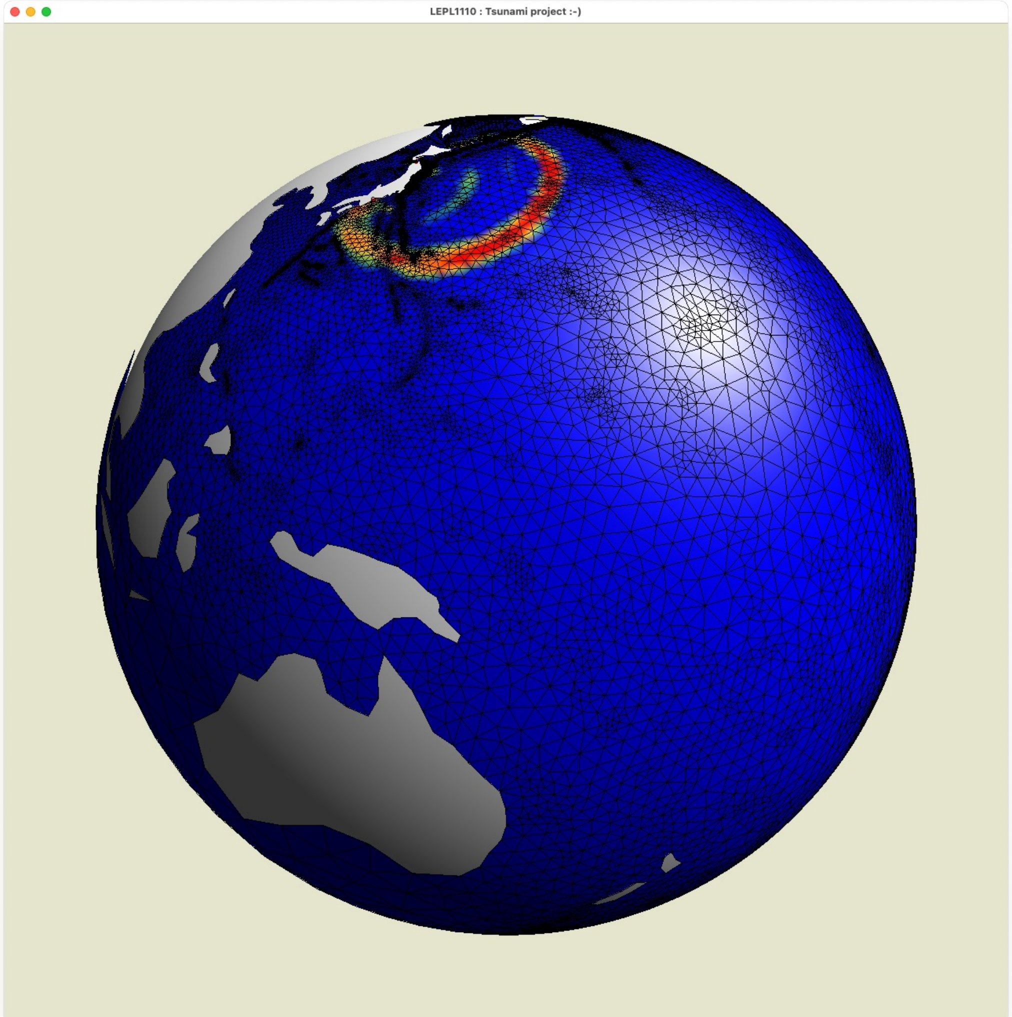
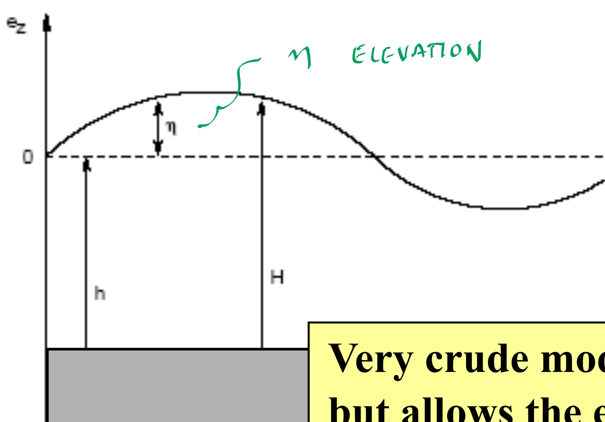


Et maintenant,
les équations du tsunami !

$$
\begin{cases}
\dfrac{\partial \eta}{\partial t} + \dfrac{\partial}{\partial x}\Big(hu\Big) + \dfrac{\partial}{\partial y}\Big(hv\Big) & = & 0 \\[2ex]
\dfrac{\partial u}{\partial t} + \dfrac{\partial}{\partial x}\Big(g\eta\Big) & = & -\gamma u + fv + \dfrac{\tau}{\rho h} \\[2ex]
\dfrac{\partial v}{\partial t} + \dfrac{\partial}{\partial y}\Big(g\eta\Big) & = & -\gamma v - fu
\end{cases}
$$

DE NOUVEAUX TERMES !

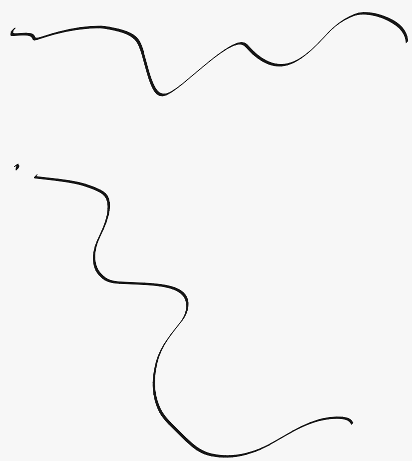
The so-called Shallow Water Equations

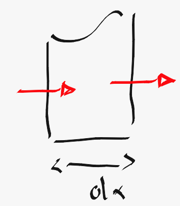


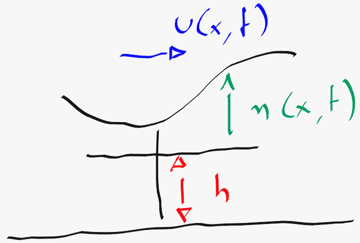
Very crude model for geophysical flows, but allows the existence of inertia-gravity waves

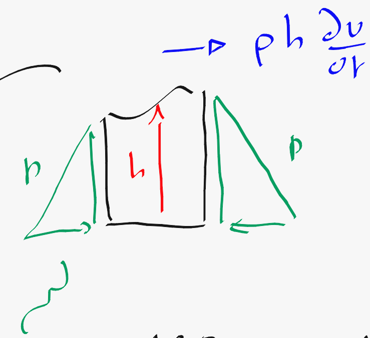
$$\frac{\partial \eta}{\partial t} = -h \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial t} = -g \frac{\partial \eta}{\partial x}$$









$$\rho g \frac{\partial}{\partial x}\left[\frac{h^2}{2}\right] = \rho g h \frac{\partial \eta}{\partial x}$$

An analytical problem as a numerical validation : Stommel :-)

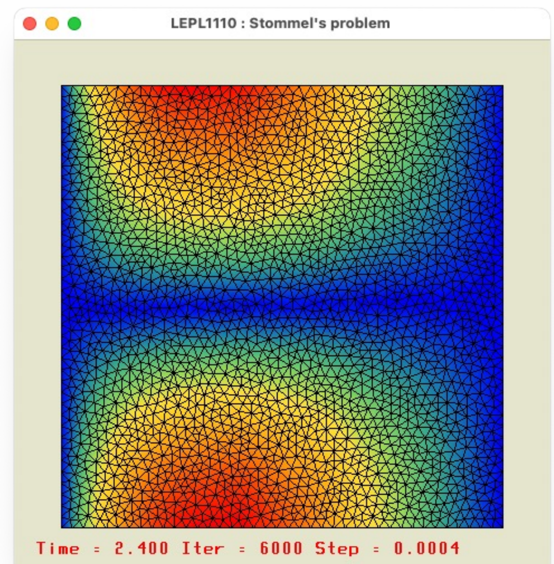
$$
\left\{
\begin{aligned}
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}\big(hu\big) + \frac{\partial}{\partial y}\big(hv\big) &= 0 \\
\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\big(g\eta\big) &= -\gamma u + fv + \frac{\tau}{\rho h} \\
\frac{\partial v}{\partial t} + \frac{\partial}{\partial y}\big(g\eta\big) &= -\gamma v - fu
\end{aligned}
\right.
$$

Forcing wind term [N m^{-2}]

Gravity [m s^{-2}]

Coriolis factor [s^{-1}]

Dissipation coefficient [s^{-1}]

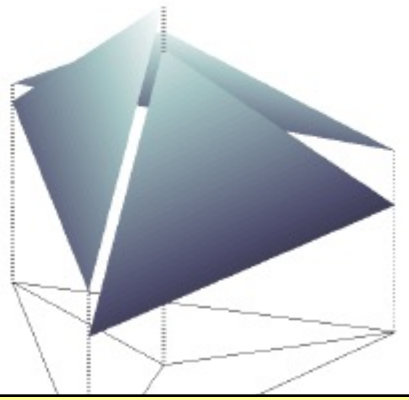


$$Fr = \frac{U}{c} = \frac{U}{\sqrt{gH}}$$

$$Ro = \frac{U}{Lf}$$

Un modèle unidimensionnel le long de l'interface…

$$\left\{\begin{array}{rcl} \dfrac{\partial \eta}{\partial t} + h\dfrac{\partial u}{\partial x} & = & 0 \\ \dfrac{\partial u}{\partial t} + g\dfrac{\partial \eta}{\partial x} & = & 0 \end{array}\right.$$



Discontinuous Galerkin

Comment calculer les flux de masse et de quantité de mouvement aux interfaces ?

Un solveur de Riemann…

$$\left\{\begin{array}{rcl}\dfrac{\partial \eta}{\partial t} + h\dfrac{\partial u}{\partial x} & = & 0\\[2ex] \dfrac{\partial u}{\partial t} + g\dfrac{\partial \eta}{\partial x} & = & 0\end{array}\right.$$

$$\begin{bmatrix}\dfrac{\partial \eta}{\partial t}\\[2ex] \dfrac{\partial u}{\partial t}\end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -h \\ -g & 0\end{bmatrix}}_{\mathbf{A}} \begin{bmatrix}\dfrac{\partial \eta}{\partial x}\\[2ex] \dfrac{\partial u}{\partial x}\end{bmatrix}$$

Discontinuous Galerkin

Comment calculer les flux de masse et de quantité de mouvement aux interfaces ?

$$\begin{bmatrix} \frac{\partial \eta}{\partial t} \\ \frac{\partial u}{\partial t} \end{bmatrix} = \begin{bmatrix} 0 & -h \\ -g & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \eta}{\partial x} \\ \frac{\partial u}{\partial x} \end{bmatrix}$$

$$\underbrace{\frac{\partial \underline{u}}{\partial t}} = \underbrace{\underline{\underline{A}}} \; \underbrace{\frac{\partial \underline{u}}{\partial x}}$$

$$\underline{\underline{A}} \cdot \underline{v} = \lambda \underline{v}$$

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = 0$$

POUR AVOIR UNE SOLUTION NON NULLE !

$$\det \begin{bmatrix} -\lambda & -h \\ -g & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 - gh = 0$$

$$\boxed{\lambda = \pm \sqrt{gh}}$$

$$\underbrace{\underline{\underline{B}}^{-1} \cdot \frac{\partial \underline{u}}{\partial t}}_{\frac{\partial \underline{v}}{\partial t}} = \underbrace{\underline{\underline{B}}^{-1} \cdot \underline{\underline{A}} \cdot \underline{\underline{B}}}_{\underline{\underline{D}}} \cdot \underbrace{\underline{\underline{B}}^{-1} \cdot \frac{\partial \underline{u}}{\partial x}}_{\frac{\partial \underline{v}}{\partial x}}$$

VALEURS PROPRES

INVARIANTS DE RIEMANN :-)

EH OUI C'EST PAS UN VECTEUR PROPRE

$$\underline{\underline{A}} \cdot \underline{v} = \lambda \underline{v}$$

$$\det(\underline{\underline{A}} - \lambda \underline{\underline{I}}) = 0$$

POUR AVOIR UNE SOLUTION NON NULLE !

$$\det\begin{bmatrix} -\lambda & -h \\ -g & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 - gh = 0$$

$$\boxed{\lambda = \pm\sqrt{gh}}$$

CALCULONS LES VECTEURS PROPRES :-)

$$\begin{bmatrix} 0 & -h \\ -g & 0 \end{bmatrix}\begin{bmatrix} r \\ s \end{bmatrix} = \sqrt{gh}\begin{bmatrix} r \\ s \end{bmatrix}$$

$$\boxed{r = 1}$$

$$-g = \sqrt{gh}\; s$$

$$\boxed{s = -\sqrt{\frac{g}{h}}}$$

$$\underline{\underline{R}} = \begin{bmatrix} 1 & 1 \\ \sqrt{\frac{g}{h}} & -\sqrt{\frac{g}{h}} \end{bmatrix}$$

$$\underline{\underline{R}}^{-1} = \begin{bmatrix} 1/2 & 1/2\sqrt{h/g} \\ 1/2 & -1/2\sqrt{h/g} \end{bmatrix}$$

$$\det(\underline{\underline{R}}) = -2\sqrt{\frac{g}{h}}$$

Calculons les valeurs propres de A pour découpler les deux équations…

$$\begin{bmatrix} \dfrac{\partial \eta}{\partial t} \\[2ex] \dfrac{\partial u}{\partial t} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -h \\[2ex] -g & 0 \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \dfrac{\partial \eta}{\partial x} \\[2ex] \dfrac{\partial u}{\partial x} \end{bmatrix}$$

$$\lambda = \pm\sqrt{gh}$$

Deux valeurs propres

Deux vecteurs propres

$$\mathbf{v} = \begin{bmatrix} 1 \\[2ex] \pm\sqrt{\dfrac{g}{h}} \end{bmatrix}$$

Effectuons un changement de variables…

$$\begin{bmatrix} r \\ s \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{\frac{h}{g}} \\ \frac{1}{2} & -\frac{1}{2}\sqrt{\frac{h}{g}} \end{bmatrix}}_{\mathbf{R}^{-1}} \begin{bmatrix} \eta \\ u \end{bmatrix}$$

r et s sont appelées
les invariants de Riemann :-)

Matrice des
deux vecteurs
propres

$$\mathbf{R} = \begin{bmatrix} 1 & 1 \\ \sqrt{\frac{g}{h}} & -\sqrt{\frac{g}{h}} \end{bmatrix}$$

Et on obtient…

$$\left(\frac{\partial \eta}{\partial t} + h\frac{\partial u}{\partial x}\right) + \sqrt{\frac{h}{g}}\left(\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x}\right) = 0$$

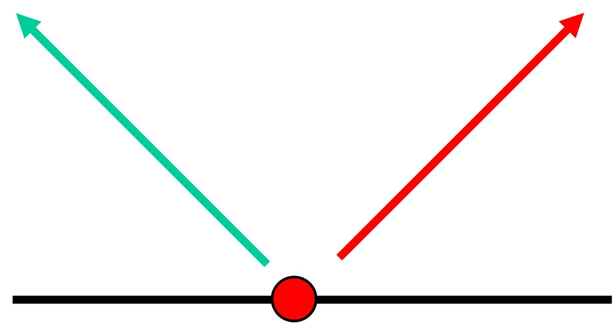
$$\frac{\partial}{\partial t}\underbrace{\left(\eta + \sqrt{\frac{h}{g}}\,u\right)}_{r} + \sqrt{gh}\,\frac{\partial}{\partial x}\underbrace{\left(\eta + \sqrt{\frac{h}{g}}\,u\right)}_{r} = 0$$

$$\left(\frac{\partial \eta}{\partial t} + h\frac{\partial u}{\partial x}\right) - \sqrt{\frac{h}{g}}\left(\frac{\partial u}{\partial t} + g\frac{\partial \eta}{\partial x}\right) = 0$$

$$\frac{\partial}{\partial t}\underbrace{\left(\eta - \sqrt{\frac{h}{g}}\,u\right)}_{s} - \sqrt{gh}\,\frac{\partial}{\partial x}\underbrace{\left(\eta - \sqrt{\frac{h}{g}}\,u\right)}_{s} = 0$$

… deux équations de transport découplées !

Les invariants de Riemann sont constants le long des courbes caractéristiques !



$$\frac{\partial r}{\partial t} + \sqrt{gh}\,\frac{\partial r}{\partial x} = 0$$

Car $\sqrt{gh} = \frac{dx}{dt}$ sur la courbe caractéristique

$$\underbrace{\frac{\partial r}{\partial t} + \frac{dx}{dt}\,\frac{\partial r}{\partial x}}_{\frac{dr}{dt}} = 0$$

$$\frac{\partial \eta}{\partial t} + h\,\frac{\partial v}{\partial x} = 0 \qquad\qquad \frac{\partial v}{\partial t} + g\,\frac{\partial \eta}{\partial x} = 0$$

$$\left[\frac{\partial \eta}{\partial t} + h\,\frac{\partial v}{\partial x}\right] + \sqrt{\frac{h}{g}}\left[\frac{\partial v}{\partial t} + g\,\frac{\partial \eta}{\partial x}\right] = 0$$

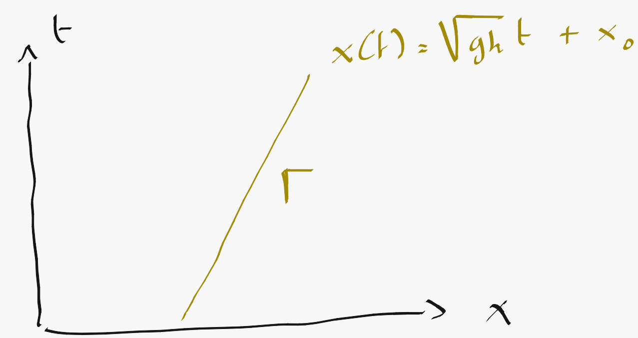
$$\frac{\partial}{\partial t}\underbrace{\left[\eta + \sqrt{\frac{h}{g}}\,v\right]}_{r} + \sqrt{gh}\;\frac{\partial}{\partial x}\underbrace{\left[\eta + \sqrt{\frac{h}{g}}\,v\right]}_{r} = 0$$

SUR UNE COURBE CARACTISTIQUE Γ

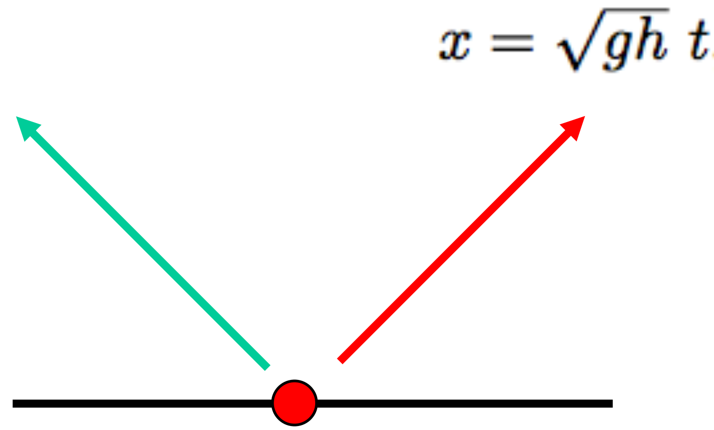
$$x(t) = \sqrt{gh}\;t$$

$$\underbrace{\frac{\partial r}{\partial t} + \underbrace{\sqrt{gh}}_{=\frac{dx}{dt}\text{ sur }\Gamma}\;\frac{\partial r}{\partial x}}_{\left.\frac{dr}{dt}\right| = 0} = 0$$

SUR LA COURBE CARACTERISTIQUE



Et on sait ce qu'il faut faire pour une équation de transport pur !

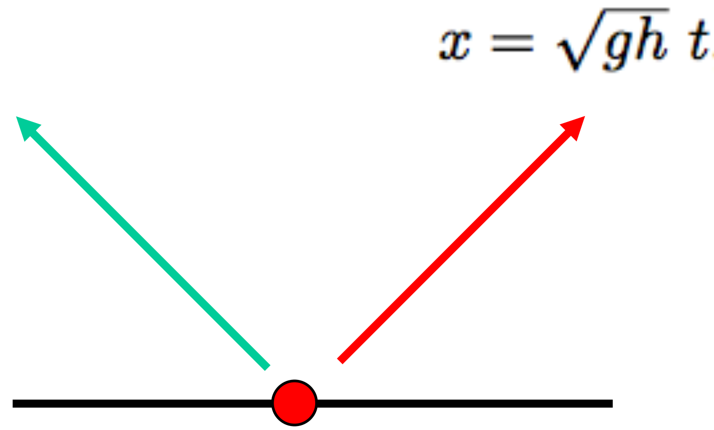


$$\left(\eta + \sqrt{\tfrac{h}{g}}\, u\right)^* \;=\; r^* \;=\; r_L \;=\; \left(\eta_L + \sqrt{\tfrac{h}{g}}\, u_L\right)$$

$$\left(\eta - \sqrt{\tfrac{h}{g}}\, u\right)^* \;=\; s^* \;=\; s_R \;=\; \left(\eta_R - \sqrt{\tfrac{h}{g}}\, u_R\right)$$

Le solveur dit de Riemann :-)

Et en termes de vitesses et d’élévation



$$\left(\eta + \sqrt{\tfrac{h}{g}}\, u\right)^* = r^* = r_L = \left(\eta_L + \sqrt{\tfrac{h}{g}}\, u_L\right)$$

$$\left(\eta - \sqrt{\tfrac{h}{g}}\, u\right)^* = s^* = s_R = \left(\eta_R - \sqrt{\tfrac{h}{g}}\, u_R\right)$$

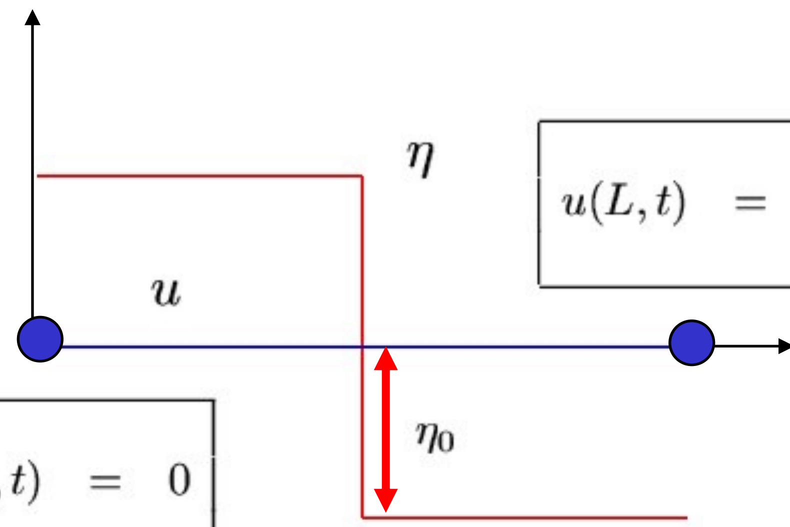
$$\eta^* = \left(\frac{r_L + s_R}{2}\right) = \left(\frac{\eta_L + \eta_R}{2}\right) + \sqrt{\frac{h}{g}}\left(\frac{u_L - u_R}{2}\right)$$

$$u^* = \sqrt{\frac{g}{h}}\left(\frac{r_L - s_R}{2}\right) = \left(\frac{u_L + u_R}{2}\right) + \sqrt{\frac{g}{h}}\left(\frac{\eta_L - \eta_R}{2}\right)$$

Le solveur dit de Riemann :-)

A 1D sharp simplified problem in a finite domain

$$
\begin{aligned}
\frac{\partial \eta}{\partial t} + H\,\frac{\partial u}{\partial x} &= 0 \\
\frac{\partial u}{\partial t} - fv + g\,\frac{\partial \eta}{\partial x} &= 0 \\
\frac{\partial v}{\partial t} + fu &= 0
\end{aligned}
$$



$$u(L,t) = 0$$

$$u(0,t) = 0$$

$$
\begin{aligned}
u(x,0) &= 0 & & \\
v(x,0) &= 0 & & \\
\eta(x,0) &= \eta_0 & & x \in [0, \tfrac{L}{2}[\\
\eta(x,0) &= -\eta_0 & & x \in]\tfrac{L}{2}, L]
\end{aligned}
$$

What is the solution ?

$$\frac{\partial \eta'}{\partial t'} + \frac{\partial u'}{\partial x'} = 0$$

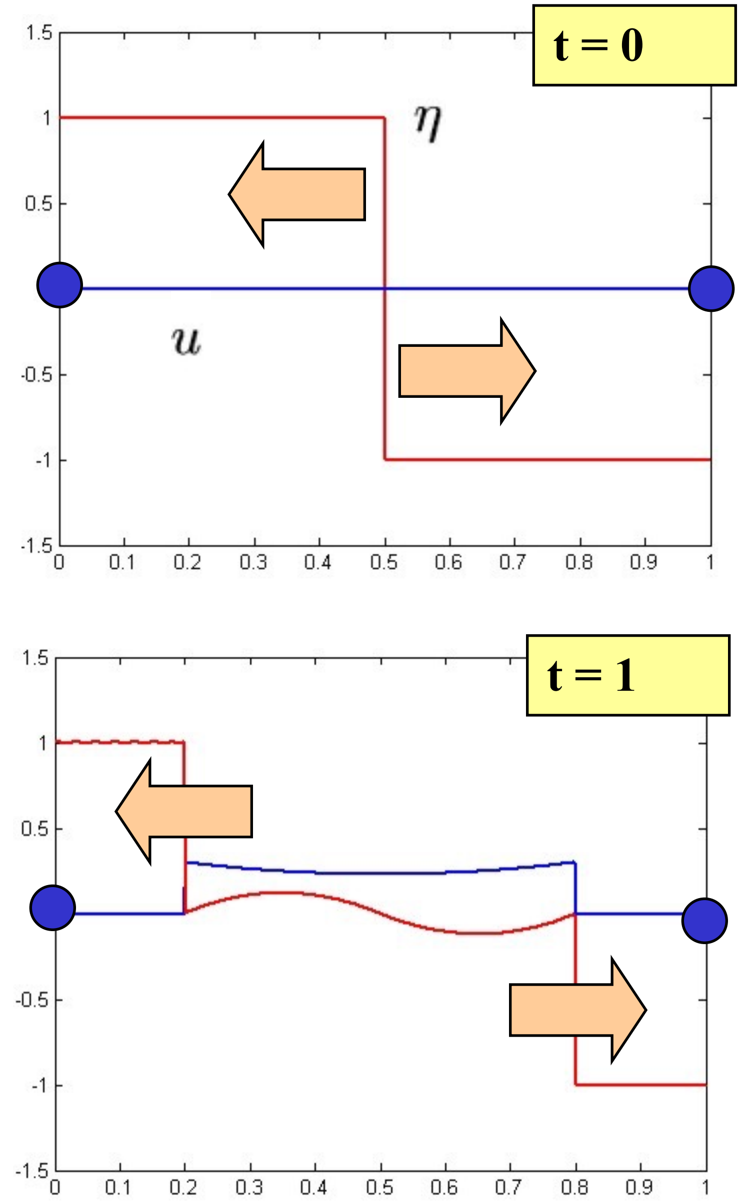
$$\frac{\partial u'}{\partial t'} - v' + \alpha^2 \frac{\partial \eta'}{\partial x'} = 0$$

$$\frac{\partial v'}{\partial t'} + u' = 0$$

$$x' = \frac{x}{L}, \qquad t' = f\,t, \qquad \eta' = \frac{\eta}{\eta_0}, \qquad \boldsymbol{u}' = \frac{H\boldsymbol{u}}{Lf\eta_0},$$

$$\alpha = \boxed{\frac{\sqrt{gH}}{f}}\,\frac{1}{L}$$

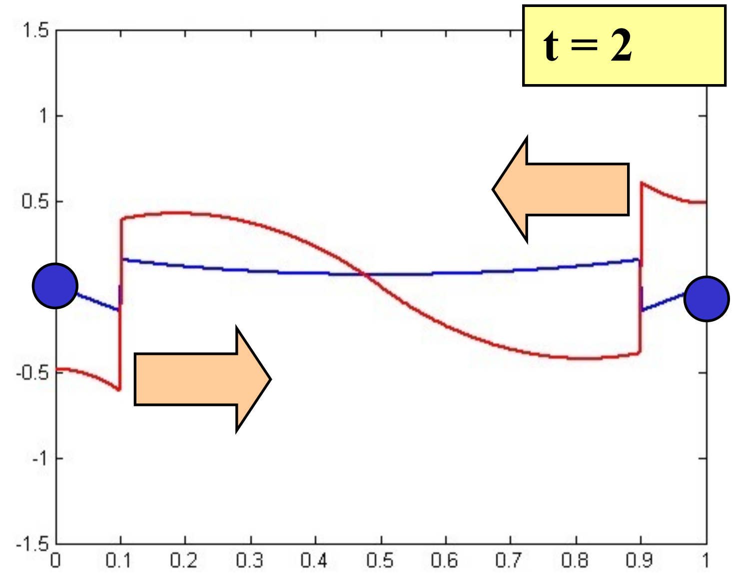
Rossby's radius

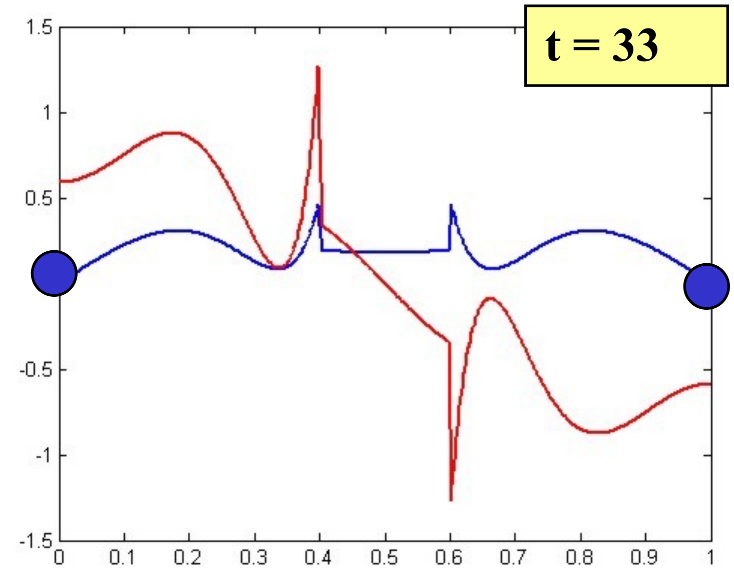


A more and more complex and interesting solution…

$$\alpha = \frac{\sqrt{gH}}{f}\,\frac{1}{L} = \frac{\sqrt{10}}{10} = 0.3162$$

$$\begin{aligned} f &= 10^{-4}\ 1/s \\ L &= 1000\ Km \\ H &= 100\ m \\ g &= 10\ m/s^2 \end{aligned}$$



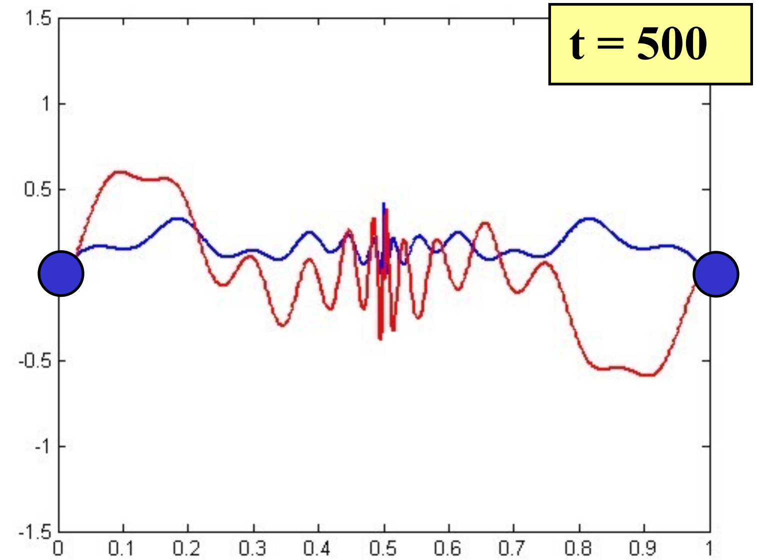


What are the equations ?

$$\frac{\partial \eta}{\partial t} + \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} - v + \alpha^2 \frac{\partial \eta}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + u = 0$$



$$\frac{\partial^2 u}{\partial t^2} - \underbrace{\frac{\partial v}{\partial t}}_{-u} + \alpha^2 \underbrace{\frac{\partial^2 \eta}{\partial t \partial x}}_{-\frac{\partial^2 u}{\partial x^2}} = 0$$

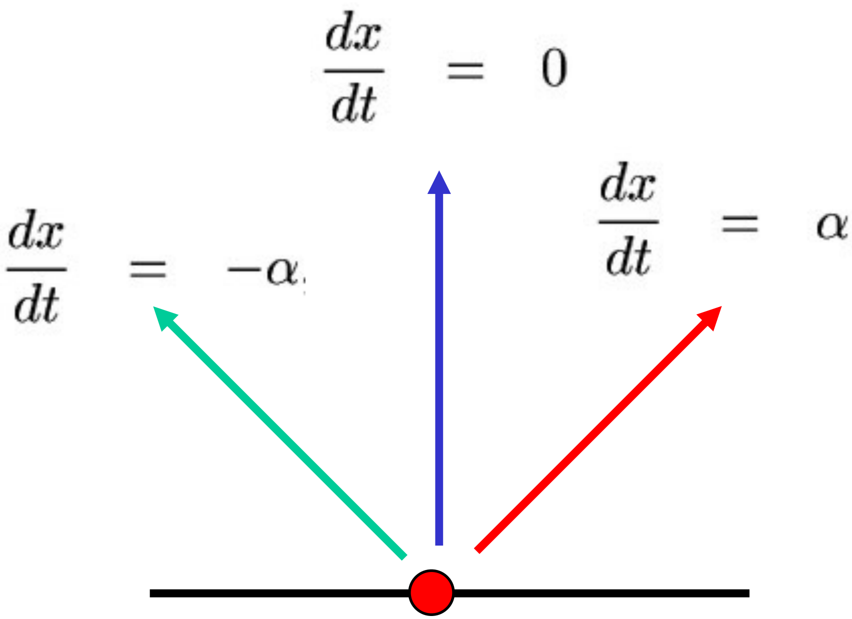
Helmholtz's Equation
Forced Wave Equation

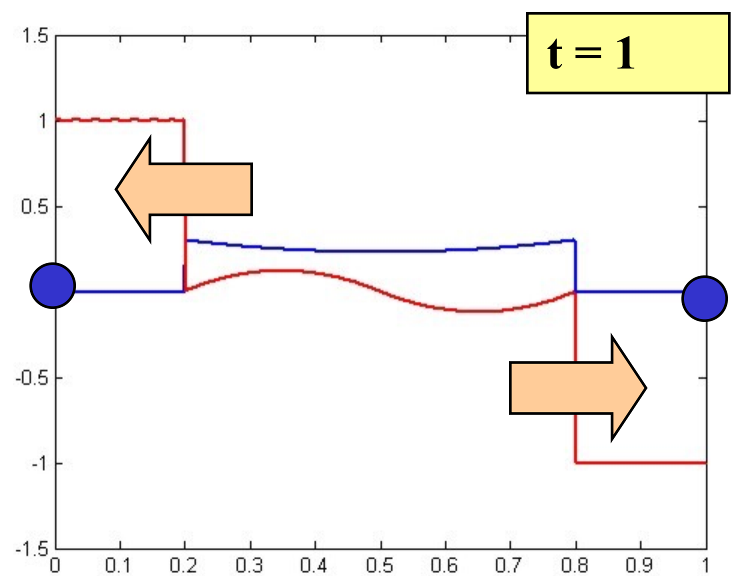
$$\frac{\partial^2 u}{\partial t^2} + u - \alpha^2 \frac{\partial^2 u}{\partial x^2} = 0$$

How does information propagate ?

$$
\begin{aligned}
\frac{d}{dt}(\alpha\eta - u) &= -v && \text{on } \frac{dx}{dt} = -\alpha, \\
\frac{d}{dt}(\alpha\eta + u) &= v && \text{on } \frac{dx}{dt} = \alpha, \\
\frac{dv}{dt} &= -u && \text{on } \frac{dx}{dt} = 0,
\end{aligned}
$$

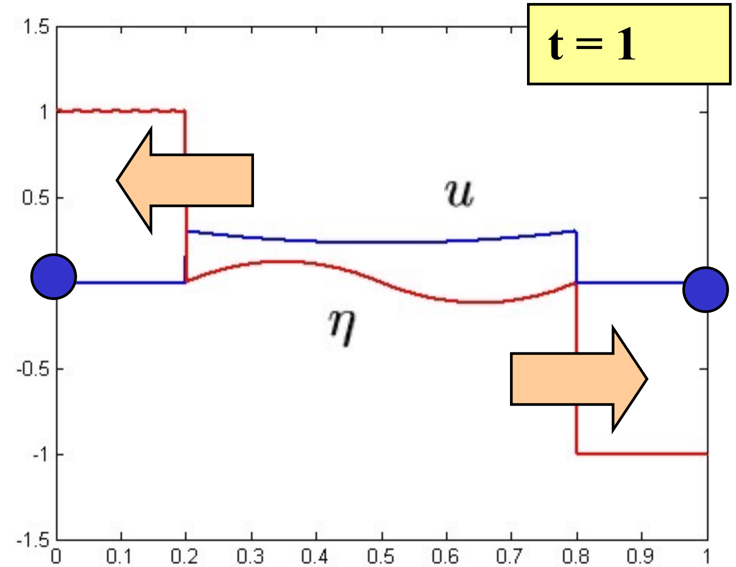
Riemann's Invariants

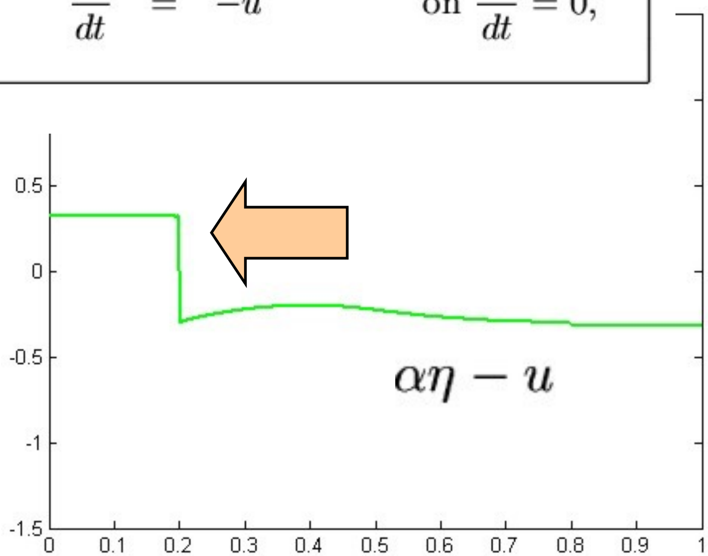


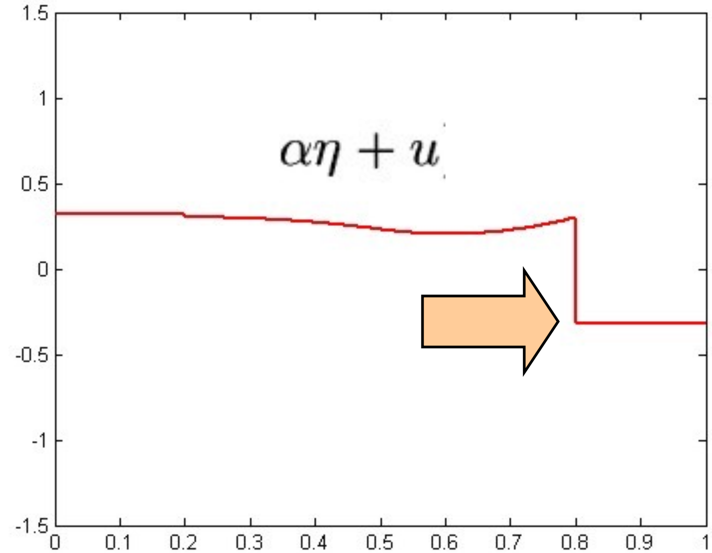


Two distinct waves…

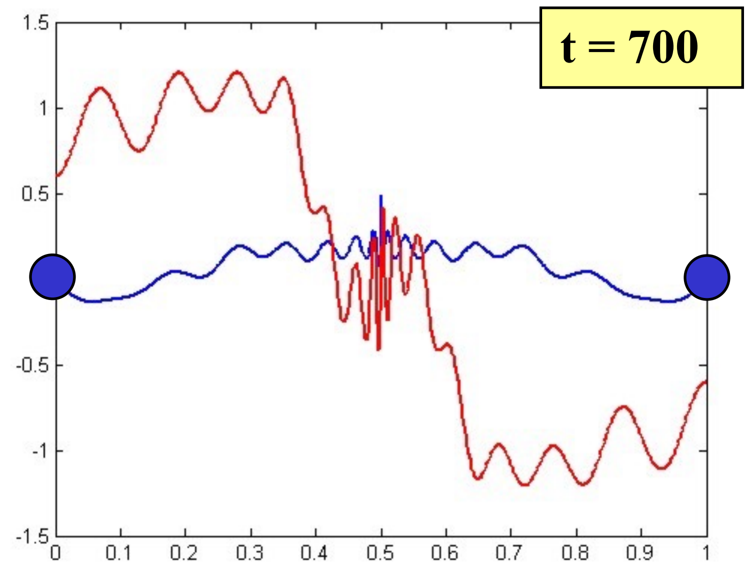
$$
\begin{aligned}
\frac{d}{dt}(\alpha\eta - u) &= -v \qquad \text{on } \frac{dx}{dt} = -\alpha,\\
\frac{d}{dt}(\alpha\eta + u) &= v \qquad \text{on } \frac{dx}{dt} = \alpha,\\
\frac{dv}{dt} &= -u \qquad \text{on } \frac{dx}{dt} = 0,
\end{aligned}
$$







An analytical solution exists !



$$\frac{\partial^2 u}{\partial t^2} + u - \alpha^2 \frac{\partial^2 u}{\partial x^2} = 0$$

Separation of the Classical Equations with the boundary conditions

$$u(x,t) = T(t)f(x)$$

$$\frac{T''}{T} = \alpha^2 \frac{f''}{f} - 1$$

$$u(x,t) = \sum_{i=1}^{\infty} \frac{4\alpha^2(-1)^{i+1}}{\omega_i} \sin(\omega_i t)\sin(k_i x)$$

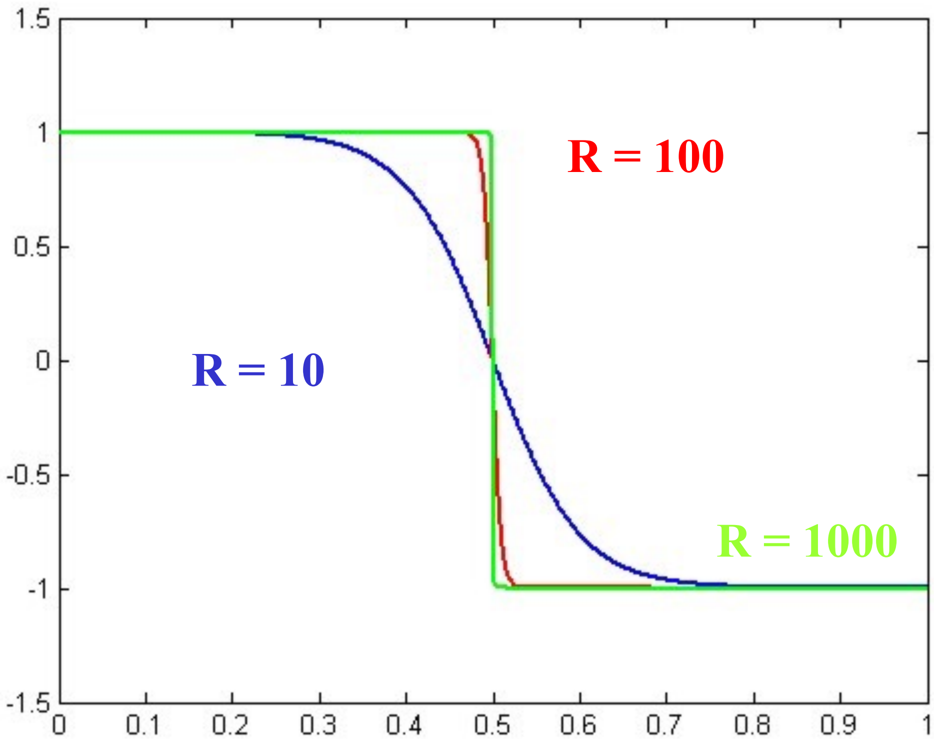
$$k_i = (2i-1)\pi$$

$$\omega_i = \sqrt{1+\alpha^2 k_i^2}$$

Analytical solution for any initial elevation data

$$
\begin{aligned}
u(x,t) &= \sum_{i=1}^{\infty} A_i \, \frac{\alpha^2 k_i}{\omega_i} \, \sin(\omega_i t) \, \sin(k_i x) \\
v(x,t) &= \sum_{i=1}^{\infty} A_i \, \frac{\alpha^2 k_i}{\omega_i^2} \left(\cos(\omega_i t) - 1 \right) \sin(k_i x) \\
\eta(x,t) &= \sum_{i=1}^{\infty} A_i \left(1 - \frac{\alpha^2 k_i^2}{\omega_i^2} \left(1 - \cos(\omega_i t) \right) \right) \cos(k_i x) \\
k_i &= (2i-1)\pi \\
\omega_i &= \sqrt{1 + \alpha^2 k_i^2} \\
A_i &= 2 \int_0^1 \eta(x,0) \cos(k_i x) \, dx
\end{aligned}
$$

A family of initial conditions…



Stiffness factor

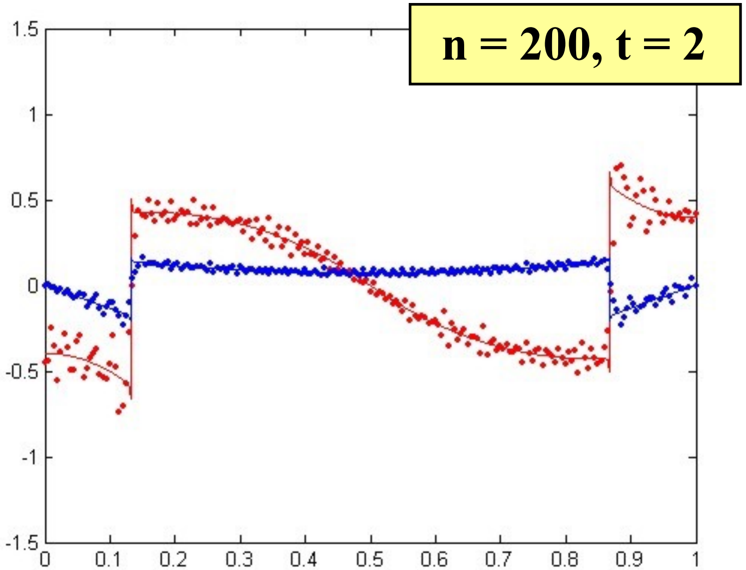
$$\eta(x,0) = -\frac{\tanh\left(R(x-0.5)\right)}{\tanh\left(0.5R\right)}$$

The Continuous Galerkin Method

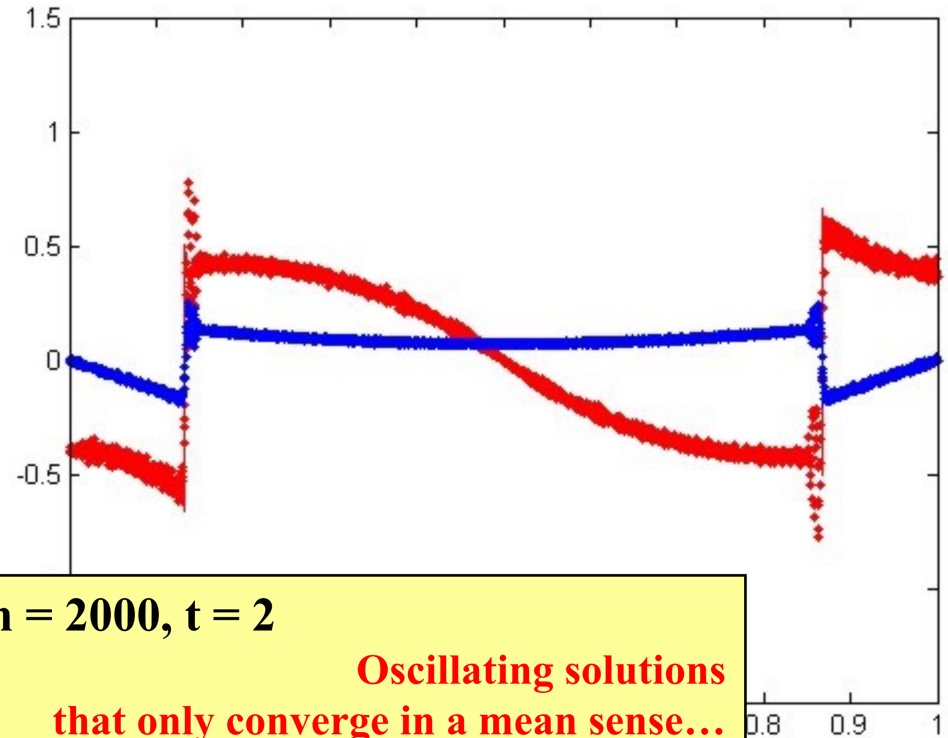
Find $\eta^h \in \mathcal{E}^h$ and $(u^h, v^h) \in \mathcal{U}^h \times \mathcal{U}^h$ such that

$$\begin{aligned}
\int_\Omega \left(\frac{\partial \eta^h}{\partial t} \hat{\eta}^h + \frac{\partial u^h}{\partial x} \hat{\eta}^h \right) dx &= 0 \qquad \forall \hat{\eta}^h \in \mathcal{E}^h, \\
\int_\Omega \left(\frac{\partial u^h}{\partial t} \hat{u}^h + v^h \hat{u}^h + \alpha^2 \frac{\partial \eta^h}{\partial x} \hat{u}^h \right) dx &= 0 \qquad \forall \hat{u}^h \in \mathcal{U}^h, \\
\int_\Omega \left(\frac{\partial v^h}{\partial t} \hat{v}^h - u^h \hat{v}^h \right) dx &= 0 \qquad \forall \hat{v}^h \in \mathcal{U}^h,
\end{aligned}$$

The Continuous Galerkin Method



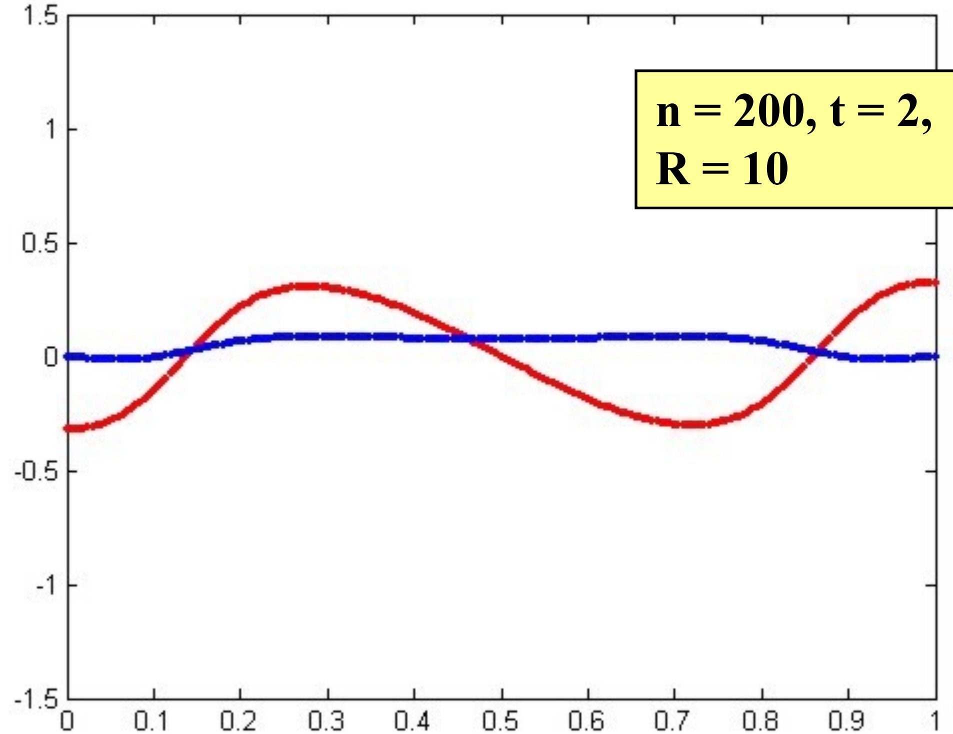
n = 200, t = 2

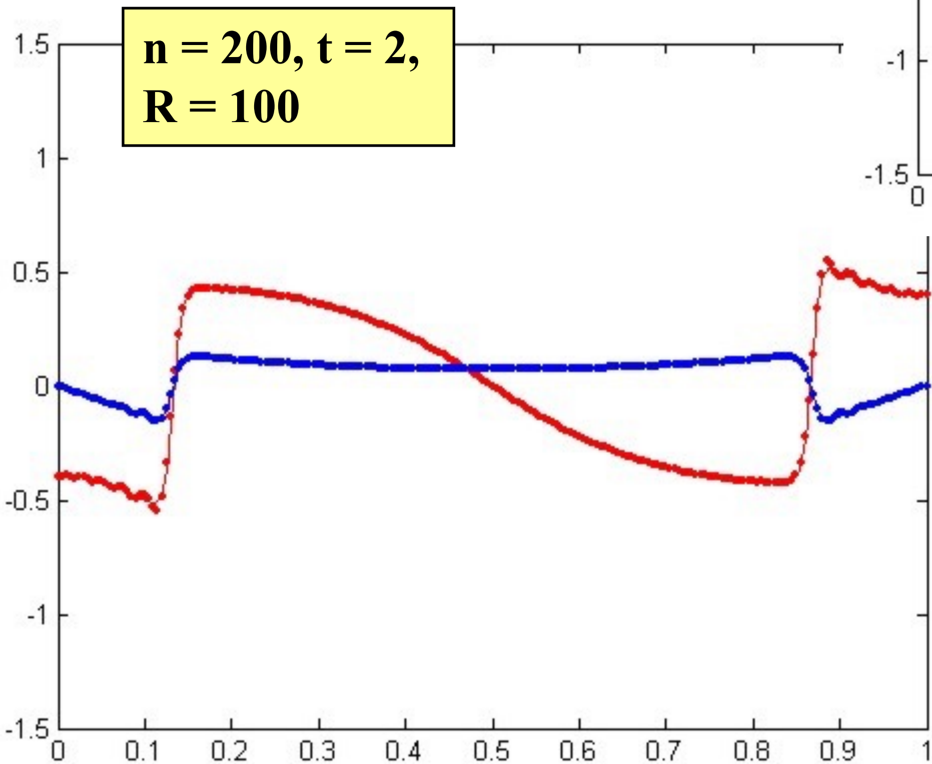


n = 2000, t = 2

Oscillating solutions that only converge in a mean sense…

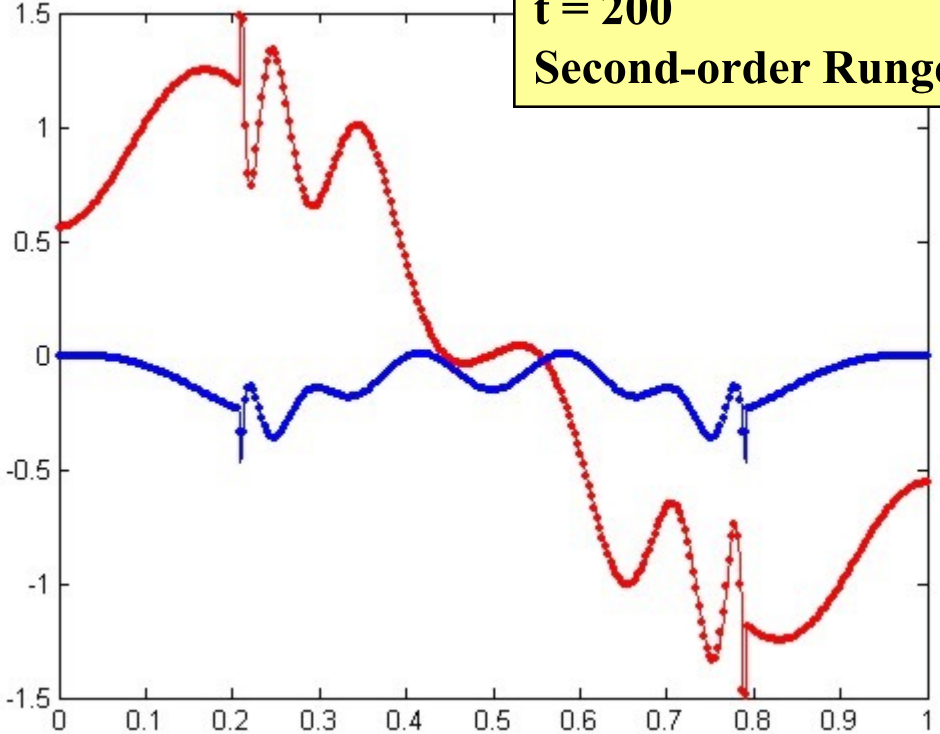
For smooth solutions, it works !



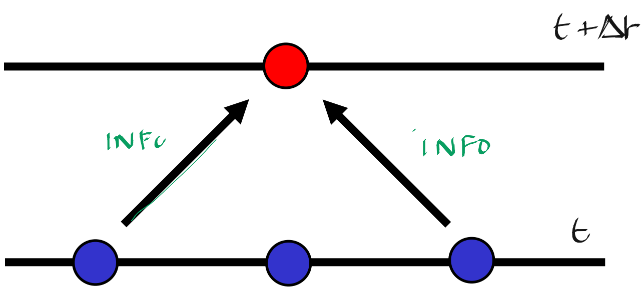


$$\eta(x,0) = -\frac{\tanh\Big(R(x-0.5)\Big)}{\tanh\Big(0.5R\Big)}$$

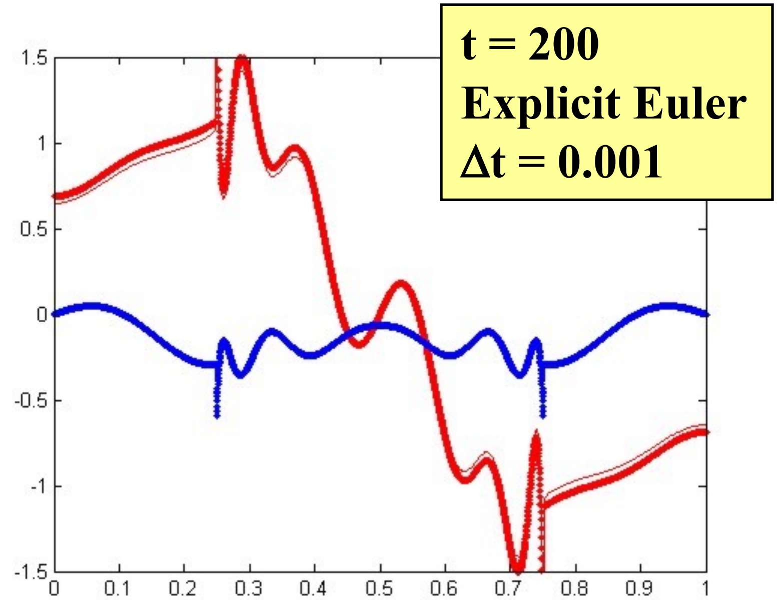
The Optimal Technique : Integrating along characteristics

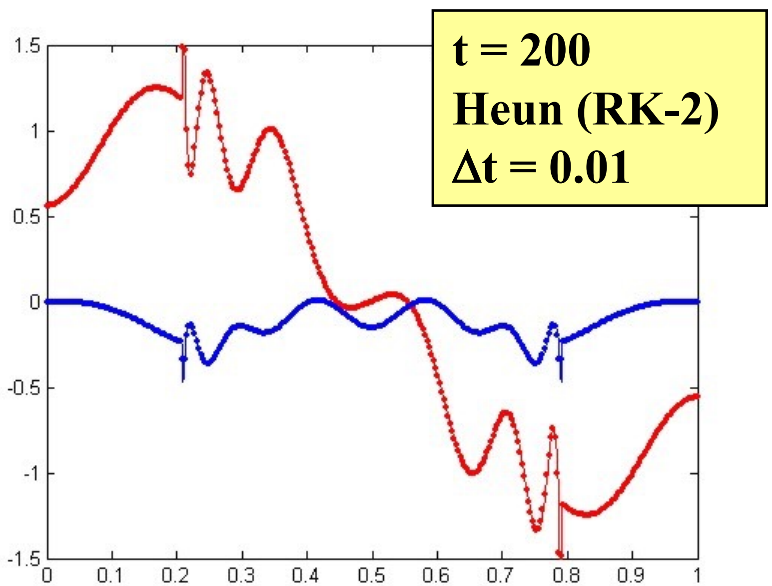


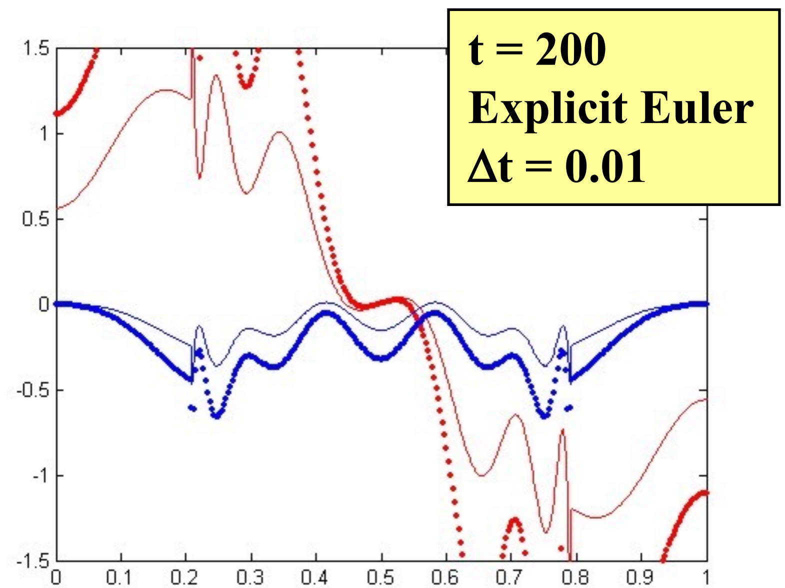
t = 200
Second-order Runge-Kutta



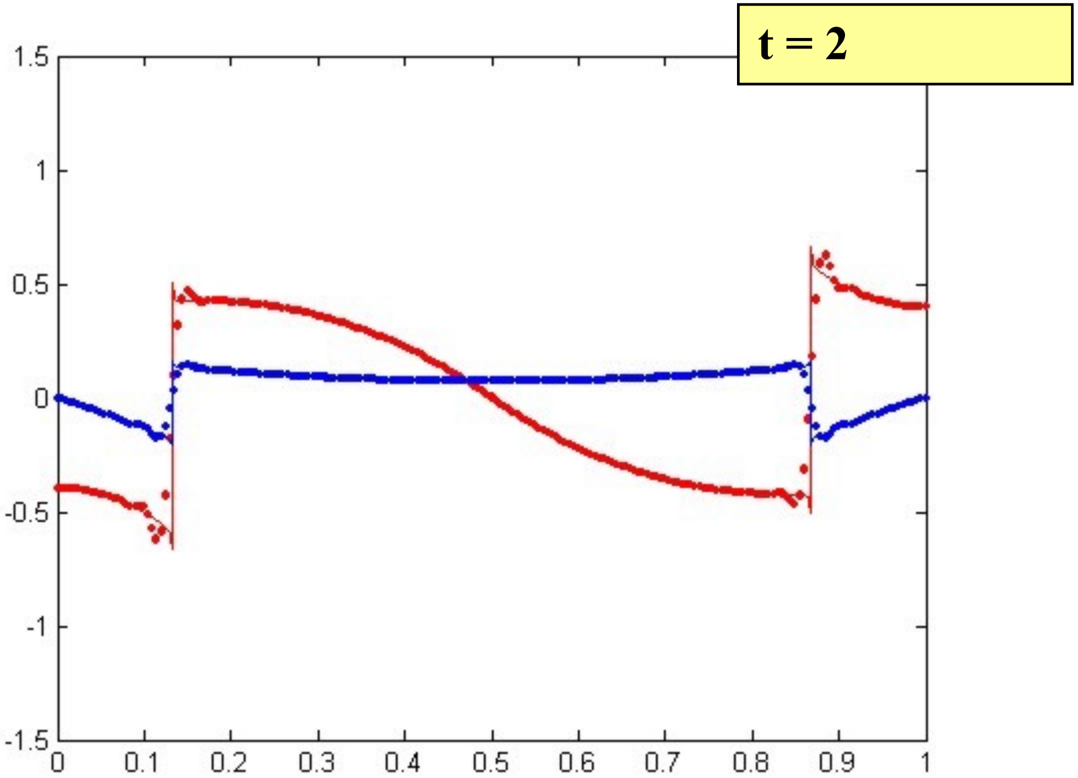
Time integration has to be accurately performed…



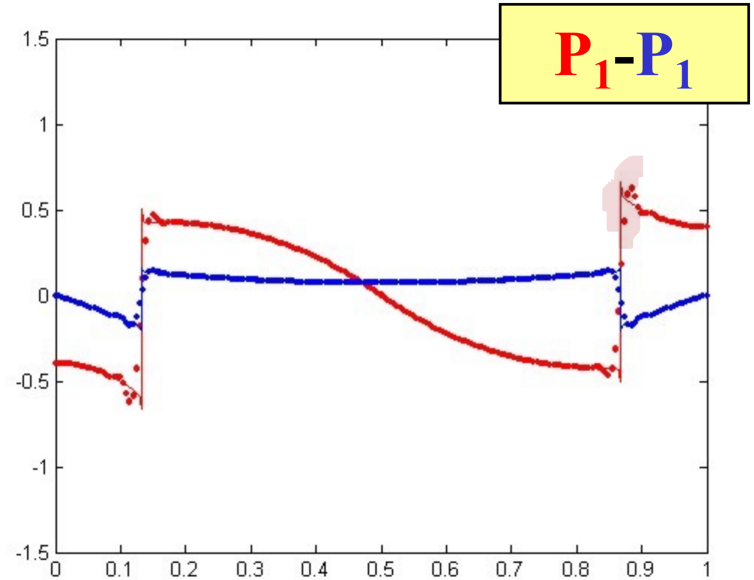


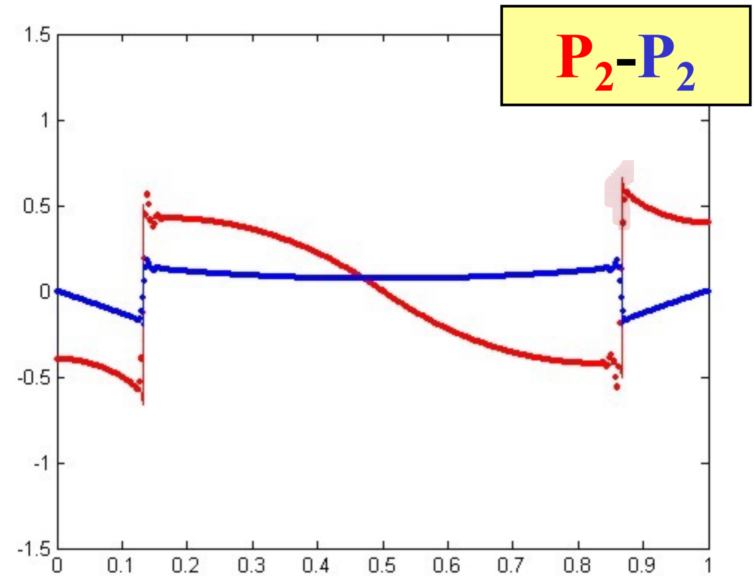


The Discontinuous Riemann-Galerkin Method



Increasing the order of shape functions…





How to estimate the local error ?

The jumps at discontinuities are proportional to the local error

$$j \cong e \cong h^{p+1}$$

The local error are also proportional to h^{p+1} where p is the order of elements and h the characteristic size.

The Discontinuous Galerkin Method provides an efficient and simple error estimator !

Adaptive strategy

$$h_{n+1} = \sqrt[p+1]{\frac{e_t}{j_n}}\, h_n$$

Target error

New requested mesh size field from the error estimator

From this new mesh size field, we can create a new adapted mesh.

How to evaluate the error estimator ?

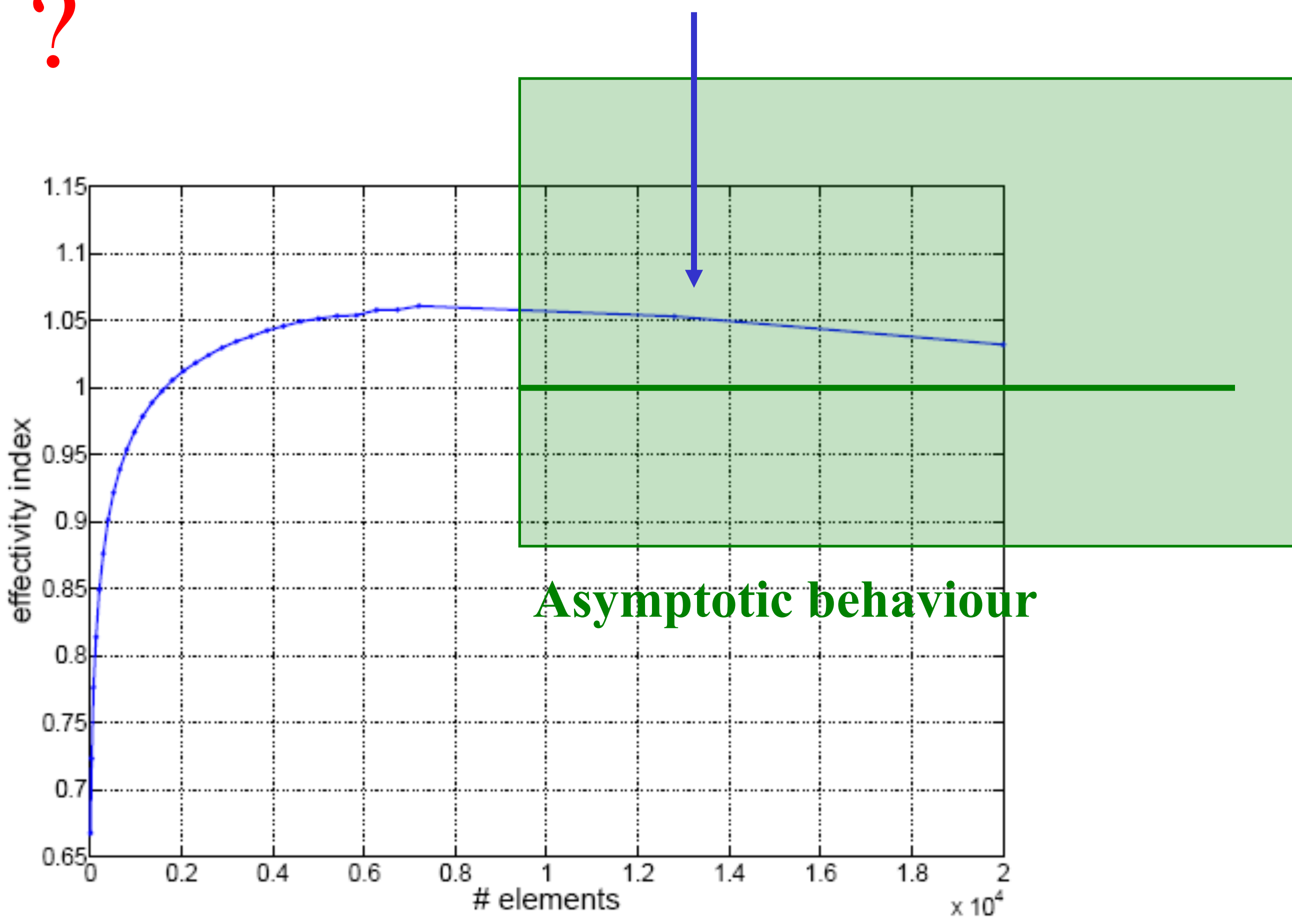
The error estimator slightly overestimates the error, but converges to the true error

Effectivity index

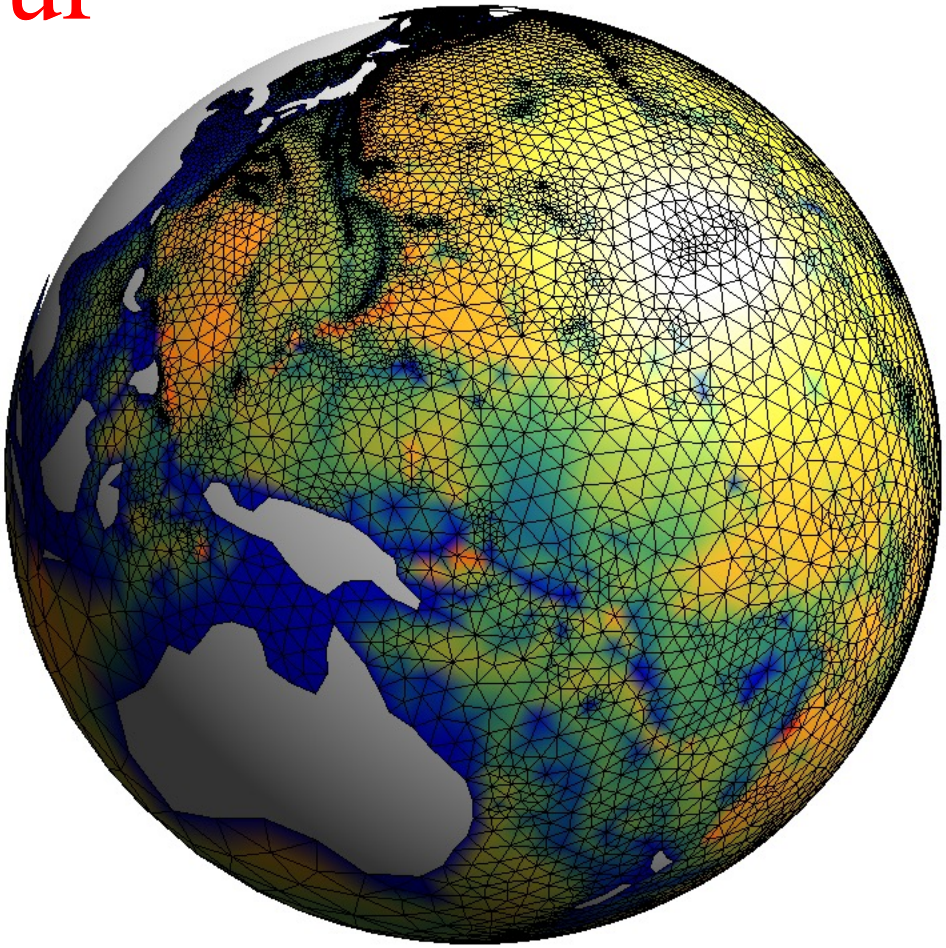
Norm of the error estimator

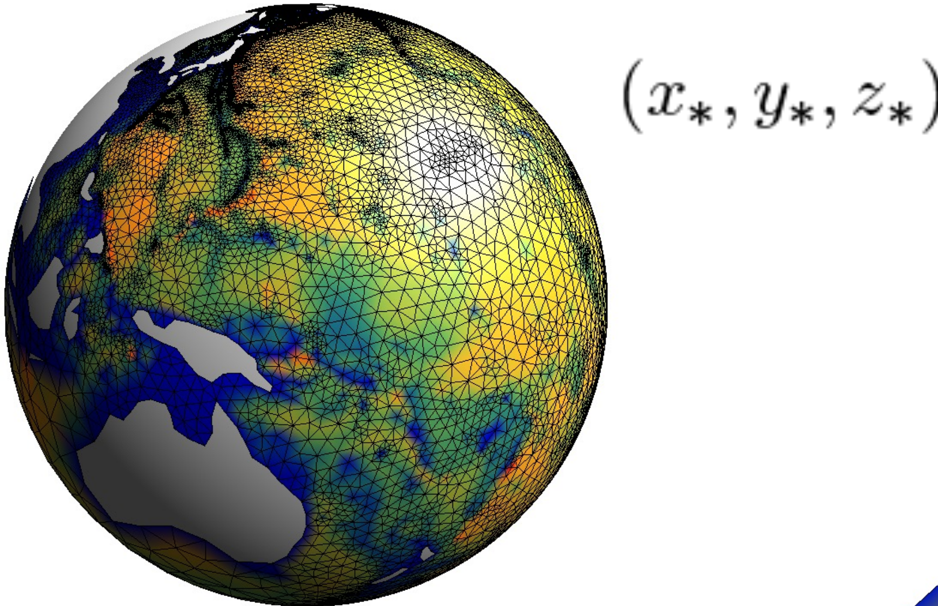
$$\theta = \frac{\|\epsilon\|}{\|e\|}$$

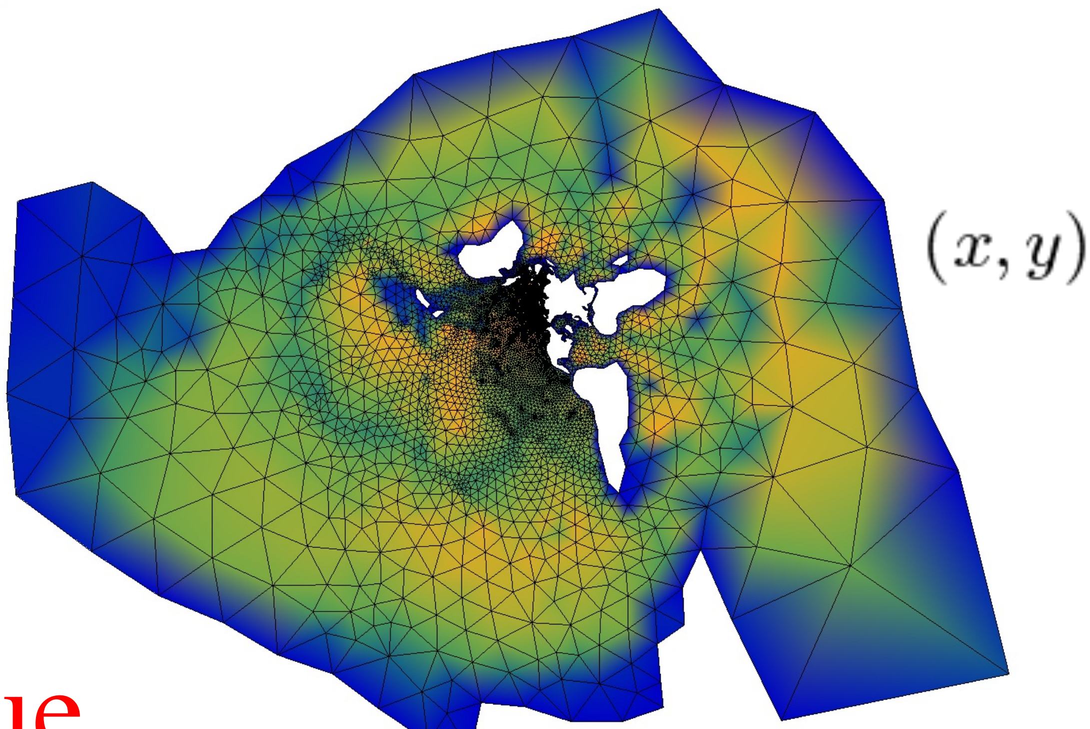
Norm of the exact error



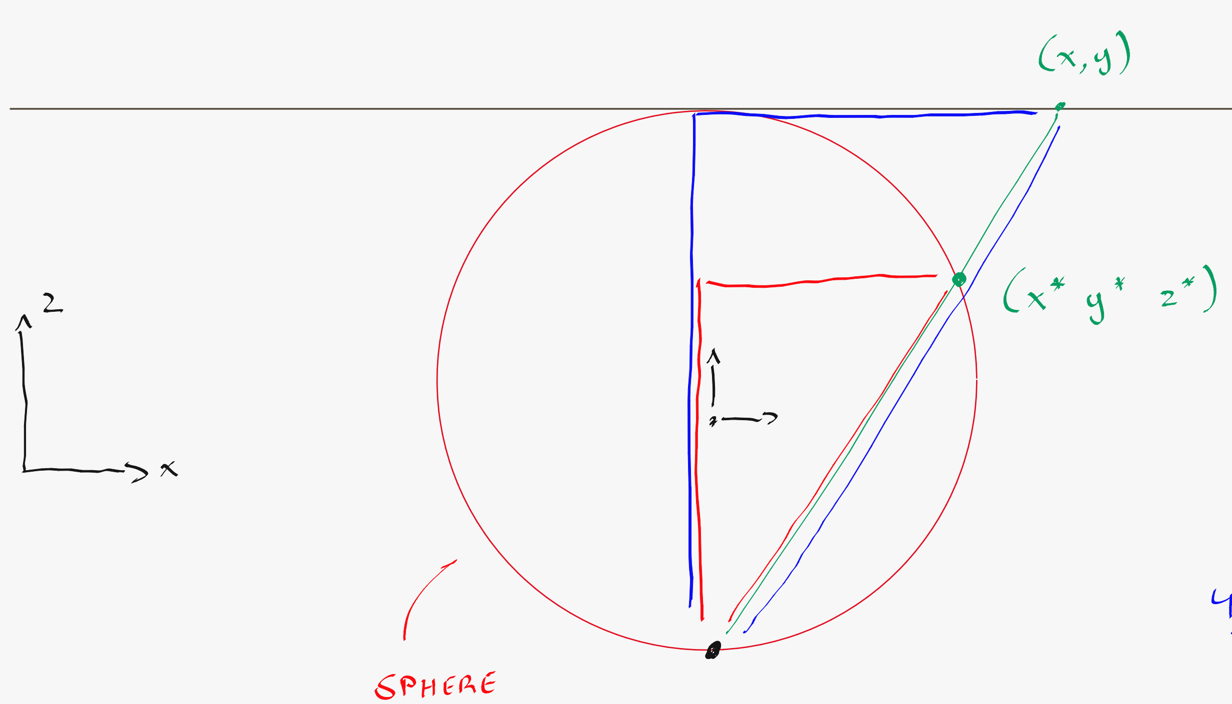
Un tout petit dernier mot sur le projet





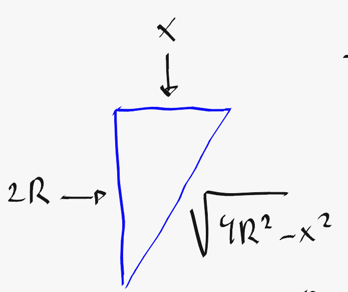


La projection stéréographique



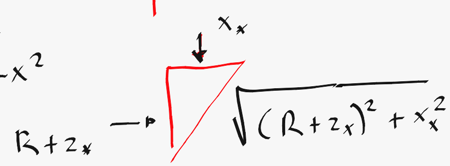
$$\frac{x}{x^*} \qquad \frac{4R^2+x^2}{2R} \cdot \frac{2R}{R+z_*} \cdot \frac{1}{2R}$$

$$\boxed{\frac{x}{x_*} = \frac{2R}{R+z_*}}$$



$$\frac{x^2}{x_*^2} = \frac{4R^2+x^2}{(R+z_*)^2 + x_*^2} = \frac{4R^2+x^2}{R^2+2Rz_* + \underbrace{z_*^2 + x_*^2}_{R^2}} = \frac{4R^2+x^2}{2R(R+z_*)}$$

$$= \frac{4R^2+x^2}{4R^2} \, \frac{x}{x^*}$$



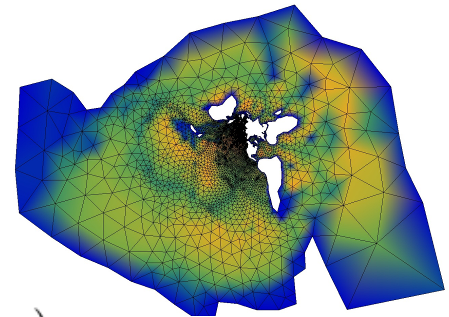
$$\frac{x}{x^*} \qquad \frac{4R^2+x^2}{2R} \quad \overbrace{\frac{2R}{R+z_*}} \quad \frac{1}{2R}$$

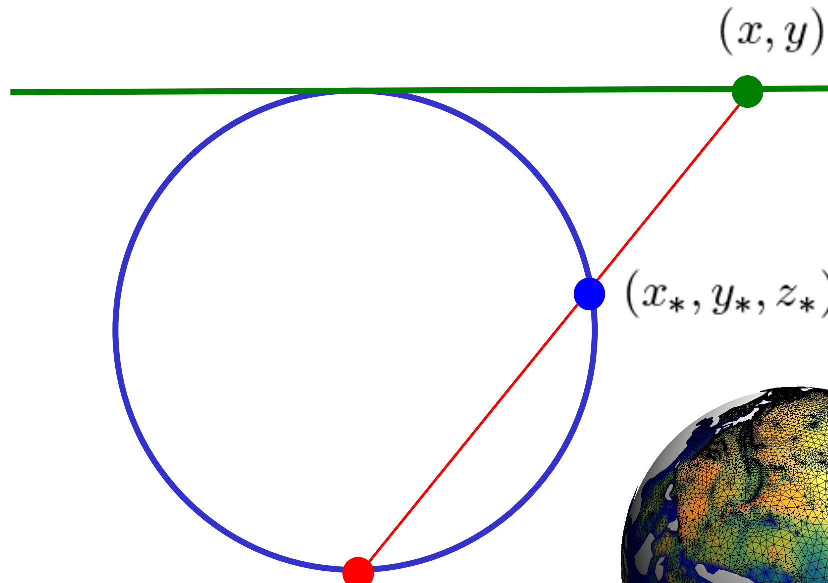
$$\frac{x^2}{x_*^2} = \frac{4R^2+x^2}{(R+z_*)^2+x_*^2} = \frac{4R^2+x^2}{R^2+2Rz_*+\underbrace{z_*^2+x_*^2}_{R^2}} = \frac{\overbrace{4R^2+x^2}}{2R(R+z_*)}$$

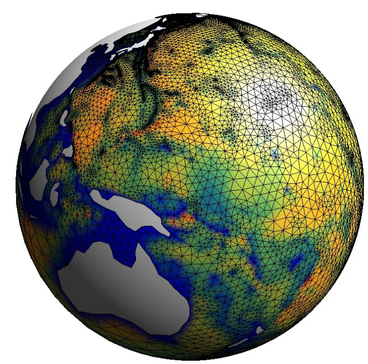
$$= \frac{4R^2+x^2}{4R^2}\,\frac{x}{x^*}$$

$$\frac{x}{x_*} = \frac{4R^2+x^2}{4R^2}$$

$$\boxed{x_* = \frac{4R^2x}{4R^2+x^2}}$$

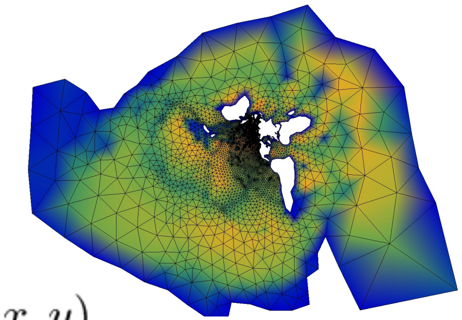


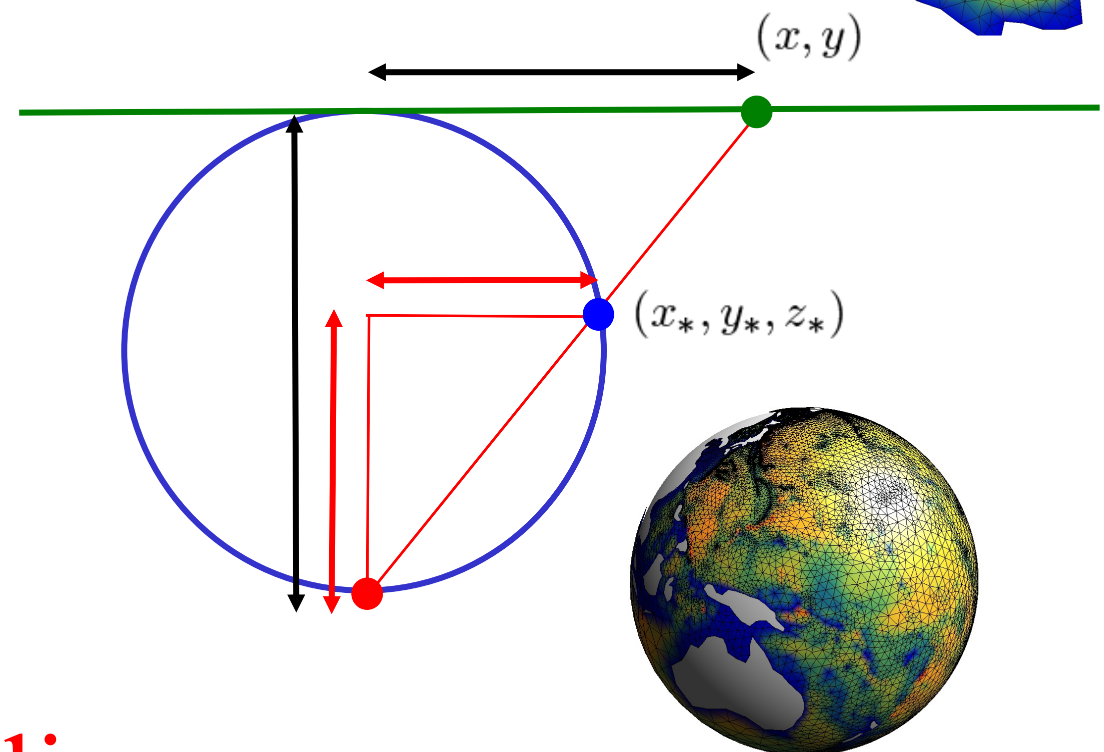




Le pôle sud est le
pôle de notre projection !

$$\frac{x}{x_*} = \frac{2R}{R + z_*}$$



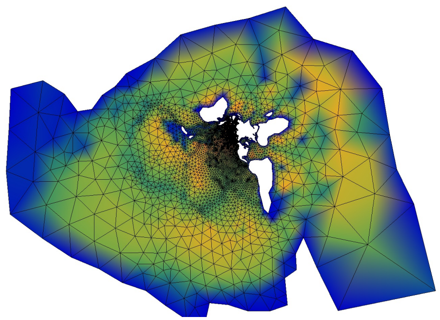


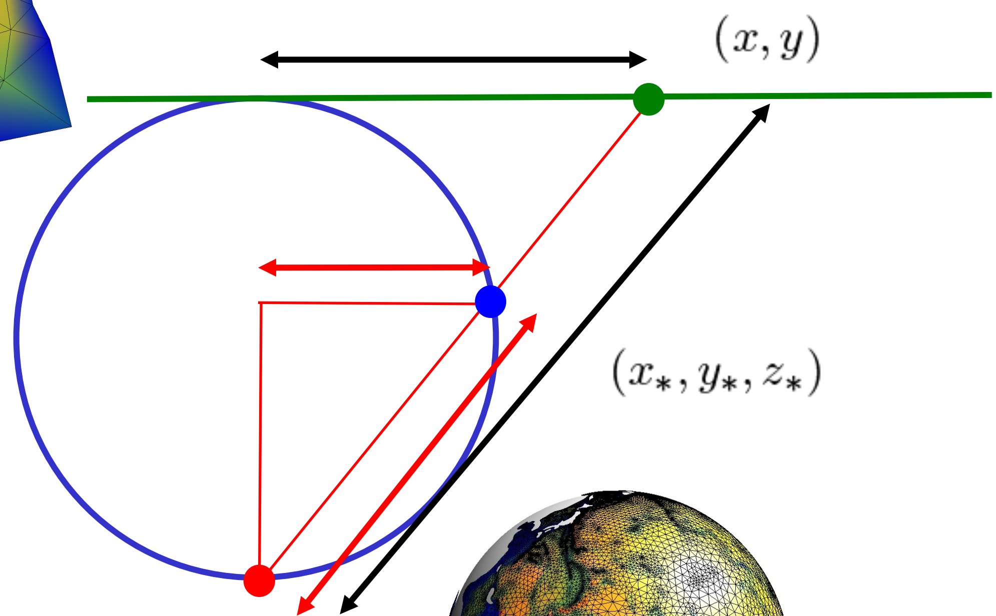
Un fifrelin de géométrie

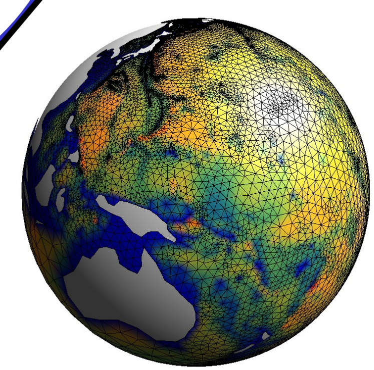
$$\frac{x^2}{x_*^2} = \frac{4R^2+x^2}{(R+z_*)^2+x_*^2} = \frac{4R^2+x^2}{R^2+\underbrace{z_*^2+x_*^2}_{R^2}+2Rz_*} = \frac{1}{2R}\frac{2R}{(R+z_*)}\frac{4R^2+x^2}{2R} = \frac{x}{2Rx_*}\frac{4R^2+x^2}{2R}$$

$$\downarrow$$

$$x_* = \frac{4R^2x}{4r^2+x^2}$$







Un fifrelin d'algèbre

Et ce truc là ?

$$
\left\{
\begin{aligned}
\frac{\partial \eta}{\partial t} + \left(\frac{4R^2 + x^2 + y^2}{4R^2}\right)\frac{\partial}{\partial x}\left(hu\right) + \left(\frac{4R^2 + x^2 + y^2}{4R^2}\right)\frac{\partial}{\partial y}\left(hv\right) &= \boxed{\frac{(xu + yv)h}{2R^2}} \\
\frac{\partial u}{\partial t} + \left(\frac{4R^2 + x^2 + y^2}{4R^2}\right)\frac{\partial}{\partial x}\left(g\eta\right) &= -\gamma u + fv \\
\frac{\partial v}{\partial t} + \boxed{\left(\frac{4R^2 + x^2 + y^2}{4R^2}\right)}\frac{\partial}{\partial y}\left(g\eta\right) &= -\gamma v - fu
\end{aligned}
\right.
$$

Rapport des longueurs

Les équations dans le plan stéréographique