Lax algebras

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Summary

The category of algebras associated to a monad provides an abstraction of *algebraic-like* structures over a given base category. Lax algebras associated to a monad allow, in turn, the study of *topological-like* structures, while taking advantage of the algebraic background provided by the monad.

Outline

The course will begin by reviewing the theory of monads, their algebras, and their associated Eilenberg–Moore and Kleisli categories. We will illustrate these concepts with various examples, including the filter and ultrafilter monads.

We will then focus on $\mathsf{V}\text{-}\mathsf{Rel}$, the category of quantale-based relations, to which certain monads on Set can be laxly extended. This will lead to the definition of lax algebras. Prototypical examples include ordered sets, generalized metric and topological spaces.

Lax algebras are *lax* Eilenberg–Moore algebras, but they can equivalently be presented as monoids in the Kleisli category of a monad. We will describe this correspondence and explain how the Kleisli presentation is "universal".

Finally, we will present recent applications of the theory in the context of topological, metric and order theories.

References

[1] M.M. Clementino, E. Colebunders, D. Hofmann, R. Lowen, R. Lucyshyn-Wright, G.J. Seal, W. Tholen. *Monoidal Topology — A Categorical Approach to Order, Metric and Topology*, in preparation (2013), approx. 500pp.