Hopf-Galois extensions and descent

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The first part of the course will concern homotopic descent. I'll begin by recalling classical monadic descent and then describe a general homotopy-theoretic framework for studying descent and completion, and the dual notions of codescent and cocompletion, in model categories enriched over monoidal model categories such as simplicial sets, chain complexes or spectra. I'll construct general descent spectral sequences, explain how to interpret them in terms of derived completion and homotopic descent and relate them to generalized Adams spectral sequences. To conclude I'll sketch a couple of applications.

Homotopic Hopf-Galois extensions will be the theme of the second part of the course. Chase and Sweedler defined Hopf-Galois extensions of commutative rings as a generalization of Galois extensions, with the coaction of a Hopf algebra replacing the action of a group. Galois extensions with respect to a group G are then Hopf-Galois extensions with respect to the dual of the group algebra of G. Rognes recently extended the notion of Hopf-Galois extensions to the category of structured ring spectra, motivated by the fundamental example of the unit map from the sphere spectrum to MU. I will outline a theory of homotopic Hopf-Galois extensions in model categories enriched over monoidal model categories that generalizes the case of structured ring spectra. I will characterize homotopic Hopf-Galois extensions in various categories of interest, showing that, for example, principal fibrations of topological monoids are Hopf-Galois extensions in the category of topological spaces.

To conclude I will describe the interesting relationships among the notions of Koszul duality, Grothendieck descent and Hopf-Galois extensions in the case of dg algebras.

References

[1] K. Hess, *Homotopic Hopf-Galois extensions: Foundations and examples*, Geometry and Topology Monographs 16 (2009) 79–132.

[2] P. Müller, *Homotopic descent over monoidal model categories*, EPFL PhD Thesis (2011).

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