Calculation of the decay of colliding turbulent vortex rings

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Abstract

Introduction

There has always been a strong relationship between progress in vortex methods and advancements in acceleration techniques that utilize this method. When the classical vortex methods became popular nearly 30 years ago, the calculation cost of the N-body solver was $O(N^2)$ for N particles. Due to this enormous calculation cost, the intention at that time was not to fully resolve the high Reynolds number fluid flow, but to somewhat mimic the dominant vortex dynamics using discrete vortex elements. The advent of fast algorithms made it possible to achieve a scaling of $O(N)$, and with the help of the rapid development in computational hardware, the calculation involving millions of vortex elements became possible. In our recent study, the efficient use of such hardware (MDGRAPE-2) for the vortex method has been demonstrated (Sheel et al. (2007)).

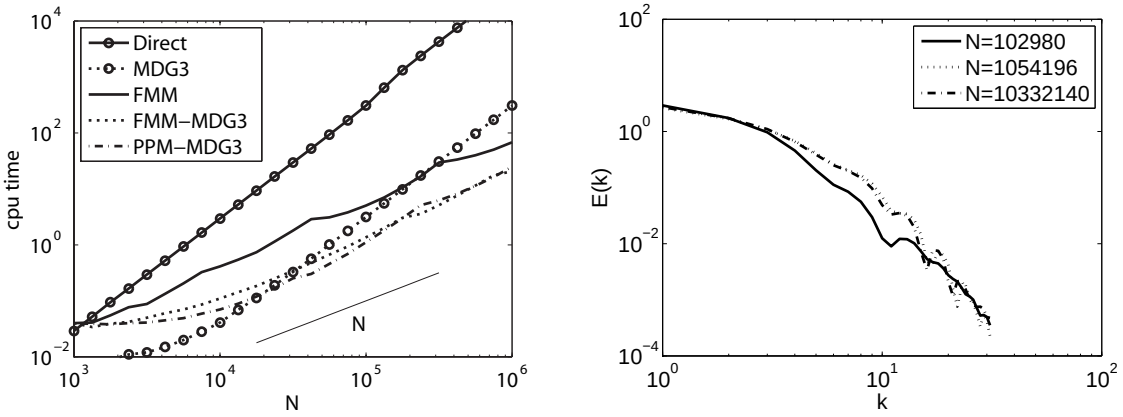
FMM on MDGRAPE-3

In our present research, we investigate the possibility of further acceleration with the simultaneous use of the FMM (Cheng et al. (1999)) by using MDGRAPE-3 (Narumi et al. (2006)). The MDGRAPE-3, successor of MDGRAPE-2, is a special-purpose computer exclusively designed for molecular dynamics simulations. The most time consuming parts of the FMM are the multipole to local translation and the direct calculation. Dividing the particles into excessively small boxes will result in an enormous amount of multipole to local translations, whereas not dividing them enough would result in a large amount of direct calculation of neighboring particles. These two steps must be balanced by changing the level of box divisions according to the number of particles being calculated.

One major problem in this sense is that the MDGRAPE-3 chip can only reduce the cost of direct calculation. The mutipole and local expansions and their translations are impossible to calculate on the MDGRAPE-3 in a straight-forward implementation of the FMM. There is a way to calculate both hot-spots of the FMM on MDGRAPE-3 by converting the multipole to local translation into an N-body interaction. It is possible by implementing the pseudo-particle multipole method (Makino (1999)) on MDGRAPE-3.

Performance

The CPU-time for all methods (when optimized) are plotted in Fig. 1(a). Direct, MDG3, FMM, FMM-MDG3, and PPM-MDG3 represent the calculation without FMM or MDGRAPE-3, with MDGRAPE-3, with FMM, with FMM and MDGRAPE-3, and with the pseudo-particle method and MDGRAPE-3, respectively. The direct calculation without the use of any acceleration technique has a high asymptotic constant and an order of $O(N^2)$. The direct calculation on MDGRAPE-3 has a lower asymptotic constant but still has a scaling of $O(N^2)$. On the contrary, the FMM only has a high asymptotic constant, but its complexity is $O(N)$. The combination of the FMM and MDGRAPE-3 results in a calculation with a low asymptotic constant and $O(N)$ complexity. The pseudo-particle method on MDGRAPE-3 has a speed comparable to the FMM on MDGRAPE-3. At $N = 10^6$ the FMM on MDGRAPE-3 is approximately 4 times faster than the FMM.



(a) CPU-time of different methods

(b) Effect of Spatial Resolution on the Energy Spectra

Figure 1: CPU-time and Energy Spectra

Application

We choose the collision of two identical turbulent vortex rings for our calculations. The following characteristics of this flow allow us to focus on the assessment of the present acceleration technique. The flow does not involve solid or periodic boundaries, and thus causes minimal complications in the implementation of the FMM itself. Also, the initial condition is simple to generate using vortex methods. Furthermore, although the initial flow field is quite simple, the collision of the rings results in a highly turbulent state, and is greatly affected by the Reynolds number of the initial situation. It is evident from previous studies, that a sufficiently large number of particles are necessary to capture the essential characteristics of the vortex ring collision. This fact allows us to demonstrate the ability to handle high Reynolds number flows by using a large number of particles, and is possible with the use of our acceleration method.

The energy spectra for different N are plotted in Fig. 1(b). The number of particles is increased from $N = 10^5$ to $N = 10^7$ with the Reynolds number $Re_\Gamma = 600$. The spectra shows a large difference between the calculations using $N \approx 10^5$ and $N \approx 10^6$ elements. This difference decreases drastically when the number of elements is increased another order of magnitude to 10^7. It is clearly shown that the spatial resolution has a large effect on these calculations and for the calculation of $Re_\Gamma = 600$, the vortex method requires $N = 10^7$ particles for an accurate calculation. This result indicates that for the calculation of further high Reynolds numbers, the vortex method requires significantly larger N, which is possible by using the proposed acceleration method. Therefore, the proposed scheme will make a large contribution to the simulations that have been previously difficult to perform with existing methods.

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