Dynamics of Particle Groups

and the Design of Mobile Sensor Networks

IAP V/22
systems & control

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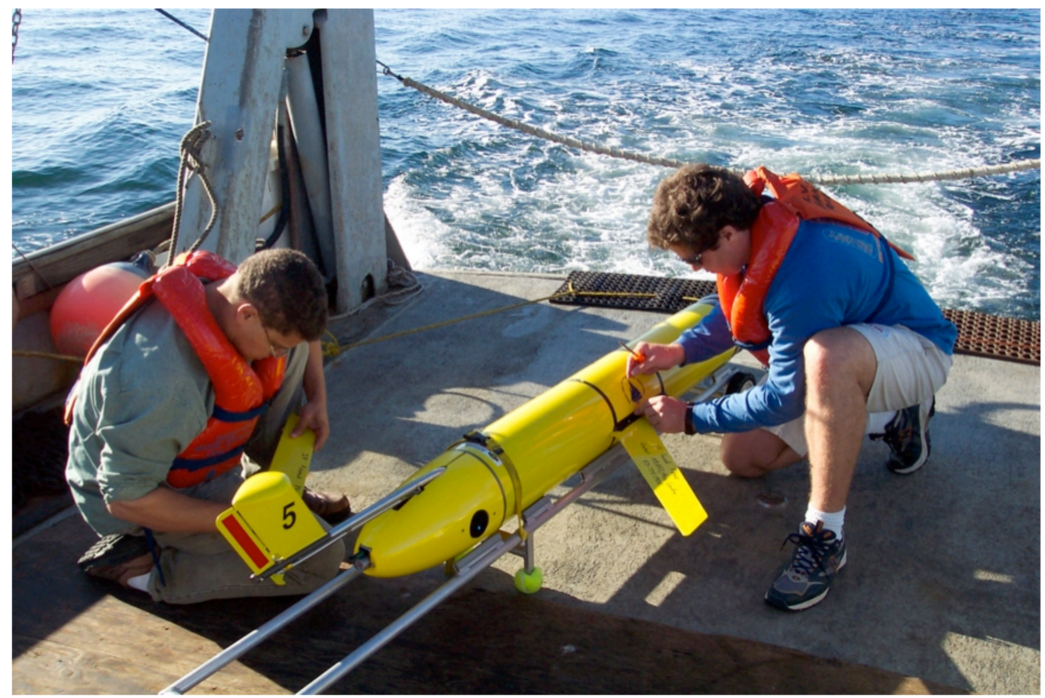
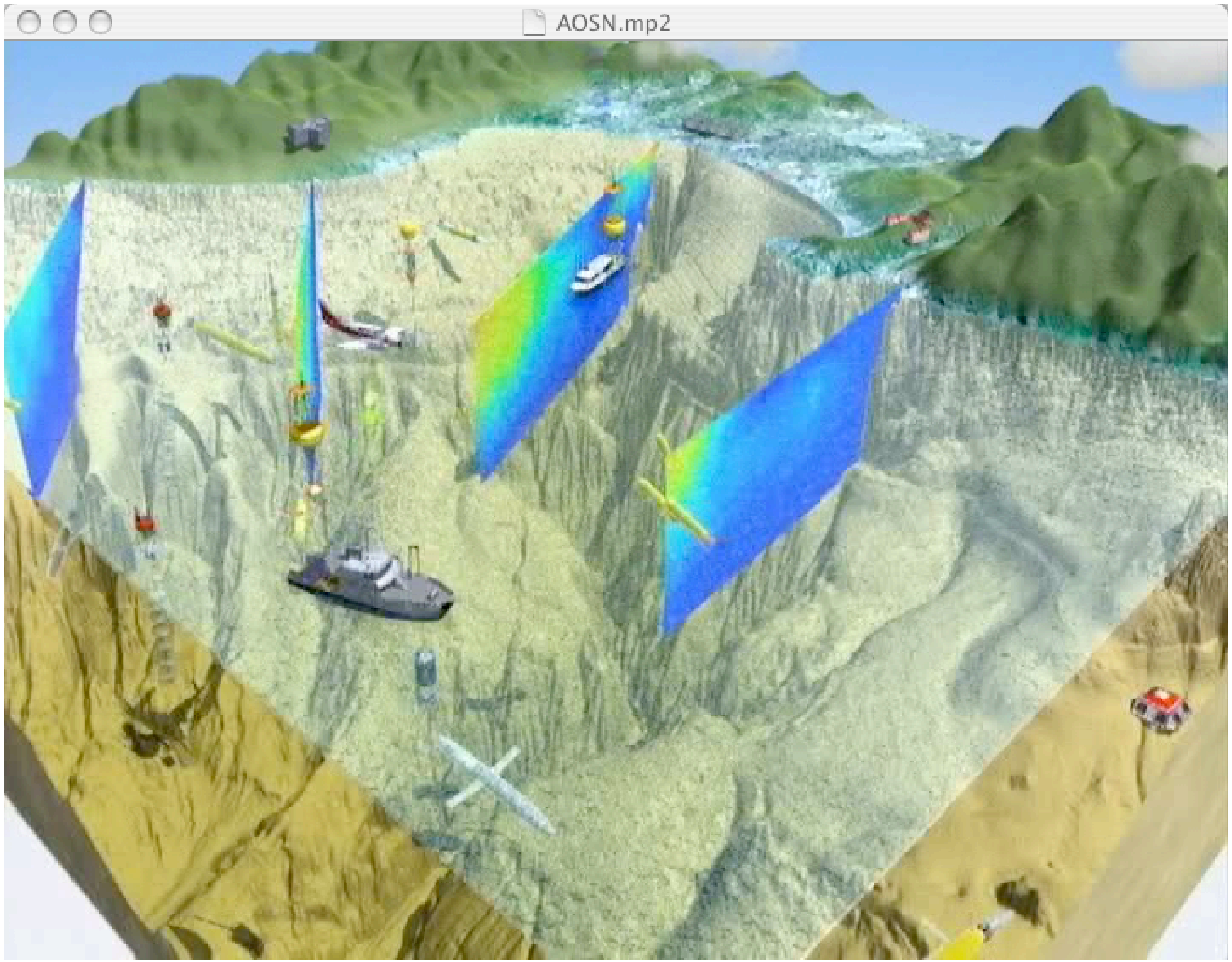
Application: Mobile Sensor Network

- Design periodic trajectories for oceanographic broad-area mapping
- Real-time closed-loop control for optimal coordination and adaptive sampling

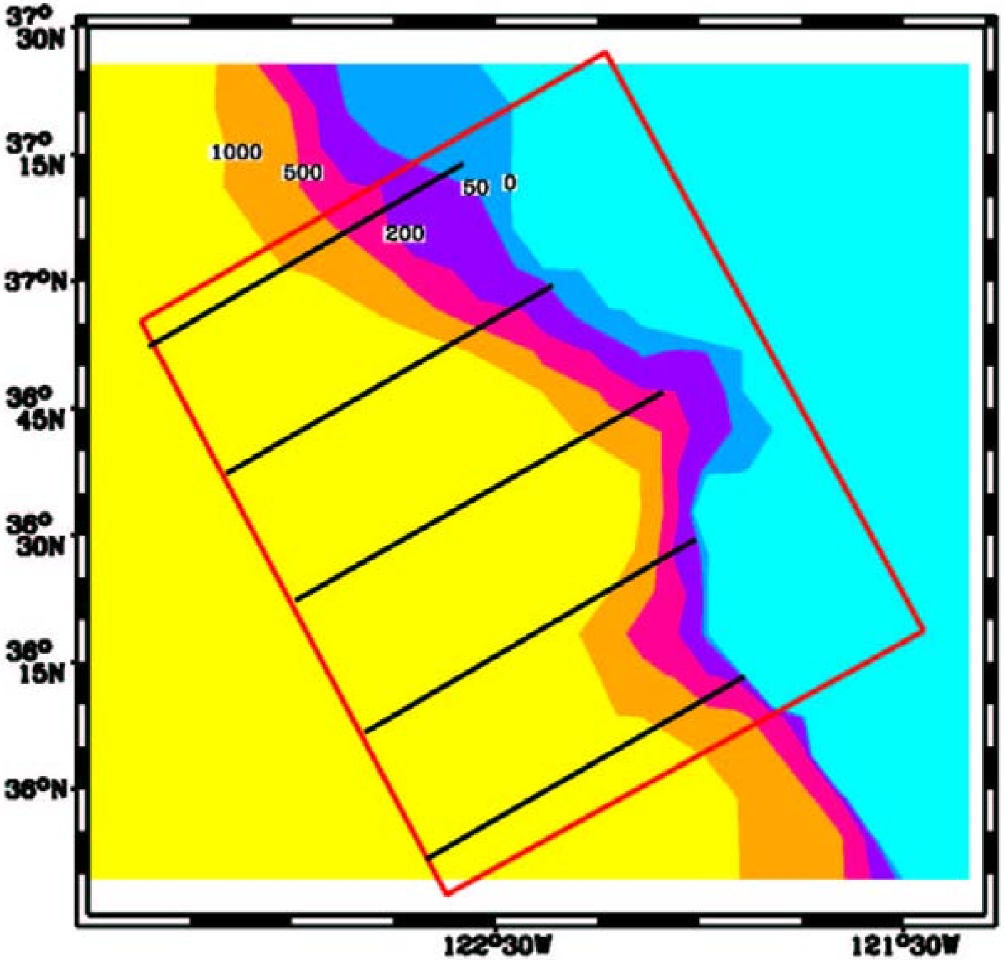


AOSN-II, Monterey Bay, CA USA 2003

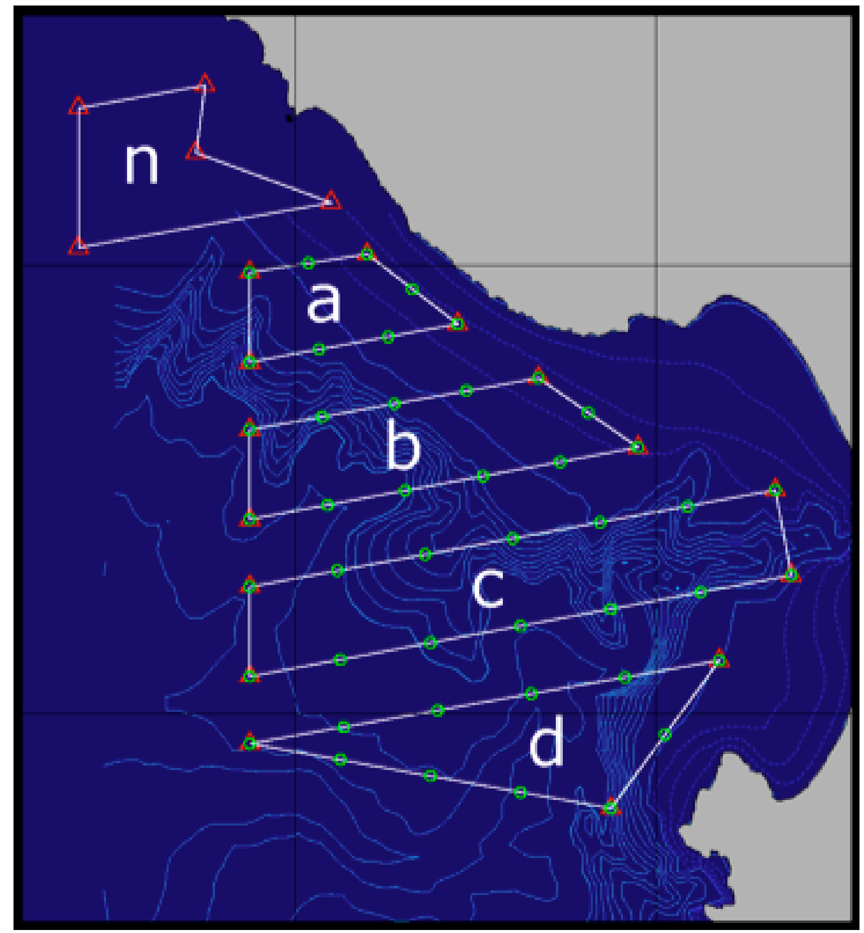
AOSN-II Monterey, CA August 2003



AOSN-II Glider Plan

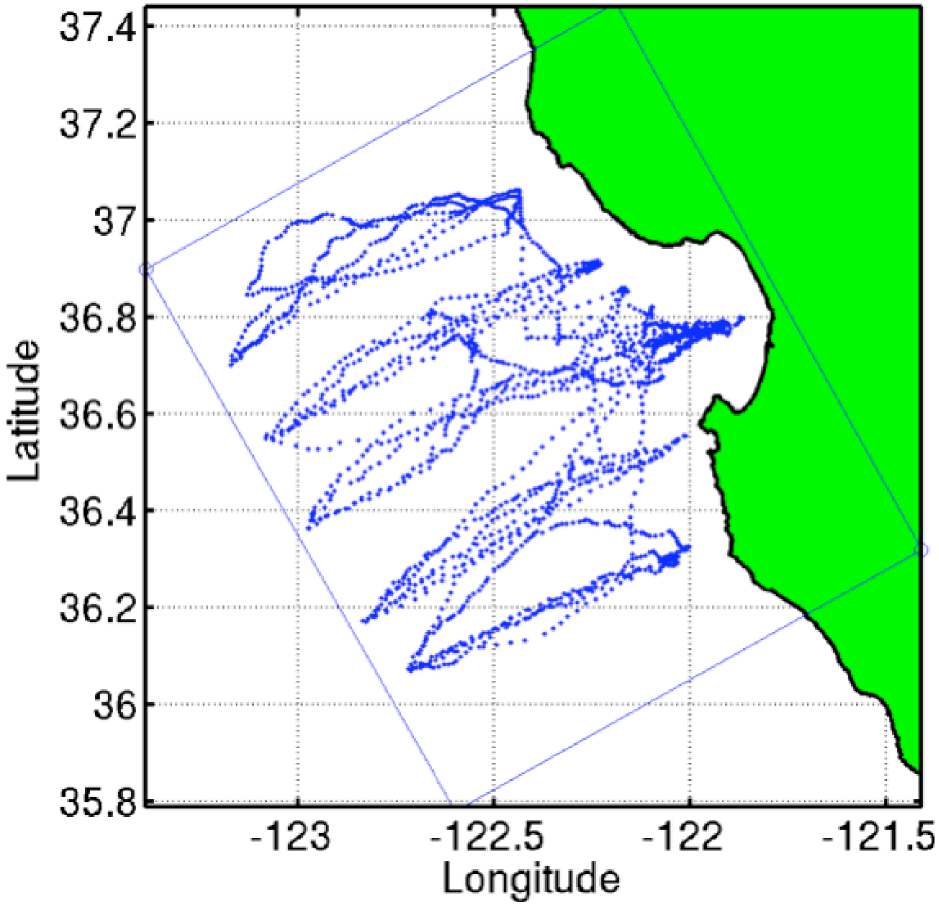


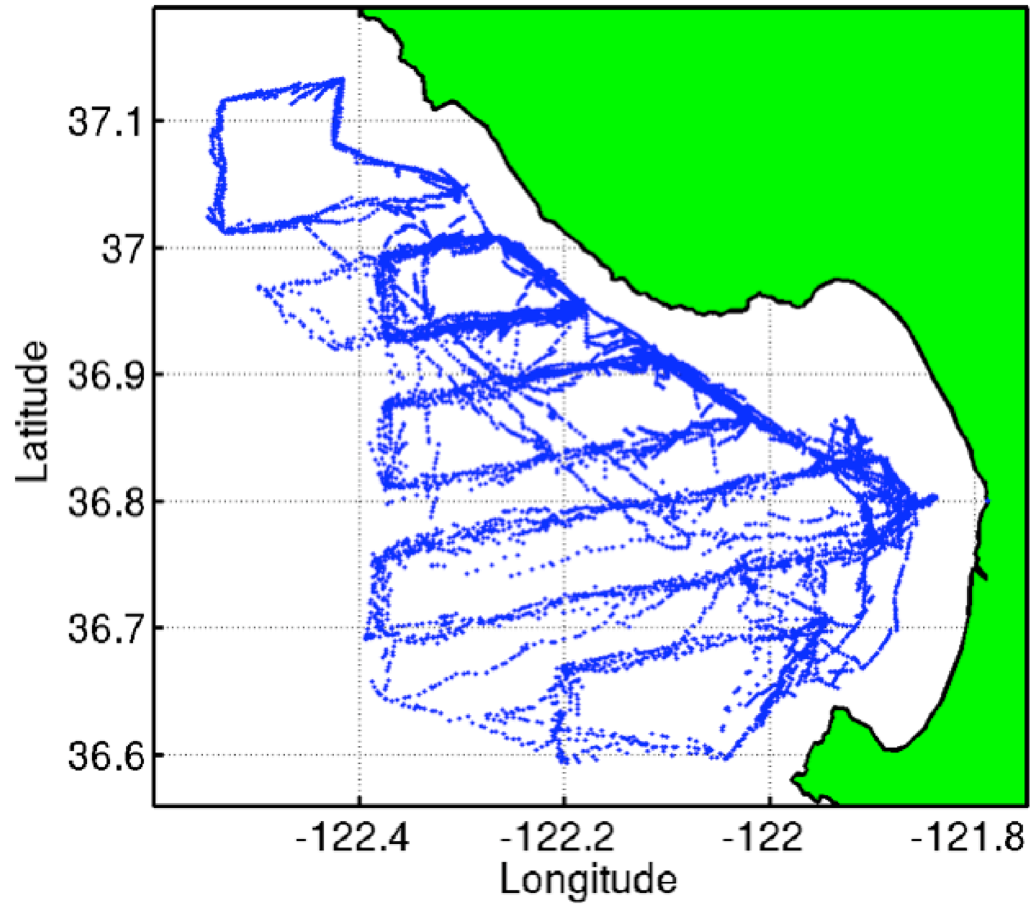
5 SIO Spray Gliders



10 WHOI Slocum Gliders

AOSN-II Glider Measurements





Overview of Talk

- Feedback control laws for stabilization of collective motion
- Quantitative metrics for evaluation/planning of measurement distributions
- Sensor network design for adaptive oceanographic sampling

Self-Propelled Particle Model

[Justh and Krishnaprasad, 2002]

All-to-all coupling of constant speed particles subject to steering controls:

$$\begin{aligned} \dot{\mathbf{r}}_k &= e^{i\theta_k} \\ \dot{\theta}_k &= u_k \end{aligned}$$

$$\mathbf{r}_k \in \mathbb{R}^2 \text{ and } \theta_k \in S^1$$

$$k = 1, \ldots, N$$

Singularly Perturbed System

[Sepulchre, Paley and Leonard, 2003]

Time scale separation decouples alignment and spacing controls:

$$\begin{aligned} \dot{\mathbf{r}}_k &= e^{i\theta_k} \\ \epsilon\dot{\theta}_k &= \epsilon u_k^{spac} + u_k^{align} \end{aligned}$$

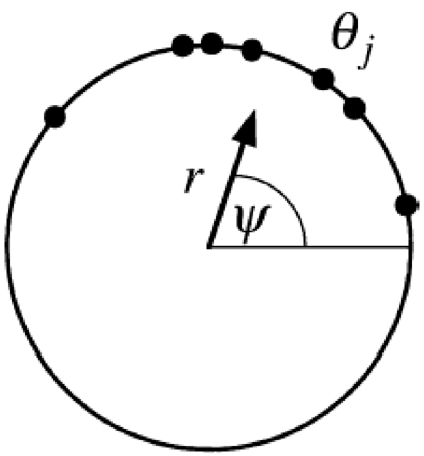
slow fast

$$|\epsilon| = \frac{1}{|K|} \ll 1$$

Complex Order Parameter

[Kuramoto, 1975]

Centroid of particles phases on unit circle:



$$p_{m\theta} = \frac{1}{N}\sum_{j=1}^{N} e^{im\theta_j} = r_m e^{i\psi_m}$$

$N p_\theta \equiv$ group linear momentum

Alignment Control

Define control in terms of gradient of scalar potential:

$$V_m = \frac{N}{2}|p_{m\theta}|^2$$

$$\nabla_k V_m = \frac{1}{N}\sum_{j=1}^{N} \sin(m(\theta_j - \theta_k))$$

Example: first harmonic only

$$K u_k^{align} = \frac{K_1}{N}\sum_{j=1}^{N} \sin(\theta_j - \theta_k)$$

Parallel Motion

Positive coupling of the first harmonic synchronizes the particle phases in the fast dynamics:

$$K_1 = 1 \Rightarrow p_\theta = 1$$

Formation Control

Choose spacing control that inserts nonlinear springs between all pairs of particles:

[Bachmayer and Leonard, 2002]

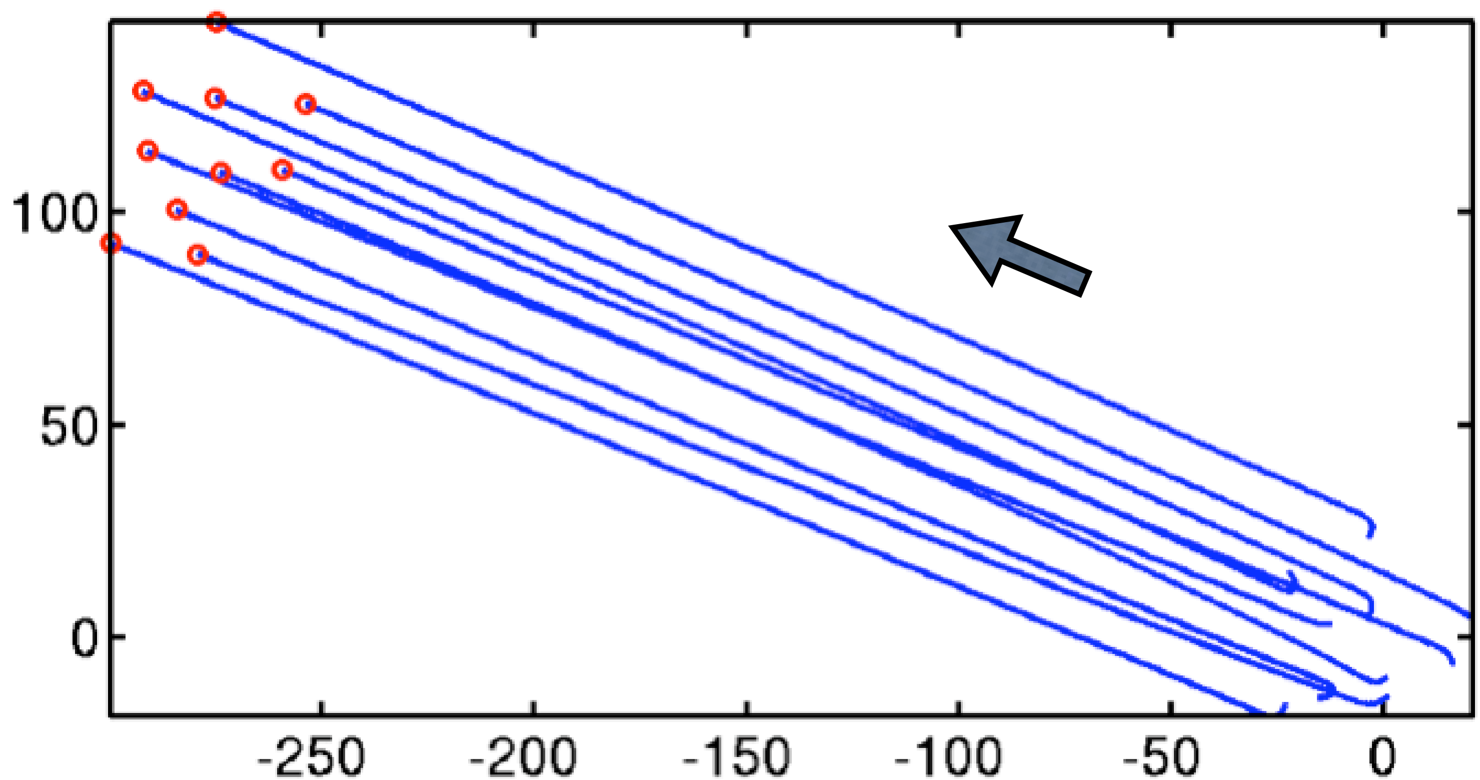
$$U_I(\mathbf{r}_{kj}) = \log \|\mathbf{r}_{kj}\| + \frac{\rho_o}{\|\mathbf{r}_{kj}\|}$$

$$u_k^{spac} = -\sum_{j \neq k}^{N} < \nabla U_I(\mathbf{r}_{kj}), ie^{i\theta_k} >$$

$$U = \sum_{j=1}^{N} \sum_{k>i} U_I(r_{jk})$$

Parallel Motion

$$N = 10, \rho_o = 10$$



Circular Motion

Negative coupling of the alignment anti-synchronizes the phases in the fast dynamics

$$K_1 = -1 \Rightarrow p_\theta = 0$$

For compatibility, the slow dynamics must remain on the balanced manifold

$$\dot{p}_\theta = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}\dot{\theta}_j = P_\theta^T u^{spac} = 0$$

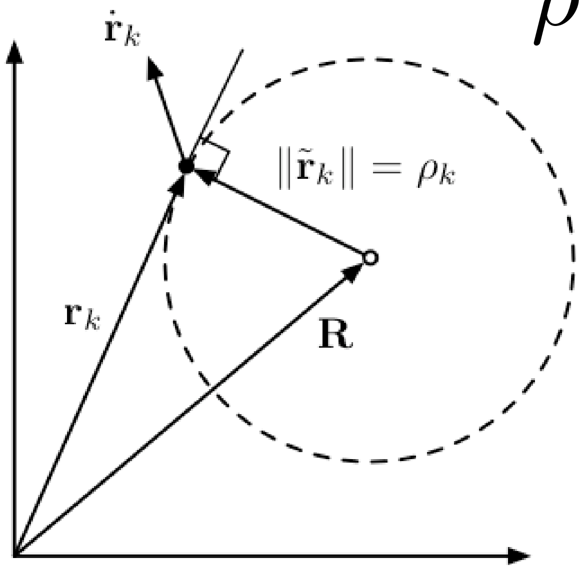
$$u^{spac} = \Pi u^{bal}, \quad \Pi = (I - P_\theta(P_\theta^T P_\theta)^{-1}P_\theta^T)$$

Beacon Control Law

In the slow dynamics, the particle center of mass is fixed

[Justh and Krishnaprasad, 2002]

$$u_k^{bal} = -f(\rho_k) < \frac{\tilde{\mathbf{r}}_k}{\rho_k}, ie^{i\theta_k} > - < \frac{\tilde{\mathbf{r}}_k}{\rho_k}, e^{i\theta_k} >$$

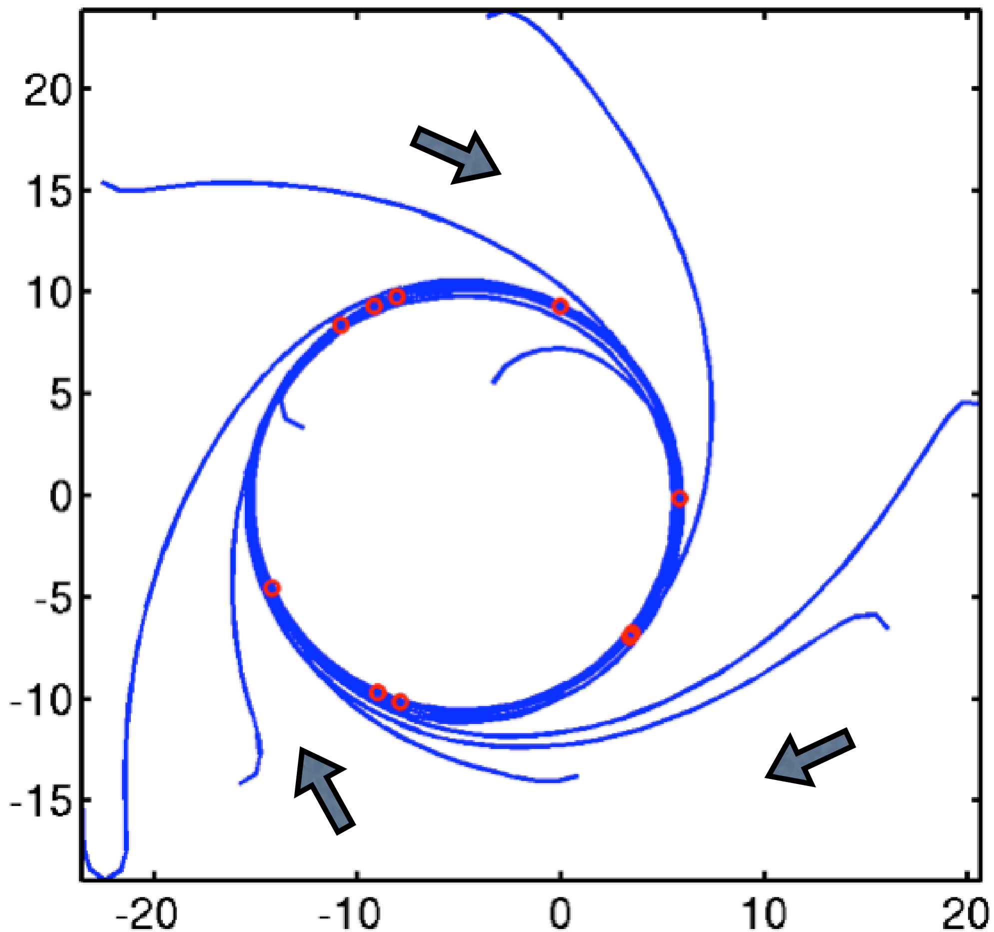


$$f(\rho_k) = 1 - \left(\frac{\rho_o}{\rho_k}\right)^2$$

$$U_k = -\log| < \frac{r_k}{\rho_k}, ie^{i\theta_k} > | + \int_{\bar{\rho}}^{\rho_k} (f(s) - \frac{1}{s})ds$$

Circular Motion

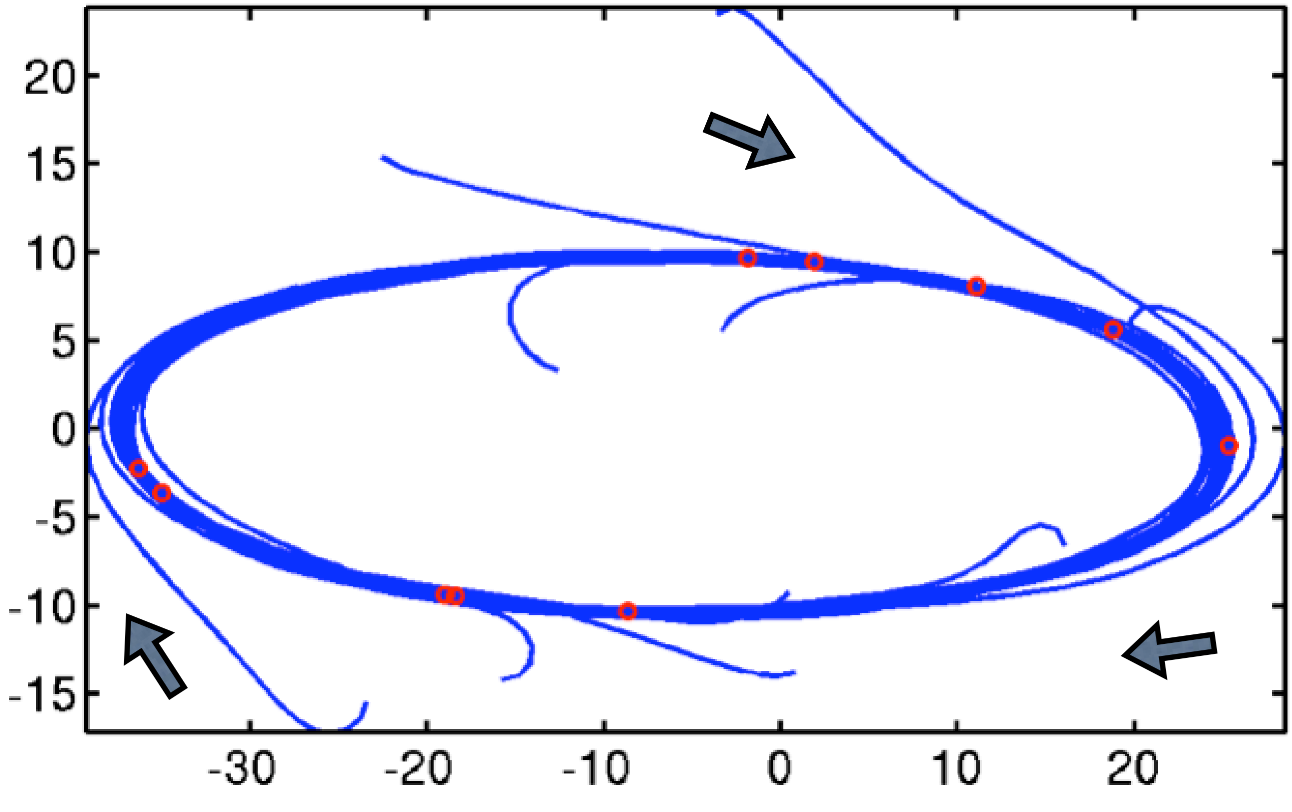
$$N = 10, \rho_o = 10$$



Extension: Shape Changes

Modify beacon control law to track elliptical trajectory:

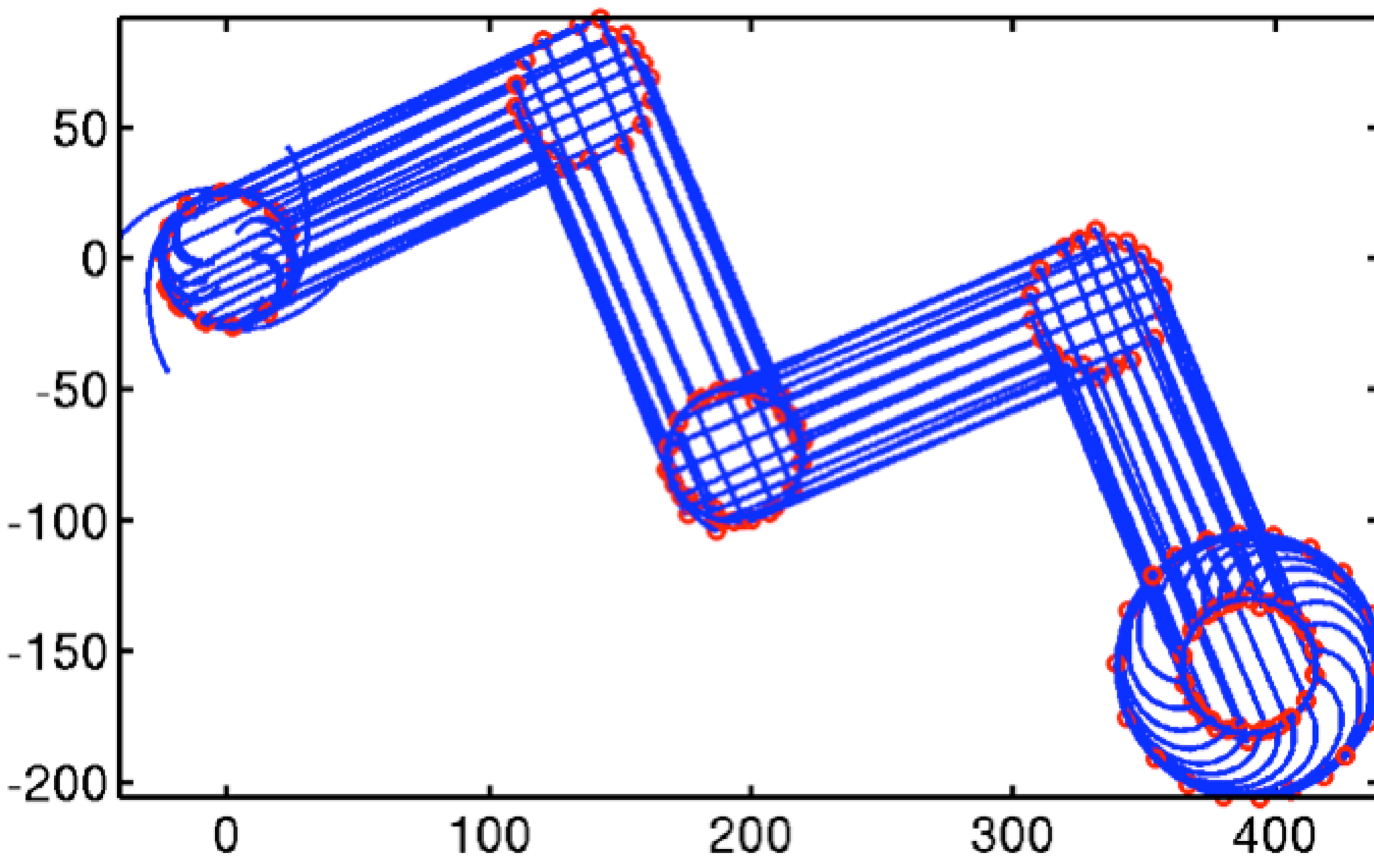
$$N = 10, a = 30, b = 10, e = \sqrt{1 - \frac{b^2}{a^2}} = 0.94$$



Extension: Trajectory Tracking

[Paley, Leonard and Sepulchre, 2004]

$$N = 20, \rho_o = 25$$

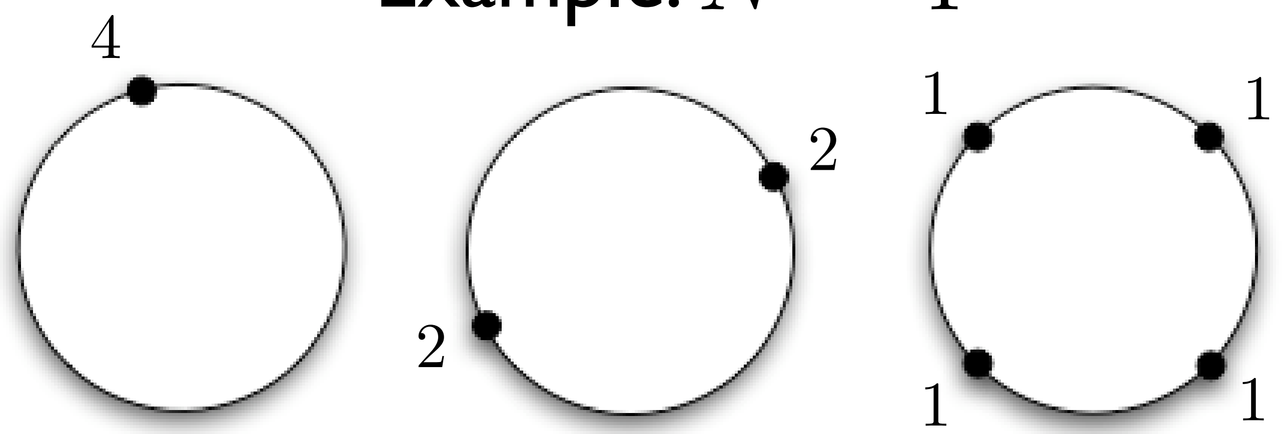


Higher Harmonics

Stabilize higher harmonics of complex order parameter:

$$K_m > 0 \Rightarrow p_{m\theta} \to 1$$
$$K_m < 0 \Rightarrow p_{m\theta} \to 0$$

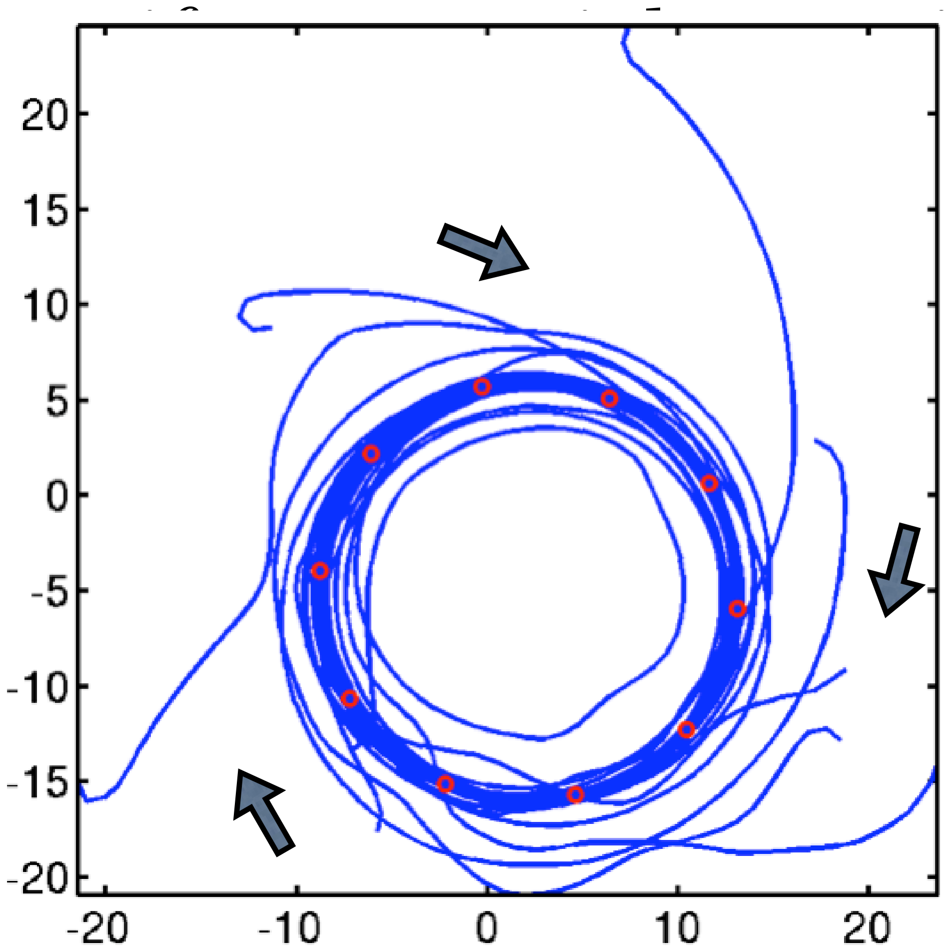
Example: $N = 4$



$p_\theta = 1$ $\quad$ $p_\theta = 0,\ p_{2\theta} = 1$ $\quad$ $p_\theta = p_{2\theta} = 0$

Stabilization of the Splay State

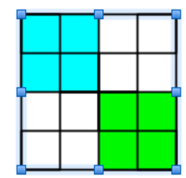
$$p_\theta = p_{2\theta} = \ldots = p_{\frac{N}{2}\theta} = 0$$

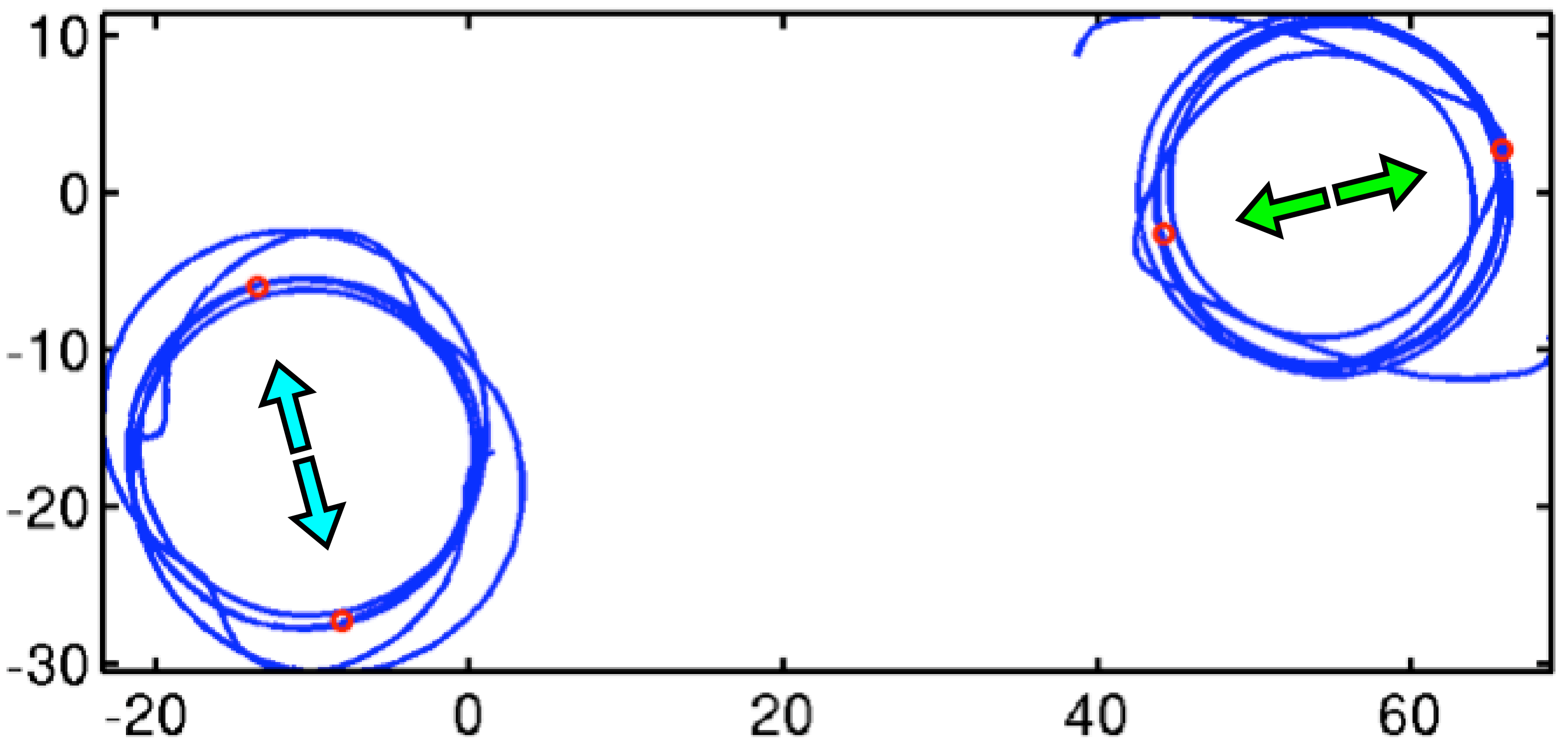


$$N = 10, \rho_o = 10$$

Extension: Topology Changes

Modify network adjacency matrix to form subgroups, while preserving splay state over all particles:

$$N = 4, \rho_o = 10, A =$$ 



Overview of Research Topics

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Objective Analysis

[Gandin, 1965]
[Bretherton, Davis and Fandry, 1976]

Used to compute a gridded error map of an estimate formed from noisy measurements of scalar field:

$$\theta(X) = \phi(X) + v$$

$$\hat{\phi}(x) = B\theta(X), \; x, X \in \mathbb{R}^2 \times \mathbb{R}^+$$

$$\varepsilon^2(x) = \overline{\left(\phi(x) - \hat{\phi}(x)\right)^2}$$

Gauss-Markov Theorem

[Liebelt, 1967]

Theorem provides a linear minimum variance unbiased estimator:

$$\mathbf{x} \in \left(\mathbb{R}^2 \times \mathbb{R}^+\right)^P, \mathbf{X} \in \left(\mathbb{R}^2 \times \mathbb{R}^+\right)^M$$

$$\mathbf{e}(\mathbf{x}) = \phi(\mathbf{x}) - \hat{\phi}(\mathbf{x}) \;\Rightarrow\; C_{\mathbf{e}} \;=\; E(\mathbf{e}\mathbf{e}^T)$$

$$C_{\mathbf{e}} = C_{\mathbf{x}} - C_{\mathbf{x}\mathbf{X}}(C_{\mathbf{X}} + C_{\mathbf{v}})^{-1} C_{\mathbf{x}\mathbf{X}}^T$$

Autocorrelation Function

[Lermusiaux, 1999]

Assume homogeneous, isotropic, stationary field:

$$C_{xX} = F(\xi, \eta)$$

$$\xi = \|(x_1 - X_1, x_2 - X_2)\|, \eta = |x_3 - X_3|$$

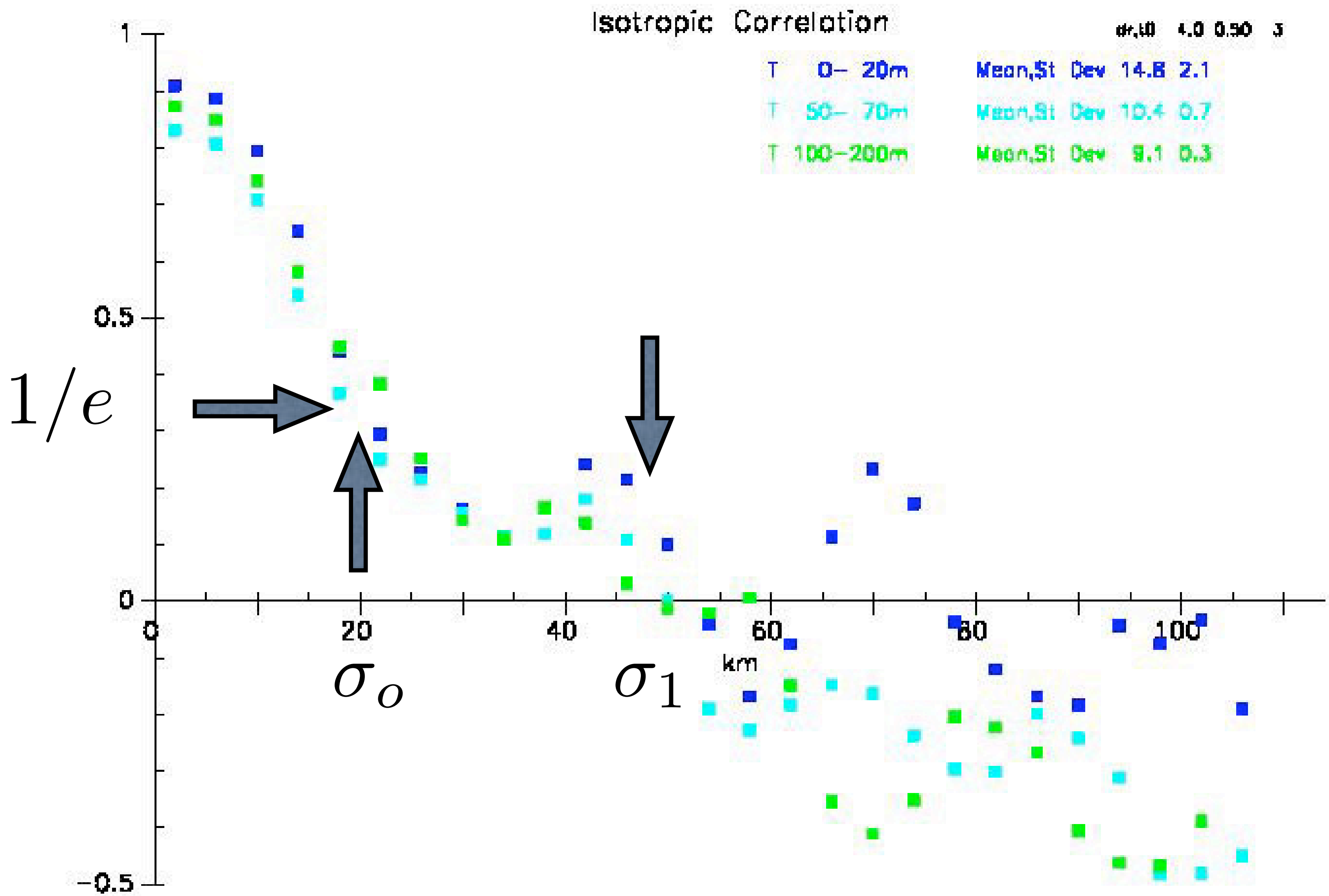
$$F(\xi, \eta) = \left(1 - \frac{\xi^2}{\sigma_1^2}\right) \exp\left[-\frac{1}{2}\left(\frac{\xi^2}{\sigma_o^2} + \frac{\eta^2}{\tau^2}\right)\right]$$

and uncorrelated, zero mean noise:

$$C_{\mathbf{v}} = E\mathbf{I}$$

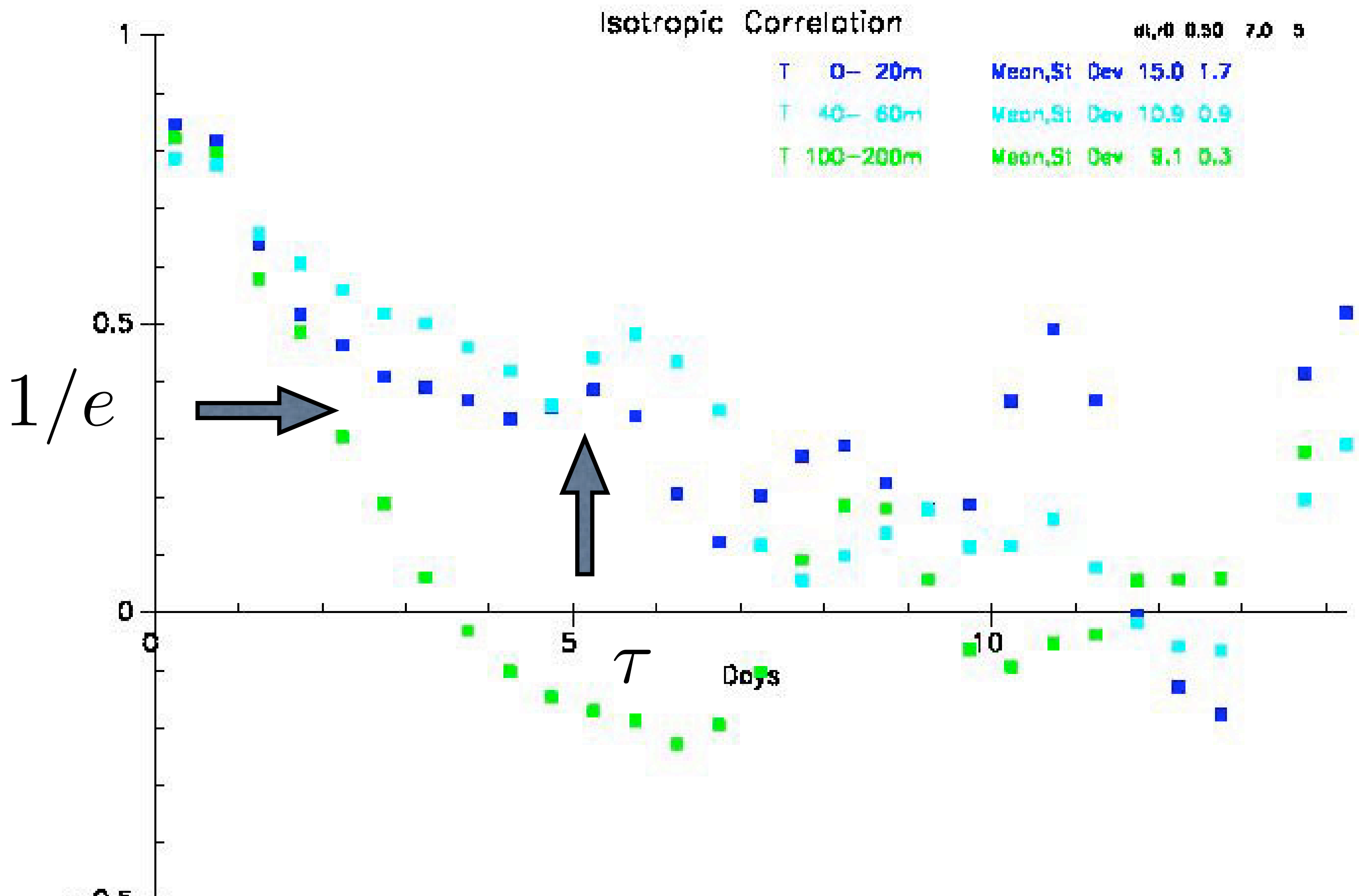
Temperature Correlation vs. Distance

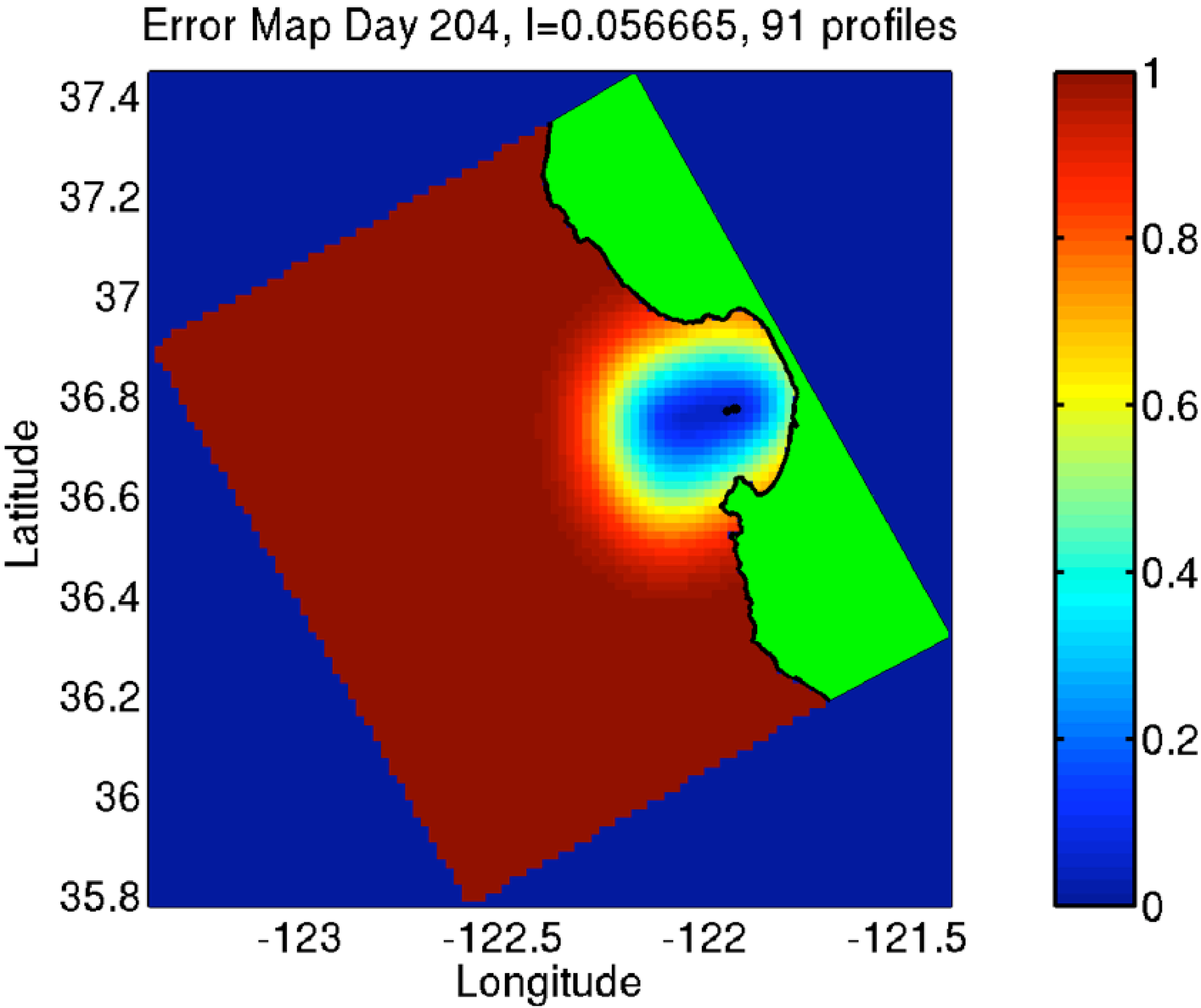
[Davis, 2003]



Temperature Correlation vs. Time

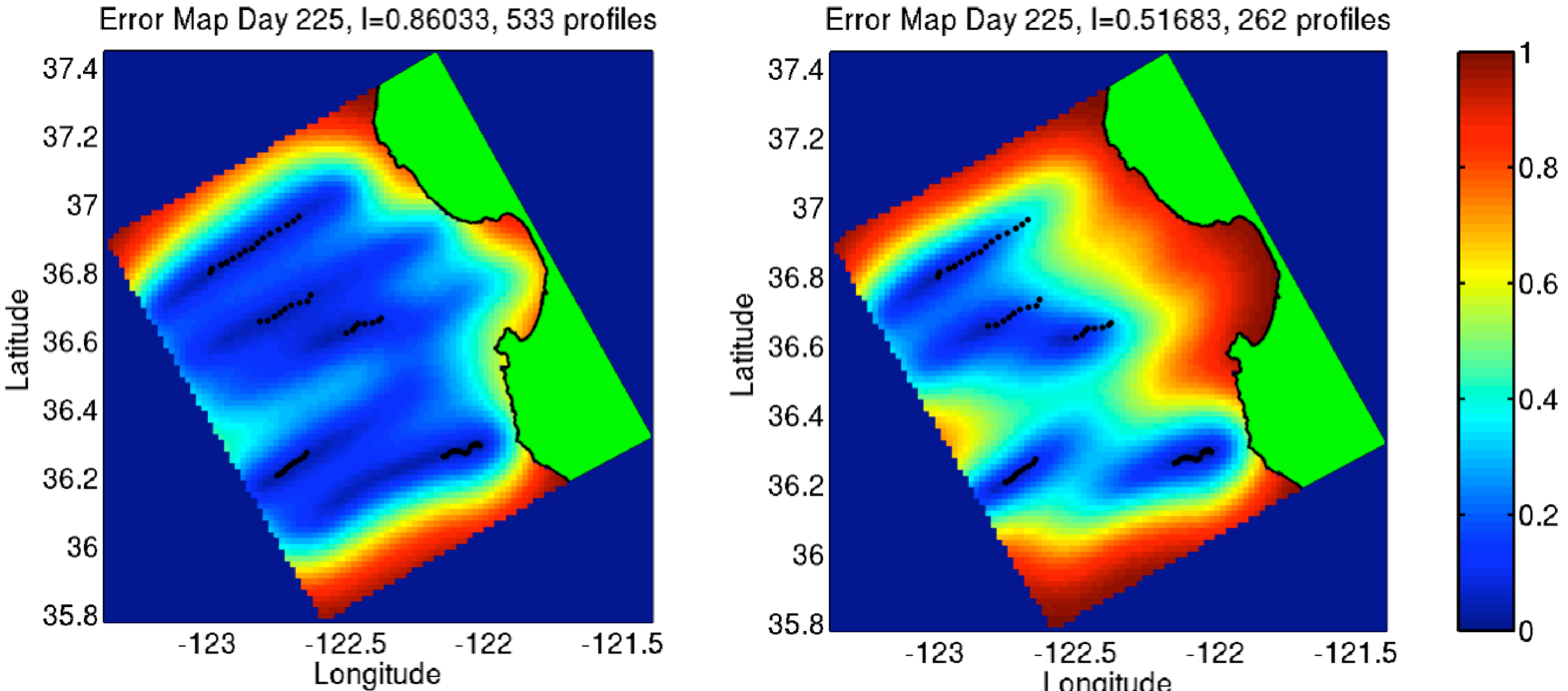
[Davis, 2003]





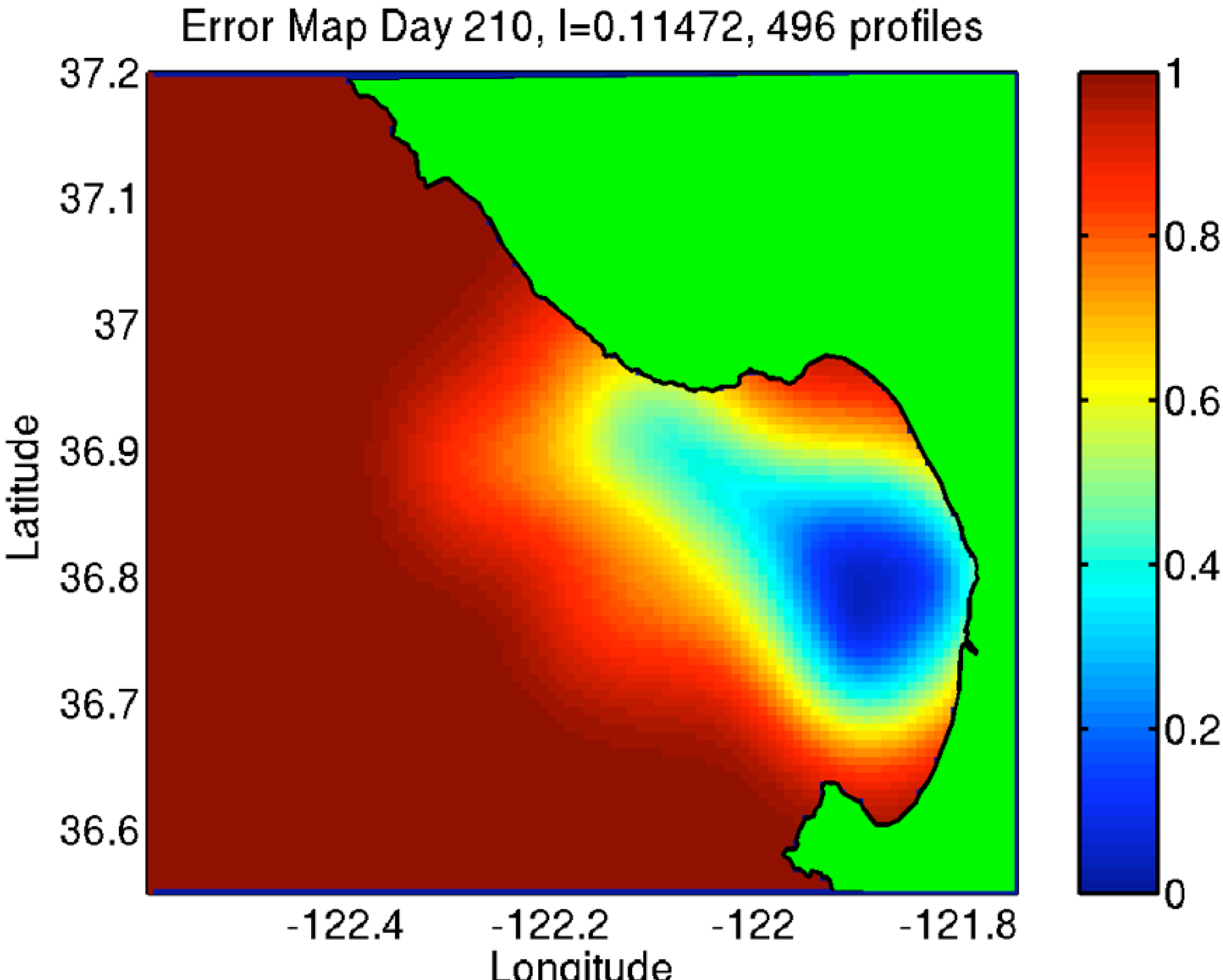
$\sigma_o = 20$ km, $\sigma_1 = 40$ km, $\tau = 5$ days, $E = 0.1$

Sensitivity to Parameters



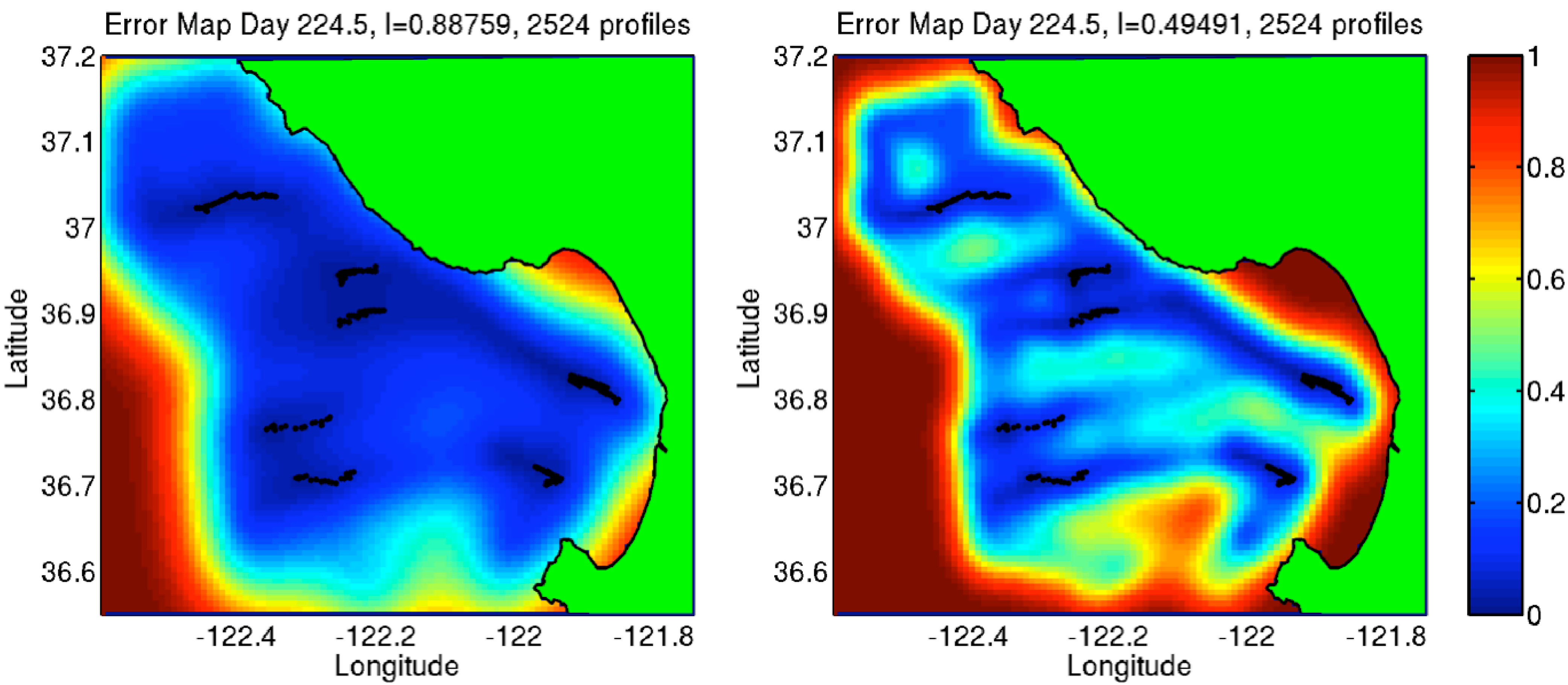
$\tau = 5$ days

$\tau = 2.5$ days



$\sigma_o = 10$ km, $\sigma_1 = 20$ km, $\tau = 2.5$ days, $E = 0.1$

Sensitivity to Parameters II



$\sigma_o = 10 \text{ km}, \sigma_1 = 20 \text{ km}$

$\sigma_o = 5 \text{ km}, \sigma_1 = 10 \text{ km}$

Scalar Entropy Metric

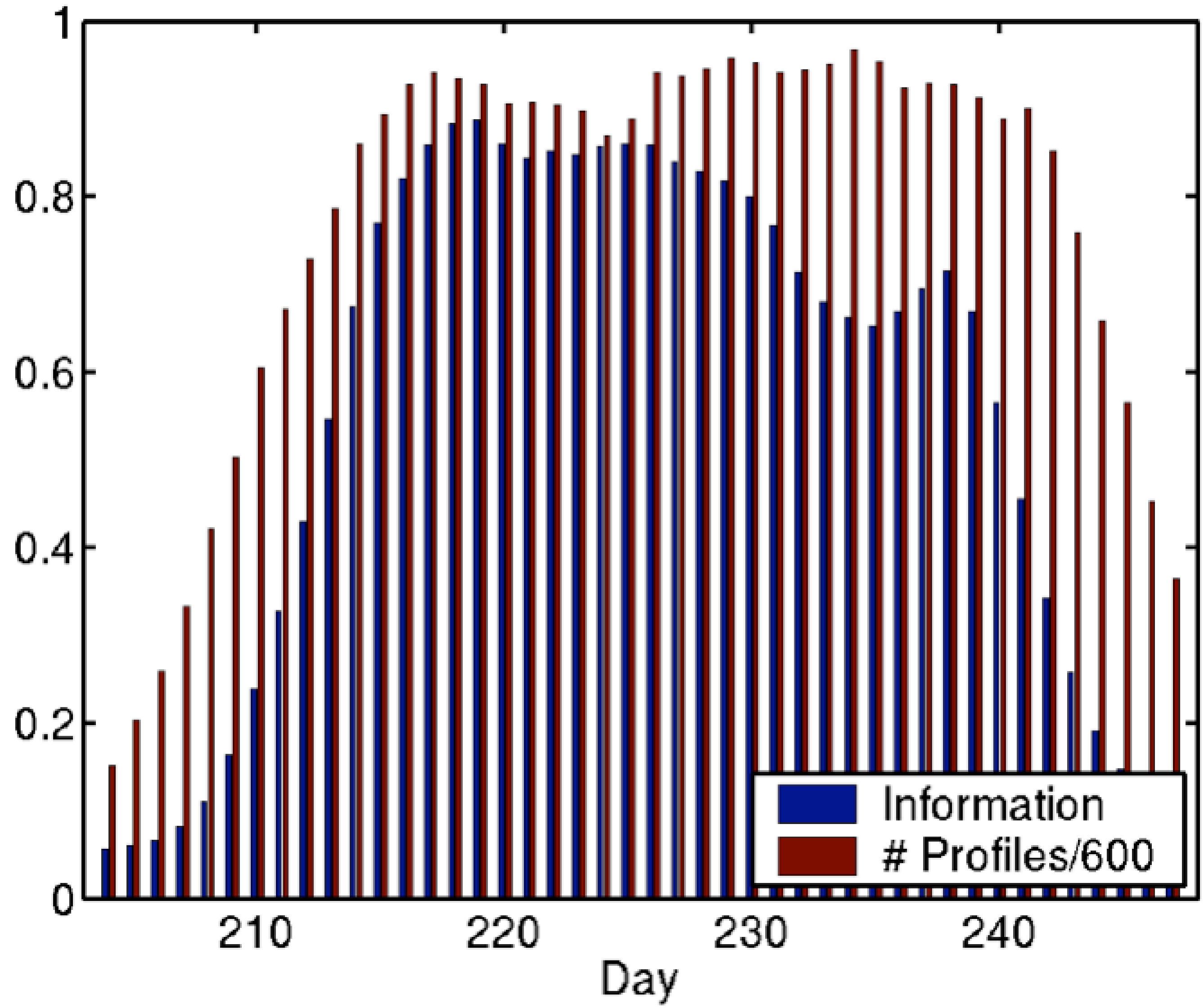
$$H(\mathbf{e}) = -E(\log Pr(\mathbf{e}))$$ [Papoulis, 2002]

$$H(\mathbf{e}) = \frac{1}{2}\log\left[(2\pi e)^P |C_{\mathbf{e}}|\right]$$ [Cover, 1991]

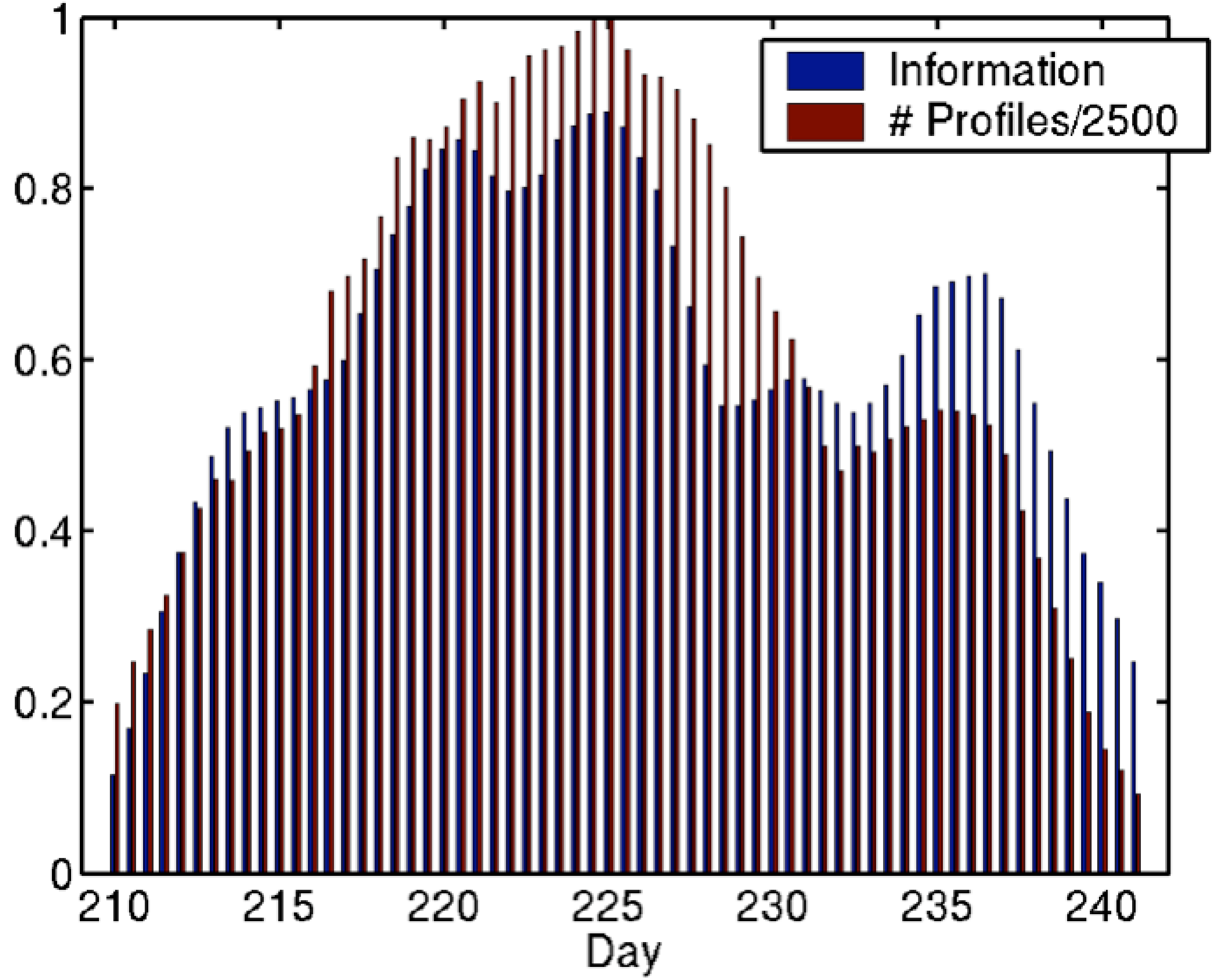
$$H(\mathbf{e}) \leq \frac{1}{2}\log\left[(2\pi e)^P \frac{tr(C_{\mathbf{e}})}{P}\right]$$

$$I(\mathbf{e}) + c \geq -\log\sqrt{\frac{tr(C_{\mathbf{e}})}{P}}$$

AOSN-II SIO Glider Performance Profile



AOSN-II WHOI Glider Performance Profile



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Dimensionless Parameters

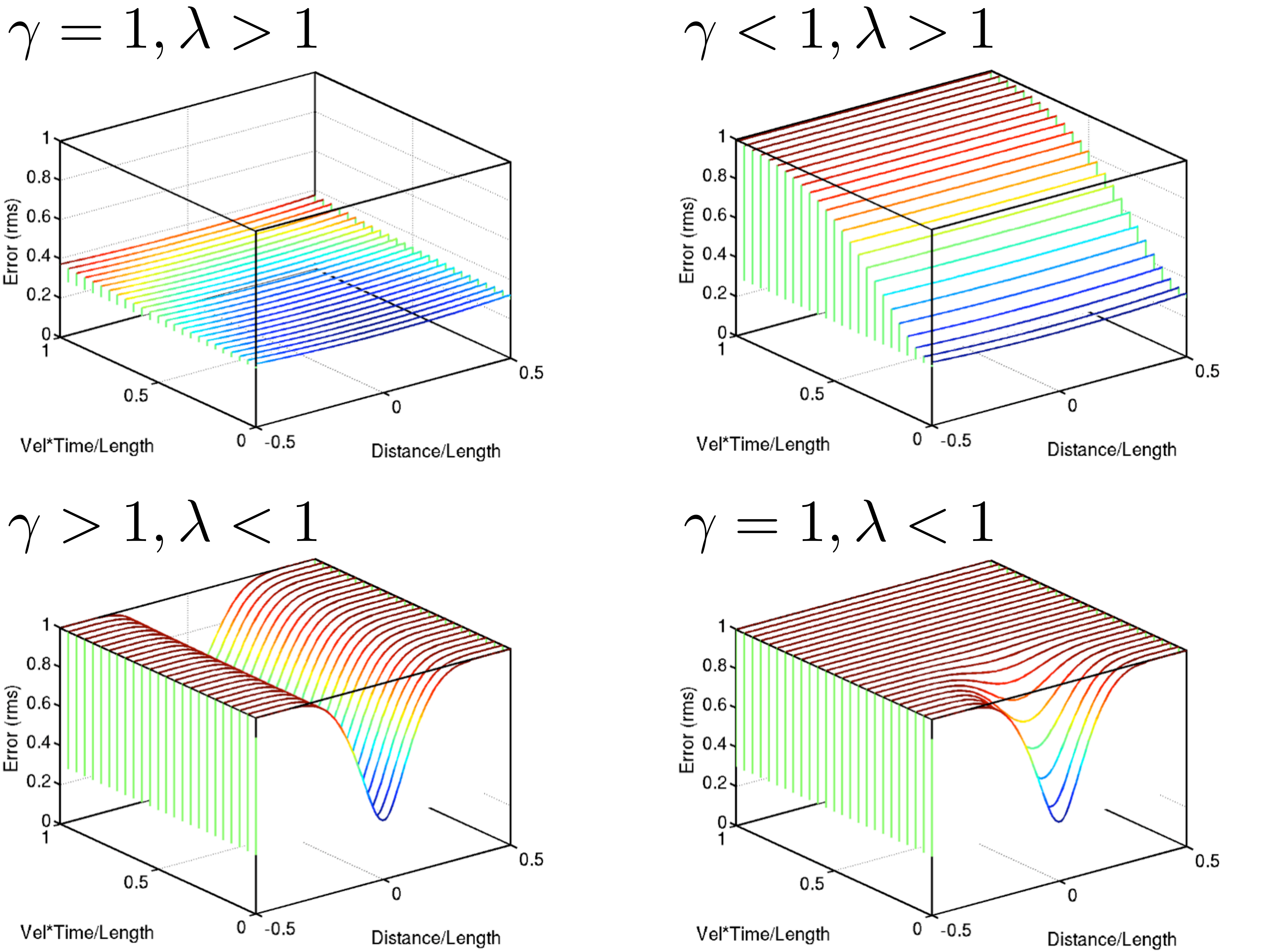
1D (space) plus time:

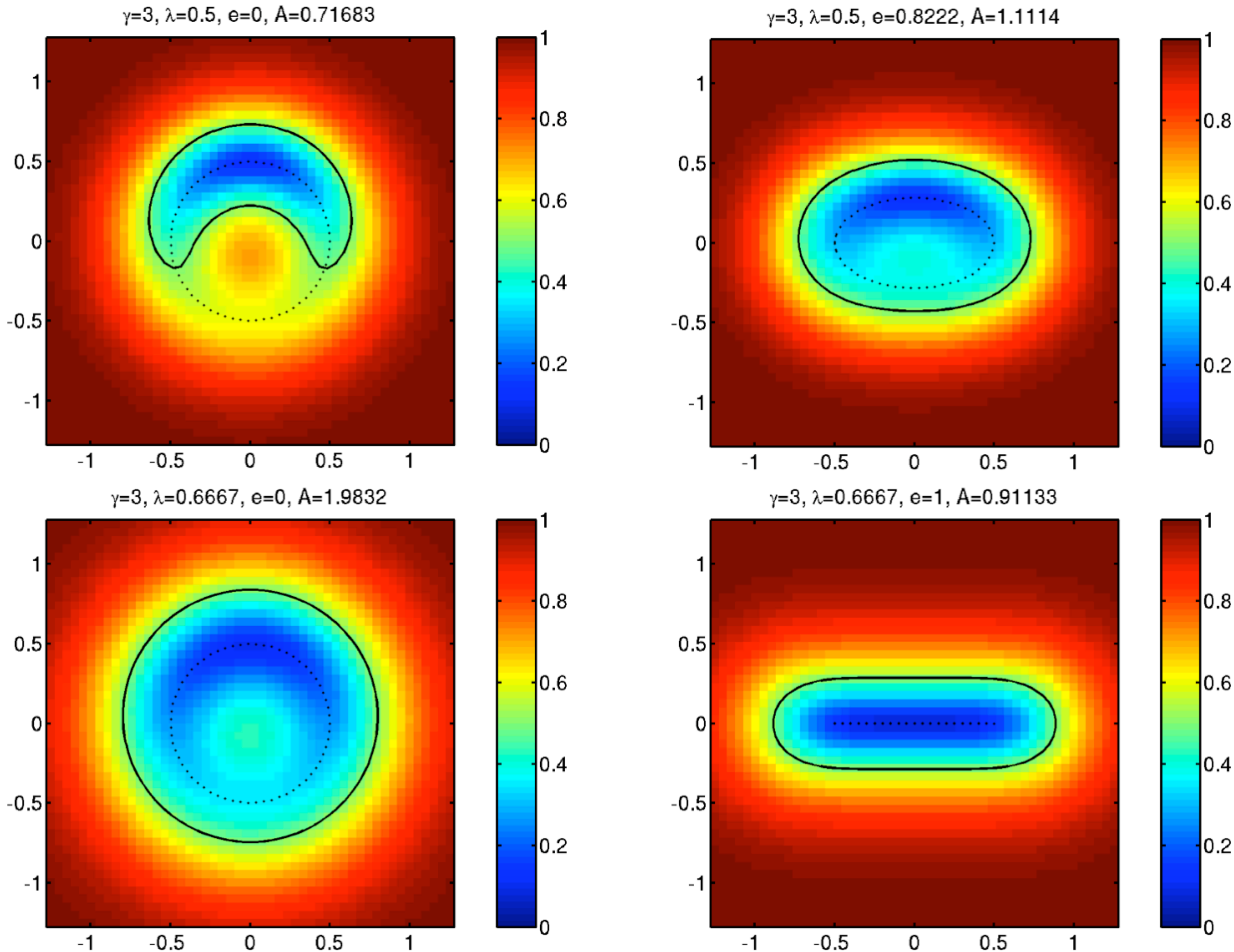
$$x \rightarrow \frac{x}{L} \qquad \lambda = \frac{\sigma}{L}$$

$$t \rightarrow \frac{vt}{L} \qquad \gamma = \frac{v\tau}{\sigma}$$

2D (space) and time:

$$L = 2a \qquad e = \sqrt{1 - \frac{b^2}{a^2}}$$


$\gamma = 1, \lambda > 1$
$\gamma < 1, \lambda > 1$
$\gamma > 1, \lambda < 1$
$\gamma = 1, \lambda < 1$
Error (rms)
Vel*Time/Length
Distance/Length
1
0.8
0.6
0.4
0.2
0
0.5
-0.5



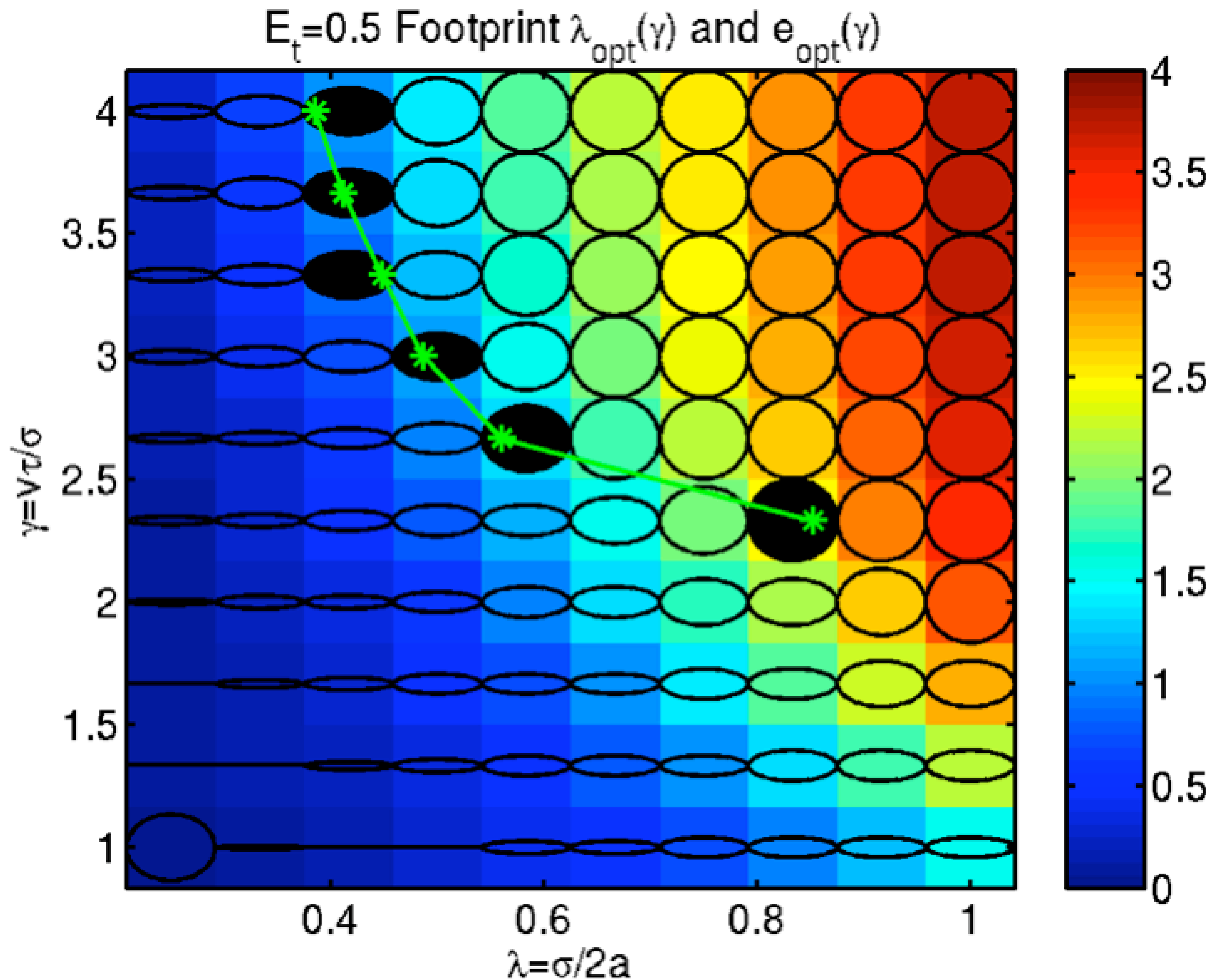
γ=3, λ=0.5, e=0, A=0.71683
γ=3, λ=0.5, e=0.8222, A=1.1114
γ=3, λ=0.6667, e=0, A=1.9832
γ=3, λ=0.6667, e=1, A=0.91133
-1
-0.5
0
0.5
1
0.2
0.4
0.6
0.8

Optimal Shape and Size

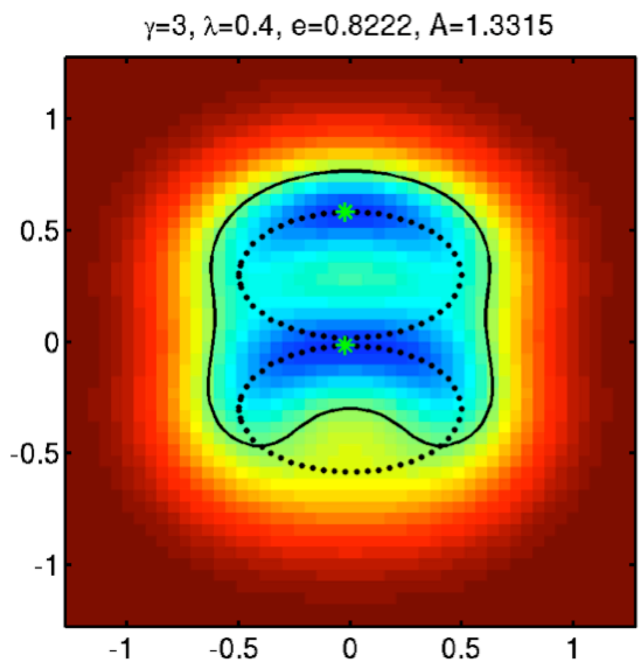
- Choose eccentricity that maximizes sensor footprint normalized area
- Choose semi-major axis that maximizes the sensor footprint area:

$$\begin{aligned}
\frac{A}{4a^2} &= p_1\lambda^2 + p_2\lambda + p_3 \\
\frac{A}{\sigma^2} &= p_1 + \frac{p_2}{\lambda} + \frac{p_3}{\lambda^2} \\
\frac{\partial A}{\partial \eta} &= \sigma^2(p_2 + 2p_3\eta) = 0 \\
\lambda_o &= \frac{1}{\eta_o} = \frac{-2p_3}{p_2}.
\end{aligned}$$

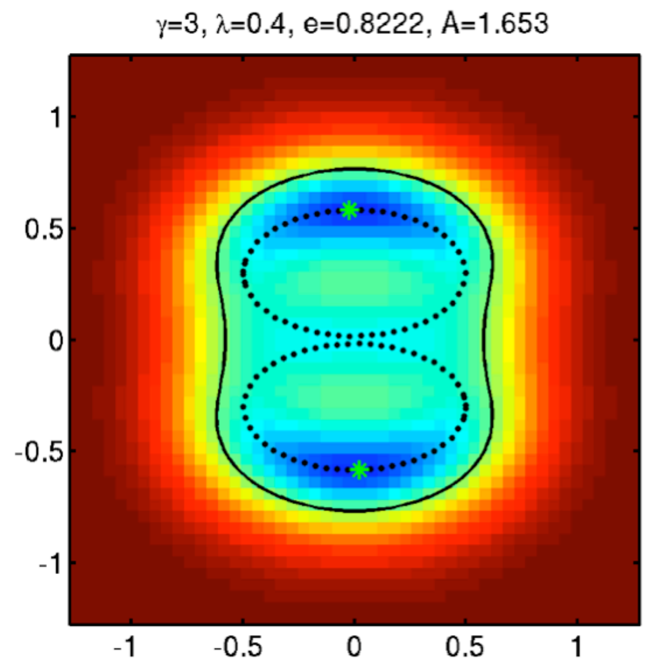
Optimal Shape and Size (N=1)



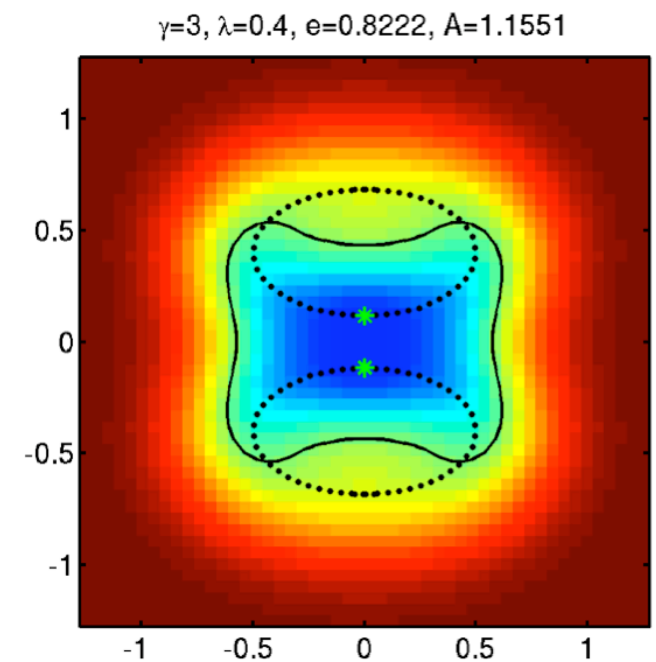
Phase-locked solutions (N=2)



Sync



Anti-Sync
(best)



Anti-Sync
(worst)

Future Work

- Symmetry breaking feedback controls to stabilize collective motion with drift, e.g.

$$\dot{\mathbf{r}}_k = \mathbf{f}(\mathbf{r}_k) + e^{i\theta_k}$$

- Study utility of Fisher information matrix eigenstructure for non-periodic trajectories
- Consider non-homogeneous fields
- Optimal experiment design literature, e.g. Ucinski, 2004