Critical phenomena for random matrices and integrable systems

Abstracts of talks

Florent Benaych-Georges (Université Paris Descartes): Extreme eigenvalues of Erdős-Rényi graphs

In this talk, we shall present some recent works with Charles Bordenave and Antti Knowles giving estimates for the extreme eigenvalues of the centered adjacency matrices of a possibly inhomogenous Erdős-Rényi graph. In the case of the Stochastic Block Model, these works allow to assess the accuracy of the spectral clustering algorithm, whereas in the case of an homogenous Erdős-Rényi graph, they establish a transition between a compact spectrum and a diffuse one, at a threshold which coincides with the connectivity threshold (the one where a giant connected component appears). The proofs of these results bring to light two classical regimes from random matrix theory: the localized one and the delocalized one.

Dimitris Cheliotis (University of Athens): Biorthogonal ensembles and triangular matrices

We will discuss the singular values of certain triangular random matrices. When their elements are i.i.d. standard complex Gaussian random variables, the squares of the singular values form a biorthogonal ensemble, and with an appropriate change in the distribution of the diagonal elements, they give the biorthogonal Laguerre ensemble. For triangular Wigner matrices, we show that the empirical distribution of the appropriately rescaled squares of the singular eigenvalues converge to a distribution with support $[0, e]$, while the rescaled largest singular eigenvalue converges to $\sqrt{e}$ under the additional assumption of mean zero and finite fourth moment for the law of the matrix elements.

Maurice Duits (KTH Stockholm): Random tilings and orthogonal polynomials in the complex plane

In this talk a connection will be explained between random tilings of planar domains and polynomials that satisfy an orthogonality relation in the complex plane. For the classical examples of the Aztec diamond and lozenge tilings of a regular hexagon, these polynomials are Jacobi polynomials with non-standard parameters. For tilings with certain periodic weightings, matrix orthogonal polynomials come into play. The benefit of this point of view is that there is good hope that the asymptotic analysis can be carried using the Riemann-Hilbert toolkit. The talk is based on joint work in progress with Arno Kuijlaars.

Tamara Grava (University of Bristol, SISSA): Painlevé equations and critical behaviour of Hamiltonian PDEs

Mario Kieburg (Bielefeld University): Matrix convolutions and Pólya ensembles

Convolutions on matrix spaces, let it be of multiplicative or additive nature, might be very challenging. Those convolutions are particularly important when building stochastic processes or constructing more realistic random matrices like lattice operators. The most prominent example is Dyson's Brownian motion on Hermitian matrices. This particular setting had the advantage that the eigenvalue statistics where analytically feasible since they satisfied the form of a polynomial ensemble and thus generated a determinantal point process. The same was found for products of matrices inside a particular class of ensembles in the past few years. Successively this class was extended from Ginibre matrices to truncated unitary matrices to Meijer G-ensembles to polynomial ensembles of derivative type. Another class of polynomial ensembles of derivative type were, very recently, found for the additive convolution of Hermitian matrices by Kuijlaars and Román. Together with Holger Kösters and Yanik-Pascal Förster we have extended this class to the additive convolution on rectangular matrices and the other two classical Lie-algebras. Thereby we addressed the question when the density is a probability distribution. We related these distributions to Pólya frequency functions and thus renamed the corresponding random matrix ensembles to Pólya ensembles. I will report on this project in my talk.

Igor Krasovsky (Imperial College London): Splitting of a gap in the spectrum of random matrices

The eigenvalues in the bulk of the spectrum of typical random matrices follow the sine kernel determinantal process. The asymptotics of the probability of a large gap, i.e. an interval without eigenvalues, and of several large gaps (except for a multiplicative constant in more than one gap case) is well known. In this talk we will present the transition between the asymptotic behaviour for one gap and that for two gaps. It corresponds to the beginning of the splitting of one gap into two. The talk is based on a joint work with Benjamin Fahs.

Arno Kuijlaars (KU Leuven): Universality for conditional measures of the sine point process

The sine process is a random point process that is obtained as a limit from the eigenvalues of many random matrices as the size tends to infinity. This phenomenon is called universality in random matrix theory, and it also holds for many orthogonal polynomial ensembles.

In this talk I want to emphasize another connection of the sine point process with orthogonal polynomials. It comes from a surprising property called number rigidty in the sense of Ghosh and Peres. This means that for almost all configurations, the number of points in an interval $[-R, R]$ is determined exactly by the points outside the interval. The conditional measures is the joint distribution of the points in $[-R, R]$ given the points outside. Bufetov showed that these are orthogonal polynomial ensembles with a weight that comes from the points outside $[-R, R]$.

I will report on recent work with Erwin Mina-Diaz (arXiv:1703.02349) where we prove a universality result for these orthogonal polynomial ensembles that in particular implies that the correlation kernel of the orthogonal polynomial ensemble tends to the sine kernel as R tends to infinity. This answers a question posed by Alexander Bufetov.

Gaultier Lambert (University of Zurich): On the law of large numbers for the maximum of the log-characteristic polynomial of Unitary invariant ensembles

I will report on some of the recent results on the maximum of log-correlated process coming from random matrix theory. In particular, I will sketch a proof that the maximum of the centered log-characteristic polynomial of an $N \times N$ unitary invariant matrix is $\log N + o(\log N)$ with high probability. This confirms the first term in a conjecture by Fyodorov and Simm. We will try to emphasize on the connection with the orthogonal polynomials Riemann-Hilbert problem and the second moment originating from the theory of branching random walks. This is joint work with Elliot Paquette.

Mylène Maida (Université de Lille 1): Concentration of measure for Coulomb gases

A Coulomb gas is the canonical Gibbs measure associated with a system of particles in electrostatic interaction. As the number of particles grows to infinity, the empirical measure of a Coulomb gas converges weakly towards an equilibrium measure, characterized by a variational principle. We obtain sub-gaussian concentration inequalities around this equilibrium measure, in the weak and Wasserstein topologies. This yields for instance a concentration inequality at the correct rate for the Ginibre ensemble. The proof relies on new functional inequalities, which are counterparts of Talagrand's transport inequality in the Coulomb interaction setting. Joint work with Djalil Chafaï and Adrien Hardy.

Nicholas Simm (University of Warwick): Regularized counting statistics and Gaussian multiplicative chaos

We define a statistic which counts the number of eigenvalues of a random matrix falling in a given interval, regularized at some appropriate scale. Varying the interval, we obtain a random process which, for large dimensional matrices, is asymptotically Gaussian and has a logarithmic covariance structure. I will discuss results about constructing the exponential of this process in terms of the theory of Gaussian multiplicative chaos, obtaining convergence in the full subcritical phase. As a by-product of the analysis, we prove a smoothed analogue of a conjecture due to Fyodorov, Hiary and Keating about integrated moments of characteristic polynomials. This is joint work with Gaultier Lambert and Dmitry Ostrovsky.

Christophe Texier (Université Paris-Sud): Truncated linear statistics associated with the eigenvalues of random matrices

Given a certain invariant random matrix ensemble characterised by the joint probability distribution of eigenvalues $P(\lambda_1, \cdots, \lambda_N)$, the study of linear statistics of the eigenvalues $L = \sum_{i=1}^{N} f(\lambda_i)$, where $f(\lambda)$ is a known function, has played an important role in many applications of random matrix theory. I will discuss the distribution of truncated linear statistics of the form $\tilde{L} = \sum_{i=1}^{N_1} f(\lambda_i)$, when the sum runs over a fraction of the eigenvalues ($N_1 < N$). By using the Coulomb gas technique, the large deviation function controlling the distribution of such sums in the limit of large N, with $0 < N_1/N < 1$ fixed, will be analysed. Two situations will be considered leading to two different universal scenarii:

1) the case where the truncated linear statistics is restricted to the largest (or smallest) eigenvalues. We have shown that the constraint that $\tilde{L} = \sum_{i=1}^{N_1} f(\lambda_i)$ is fixed drives an infinite order phase transition in the underlying Coulomb gas. This transition corresponds to a change in the density of the gas, from a density defined on two disjoint intervals to a single interval. In this latter case the density presents a logarithmic divergence inside the bulk.

2) the second situation is the case without further restriction on the ordering of the eigenvalues contributing to the truncated linear statistics (this can be viewed as a new ensemble which is related, but not equivalent, to the "thinned ensembles" introduced by Bohigas and Pato). In

this case, a region opens in the phase diagram of the Coulomb gas, where the large deviation function is mostly controlled by entropy (in particular this induces a change in the scaling of the relative fluctuations of the truncated linear statistics, from the usual $1/N$ for $N_1 = N$, to $1/\sqrt{N}$ when $N_1 < N$). Our analysis relies on the mapping on a problem of N_1 fictitious non-interacting fermions in N energy levels, which can exhibit both positive and negative effective (absolute) temperatures.

- Aurélien Grabsch, Satya N. Majumdar and Christophe Texier, Truncated linear statistics associated with the top eigenvalues of random matrices, *J. Stat. Phys.* **167**(2), 234–259 (2017).
- Auréien Grabsch, Satya N. Majumdar and Christophe Texier, Truncated linear statictics of the eigenvalues of random matrices II. Partial sums of proper time delays in chaotic quantum dots, *J. Stat. Phys.* **167**(6), 1452–1488 (2017).

Pierre van Moerbeke (Université catholique de Louvain, Brandeis University): Tilings of nonconvex domains and a new universality class

We show that tilings of nonconvex domains lead asymptotically to new universality classes. In particular, such new statistics is seen for lozenge tilings of hexagons with cuts, by taking the limit when the size of both, the hexagon and the cuts, tend to infinity, while keeping certain geometric data fixed. (joint work with Mark Adler and Kurt Johansson)

Christian Webb (Aalto University): Multiplicative chaos in random matrix theory and number theory

I will give an overview of recent developments on the connection between global statistics of large random matrices, number theory, and random geometry. More precisely I will discuss how on the global scale, the asymptotic behavior of characteristic polynomials of e.g. CUE matrices can be described in terms of random fractal measures known as Gaussian multiplicative chaos measures and how similar objects also arise when studying the statistical behavior of the Riemann zeta function on the critical line.