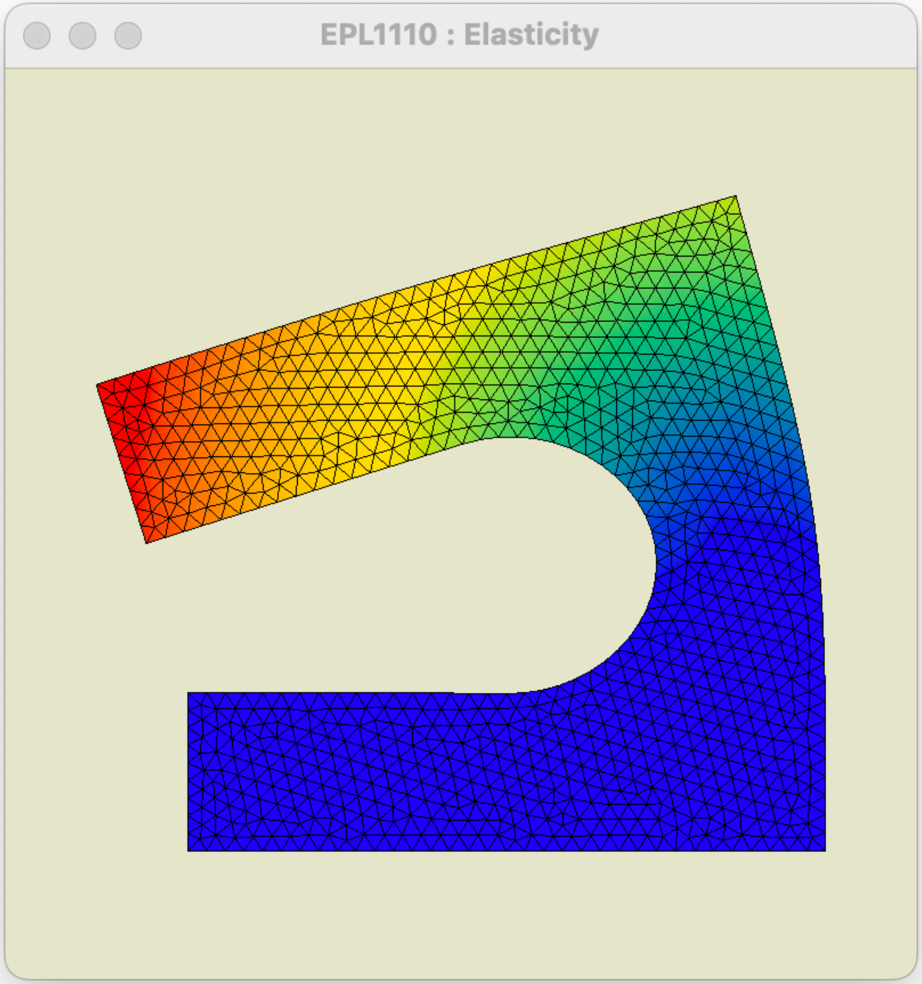
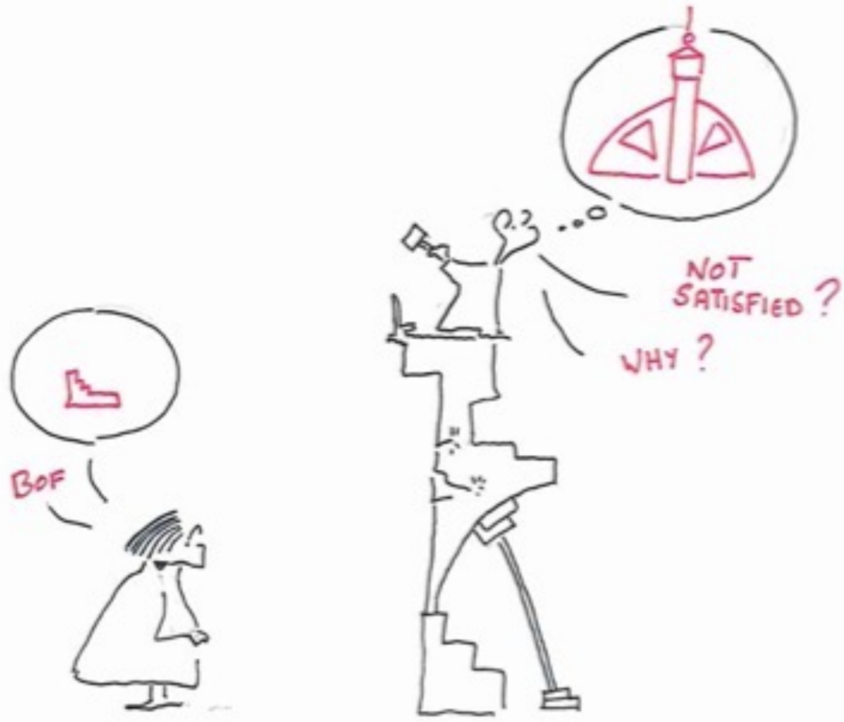
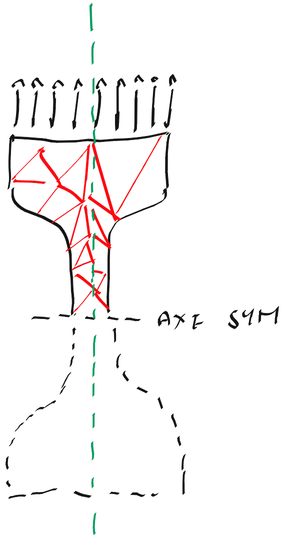
Projet 2022-23





J

Questions ?

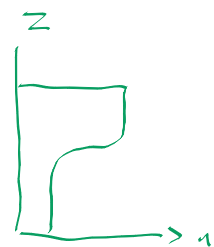


Ecrire un code informatique efficace pour l'élasticité linéaire plane

Tensions planes et déformations planes
Triangles linéaires
Quads bilinéaires
Problèmes axisymétriques
Conditions essentielles en xy et en normale/tangentielle
Conditions naturelles en xy et en normale/tangentielle

50%

2 parties dans le projet !



Définir un problème original !
Le résoudre avec votre code !
Analyser le résultat !

50%

Projet 2022-23

INPUT

mesh.txt

```
Number of nodes 335
      0 :  0.0000000e+00  1.0000000e+00
      1 :  0.0000000e+00  0.0000000e+00
      2 :  1.0000000e+00  1.0000000e+00
      3 :  1.0000000e+00  7.5000000e-01
      4 :  5.0000000e-01  7.5000000e-01
      5 :  5.0000000e-01  2.5000000e-01
      6 :  1.0000000e+00  2.5000000e-01
      7 :  1.0000000e+00  0.0000000e+00
      8 :  0.0000000e+00  9.5000000e-01
      9 :  0.0000000e+00  9.0000000e-01
```

problem.txt

```
Type of problem : planar strains
Young modulus   :  2.1100000e+11
Poisson ratio   :  3.0000000e-01
Mass density    :  7.8500000e+03
Gravity         :  9.8100000e+00
```

myFem

OUTPUT

result.txt

```
Elastic deformation 335
      0 :  0.0002330e-03  1.0000000e+00
      1 :  0.0000000e+00  0.0000000e+00
```

Si !

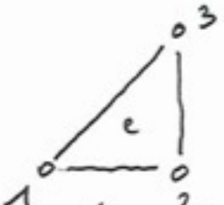
```
for (iElem = 0; iElem < theMesh->nElem; iElem++) {
    for (j=0; j < nLocal; j++) {
        map[j]  = theMesh->elem[iElem*nLocal+j];
        mapX[j] = 2*map[j];
        mapY[j] = 2*map[j] + 1;
        x[j]    = theNodes->X[map[j]];
        y[j]    = theNodes->Y[map[j]];}

    for (iInteg=0; iInteg < theRule->n; iInteg++) {
        double xsi    = theRule->xsi[iInteg];
        double eta    = theRule->eta[iInteg];
        double weight = theRule->weight[iInteg];
        femDiscretePhi2(theSpace,xsi,eta,phi);
        femDiscreteDphi2(theSpace,xsi,eta,dphidxsi,dphideta);

        double dxdxsi = 0.0;
        double dxdeta = 0.0;
        double dydxsi = 0.0;
        double dydeta = 0.0;
        for (i = 0; i < theSpace->n; i++) {
            dxdxsi += x[i]*dphidxsi[i];
            dxdeta += x[i]*dphideta[i];
            dydxsi += y[i]*dphidxsi[i];
            dydeta += y[i]*dphideta[i]; }
        double jac = fabs(dxdxsi * dydeta - dxdeta * dydxsi);

        for (i = 0; i < theSpace->n; i++) {
            dphidx[i] = (dphidxsi[i] * dydeta - dphideta[i] * dydxsi) / jac;
            dphidy[i] = (dphideta[i] * dxdxsi - dphidxsi[i] * dxdeta) / jac; }
        for (i = 0; i < theSpace->n; i++) {
            for(j = 0; j < theSpace->n; j++) {
                A[mapX[i]][mapX[j]] += (dphidx[i] * a * dphidx[j] +
                                        dphidy[i] * c * dphidy[j]) * jac * weight;
                A[mapX[i]][mapY[j]] += (dphidx[i] * b * dphidy[j] +
                                        dphidy[i] * c * dphidx[j]) * jac * weight;
                A[mapY[i]][mapX[j]] += (dphidy[i] * b * dphidx[j] +
                                        dphidx[i] * c * dphidy[j]) * jac * weight;
                A[mapY[i]][mapY[j]] += (dphidy[i] * a * dphidy[j] +
                                        dphidx[i] * c * dphidx[j]) * jac * weight; }}
         for (i = 0; i < theSpace->n; i++) {
            B[mapY[i]] -= phi[i] * g * rho * jac * weight; }}}

int *theConstrainedNodes = theProblem->constrainedNodes;
for (int i=0; i < theSystem->size; i++) {
    if (theConstrainedNodes[i] != -1) {
        double value = theProblem->conditions[theConstrainedNodes[i]]->value;
        femFullSystemConstrain(theSystem,i,value); }}

return femFullSystemEliminate(theSystem);
```

COMPUTING A LOCAL ELASTICITY MATRIX

$$\begin{bmatrix} A\langle \tau_{i,x}\,\tau_{j,x}\rangle + C\langle \tau_{i,y}\,\tau_{j,y}\rangle & B\langle \tau_{i,x}\,\tau_{j,y}\rangle + C\langle \tau_{i,y}\,\tau_{j,x}\rangle \\ B\langle \tau_{i,y}\,\tau_{j,x}\rangle + C\langle \tau_{i,x}\,\tau_{j,y}\rangle & A\langle \tau_{i,y}\,\tau_{j,y}\rangle + C\langle \tau_{i,x}\,\tau_{j,x}\rangle \end{bmatrix}$$

$\underline{\underline{A}}^e_{11}$

$A_{xx11} = A$
$A_{yy11} = C$
$A_{xy11} = 0$
$A_{yx11} = 0$



	$\tau_{i,x}$	$\tau_{i,y}$
1	-1	0
2	1	-1
3	0	1

$$\underline{\underline{A}}^e_{y} = \begin{bmatrix} A & 0 & -A & B & 0 & -B \\ & C & C & -C & -C & 0 \\ & & A+C & -B-C & -C & B \\ & & & A+C & C & -A \\ & & & & C & 0 \\ & & & & & A \end{bmatrix} \frac{1}{2}$$

Que peut-on utiliser pour écrire le projet ?

FEM.C
FEM.H

AVEC TOUT CE QUI EST DISPONIBLE DANS LES DEVOIRS PRECEDENTS

C'EST AUTORISÉ

CE QUI EST INTERDIT !

TOUT USAGE DE LIBRAIRIES EXTERNES SAUF BLAS

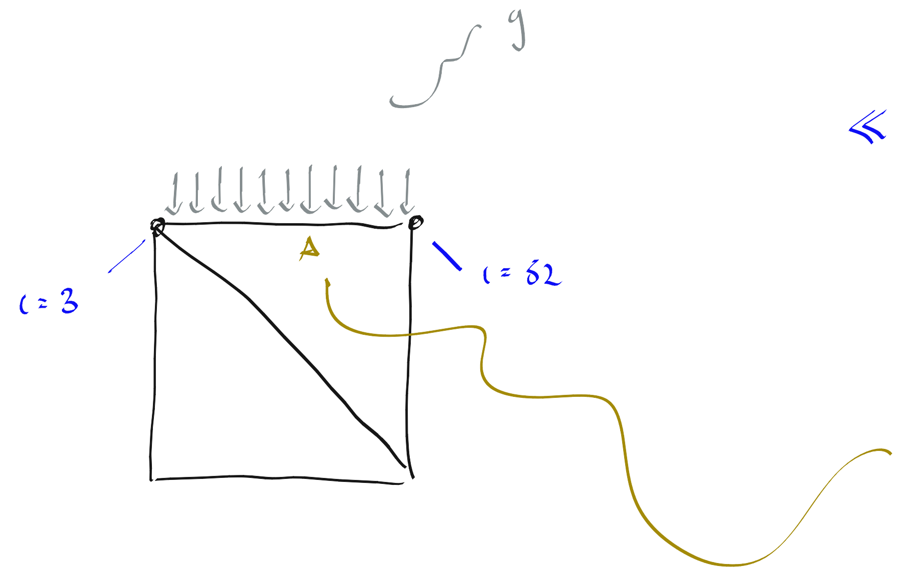
CE QUI N'EST PAS DEMANDÉ !

- UN CODE //

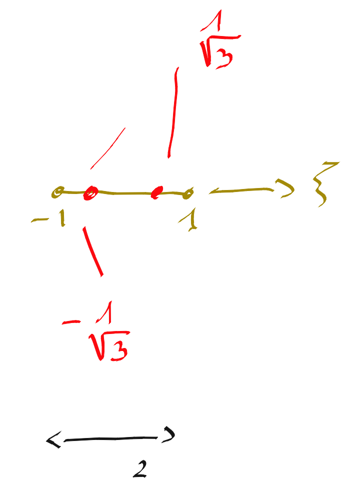
BONUS

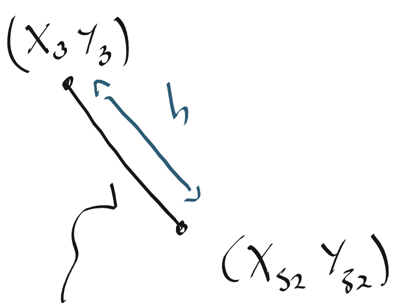
- IMPLEMENTER LA RENUMEROTATION DES NOEUDS
- LE SOLVEUR FRONTAL
- UN SOLVEUR SKYLINE

Conditions de Neumann !



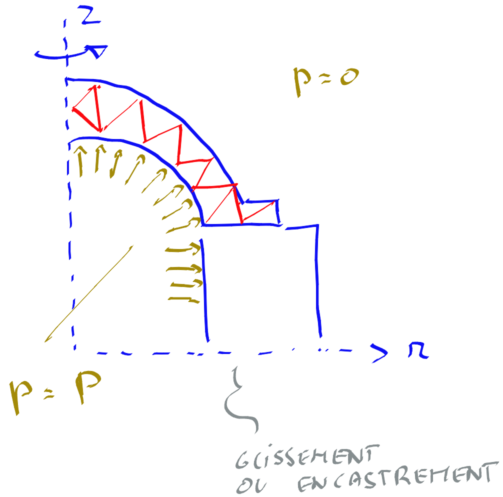
« $g_x \tau_i$ »

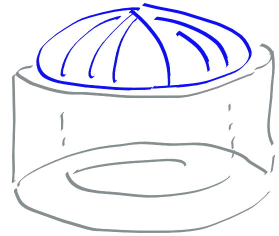




$$\sqrt{(X_{62}-X_3)^2+(Y_{62}-Y_3)^2}$$

En normale et tangent !



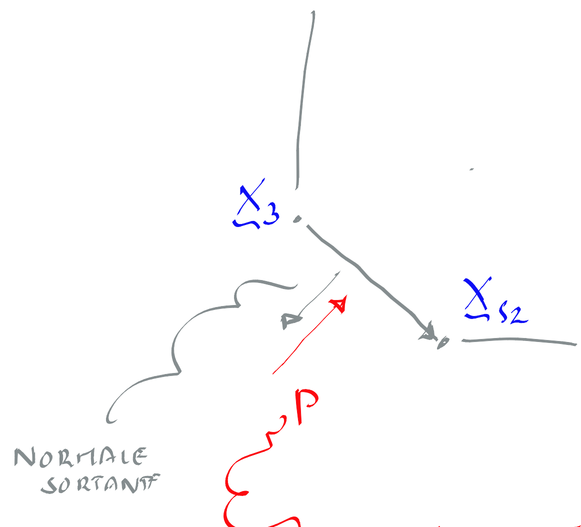


$$\begin{bmatrix} n_x \\ n_y \end{bmatrix} \qquad \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

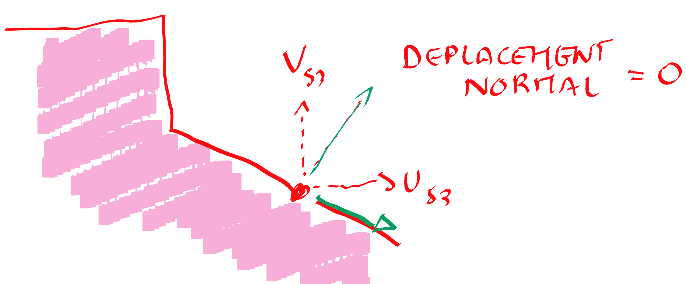
$$\underset{\sim}{n} \cdot \underset{\sim}{t} = 0$$

$$n_x = -t_y$$
$$n_y = t_x$$

$$\underset{\sim}{t} = \begin{bmatrix} X_{s2} - X_3 \\ Y_{s2} - Y_3 \end{bmatrix}$$



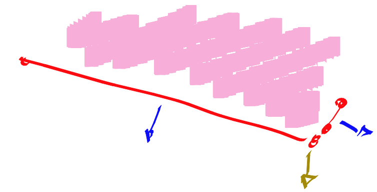
$$\begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \end{bmatrix} p$$

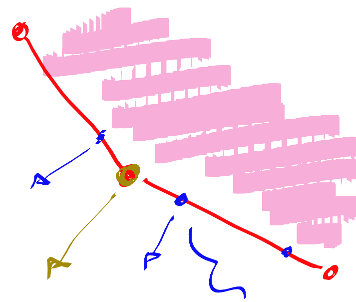


$$U_{s3} + V_{s3} = 0$$

$$\begin{bmatrix} \times & \times & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times & \times & \times \end{bmatrix} \begin{bmatrix} U_{s3} \\ V_{s3} \end{bmatrix} = \begin{bmatrix} \times \\ \times \end{bmatrix} \begin{matrix} \} \; U_t \\ \} \; U_n \end{matrix}$$

En normale et tangent !



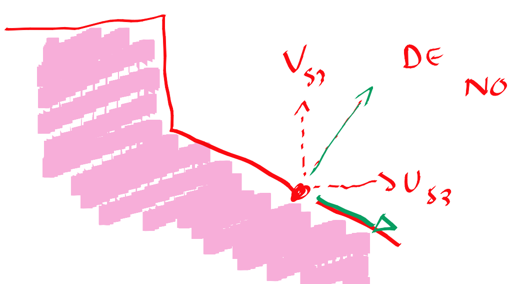


COMMENT ON TROUVE LA NORMALE ?

POINT D'INTEGRATION :-)

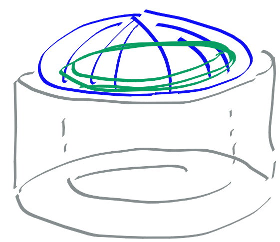
FAIRE UNE COMBINAISON LINEAIRE (MOYENNE ?) DES NORMALES DES VOISINS

UNE MOYENNE PROPORTIONNELLE A LA LONGUEUR DES ELEMENTS !

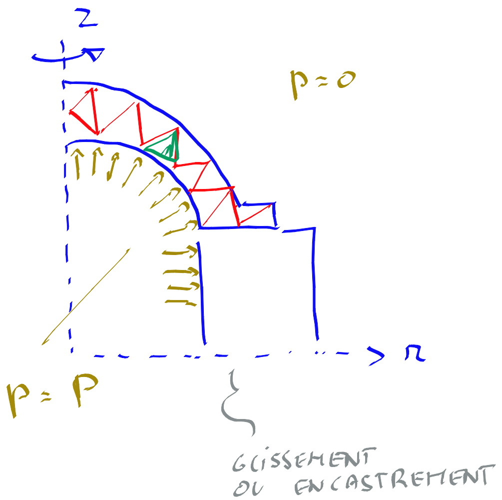


$U_{S3} + V_{S3} = 0$

Et l'axisymétrique ?



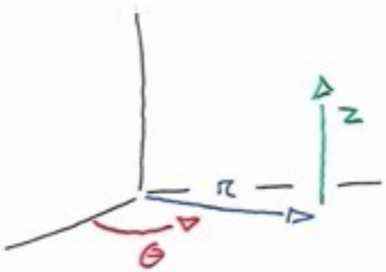
$$\frac{\partial}{\partial z} = 0$$



AXISYMMETRIC PROBLEMS

AXISYMMETRY!

$$u_\theta = 0 \qquad \frac{\partial}{\partial\theta} = 0$$



$$\underset{\sim}{u} = \begin{bmatrix} u_r \\ u_\theta \\ u_z \end{bmatrix} = \begin{bmatrix} v(r,z) \\ 0 \\ v(r,z) \end{bmatrix}$$

A, B, C OF PLANAR DEFORMATIONS !

$$\underset{\approx}{\varepsilon} = \begin{bmatrix} u_{r,r} & 0 & (u_{r,z}+u_{z,r})/2 \\ & u_r/r & 0 \\ & & u_{z,z} \end{bmatrix}$$

$$\underset{\approx}{\sigma} = \begin{bmatrix} A\varepsilon_{rr} + B(\varepsilon_{\theta\theta}+\varepsilon_{zz}) & 0 & 2C\varepsilon_{rz} \\ & A\varepsilon_{\theta\theta} + B(\varepsilon_{rr}+\varepsilon_{zz}) & 0 \\ & & A\varepsilon_{zz} + B(\varepsilon_{rr}+\varepsilon_{\theta\theta}) \end{bmatrix}$$

$$\underline{\underline{\varepsilon}}\left(\begin{bmatrix}\tau_i & 0 & 0\end{bmatrix}\right) = \begin{bmatrix}\tau_{i,r} & 0 & \tau_{i,z}/2 \\ & \tau_i/r & 0 \\ & & 0\end{bmatrix}$$

$$\underline{\underline{\varepsilon}}\left(\begin{bmatrix}0 & 0 & \tau_i\end{bmatrix}\right) = \begin{bmatrix}0 & 0 & \tau_{i,r}/2 \\ & 0 & 0 \\ & & \tau_{i,z}\end{bmatrix}$$

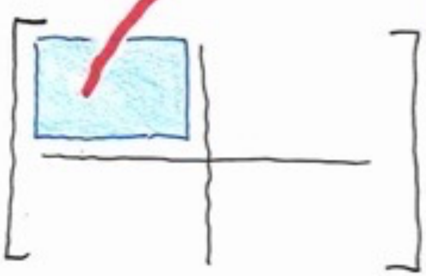
$$2\pi \underbrace{\left[\begin{array}{c|c} \left\langle r\, \underline{\underline{\varepsilon}}\begin{pmatrix}\tau_i\\0\\0\end{pmatrix} : \underline{\underline{\sigma}}\begin{pmatrix}\tau_j\\0\\0\end{pmatrix}\right\rangle & \left\langle r\, \underline{\underline{\varepsilon}}\begin{pmatrix}\tau_i\\0\\0\end{pmatrix} : \underline{\underline{\sigma}}\begin{pmatrix}0\\0\\\tau_j\end{pmatrix}\right\rangle \\ \hline \left\langle r\, \underline{\underline{\varepsilon}}\begin{pmatrix}0\\0\\\tau_i\end{pmatrix} : \underline{\underline{\sigma}}\begin{pmatrix}\tau_j\\0\\0\end{pmatrix}\right\rangle & \left\langle r\, \underline{\underline{\varepsilon}}\begin{pmatrix}0\\0\\\tau_i\end{pmatrix} : \underline{\underline{\sigma}}\begin{pmatrix}0\\0\\\tau_j\end{pmatrix}\right\rangle \end{array}\right]}_{\underline{\underline{A}}_{ij}}$$

$\langle\ \rangle$

$\langle 2\pi r\ \ \rangle = \langle\ \ \rangle_{Ax_i}$?

$$\underline{\underline{\varepsilon}}\begin{pmatrix}\tau_i\\0\\0\end{pmatrix} = \begin{bmatrix}\tau_{i,r} & 0 & \tau_{i,z}/2\\ & \tau_i/r & 0\\ & & 0\end{bmatrix}$$

$$\underline{\underline{\sigma}}\begin{pmatrix}\tau_j\\0\\0\end{pmatrix} = \begin{bmatrix}A\tau_{j,r} + B\tau_j/r & 0 & C\tau_{j,z}\\ & A\tau_j/r + B\tau_{j,r} & 0\\ & & B\tau_j/r + B\tau_{j,r}\end{bmatrix}$$



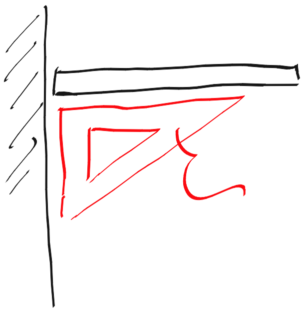
$$\boxed{\underline{\underline{\varepsilon}}\begin{pmatrix}\tau_i\\0\\0\end{pmatrix} : \underline{\underline{\sigma}}\begin{pmatrix}\tau_j\\0\\0\end{pmatrix}} = \left\langle \tau_{i,r}\left(A\tau_{j,r} + B\tau_j/r\right) r \right\rangle + \left\langle \tau_{i,z}\, C\, \tau_{j,z}\, r \right\rangle + \left\langle \frac{\tau_i}{r}\left(A\frac{\tau_j}{r} + B\tau_{j,r}\right) r \right\rangle$$

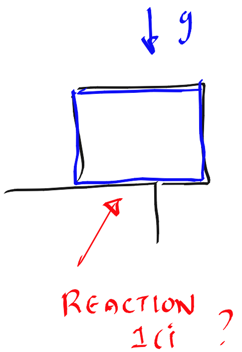
$$\underline{\underline{A}}_{ij}$$

IT IS ALMOST LIKE A 2D PROBLEM !

$$
\begin{bmatrix}
\begin{array}{l}
A \langle \tau_{i,r}\, \tau_{j,r}\, r \rangle \\
\quad + C \langle \tau_{i,z}\, \tau_{j,z}\, r \rangle \\
\quad\quad + B \langle \tau_{i,r}\, \tau_j \rangle \\
\quad\quad + \langle \underbrace{\tau_i}_{\varepsilon_{\theta\theta}/r} \underbrace{\left(B\, \tau_{j,r} + A \frac{\tau_j}{r} \right)}_{\sigma_{\theta\theta}} \rangle
\end{array}
&
\begin{array}{l}
B \langle \tau_{i,r}\, \tau_{j,z}\, r \rangle \\
\quad + C \langle \tau_{i,z}\, \tau_{j,r}\, r \rangle \\
\quad\quad + B \langle \tau_i\, \tau_{j,z} \rangle
\end{array}
\\
\begin{array}{l}
B \langle \tau_{i,z}\, \tau_{j,r}\, r \rangle \\
\quad + C \langle \tau_{i,r}\, \tau_{j,z}\, r \rangle \\
\quad\quad + B \langle \tau_{i,z}\, \tau_j \rangle
\end{array}
&
\begin{array}{l}
A \langle \tau_{i,z}\, \tau_{j,z}\, r \rangle \\
\quad + C \langle \tau_{i,r}\, \tau_{j,r}\, r \rangle
\end{array}
\end{bmatrix}
$$

Comment trouver
un problème ?





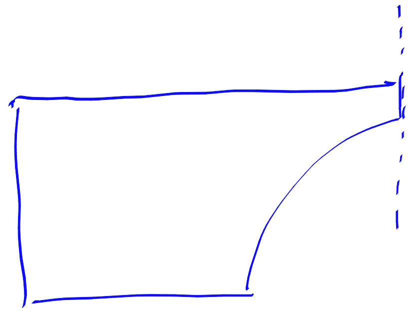
Comment contacter
son assistant
de référence ?

Format
du fichier
problem.txt ?

problem.txt

```
Type of problem : planar strains
Young modulus   :  2.1100000e+11
Poisson ratio   :  3.0000000e-01
Mass density    :  7.8500000e+03
Gravity         :  9.8100000e+00
```

Et le calcul
des tensions ?



$J(\underset{\sim}{u}^h) =$ MINIMUM OF THE POTENTIAL ENERGY

$\underbrace{\text{POTENTIAL ENERGY}}_{\text{DEFORMATION ENERGY} \,-\, \text{WORK OF EXTERNAL FORCES}}$

$\underset{\sim}{u}^h$ MINIMIZES THE ENERGY

DISPLACEMENTS FORMULATION

BUT $\underline{\underline{\sigma}}(\underset{\sim}{u}^h)$

DISCONTINUOUS ! → SEVERAL VALUES AT EACH VERTEX !

TYPICALLY CONTINUOUS LINEAR OR QUADRATIC

MUST BE VIEWED AS A LEAST-SQUARE FIT OF THE EXACT SOLUTION $\underline{\underline{\sigma}}(\underset{\sim}{u})$

→ ALLOWS US TO USE THE BEST STRATEGY FOR THE INTERPRETATION OF $\underline{\underline{\sigma}}(\underset{\sim}{u}^h)$

$\underline{\underline{\sigma}}(\underline{u}^h)$ = SOLUTION OF A LEAST-SQUARES FIT PROBLEM

$a(\hat{\underline{u}}, \underline{u}) = b(\hat{\underline{u}})$
$\forall \hat{\underline{u}} \in \hat{\mathcal{U}}$

$a(\hat{\underline{u}}^h, \underline{u}^h) = b(\hat{\underline{u}}^h)$
$\forall \underline{u}^h \in \hat{\mathcal{U}}^h$

CFR $\hat{\mathcal{U}}^h \subset \hat{\mathcal{U}}$!

CFR LINEARITY !

? $\underline{u}^h$

$a(\hat{\underline{u}}^h, \underline{u}^h - \underline{u}) = 0$
$\forall \hat{\underline{u}}^h \in \hat{\mathcal{U}}^h$

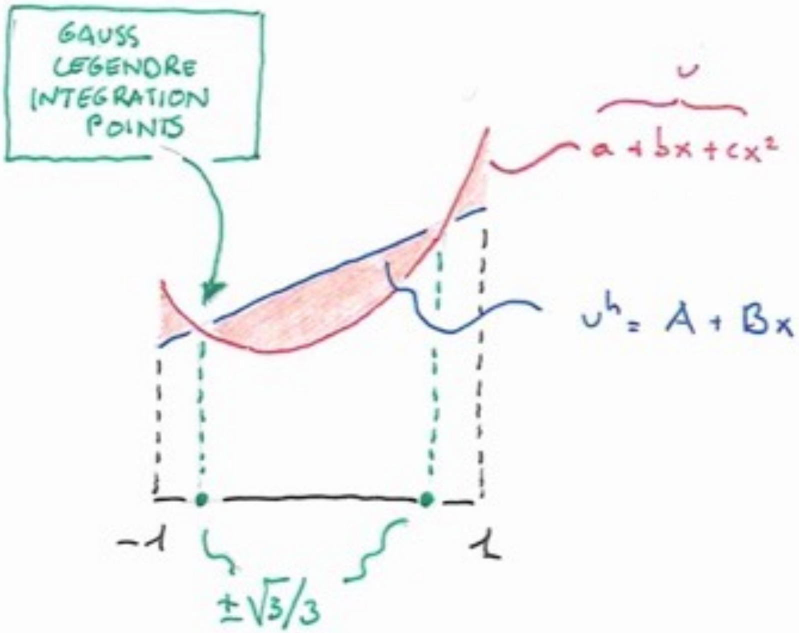
$\Updownarrow$

$\hat{b}(\hat{\underline{u}}^h) = a(\hat{\underline{u}}^h, \underline{u})$
$\hat{a}(\hat{\underline{u}}^h, \underline{u}^h) = a(\hat{\underline{u}}^h, \underline{u}^h)$

? $\underline{u}^h$

$$G(\underline{u}^h) = \min_{\underline{v}^h \in \mathcal{U}^h} \underbrace{\frac{1}{2} \left\langle \left(\underline{\underline{\varepsilon}}(\underline{v}^h) - \underline{\underline{\varepsilon}}(\underline{u})\right) : \underline{\underline{\underline{\underline{C}}}} : \left(\underline{\underline{\varepsilon}}(\underline{v}^h) - \underline{\underline{\varepsilon}}(\underline{v})\right) \right\rangle}_{G(\underline{v}^h)}$$

WHAT ARE THE "BEST VALUES" OF A LEAST-SQUARES POLYNOMIAL FIT ?



? A, B SUCH THAT

$$G(A,B) = \min_{\hat{A}\hat{B}} \int_{-1}^{1} (\hat{A} + \hat{B}x - a - bx - cx^2)^2 \, dx$$

$$\begin{cases} B \int_{-1}^{1} (A + Bx - a - bx - cx^2)\,x \, dx = 0 \\ A \int_{-1}^{1} (A + Bx - a - bx - cx^2) \, dx = 0 \end{cases}$$

$$\begin{cases} B^2 \frac{2}{3} - Bb\frac{2}{3} = 0 \longrightarrow B = b \\ 2A^2 - 2Aa - Ac\frac{2}{3} = 0 \longrightarrow A = a + \frac{c}{3} \end{cases}$$

$$\frac{c}{3} - cx^2 = 0 \rightarrow x = \pm\frac{\sqrt{3}}{3}$$