Optimisation and accuracy of FMM for vortex method calculation

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Abstract

The fast multipole method (FMM) has been applied to accelerate the vortex method calculation. It has been observed that optimization is an important issue before applying FMM for any calculation. The most time consuming parts of FMM are multipole to local translation (M2L) and the direct calculation. An optimum level of box division has been determined according to the number of particles being calculated. The most important issue regarding the simultaneous use of the FMM and MDGRAPE-3 is the balance of the workload between the M2L translation and the direct calculation. MDGRAPE-3 can handled only the direct calculation, hence an optimum level has been determined. It has been observed that the optimum level of box division of the FMM on MDGRAPE-3 is approximately two levels lower than that of the FMM without MDGRAPE-3 because only the direct summation is accelerated. The accuracy of FMM calculation is dependent on its order of multipole moment. The order of multiple moment has been determined for the highest accuracy of vortex method calculation which is confirmed to below $10^{-5}$ for entire calculations.

Keyword: Fast Multipole Method (FMM), Special-purpose Computer, Optimum Level, Vortex Method

Figure 1. Accuracy of FMM

Figure 2. Optimum level of FMM

Figure 1 represent $L^2$ norm error for Biot-Savart calculation for different moments. For $p=7$, initially the error is above $10^{-6}$ and it was increased for larger $N=10^4$ which is below $10^{-4}$. The error has been decreased when order of moment increases accordingly. It was below $10^{-7}$ initially for $p=9$ and 10, also it is below $10^{-6}$ for $p=10$.

The optimum box level has been investigated for the FMM. The cpu-time of the Biot-Savart calculation is plotted against the number of elements for different box levels in Fig. 2. L is the level of the oct-tree box division, and N is the number of elements.

Reference: