model is inadequate, and engineers must handle discrete-time equipment replacement models, which, although conceptually close to continuous-time vintage models, require solving nonlinear integer programming problems. The technical difficulties are substantial, and specific to the treatment of integer equations and optimization.

In addition to age, variables describing heterogeneity such as size or stage are commonly necessary in the description of animal and plant populations. The age-structured optimal control problem consists of finding the optimal maintenance and exploitation, be it for the cattle, fishery, forestry, or biofuel industry. Similarly, the optimal vaccination of a population at risk of a disease gains realism when age structure is considered because the incidence of many diseases varies with the age of the susceptible. We think, for example, of measles whose lethality is highest among the young, or the seasonal flu which kills old people preferentially. In general, the objective of engineers is to maximize the harvesting return of a farming, the benefit of a firm, or the efficiency of a procedure at a given time horizon. One control variable is the scrapping time, the time at which machines are withdrawn from the system or at which workers are withdrawn from the labor force. One other control variable is fertility, a generic term which covers the operations of sowings for cultivation, recruitment for personnel, susceptibility for epidemiology, and investment for machinery. It also covers the actual meaning of fertility for animal and human populations. This is the point, in my opinion, which disrupts the claimed unity of all decay-renewal models under the single hat of vintage models. I doubt that human populations can be controlled the way machines or animals are. The gap created by history, psychology, sociology, forbids one to believe in mechanistic relationships between human fertility and economics. This is more sensitive with regard to the problematic of control.

For human populations indeed, it is doubtful that fertility is an easily tractable control. For example, the authors of chapter 1 make population growth depend on policy decisions. Specifically, fertility $g(m)$ is assumed to depend on "population policy expenditure per capita" m, with $g'(m) > 0$, $g''(m) < 0$. Without entering into this old and abundantly commented theme in demography, the link between policy and fertility is far from simple. People seldom respond to policy expenditure. One famous example is the controversy about the efficiency of pronatalist policies in France since 1945. The results are mitigated (TFR = 2.00 in 2008), whereas Sweden, which developed familialist policies, obtained comparable fertility rates (TFR = 1.91 in 2008). Another example is Egypt, where fertility declined slowly despite heavy spending on family planning, compared to Morocco where the decline was quicker with less investment into family planning (Bonneuil and Dassouki, 2006, 2007). In the nineteenth century, the *départements* that pioneered the French demographic transition were neither the richest nor the most urbanized (Bonneuil, 1997). Instead of a relationship with expenditure, data point toward the efficacy of measures based on the organization of women's time, such as the introduction of affordable child care facilities and a more flexible relationship between family and work (this is the case in France, in the Scandinavian countries, and in the Netherlands) (Bonneuil, 2010). However, this is not the topic of chapter 1, which is content in pursuing an academic debate between theoreticians of population control by economics. Therefore, I am not sure that economic models improve our understanding of society when they rely on presumptions contradicted by data, even if they are supported by other purely theoretical literature.

Nevertheless, this book brings substantial contributions to age-structured population dynamics. It should interest researchers from various fields dealing with decay, aging, and renewal, in the tradition of Euler and Lotka, although the materials (e.g., humans, machines, animals) are not commutable and each deserve a specific understanding and contextualization.

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