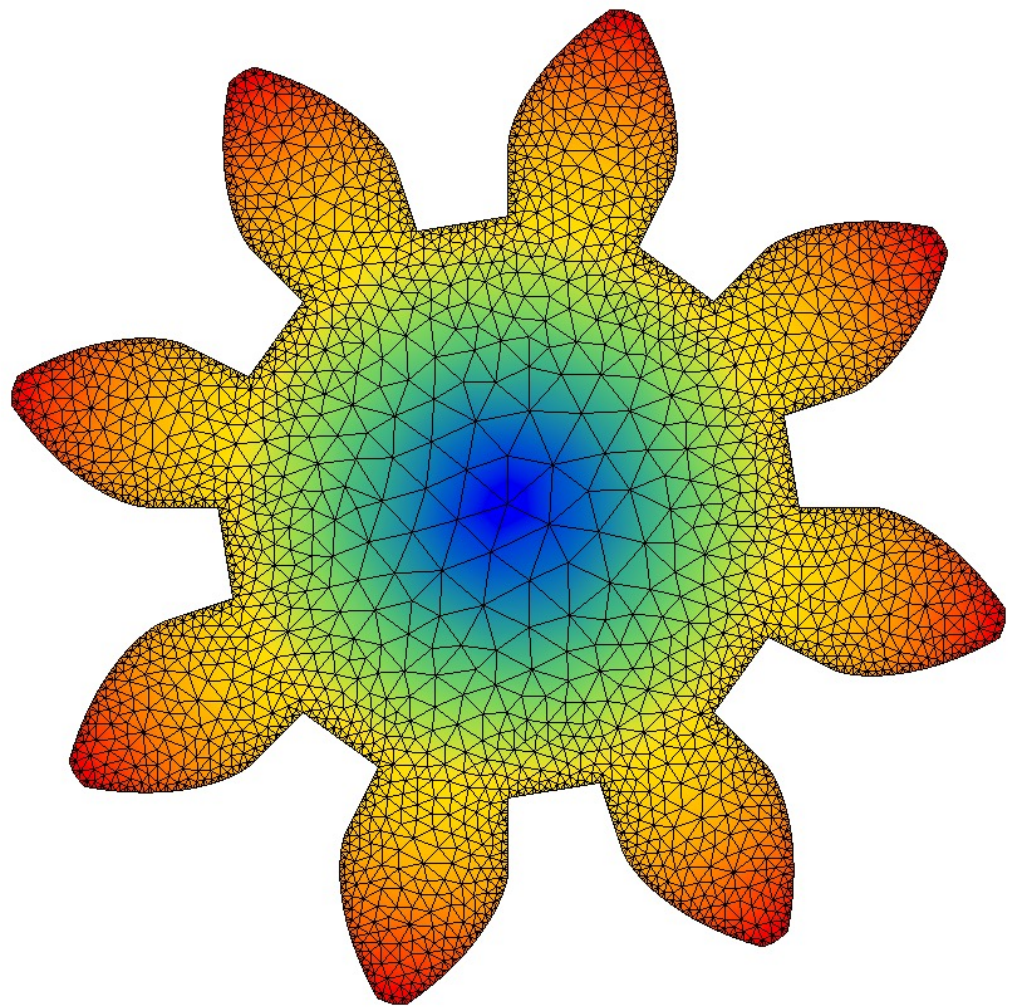
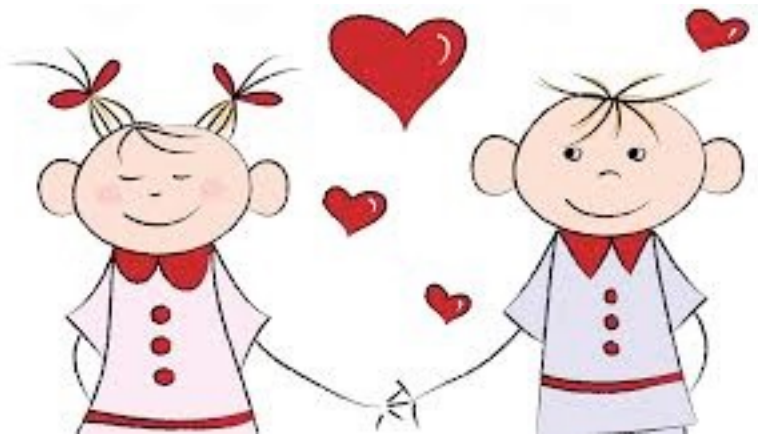




Ne pas oublier de trouver votre Valentin(e) !



161 : Lassore, Romain
162 : Leblanc, Christophe
163 : Leboutte, Pierre
164 : Lebras, Floriane
165 : Lebrun, Léa
166 : Lecomte, Antonin
167 : Lederer, Louis
168 : Leemans, Jade
169 : Lefebvre, Camille
170 : Legrand, Mattéo
171 : Lemaigre, Alexandre
172 : Lemy Storme, Noam
173 : Lentz, Bernard
174 : Lequeu, Sophie
175 : Lesire, Célestin
176 : Levaque, Jonathan
177 : Liesse, Simon
178 : Lizée, Simon
179 : Luyckx, Nicolas
180 : Maissin, Matteo
181 : Makhoulouf, Rebecca
182 : Malevé, Hugo
183 : Marotta, Romain
184 : Marsily, Victor
185 : Martin, Antoine
186 : Mary, Théo
187 : Massart, Edouard
188 : Mathy, Sacha
189 : Mauro, Matteo
190 : Maus de Rolley, Sosthène
191 : Mbengue, Khaly
192 : Meli Nguenkeung, François
193 : Mesaros, Alex
194 : Mesbahi, Adam
195 : Mestach, Cyprien
196 : Michaux, Bastien
197 : Michiels, Edouard
198 : Monfils, Julien
199 : Moons, Perrine

FSA 1 BA	(Mardi-BA91)	(groupe : 103)
FSA 1 BA	(Mardi-BA91)	(groupe : 95)
FSA 1 BA	(Mardi-BA91)	(groupe : 7)
FSA 1 BA	(Mardi-BA91)	(groupe : 8)
FSA 1 BA	(Mardi-BA03)	(groupe : 145)
FSA 1 BA	(pas de séance :-)	(pas de groupe :-)
FSA 1 BA	(Mardi-BA03)	(groupe : 80)
FSA 1 BA	(Mardi-BA91)	(groupe : 14)
FSA 1 BA	(Mardi-BA03)	(groupe : 93)
FSA 1 BA	(Mardi-BA91)	(groupe : 138)
FSA 1 BA	(Mardi-BA91)	(groupe : 29)
FSA 1 BA	(Mardi-BA03)	(groupe : 79)
FSA 1 BA	(Jeudi-BA91)	(groupe : 108)
MAP 2 MS/G	(pas de séance :-)	(pas de groupe :-)
FSA 1 BA	(Mardi-BA91)	(groupe : 131)
FSA 1 BA	(Mardi-BA03)	(groupe : 37)
FSA 1 BA	(Mardi-BA91)	(groupe : 86)
FSA 1 BA	(Mardi-BA91)	(groupe : 43)
FSA 1 BA	(Mardi-BA03)	(groupe : 119)
FSA 1 BA	(Mardi-BA91)	(groupe : 101)
FSA 1 BA	(Mardi-BA91)	(groupe : 59)
FSA 1 BA	(Mardi-BA91)	(groupe : 51)
FSA 1 BA	(Mardi-BA91)	(groupe : 39)
FSA 1 BA	(Mardi-BA91)	(groupe : 87)
FSA 1 BA	(Mardi-BA91)	(groupe : 16)
FSA 1 BA	(Mardi-BA91)	(groupe : 33)
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FSA 1 BA	(Mardi-BA91)	(groupe : 95)
FSA 1 BA	(Mardi-BA91)	(groupe : 92)
FSA 1 BA	(Mardi-BA03)	(groupe : 67)
FSA 1 BA	(Mardi-BA91)	(groupe : 92)
FSA 1 BA	(Mardi-BA91)	(groupe : 136)
FSA 1 BA	(Mardi-BA91)	(groupe : 83)
FSA 1 BA	(Mardi-BA91)	(pas de groupe :-)
FSA 1 BA	(Mardi-BA91)	(pas de groupe :-)
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FSA 1 BA	(Mardi-BA91)	(groupe : 113)
FSA 1 BA	(Mardi-BA91)	(groupe : 1)
FSA 1 BA	(Mardi-BA91)	(groupe : 76)

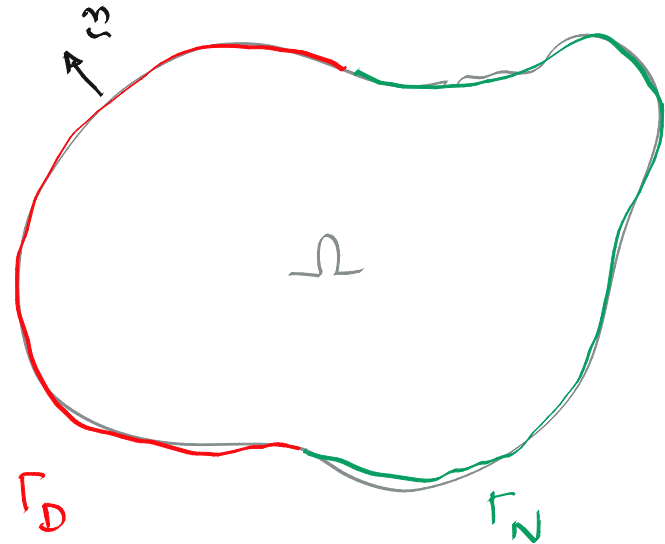
**Plus sérieusement,
A partir de ce soir, il sera possible de rester célibataire !
Mais, ce n'est pas la meilleure idée !**

Typical elliptic boundary value problem

$$\nabla \cdot \nabla u + f = 0 \quad \text{sur } \Omega$$

$$\nabla \cdot \nabla u = g \quad \text{sur } \Gamma_N$$

$$u = t \quad \text{sur } \Gamma_D$$



Conditions essentielles
Conditions de Dirichlet

$$\partial\Omega = \Gamma_D \cup \Gamma_N$$

Conditions naturelles
Conditions de Neumann

En général, la fonction n'est pas connue...

Mais, c'est la solution d'une équation aux dérivées partielles !

Commençons par une équation de Poisson

Some nice spaces and notations...

$$\mathcal{U} = \{ u \dots \text{telh que } u = t \text{ sur } \Gamma_D \}$$

$$\hat{\mathcal{U}} = \{ \hat{u} \dots \text{telh que } u = 0 \text{ sur } \Gamma_D \}$$

ESPACE DES CANDIDATS SOLUTION
 ESPACE DES FONCTIONS TEST

$$\langle f g \rangle = \int_{\Omega} f g \, d\Omega$$

$$\ll f g \gg = \int_{\partial\Omega} f g \, ds$$

$$\ll f g \gg_N = \int_{\Gamma_N} f g \, ds$$

Ne faisons pas comme les mathématiciens...
 On ne va pas être rigoureux maintenant !
 On fera cela plus tard...

D'ailleurs, est-ce que cela est utile ?
 Et pourtant, oui !

... to do calculus !

Three tips !

$$\underbrace{\langle \hat{u} }_{\in \hat{U}} \underbrace{\nabla \cdot \nabla u}_{\in U} \rangle = \underbrace{\langle \nabla \cdot (\hat{u} \nabla u) \rangle}_{\text{Tip 1}} - \underbrace{\langle (\nabla \cdot \hat{u}) (\nabla u) \rangle}_{\text{Tip 2}}$$

[Tip 1]

$$(fg)' = fg' + f'g$$

[TIP 2]

$$\langle \nabla \cdot a \rangle = \langle\langle \eta \cdot a \rangle\rangle$$

$$\langle\langle (\eta \cdot \nabla u) \hat{u} \rangle\rangle$$

[TIP 3]

$$\hat{u} = 0 \text{ sur } \Gamma_D$$

$$\underbrace{\langle\langle (\eta \cdot \nabla u) \hat{u} \rangle\rangle}_g$$

And it is all !

Strong formulation



? u tel que

$$\nabla_x \cdot \nabla_x u + f = 0 \quad \text{sur } \Omega$$

$$m \cdot \nabla_x u = g \quad \text{sur } \Gamma_N$$

$$u = t \quad \text{sur } \Gamma_D$$

FORMULATION
FORTE

FORMULATION
FAIBLE

? $u \in U$

$$\underbrace{\langle \nabla_x \hat{u}, \nabla_x u \rangle}_{a(\hat{u}, u)} = \underbrace{\langle\langle g, \hat{u} \rangle\rangle_N}_{b(\hat{u})} + \underbrace{\langle f, \hat{u} \rangle}_{c(\hat{u})} \quad \forall \hat{u} \in \hat{U}$$



$a(\hat{u}, u)$

$b(\hat{u})$

$$? u \in U \quad J(u) = \min_{v \in U} \left(\underbrace{\frac{1}{2} a(v, v)}_{J(v)} - b(v) \right)$$

PROBLEME DE MINIMUM



Weak formulation

La formulation faible c'est un problème de minimum !

FORMULATION
FAIBLE



$$\begin{aligned} &? v \in U \\ &\underbrace{\langle \nabla \hat{v}, \nabla v \rangle}_{a(\hat{v}, v)} = \underbrace{\ll g \hat{v} \gg_N}_{b(\hat{v})} + \underbrace{\langle f \hat{v} \rangle}_{b(\hat{v})} \quad \forall \hat{v} \in \hat{U} \end{aligned}$$



$$? v \in U \quad J(v) = \min_{r \in U} \underbrace{\left(\frac{1}{2} a(r, r) - b(r) \right)}_{J(r)}$$

PROBLEME DE MINIMUM

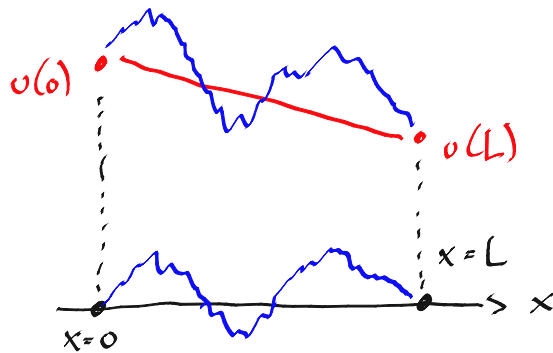


$$\alpha = J(v) \quad \left(\begin{array}{l} \uparrow \\ \in \mathbb{R} \end{array} \right) \quad \left(\begin{array}{l} \in \mathcal{U} \end{array} \right)$$

S_v VARIATION

$$\begin{array}{c} \in \mathcal{U} \\ \swarrow \\ v + \varepsilon \hat{v} \\ \swarrow \quad \searrow \\ \in \mathbb{R} \quad \in \hat{\mathcal{U}} \end{array}$$

$$\begin{aligned} \frac{d^2 J}{dx^2} &= 0 \\ v(0) &= v_0 \\ v(L) &= v_L \end{aligned}$$



$\leftarrow \Omega \rightarrow$

$$\underbrace{\left. \frac{dJ(v + \varepsilon \hat{v})}{d\varepsilon} \right|_{\varepsilon=0}}_{\delta J = 0} \quad \forall \hat{v} \in \hat{\mathcal{U}}$$

Variational
Calculus

$$J(u) = \frac{1}{2} \langle \nabla u, \nabla u \rangle - \langle f, u \rangle$$

~~$\langle \nabla u, \nabla u \rangle$~~
= 0
SPOG

$$\left. \frac{dJ(u + \varepsilon \hat{v})}{d\varepsilon} \right|_{\varepsilon=0} = 0 \quad \forall \hat{v} \in \hat{U}$$

$$J(u + \varepsilon \hat{v}) = \frac{1}{2} \langle \nabla u, \nabla u \rangle + \varepsilon \langle \nabla u, \nabla \hat{v} \rangle + \frac{1}{2} \varepsilon^2 \langle \nabla \hat{v}, \nabla \hat{v} \rangle - \langle f, u \rangle - \varepsilon \langle f, \hat{v} \rangle$$

$$\left. \frac{dJ(u + \varepsilon \hat{v})}{d\varepsilon} \right|_{\varepsilon=0} = \langle \nabla u, \nabla \hat{v} \rangle - \langle f, \hat{v} \rangle \quad \forall \hat{v} \in \hat{U}$$



$$\underbrace{\left. \frac{dJ(u + \varepsilon \hat{v})}{d\varepsilon} \right|_{\varepsilon=0}}_{=0} = \langle \nabla u, \nabla \hat{v} \rangle - \langle f, \hat{v} \rangle \quad \forall \hat{v} \in \hat{U}$$



$$\begin{aligned}
 J(u + \hat{v}) &= \underbrace{\frac{1}{2} \langle \nabla u, \nabla u \rangle}_{- \langle f, u \rangle} + \underbrace{\langle \nabla u, \nabla \hat{v} \rangle}_{- f(\hat{v})} + \underbrace{\frac{1}{2} \langle \nabla \hat{v}, \nabla \hat{v} \rangle}_{\geq 0} \\
 &\geq J(u)
 \end{aligned}$$

$\quad \quad \quad = 0$
 $\langle \nabla u, \nabla \hat{v} \rangle = \langle f, \hat{v} \rangle \quad \forall \hat{v} \in \hat{U}$



The Finite Element is a variational method

$$u \simeq u^h \in \mathcal{U}^h \subset \mathcal{U}$$

$$= \sum_{i=1}^m U_i \tau_i$$

VALEURS
NODALES
INCONNUES

FONCTIONS
DE FORME
DEFINIES
A PRIORI
 $\in \mathcal{U}$

$$J(u^h) = \min_{v \in \mathcal{U}^h} \underbrace{\frac{1}{2} a(v, v) - \underbrace{b(v)}_{J(v)}}$$

$$J(u) \leq J(u^h)$$

$$J(u^h) = \frac{1}{2} \langle (\nabla \sum U_i \tau_i), (\nabla \sum U_j \tau_j) \rangle + \langle f, (\sum U_i \tau_i) \rangle$$

$$\frac{1}{2} \sum_i \sum_j U_i U_j \underbrace{\langle \nabla \tau_i, \nabla \tau_j \rangle}_{A_{ij}} + \sum U_i \underbrace{\langle f, \tau_i \rangle}_{B_i}$$

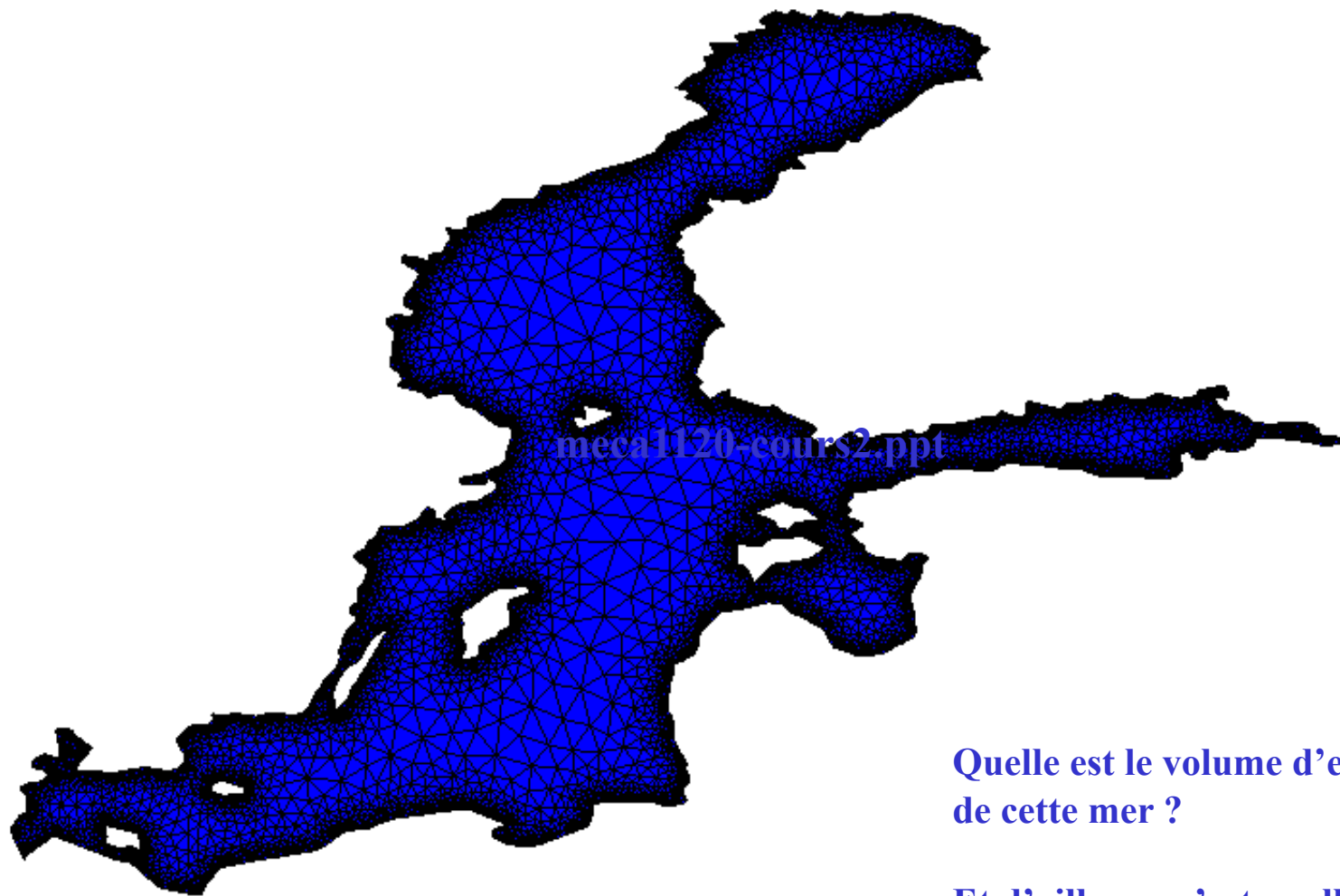
The discrete system
is just a linear system !

$$J(u^h) = \frac{1}{2} \langle (\nabla \sum_i u_i \tau_i), (\nabla \sum_j u_j \tau_j) \rangle + \langle f, (\sum_i u_i \tau_i) \rangle$$

$$\frac{1}{2} \sum_i \sum_j u_i u_j \underbrace{\langle \nabla \tau_i, \nabla \tau_j \rangle}_{A_{ij}} + \sum_i u_i \underbrace{\langle f, \tau_i \rangle}_{B_i}$$

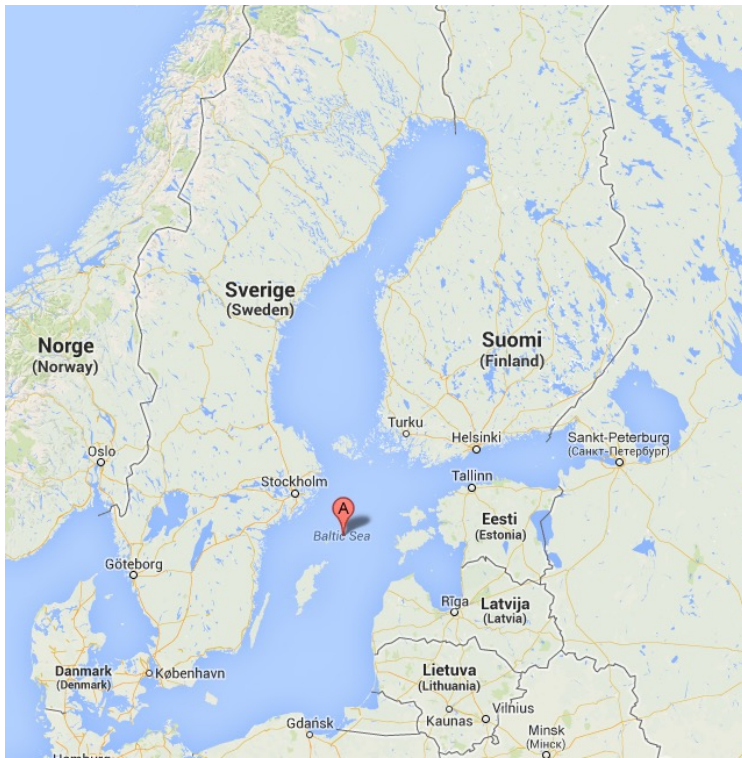
$$\frac{\partial J}{\partial u_i} = 0$$

$$\sum_j A_{ij} u_j = B_i$$

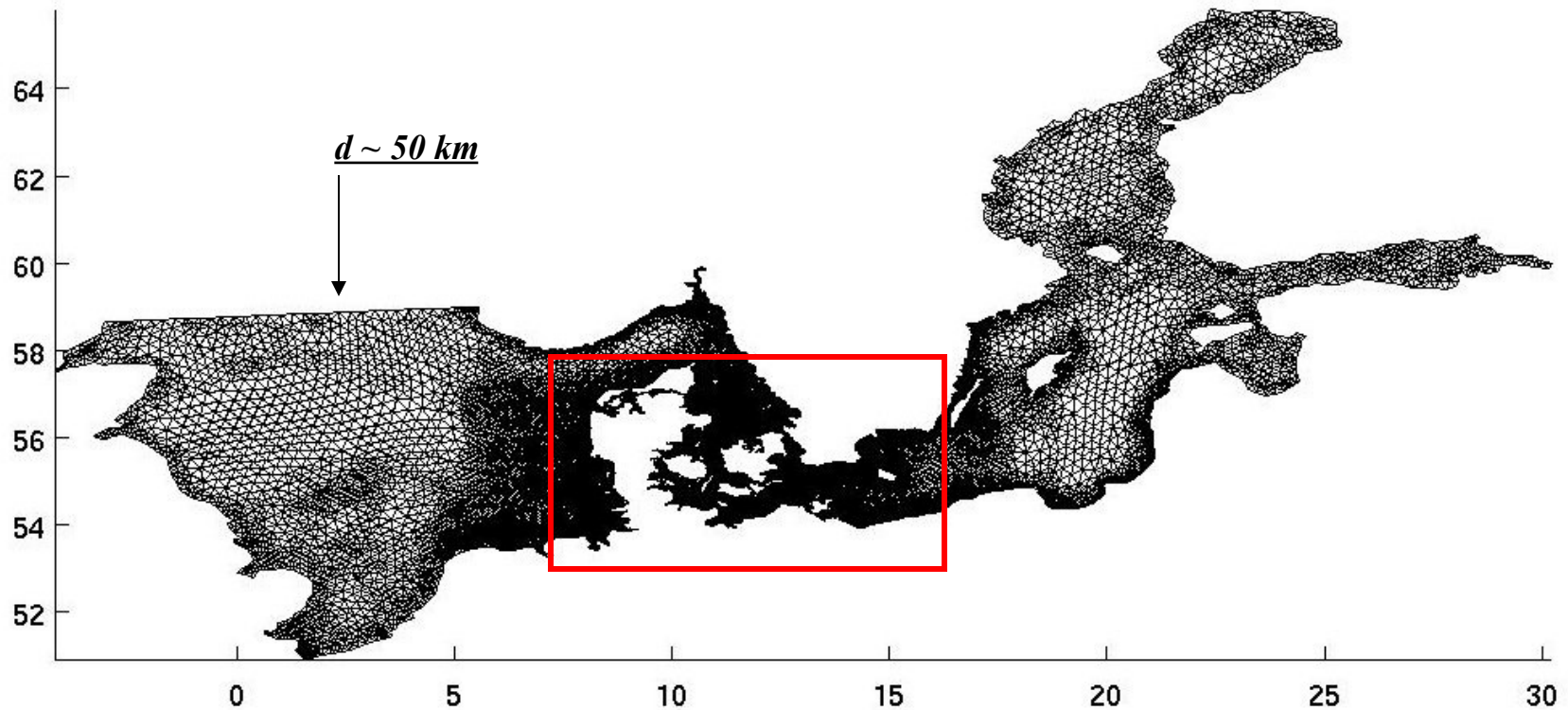


Quelle est le volume d'eau
de cette mer ?

Et d'ailleurs, c'est quelle mer ?

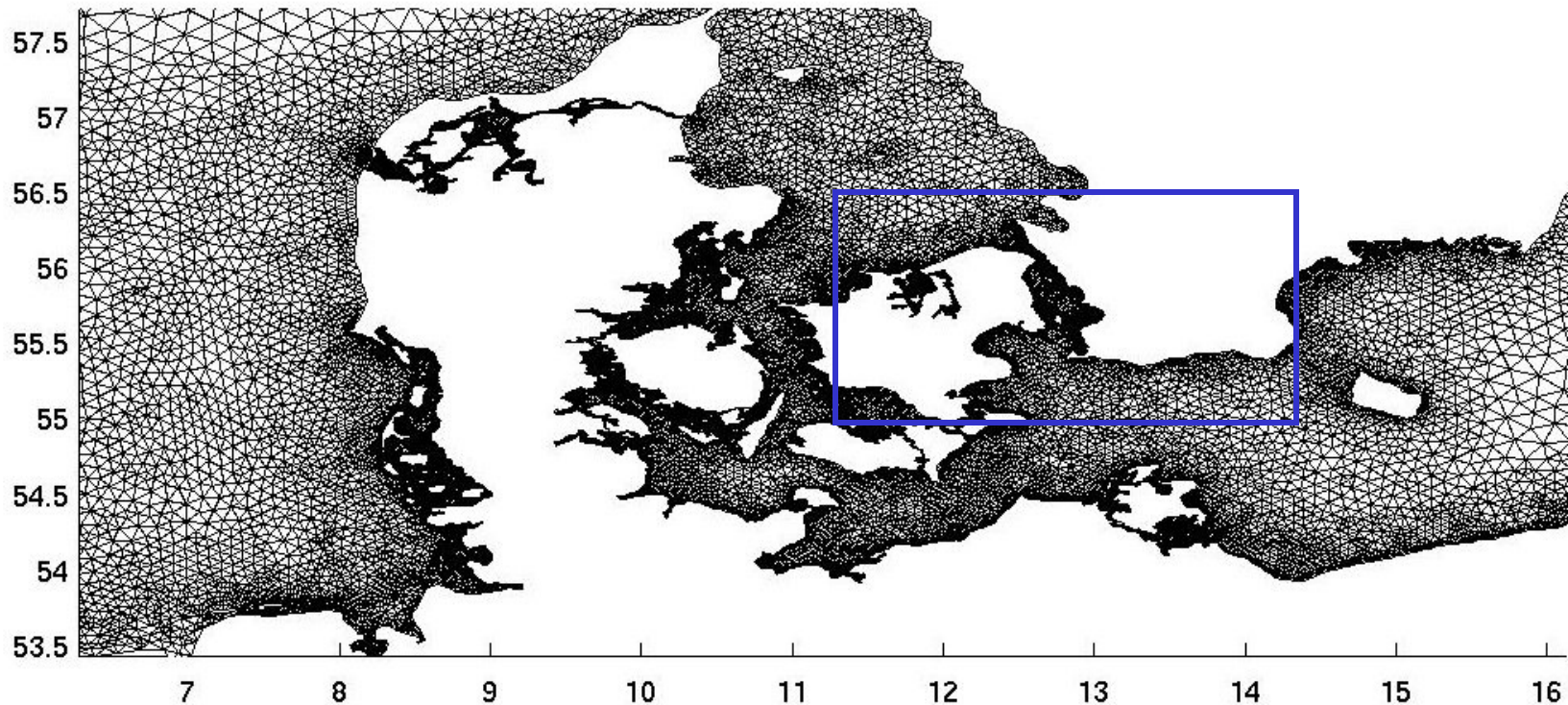


Pourquoi des maillages non structurés ?



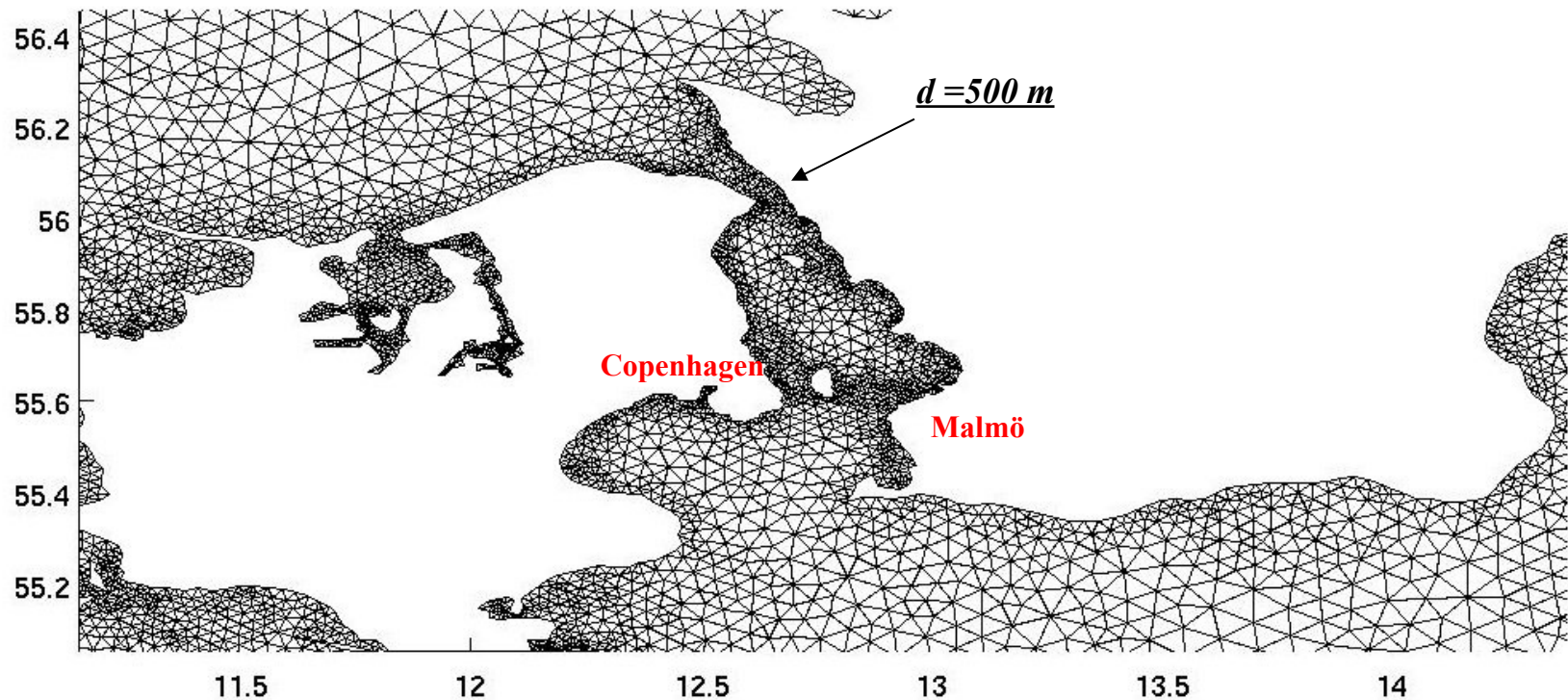
Courtesy of Dr. N. Kliem

Pourquoi des maillages non structurés ?

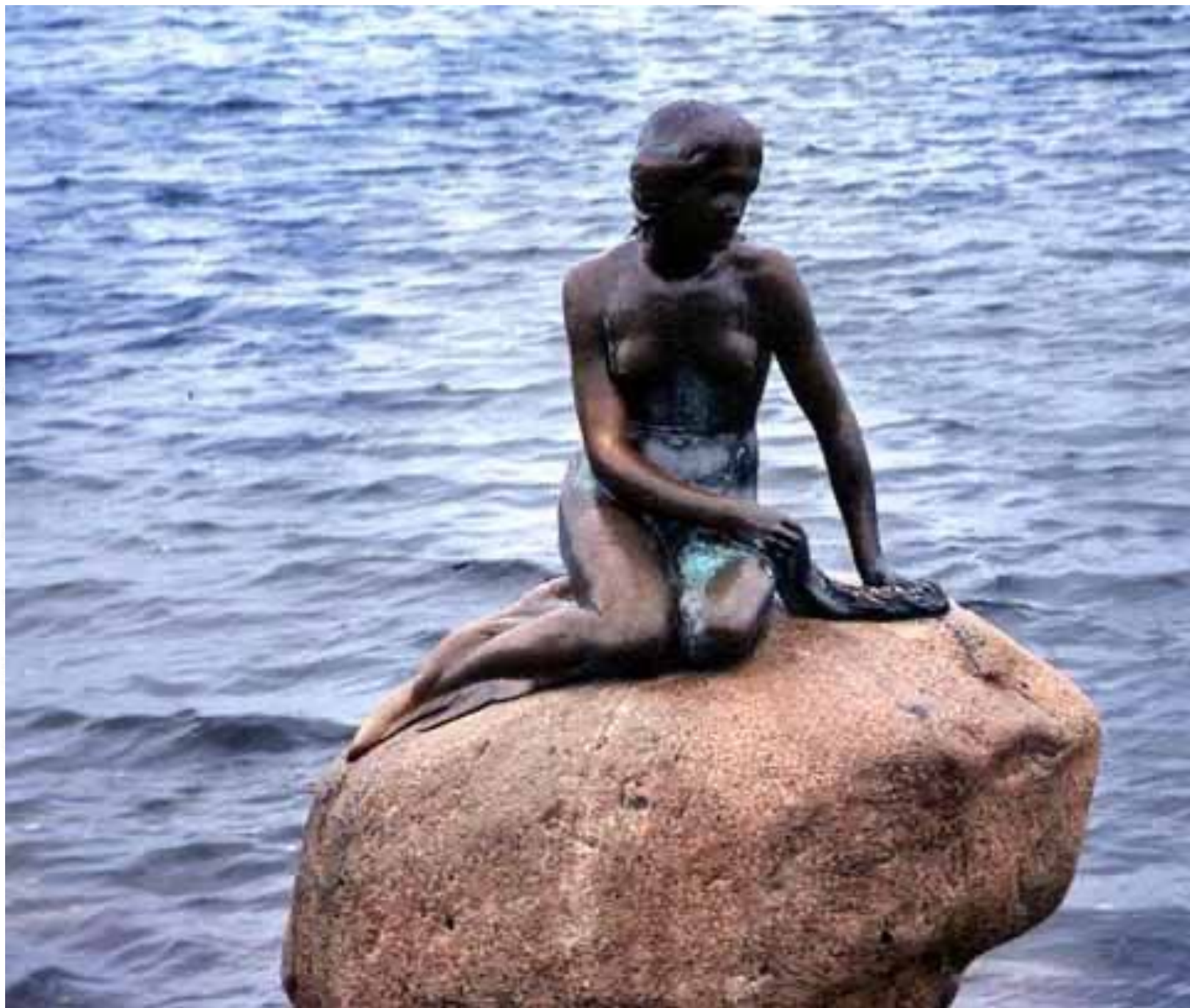


Courtesy of Dr. N. Kliem

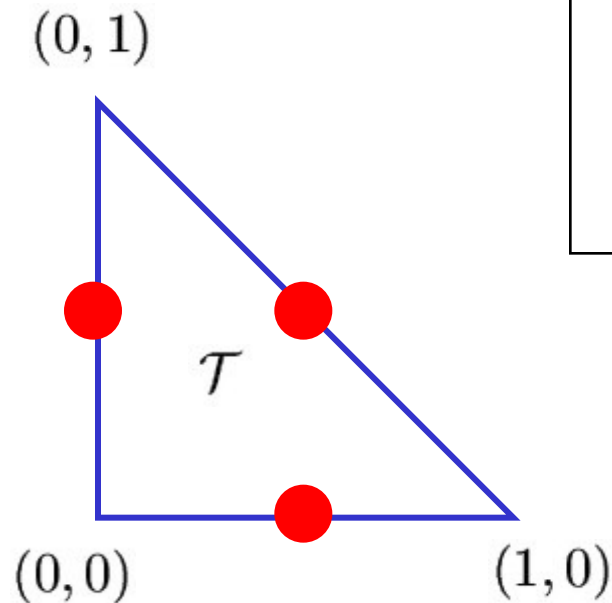
Pourquoi des maillages non structurés ?



Courtesy of Dr. N. Kliem



Intégration sur un triangle : Règle de Hammer à 3 points

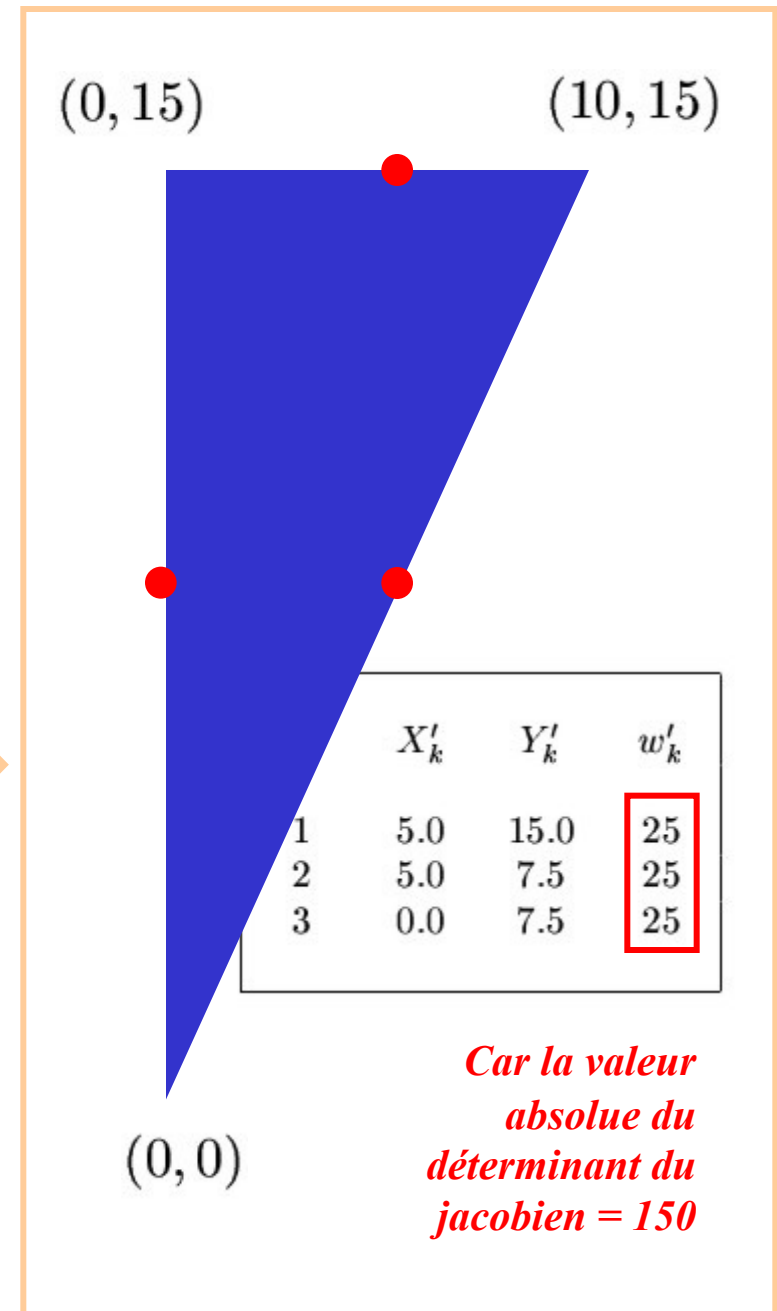
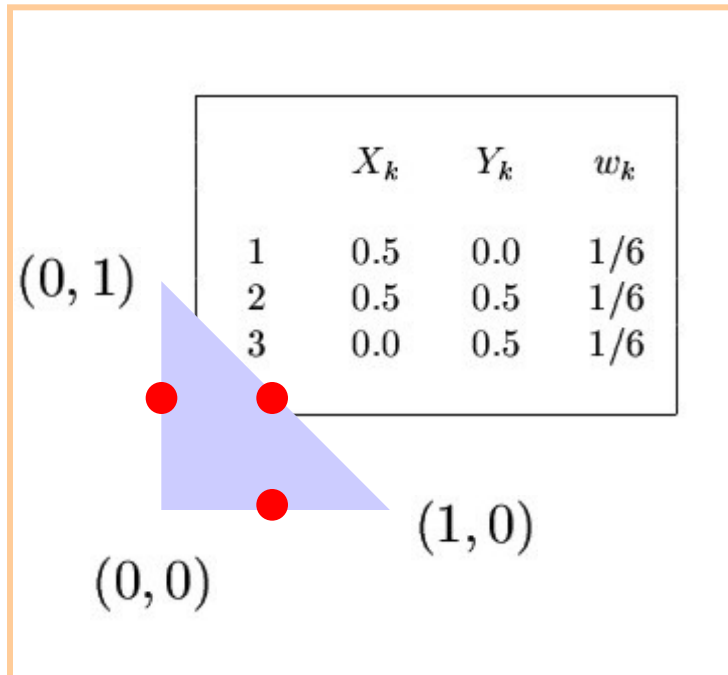


$$\underbrace{\int_{\mathcal{T}} f(x, y) \, dx \, dy}_I \approx \underbrace{\sum_{k=1}^3 w_k f(X_k, Y_k)}_{I^h}$$

	X_k	Y_k	w_k
1	0.5	0.0	1/6
2	0.5	0.5	1/6
3	0.0	0.5	1/6

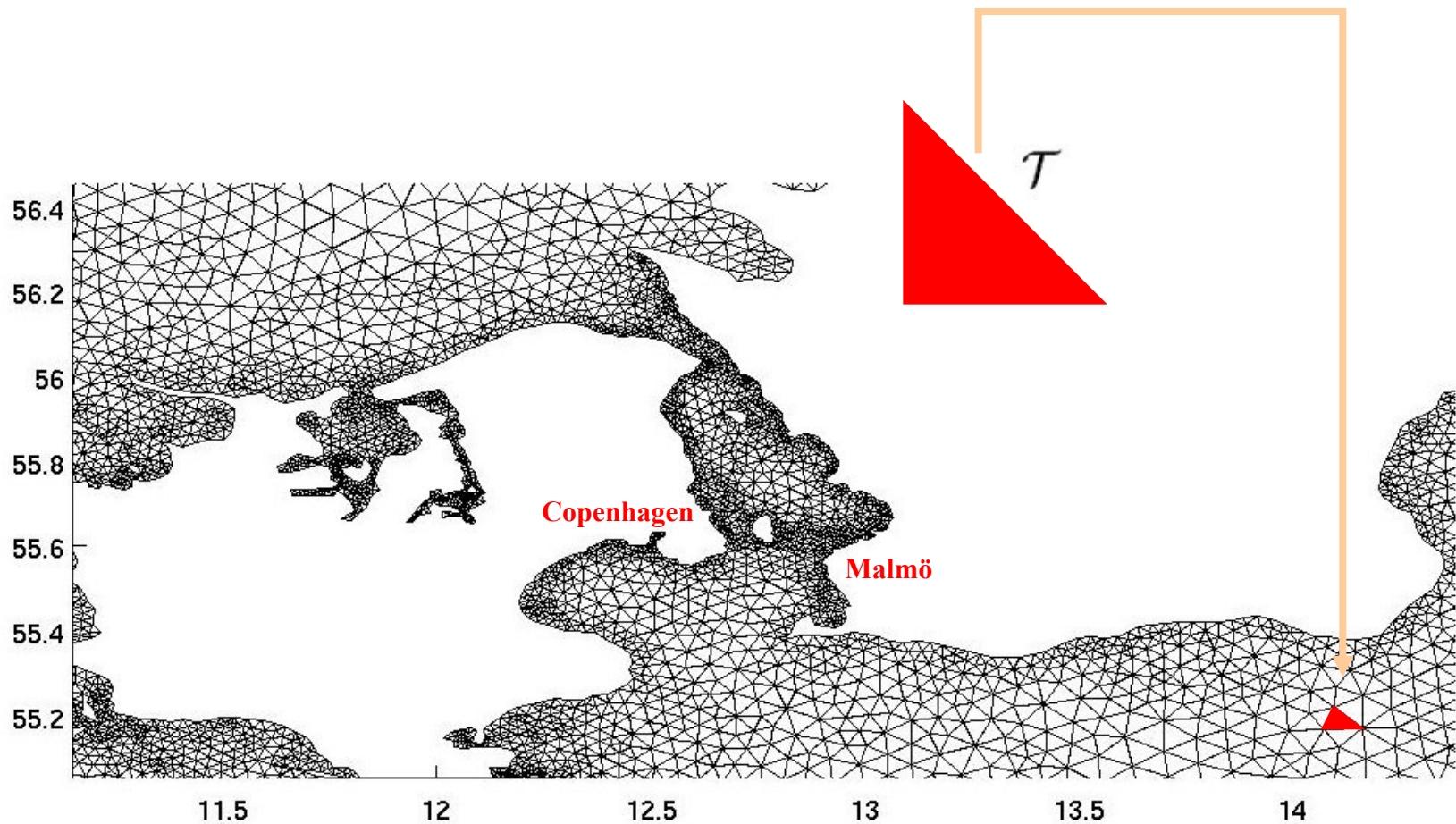
Et un autre triangle ?

$$\begin{aligned}x' &= 10x \\ y' &= 15 - 15y\end{aligned}$$

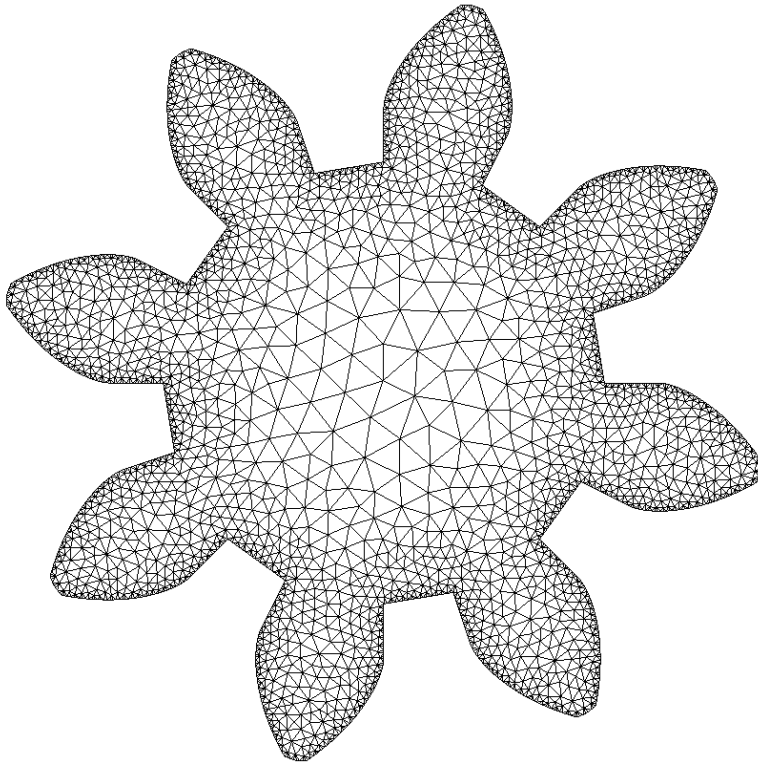


Application to Finite Elements

*Each triangle can be transformed in
the parent element through a linear
transformation.*



Définir un maillage

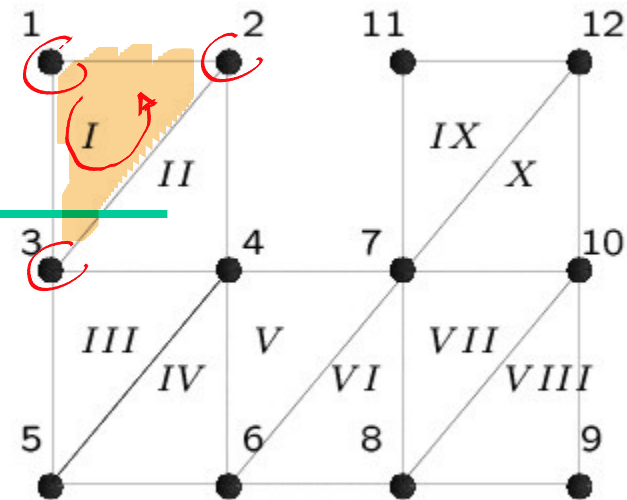


Coordonnées de noeuds
Tableau d'appartenance des triangles
Nombre de noeuds : nNode
Nombre de triangles : nElem

```
typedef struct  
{  
    int *elem;  
    double *X;  
    double *Y;  
    int nElem;  
    int nNode;  
} femMesh;
```

$$\bar{\Omega} = \bigcup_{e=1}^{N_1} \{\bar{\Omega}_e\}, \quad \Omega_e \cap \Omega_f = \emptyset, \quad \text{si } e \neq f.$$

Et concrètement c'est quoi un maillage ?



Triangle	Sommets		
1	1	3	2
2	3	4	2
3	5	4	3
4	5	6	4
5	6	7	4
6	6	8	7
7	8	10	7
8	8	9	10
9	7	12	11
10	10	12	7

Sommet	X_i	Y_i
1	0.0	2.0
2	1.0	2.0
3	0.0	1.0
4	1.0	1.0
5	0.0	0.0
6	1.0	0.0
7	2.0	1.0
8	2.0	0.0
9	3.0	0.0
10	3.0	1.0
11	2.0	2.0
12	3.0	2.0

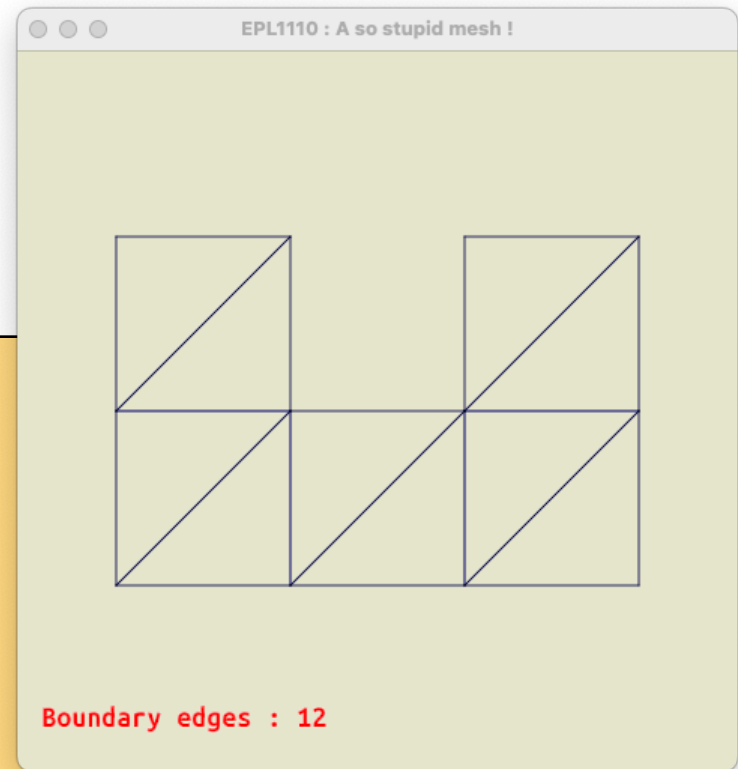
Et vraiment concrètement ?

Number of nodes 12

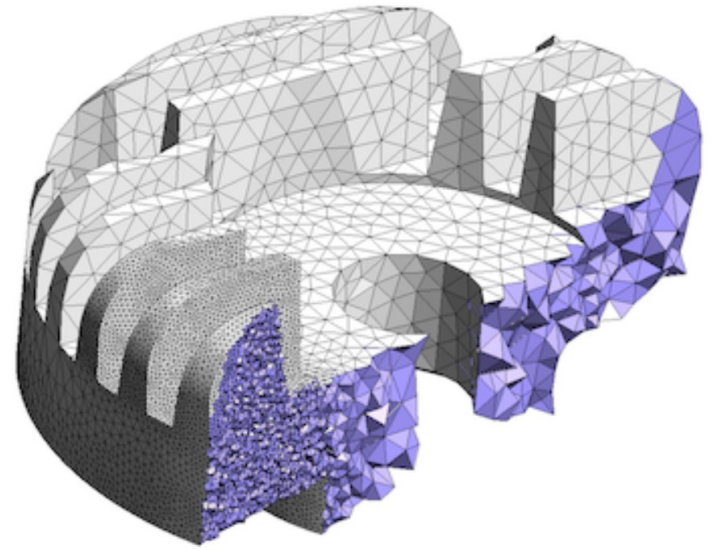
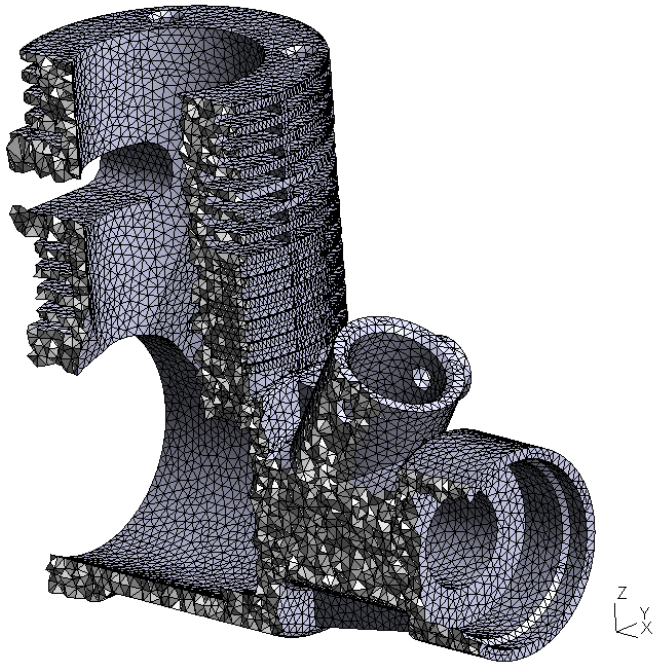
0 :	0.0000000e+00	2.0000000e+00
1 :	1.0000000e+00	2.0000000e+00
2 :	0.0000000e+00	1.0000000e+00
3 :	1.0000000e+00	1.0000000e+00
4 :	0.0000000e+00	0.0000000e+00
5 :	1.0000000e+00	0.0000000e+00
6 :	2.0000000e+00	1.0000000e+00
7 :	2.0000000e+00	0.0000000e+00
8 :	3.0000000e+00	0.0000000e+00
9 :	3.0000000e+00	1.0000000e+00
10 :	2.0000000e+00	2.0000000e+00
11 :	3.0000000e+00	2.0000000e+00

Number of triangles 10

0 :	0	2	1
1 :	2	3	1
2 :	4	3	2
3 :	4	5	3
4 :	5	6	3
5 :	5	7	6
6 :	7	9	6
7 :	7	8	9
8 :	6	11	10
9 :	9	11	6



Et vraiment très très
très concrètement ?

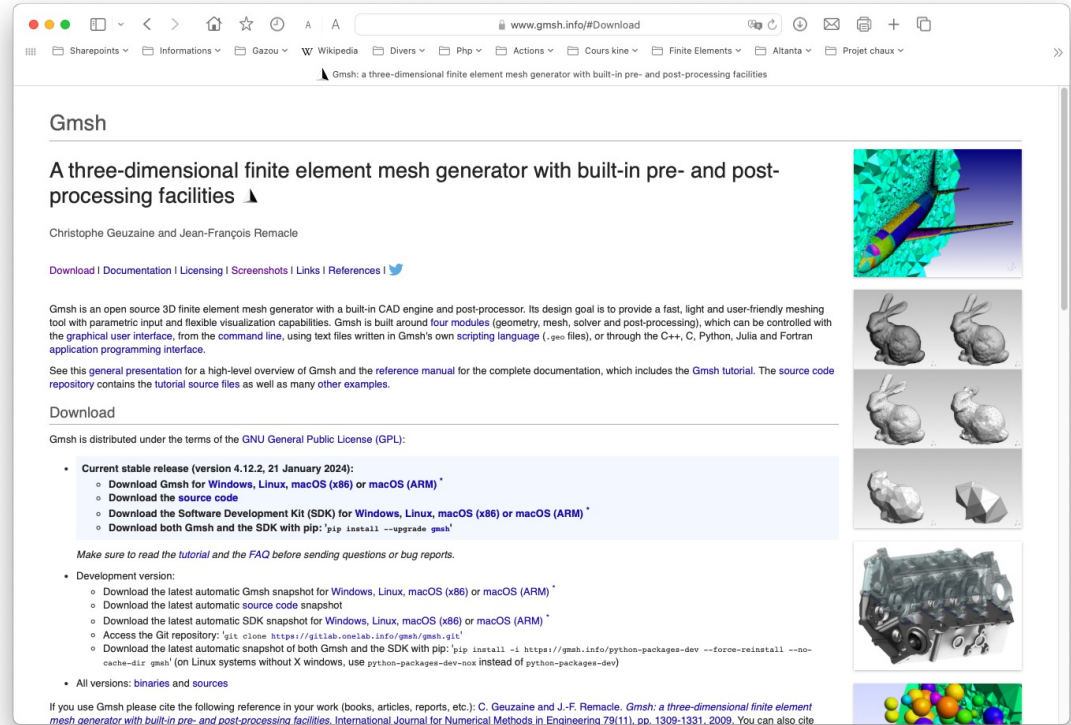


**Il existe des algorithmes pour générer
automatiquement les maillages.**

Cours de géométrie numérique LMECA2170

<http://www.geuz.org/gmsh/>

<https://www.gmsh.info>



The screenshot shows the Gmsh website with the following content:

Gmsh

A three-dimensional finite element mesh generator with built-in pre- and post-processing facilities

Christophe Geuzaine and Jean-François Remacle

[Download](#) | [Documentation](#) | [Licensing](#) | [Screenshots](#) | [Links](#) | [References](#) | [Twitter](#)

Gmsh is an open source 3D finite element mesh generator with a built-in CAD engine and post-processor. Its design goal is to provide a fast, light and user-friendly meshing tool with parametric input and flexible visualization capabilities. Gmsh is built around four modules (geometry, mesh, solver and post-processing), which can be controlled with the graphical user interface, from the **command line**, using text files written in Gmsh's own **scripting language** (.geo files), or through the C++, C, Python, Julia and Fortran application programming interface.

See this [general presentation](#) for a high-level overview of Gmsh and the [reference manual](#) for the complete documentation, which includes the [Gmsh tutorial](#). The [source code repository](#) contains the [tutorial source files](#) as well as many other [examples](#).

Download

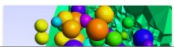
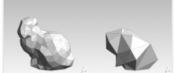
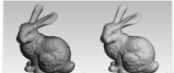
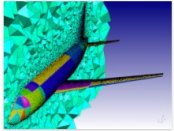
Gmsh is distributed under the terms of the [GNU General Public License \(GPL\)](#):

- **Current stable release (version 4.12.2, 21 January 2024):**
 - [Download Gmsh for Windows, Linux, macOS \(x86\) or macOS \(ARM\)](#)
 - [Download the source code](#)
 - [Download the Software Development Kit \(SDK\) for Windows, Linux, macOS \(x86\) or macOS \(ARM\)](#)
 - [Download both Gmsh and the SDK with pip: 'pip install --upgrade gmsh'](#)

Make sure to read the tutorial and the FAQ before sending questions or bug reports.

- **Development version:**
 - Download the latest automatic Gmsh snapshot for Windows, Linux, macOS (x86) or macOS (ARM)
 - Download the latest automatic source code snapshot
 - Download the latest automatic SDK snapshot for Windows, Linux, macOS (x86) or macOS (ARM)
 - Access the Git repository: 'git clone https://gitlab.onelab.info/gmsh/gmsh.git'
 - Download the latest automatic snapshot of both Gmsh and the SDK with pip: 'pip install -i https://gmsh.info/python-packages-dev --force-reinstall --no-cache-dir gmsh' (on Linux systems without X windows, use python-packages-dev-nox instead of python-packages-dev)
- [All versions: binaries and sources](#)

If you use Gmsh please cite the following reference in your work (books, articles, reports, etc.): C. Geuzaine and J.-F. Remacle. *Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities*. International Journal for Numerical Methods in Engineering 79(11), pp. 1309-1331, 2009. You can also cite



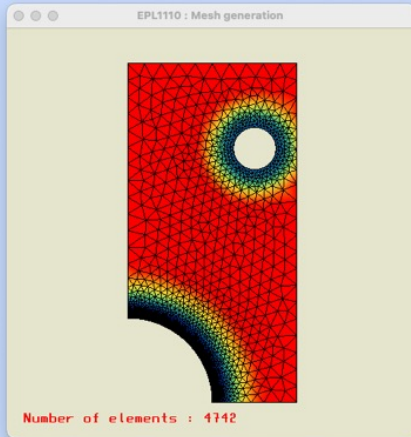
Installer
le Software Development Kit
de gmsh

<https://www.gmsh.info>

- **Current stable release (version 4.12.2, 21 January 2024):**
 - Download Gmsh for **Windows, Linux, macOS (x86) or macOS (ARM)** *
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 - Download both Gmsh and the SDK with pip: `'pip install --upgrade gmsh'`

Make sure to read the [tutorial](#) and the [FAQ](#) before sending questions or bug reports.

Il faut choisir la bonne version !

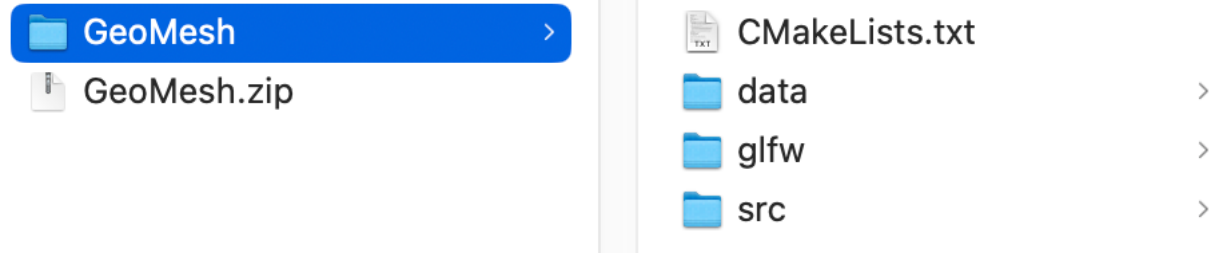


Devoir 2 : GeoMesh (12/02/2024)

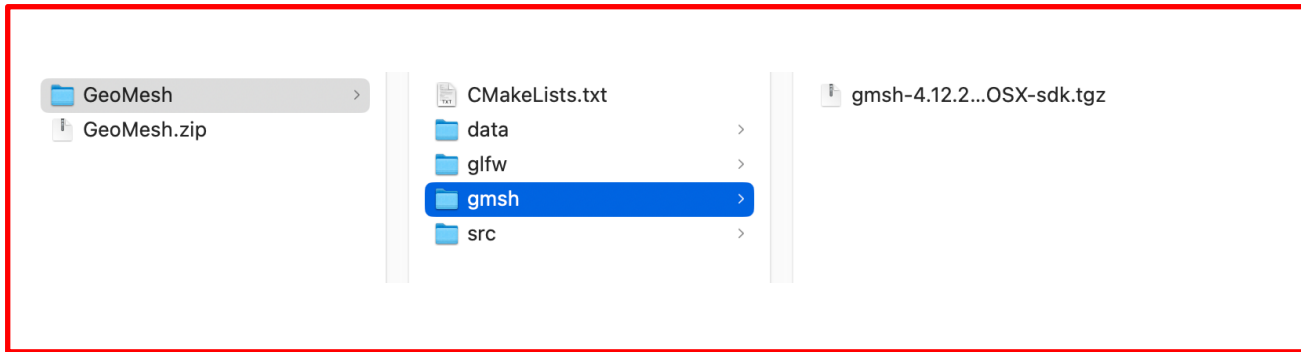
Projet à télécharger : [GeoMesh.zip](#)

Enoncé du devoir : [GeoMesh.pdf](#)

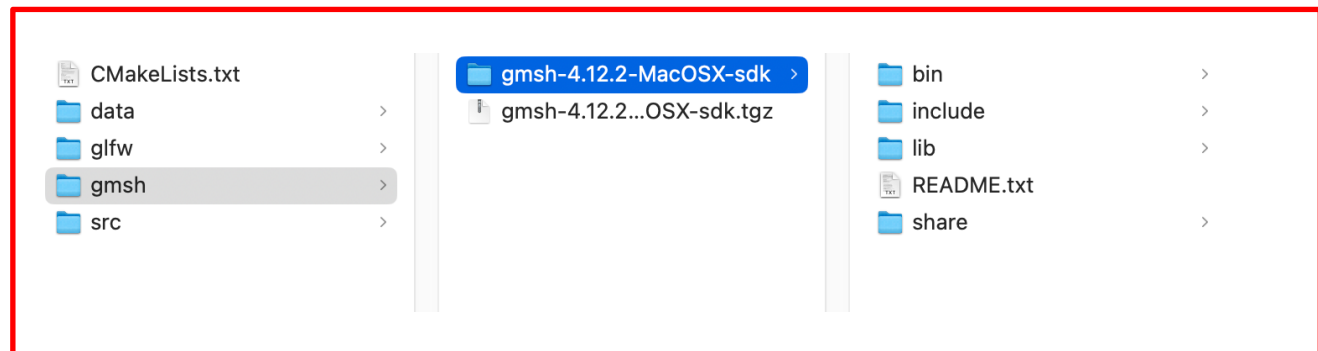
Deadline : **Lundi 26 février 2024 à 23h59**



Comment compiler le second devoir ?



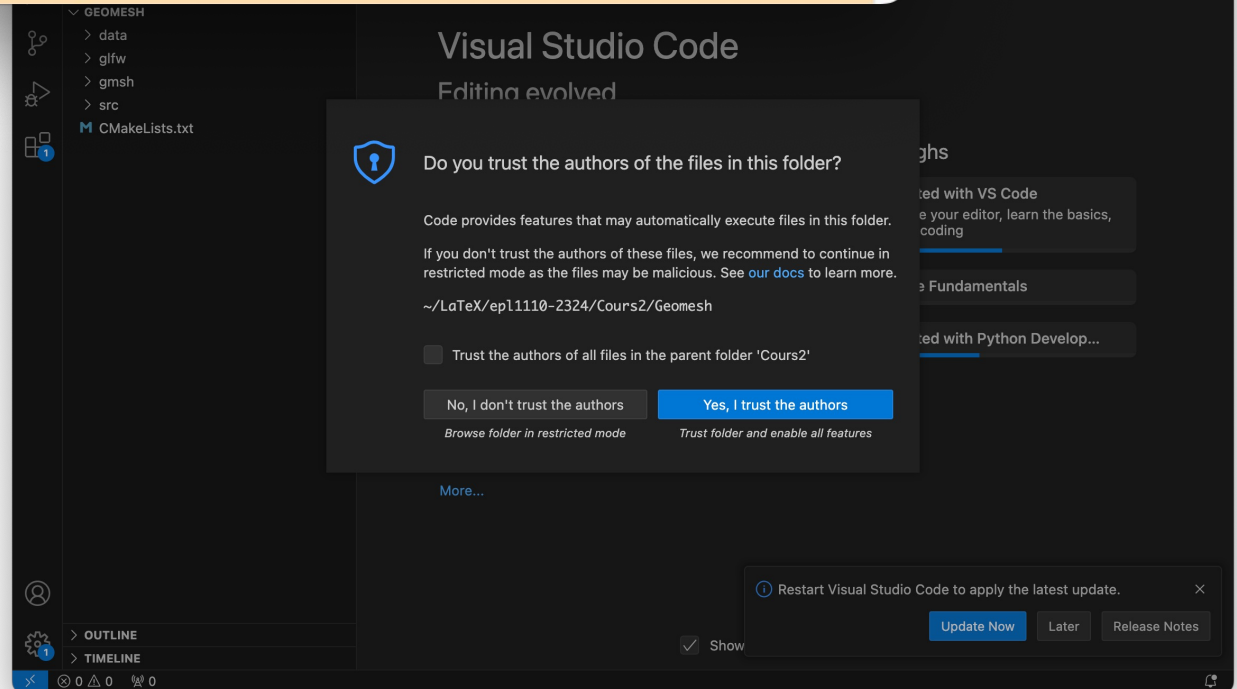
Ajouter la librairie gms
dans un répertoire gms



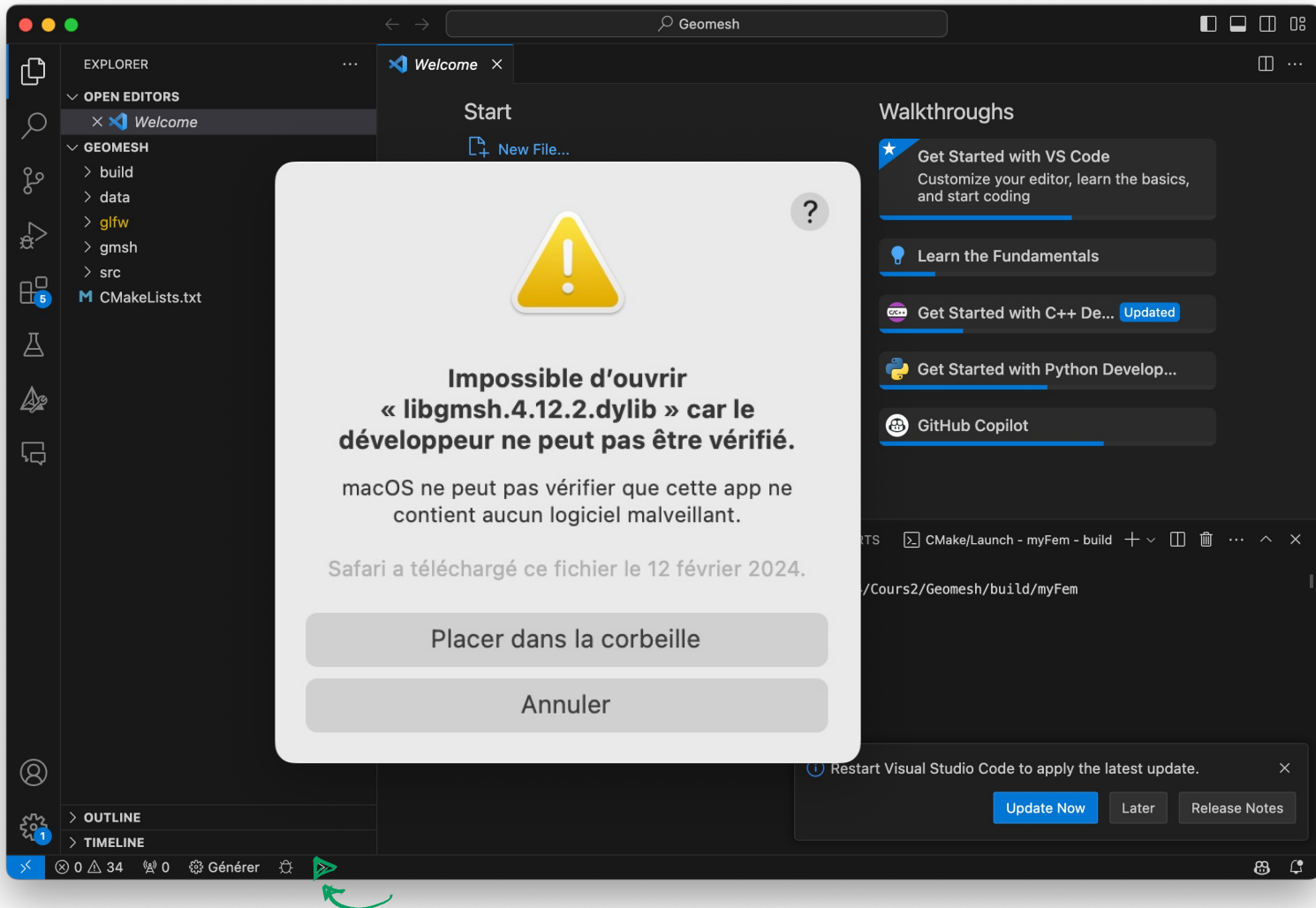
Et lancer vscode

```
Cours2 — -bash — 74x7

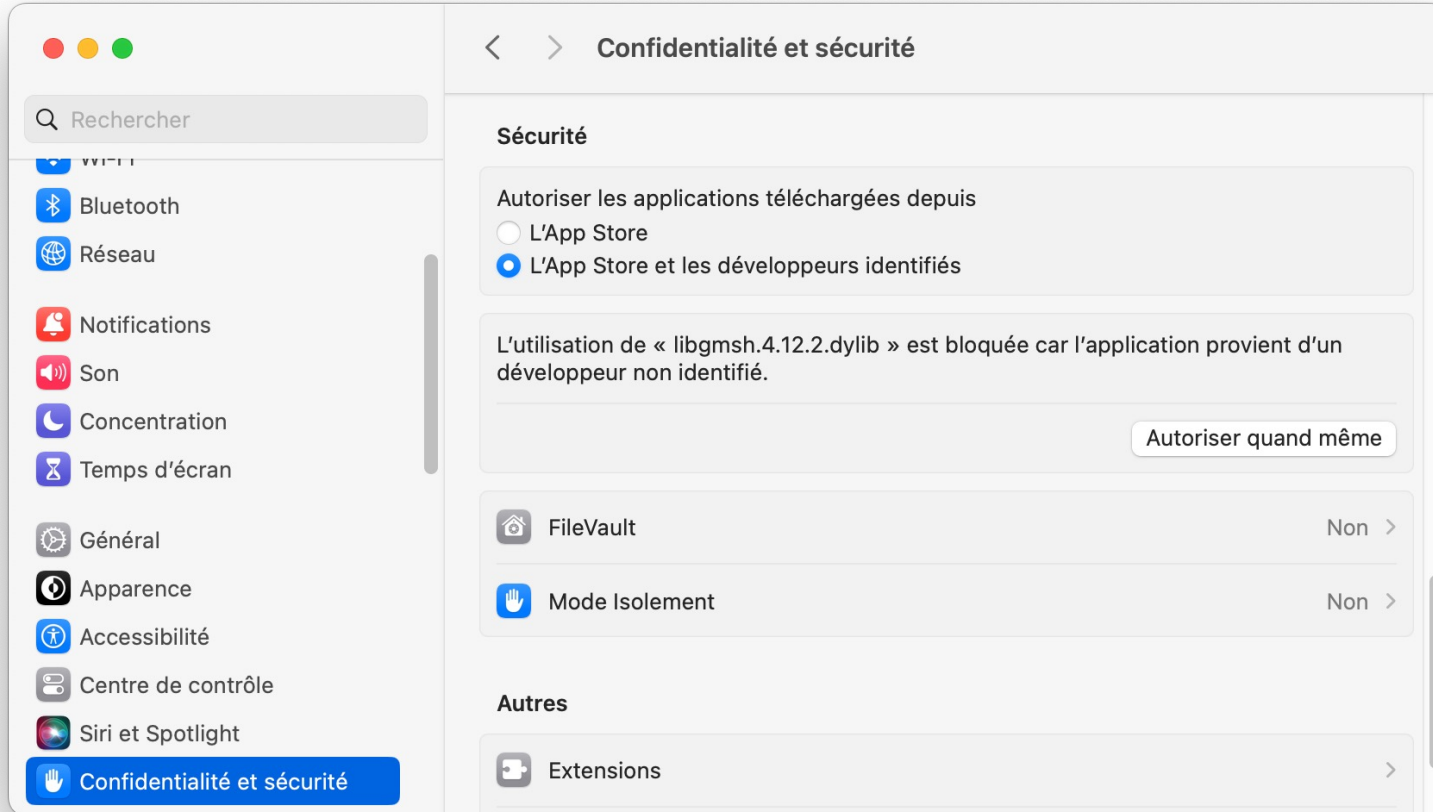
(base) mac-1U0-352:Cours2 v1$ ls
GeoMesh          GeoMesh.zip
(base) mac-1U0-352:Cours2 v1$ ls Geomesh
CMakeLists.txt  glfw          src
data            gmesh
(base) mac-1U0-352:Cours2 v1$ code Geomesh
(base) mac-1U0-352:Cours2 v1$
```



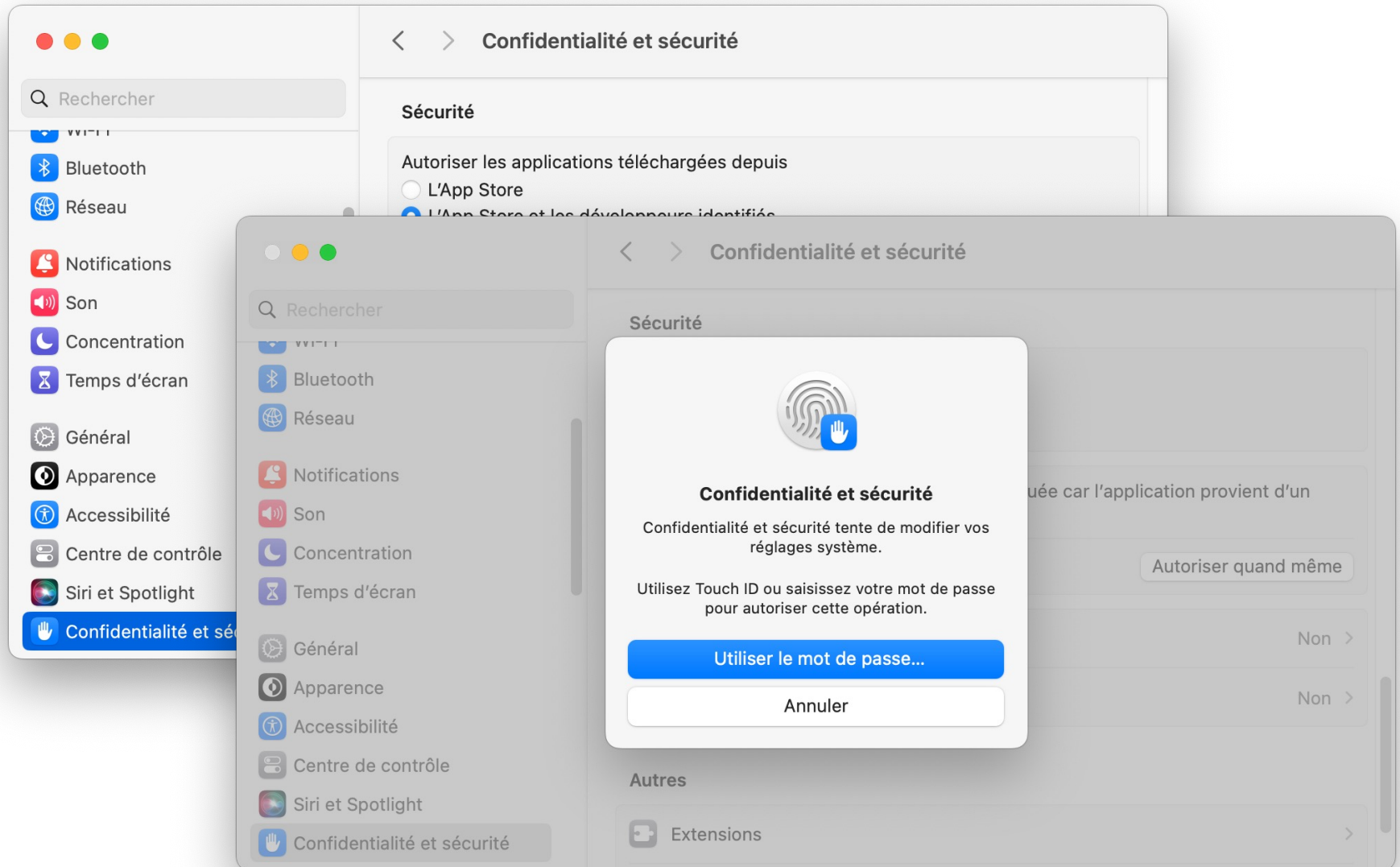
Et zou !



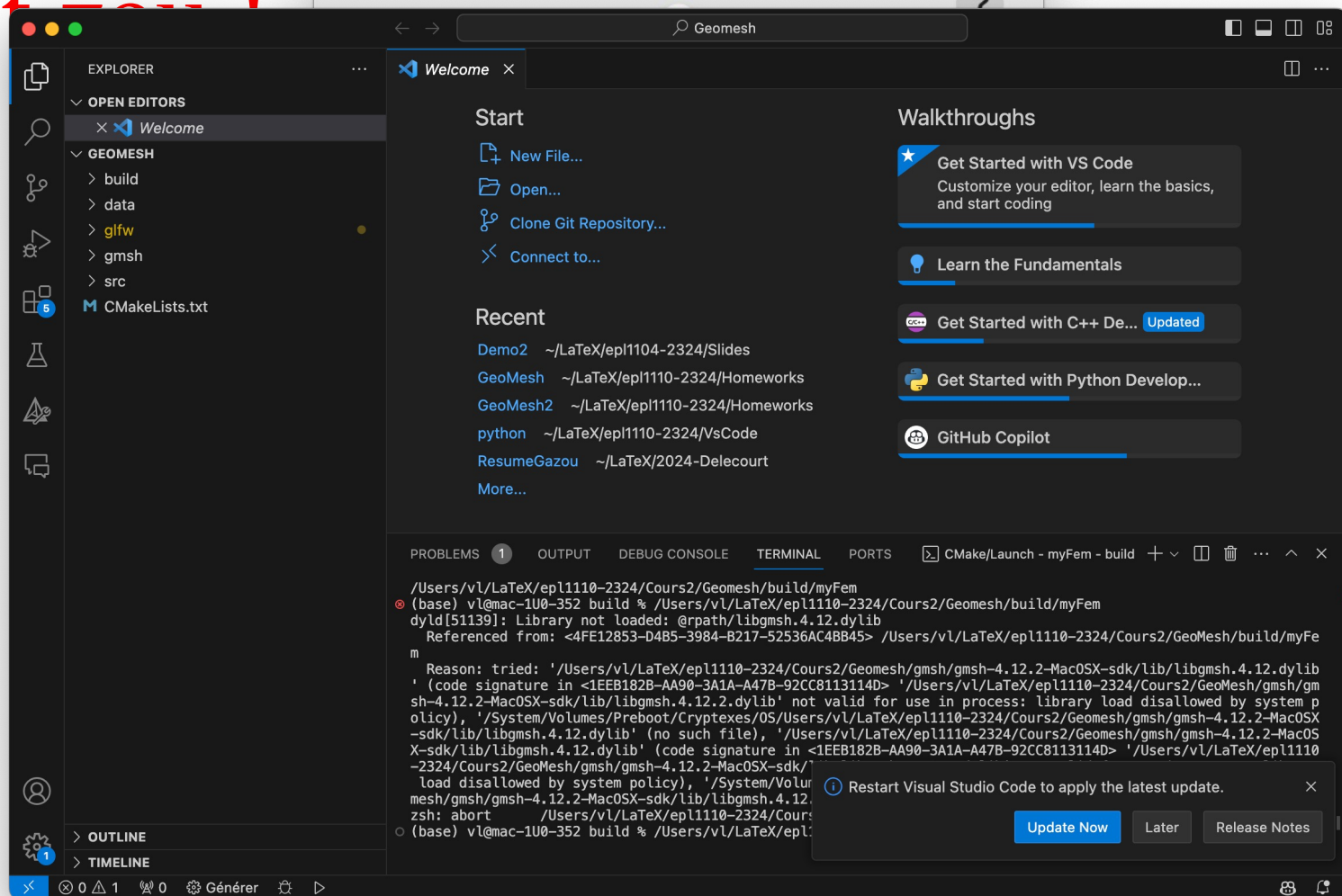
Non, non :
Remacle n'est pas malveillant !



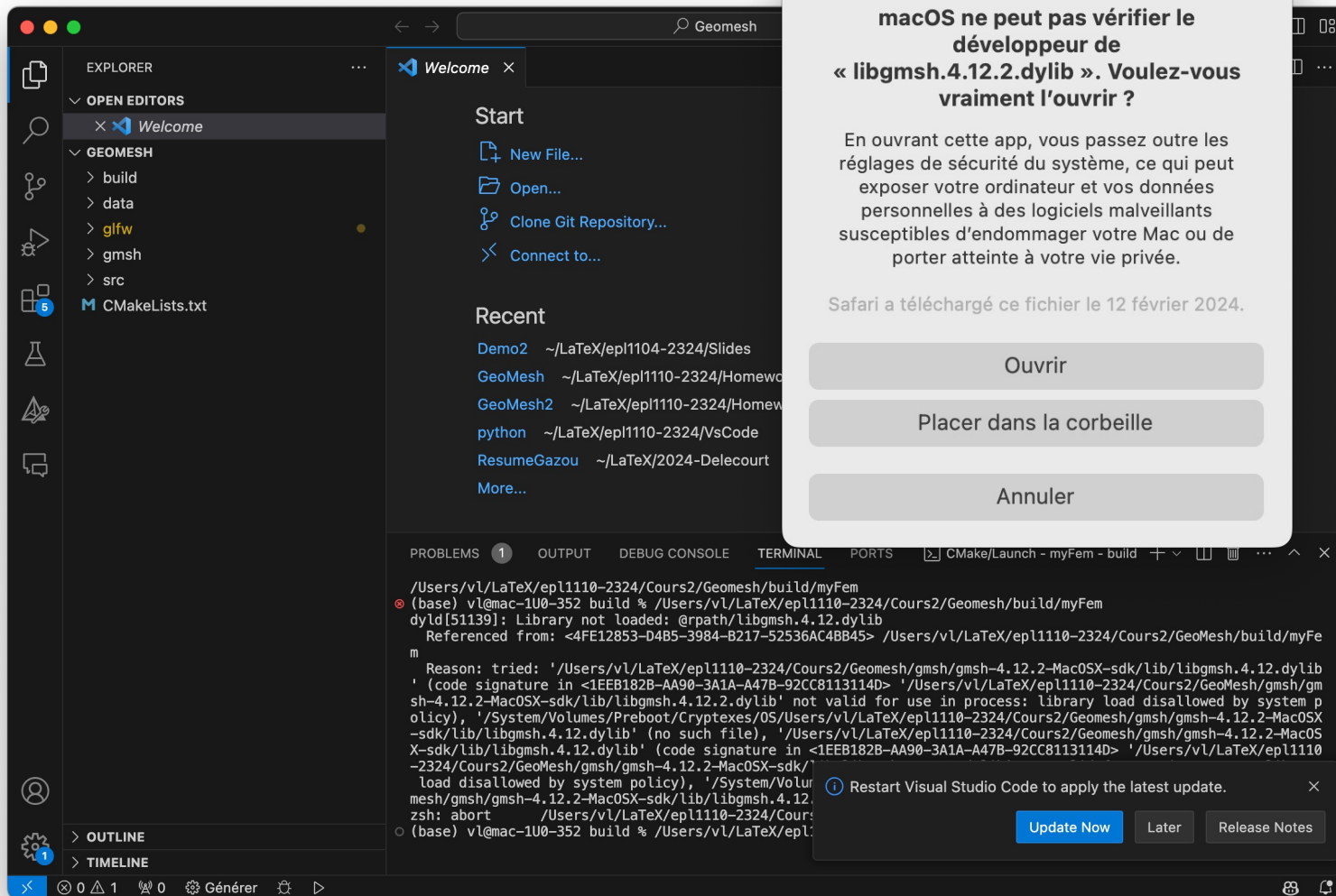
Non, non :
Remacle n'est pas malveillant !



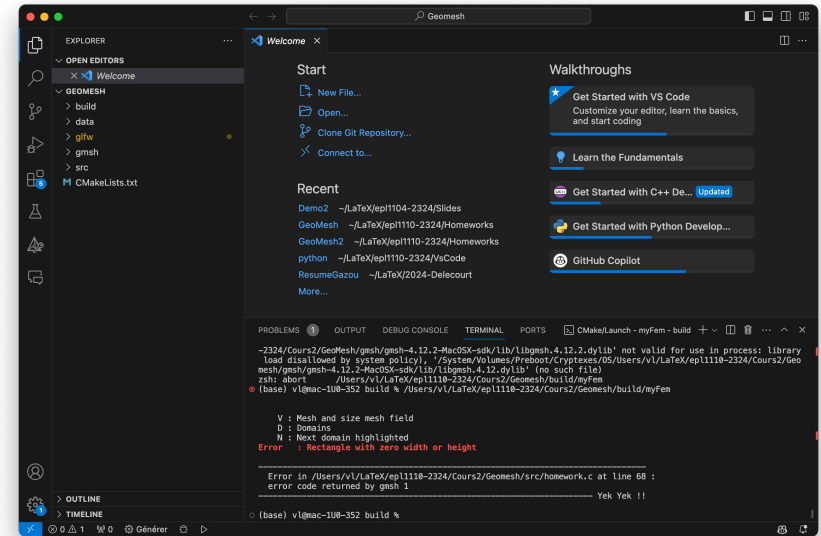
Errors!



Et zou !



Et bof !



```
PROBLEMS 1 OUTPUT DEBUG CONSOLE TERMINAL PORTS CMake/Launch - myFem - build + - [ ] [X] ... ^ X

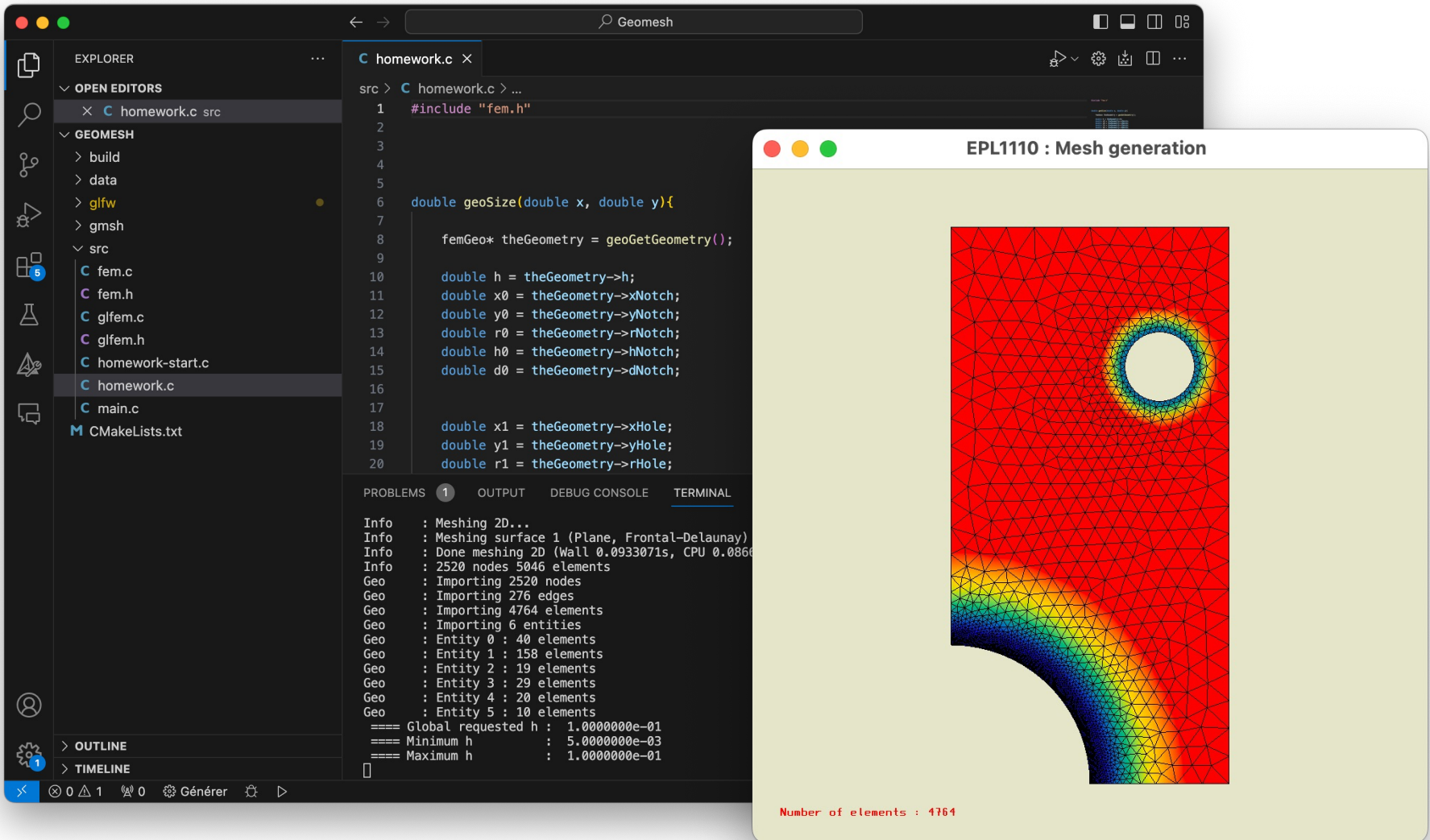
-2324/Cours2/GeoMesh/gmsh/gmsh-4.12.2-MacOSX-sdk/lib/libgmsh.4.12.2.dylib' not valid for use in process: library
load disallowed by system policy), '/System/Volumes/Preboot/Cryptexes/OS/Users/vl/LaTeX/epl1110-2324/Cours2/Geo
mesh/gmsh/gmsh-4.12.2-MacOSX-sdk/lib/libgmsh.4.12.dylib' (no such file)
zsh: abort      /Users/vl/LaTeX/epl1110-2324/Cours2/Geomesh/build/myFem
(base) vl@mac-1U0-352 build % /Users/vl/LaTeX/epl1110-2324/Cours2/Geomesh/build/myFem

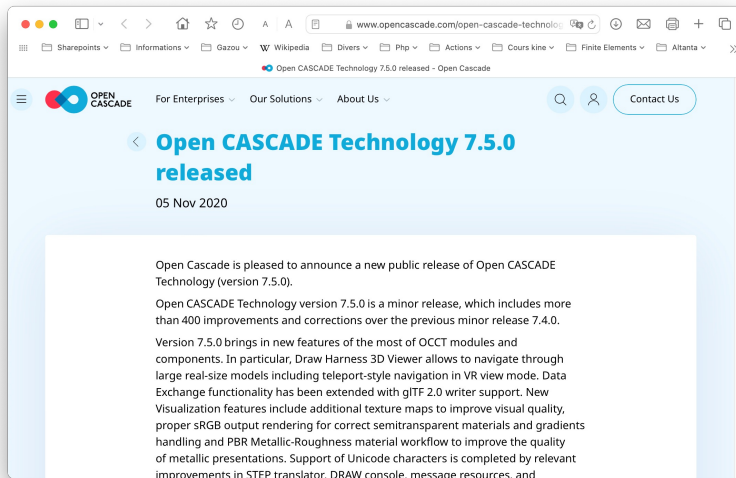
V : Mesh and size mesh field
D : Domains
N : Next domain highlighted
Error   : Rectangle with zero width or height

-----
Error in /Users/vl/LaTeX/epl1110-2324/Cours2/Geomesh/src/homework.c at line 68 :
error code returned by gmsh 1
----- Yek Yek !!

(base) vl@mac-1U0-352 build %
```

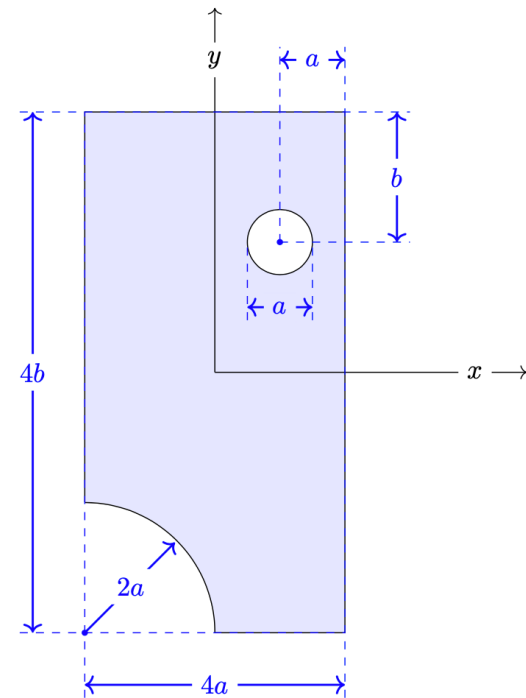
Et après avoir fait le devoir !





Définir la géométrie avec OpenCascade !

```
double Lx = 1.0;
double Ly = 2.0;
theGeometry->LxPlate = Lx;
theGeometry->LyPlate = Ly;
theGeometry->xPlate = 0.0;
theGeometry->yPlate = 0.0;
theGeometry->xHole = Lx / 4.0;
theGeometry->yHole = Ly / 4.0;
theGeometry->rHole = Lx / 8.0;
theGeometry->xNotch = -Lx / 2.0;
theGeometry->yNotch = -Ly / 2.0;
theGeometry->rNotch = Lx / 2.0;
```



Utiliser la géométrie constructive !

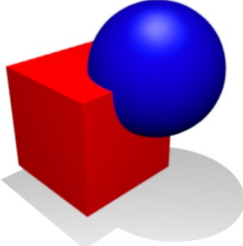
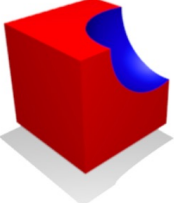
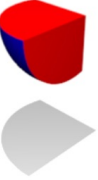
fr.wikipedia.org/wiki/Géométrie_de

Sharepoints Informations Gazou Wikipedia Divers Php Actions Cours kine

W Géométrie de construction de solides — Wikipédia

- Notation
- Géométrie

Opérations booléennes [\[modifier \]](#) [\[modifier le code \]](#)

Union (ou addition)	Différence (ou soustraction)	Intersection
		
L'assemblage des deux objets.	La soustraction d'un objet de l'autre.	La partie commune aux deux objets.

Union (ou addition) [\[modifier \]](#) [\[modifier le code \]](#)

Le résultat est l'assemblage des deux objets. Il y a parfois la possibilité de réaliser cette opération sur plus de deux objets.

Différence (ou soustraction) [\[modifier \]](#) [\[modifier le code \]](#)

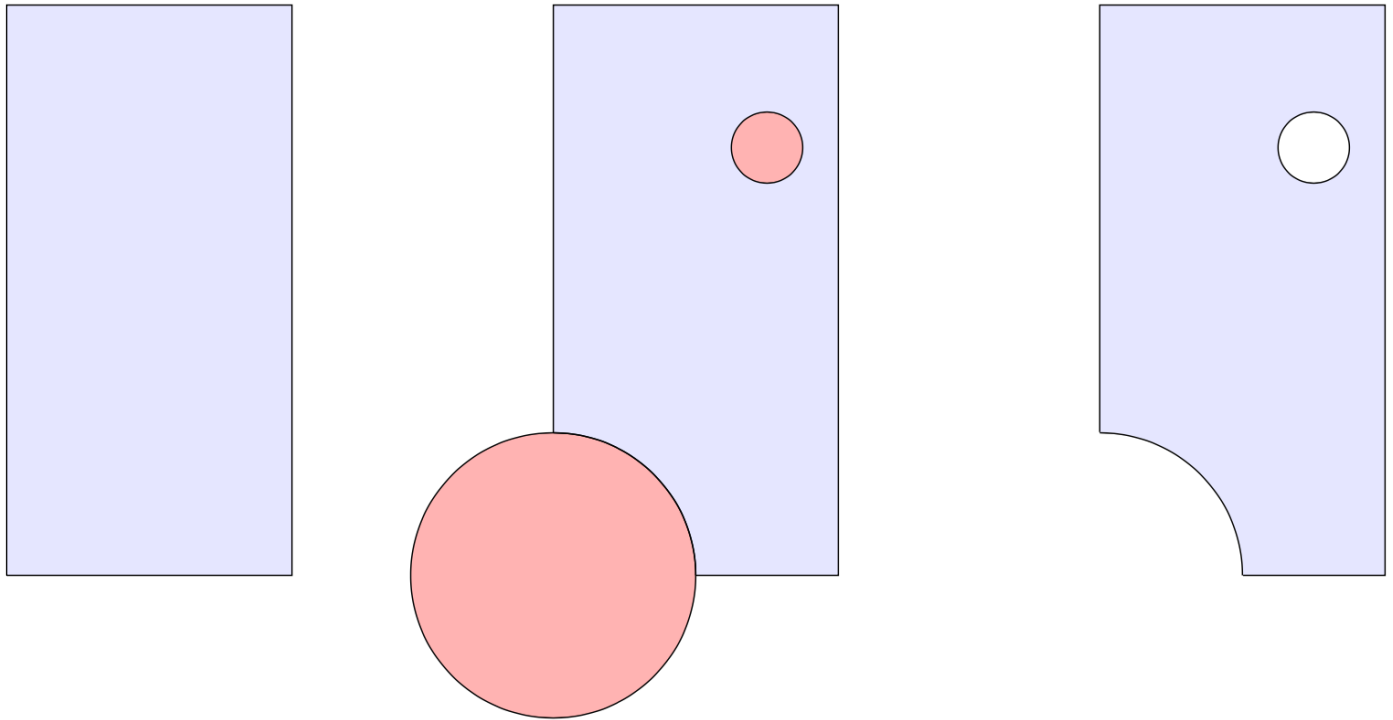
Le résultat est le premier objet moins la partie commune avec le second. Avec certains logiciels ([POV-Ray](#) par exemple), il est possible d'inverser un objet (ce qui revient à faire la soustraction inverse)...

Intersection [\[modifier \]](#) [\[modifier le code \]](#)

Le résultat est la partie commune aux deux objets.

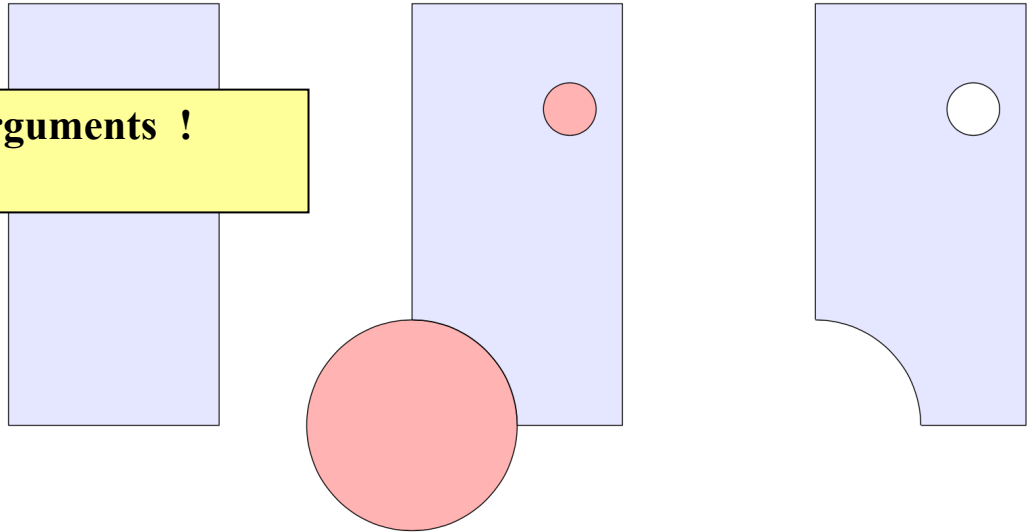
Utiliser la géométrie constructive !

- Créer le rectangle avec la fonction `gmshModelOccAddRectangle`.
- Créer les deux disques avec la fonction `gmshModelOccAddDisk`.
- Retirer chaque disque du rectangle avec la fonction `gmshModelOccCut`.



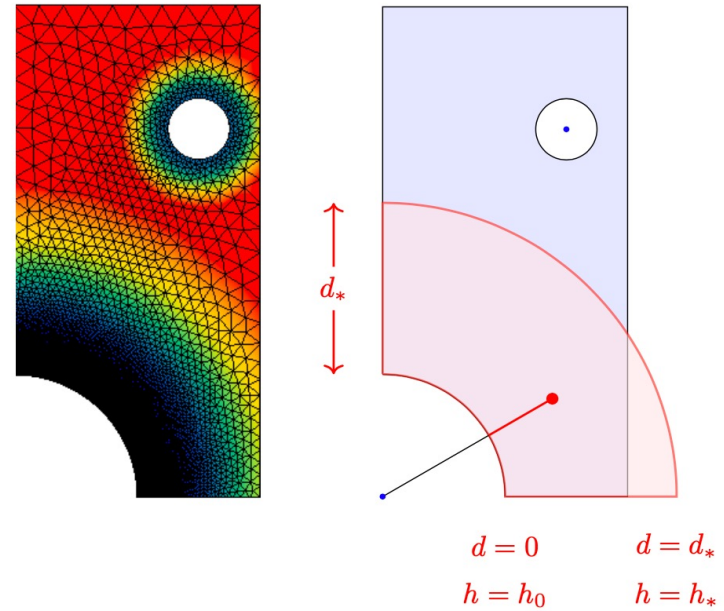
Utiliser l'API de Gmsh pour accéder à OpenCascade !

**Il faut juste trouver les arguments !
Avec l'aide de copilot ?**



```
int ierr;  
int idPlate = gmshModelOccAddRectangle(____, ____', ____', ____', ____', ____', ____', &ierr);  
int idNotch = gmshModelOccAddDisk(____, ____', ____', ____', ____', ____', NULL, 0, NULL, 0, &ierr);  
int idHole = gmshModelOccAddDisk(____, ____', ____', ____', ____', ____', NULL, 0, NULL, 0, &ierr);  
  
int plate[] = {____, ____};  
int notch[] = {____, ____};  
int hole[] = {____, ____};  
gmshModelOccCut(____, ____', ____', ____', NULL, NULL, NULL, NULL, NULL, NULL, -1, 1, 1, &ierr);  
gmshModelOccCut(____, ____', ____', ____', NULL, NULL, NULL, NULL, NULL, NULL, -1, 1, 1, &ierr);
```

Construire le maillage avec une carte de taille !



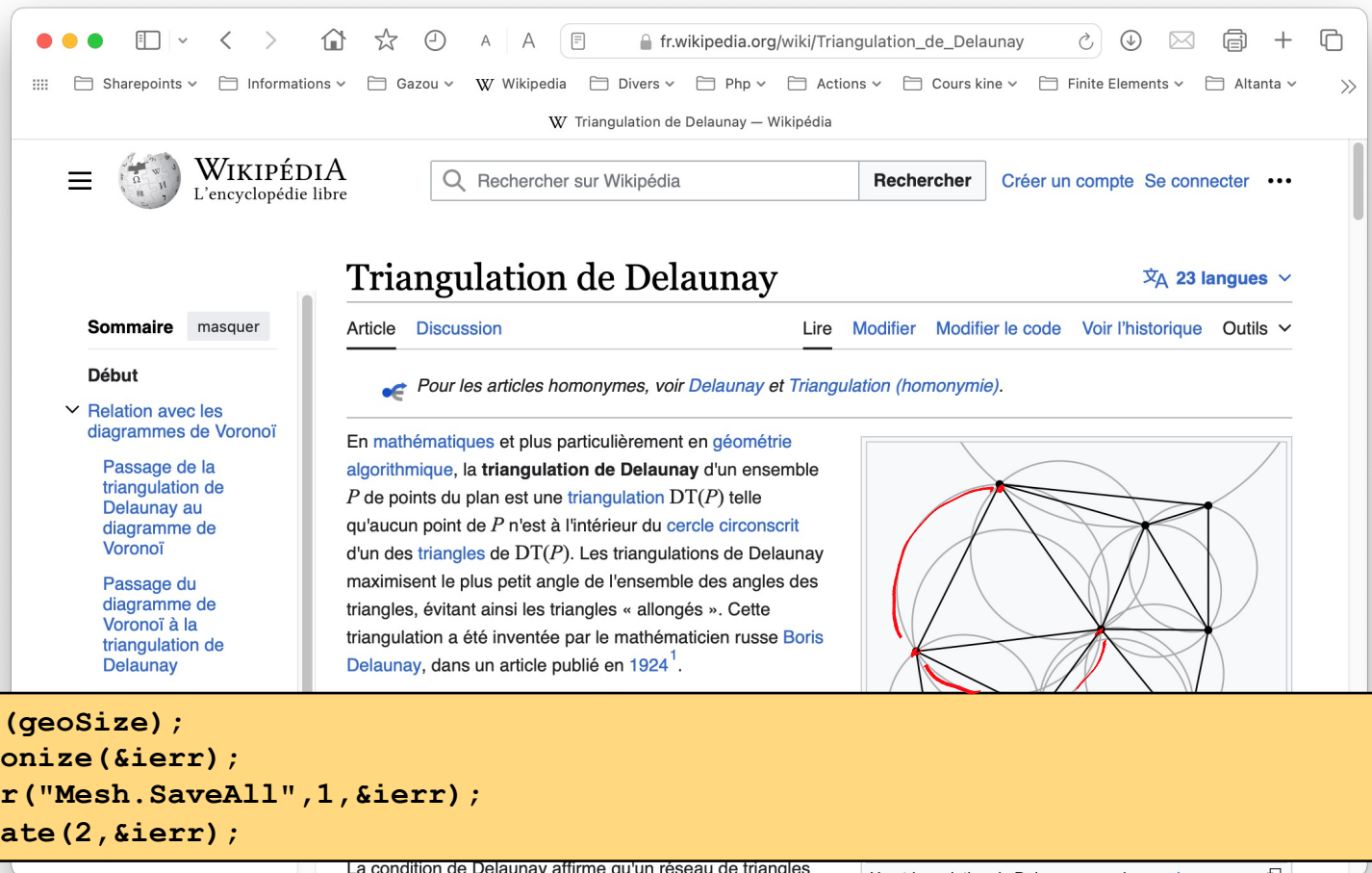
Les données pour construire la carte de taille se résument en cinq paramètres.

```
double h = theGeometry->h;  
double h0 = theGeometry->hNotch;  
double d0 = theGeometry->dNotch;  
double h1 = theGeometry->hHole;  
double d1 = theGeometry->dHole;
```

↓

```
geoSetSizeCallback(geoSize) ;  
gmshModelOccSynchronize(&ierr) ;  
gmshOptionSetNumber("Mesh.SaveAll",1,&ierr) ;  
gmshModelMeshGenerate(2,&ierr) ;
```

Ensuite, on construit le plus joli maillage possible avec la densité spécifiée par la carte de taille avec une triangulation de Delaunay !

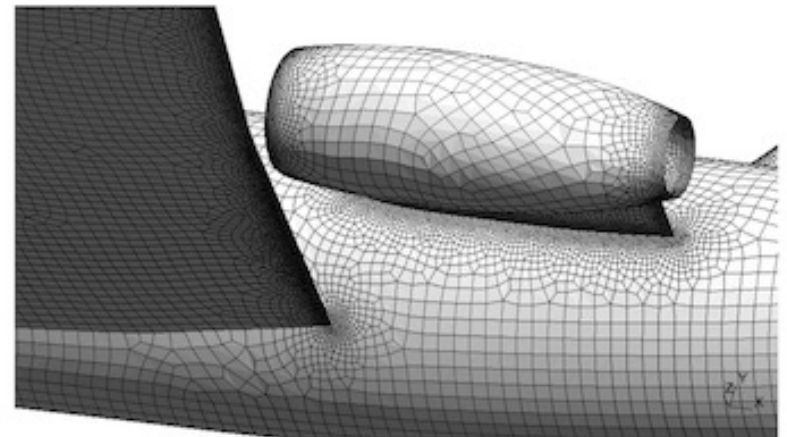
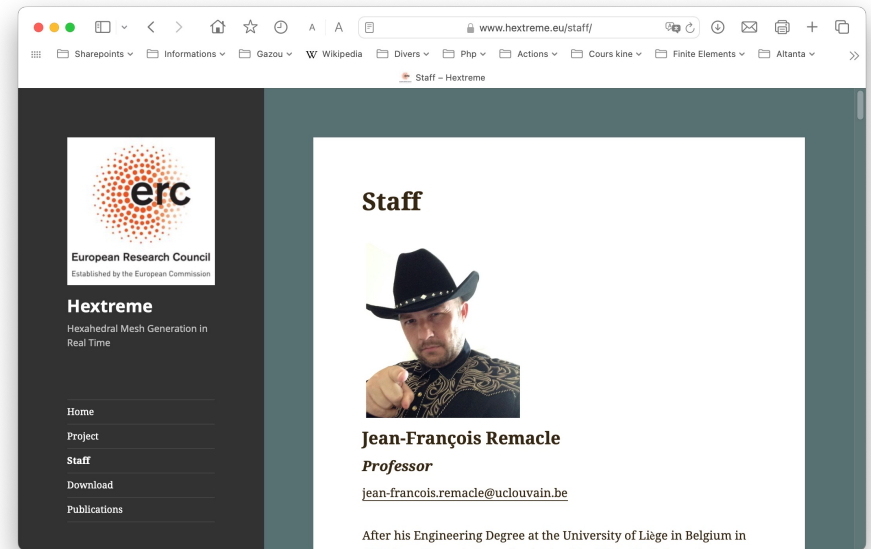
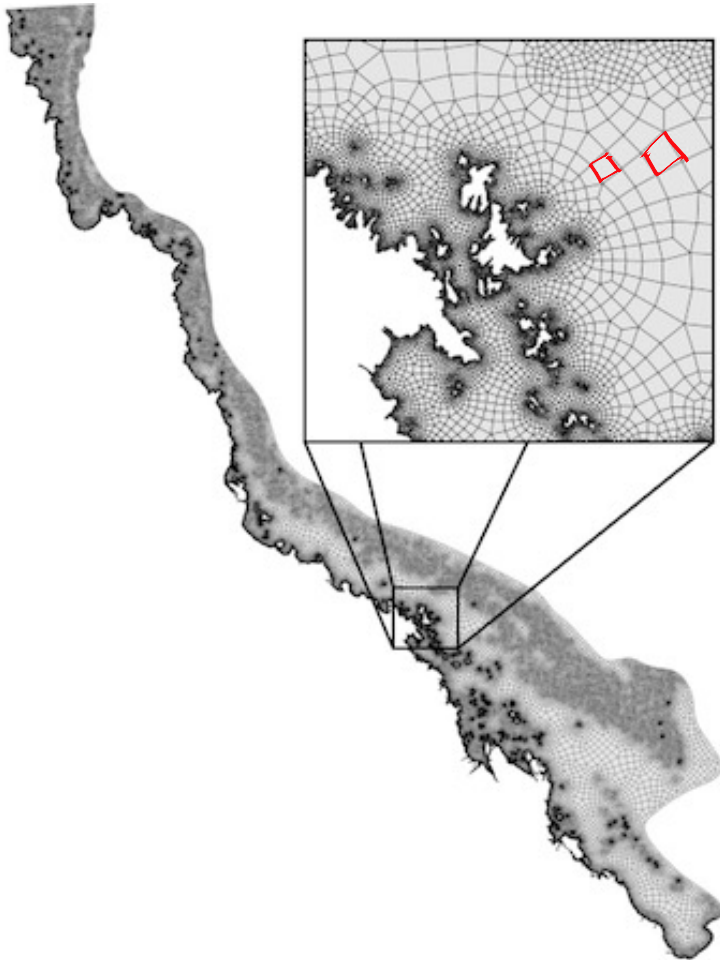


The screenshot shows the Wikipedia page for 'Triangulation de Delaunay'. The page title is 'Triangulation de Delaunay' with 23 languages available. The article text explains that in mathematics, particularly in geometry, a Delaunay triangulation of a set of points P in the plane is a triangulation $DT(P)$ such that no point of P lies inside the circumcircle of any triangle in $DT(P)$. It mentions that this triangulation maximizes the minimum angle of the triangles, avoiding 'sliver' triangles, and was invented by the Russian mathematician Boris Delaunay in 1924.

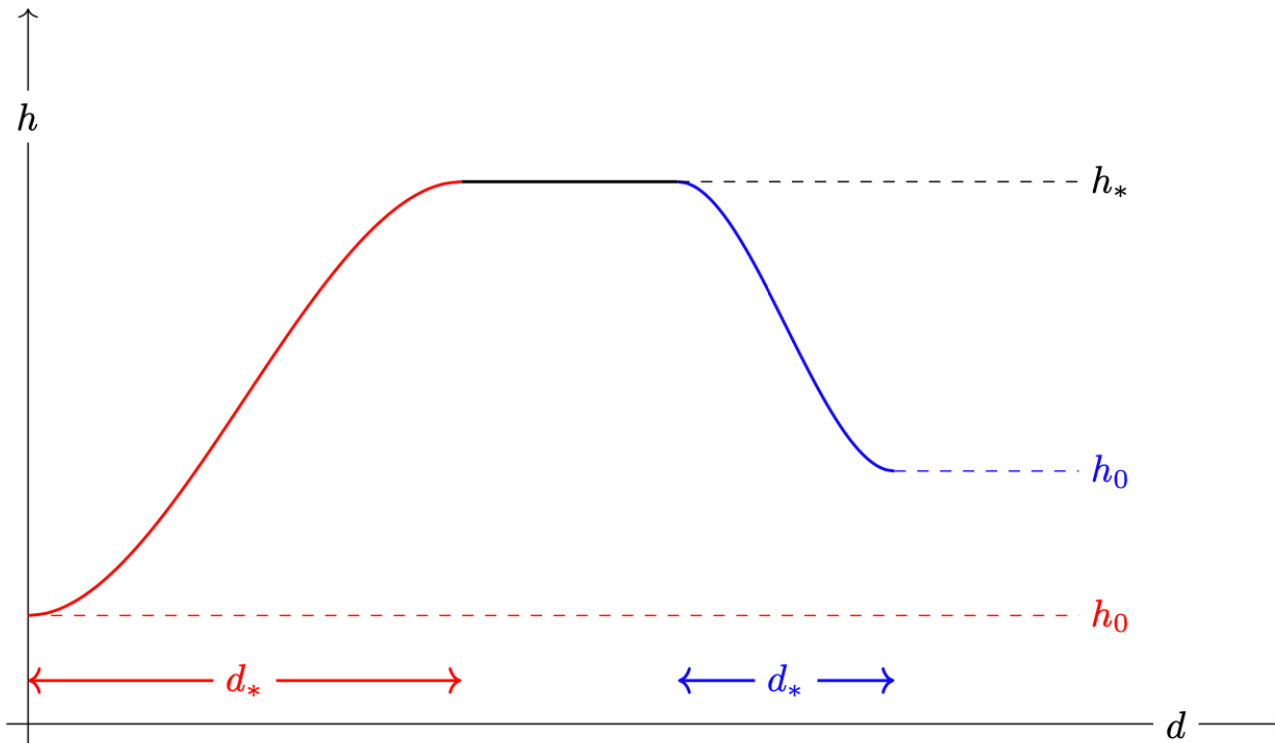
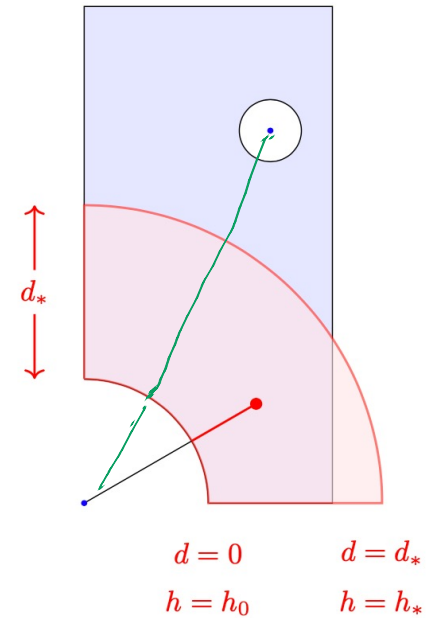
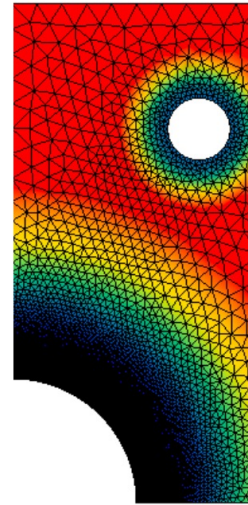
Below the article text, there is a diagram illustrating a Delaunay triangulation. It shows a set of points connected by lines to form triangles. Several circles are drawn around the points, representing the circumcircles of the triangles. The diagram highlights that no point lies inside any of these circumcircles, which is the defining property of a Delaunay triangulation.

```
geoSetSizeCallback (geoSize) ;
gmshModelOccSynchronize (&ierr) ;
gmshOptionSetNumber ("Mesh.SaveAll", 1, &ierr) ;
gmshModelMeshGenerate (2, &ierr) ;
```

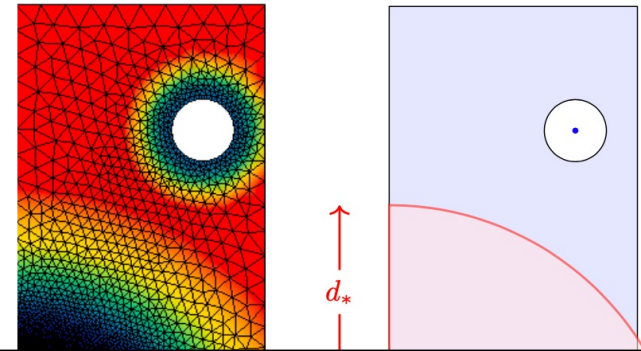
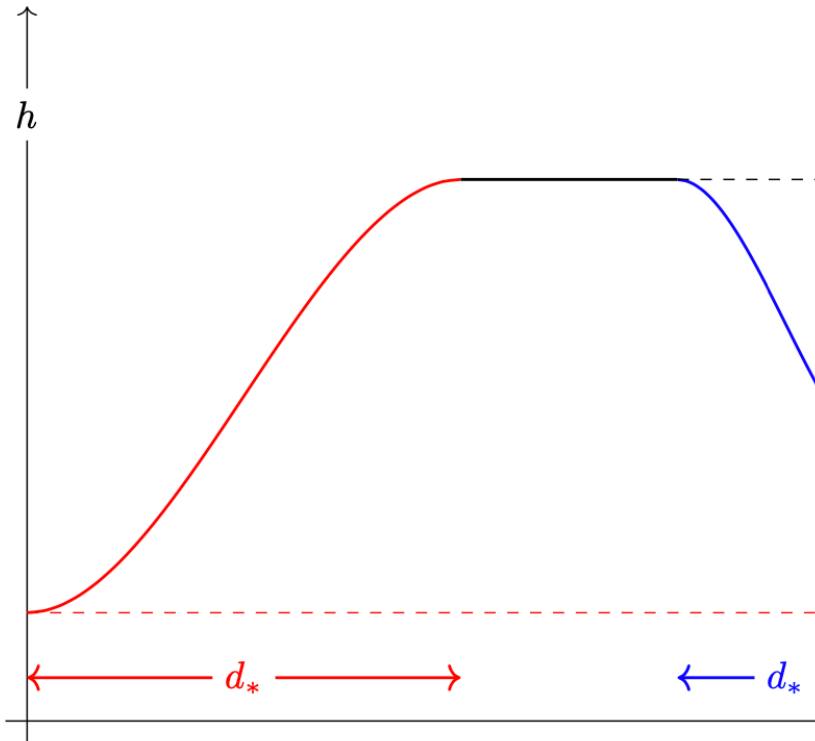
Triangles, quadrilatères, tétraèdres, hexaèdres ?



Définir une carte de taille !



Définir une carte de taille !



```
double geoSize(double x,double y)
{
    femGeo* theGeometry = geoGetGeometry();

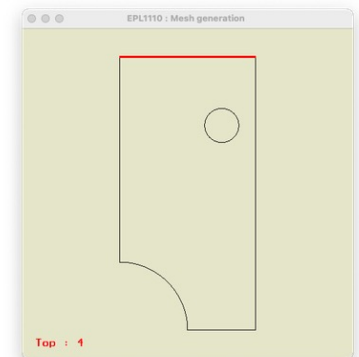
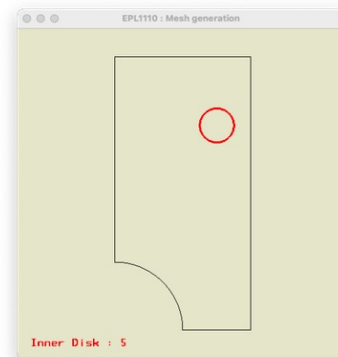
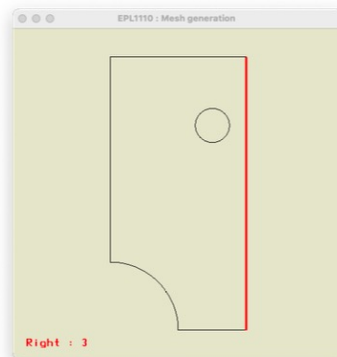
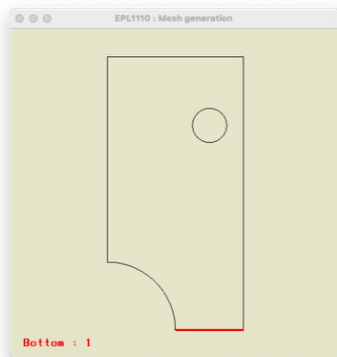
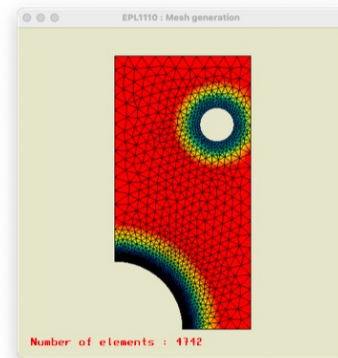
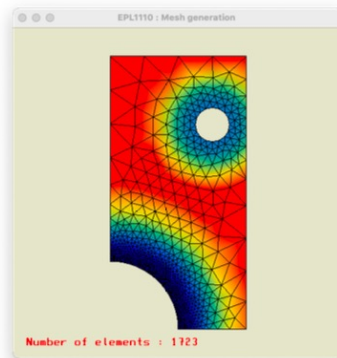
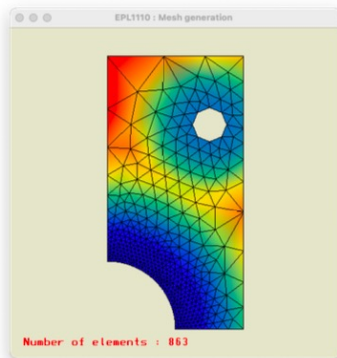
    double h = theGeometry->h;
    double x0 = theGeometry->xNotch;
    double y0 = theGeometry->yNotch;
    double r0 = theGeometry->rNotch;
    double h0 = theGeometry->hNotch;
    double d0 = theGeometry->dNotch;

    double x1 = theGeometry->xHole;
    double y1 = theGeometry->yHole;
    double r1 = theGeometry->rHole;
    double h1 = theGeometry->hHole;
    double d1 = theGeometry->dHole;

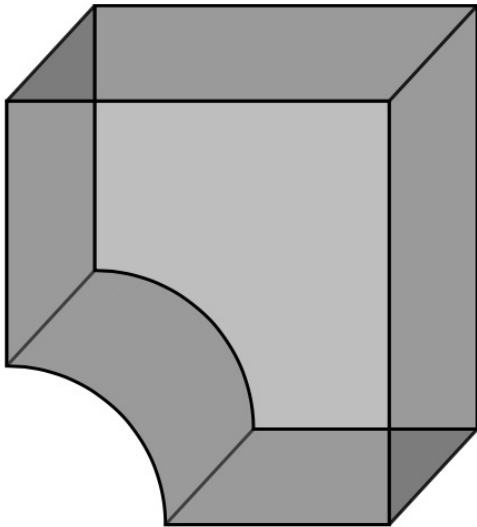
    //
    //      A modifier !
    //

    return h;
}
```

Et tout cela pourquoi ?



**La géométrie est une construction abstraite
définie par une structure hiérarchique
qui décrit la connectivité.**



Construction abstraite :

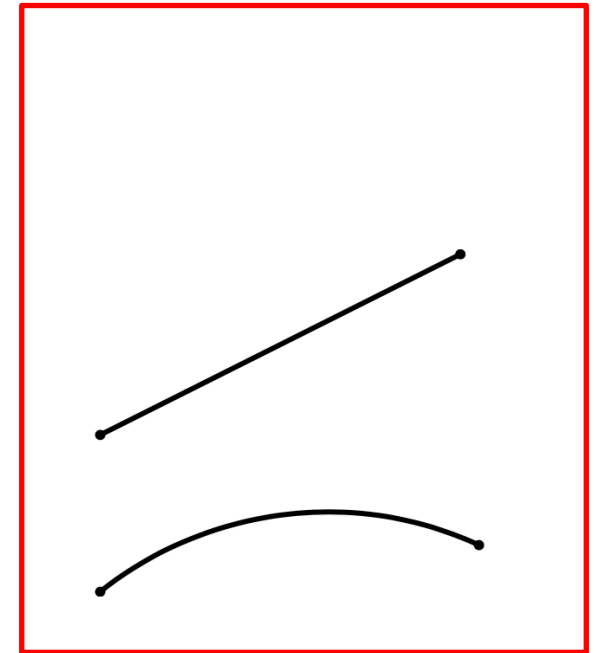
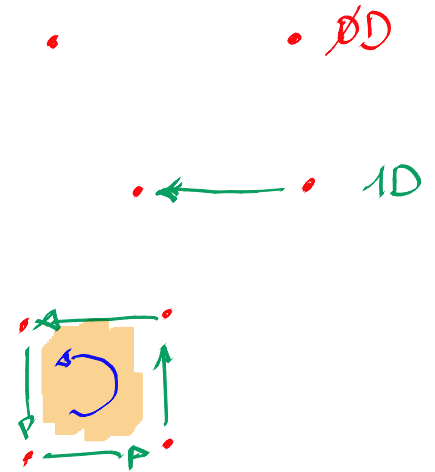
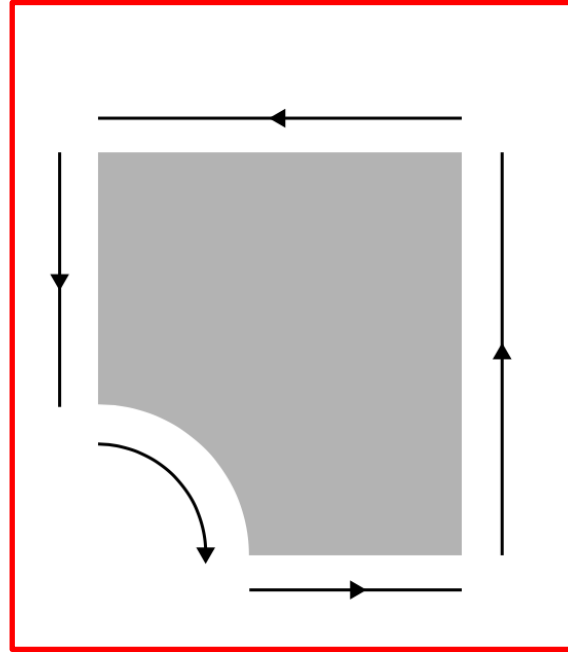
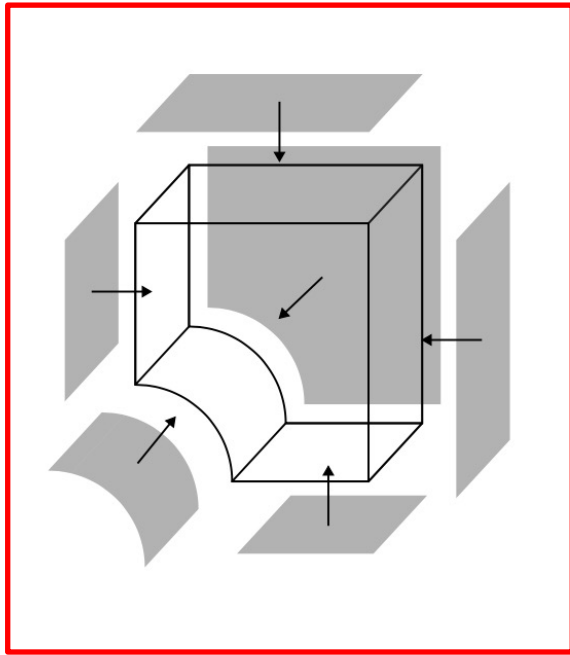
$\mathcal{V}$: volume décrit par ses surfaces

$\mathcal{S}$: surface décrit par ses arêtes

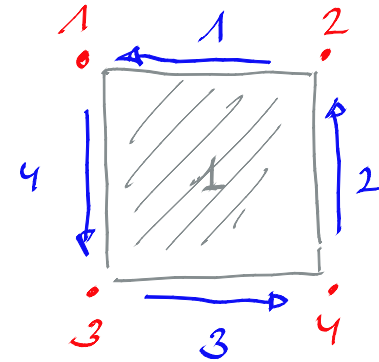
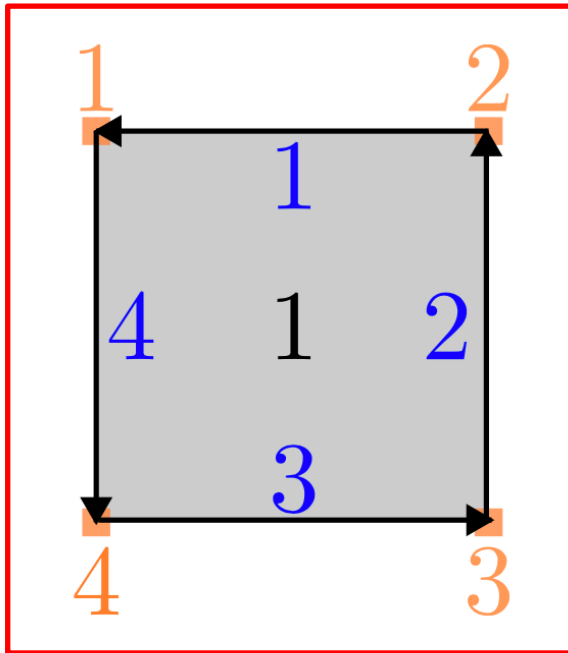
$\mathcal{A}$: arête décrit par ses noeuds

$\mathcal{N}$: noeud décrit par un indice

Topologie d'une géométrie

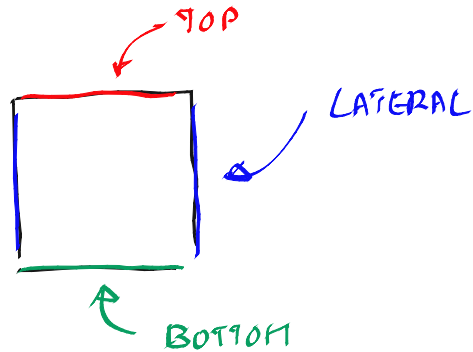


Entités
topologiques



Dim	Tag	Dim	Tag	Dim	Tag
0	1	1	1	2	1
0	2	1	2		
0	3	1	3		
0	4	1	4		

Exemple simple !



Trois tableaux
décrivent la géométrie

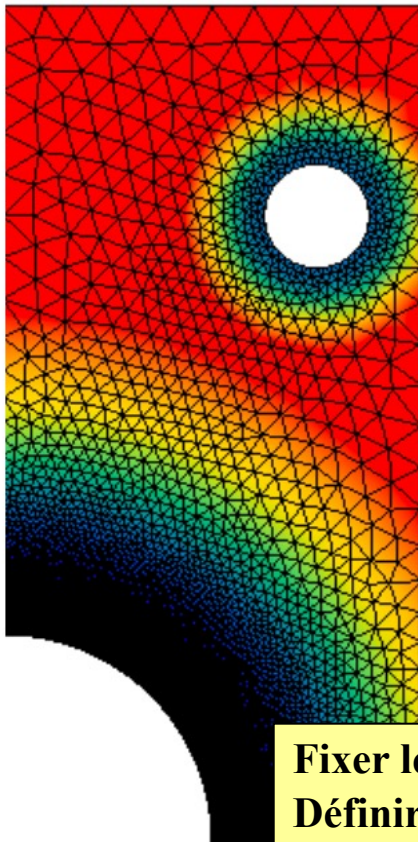
Dim	Tag	x	y
0	1	-1.0	1.0
0	2	1.0	1.0
0	3	1.0	-1.0
0	4	-1.0	-1.0

Dim	Tag	Dim	Tag	Dim	Tag
0	1	1	1	2	1
0	2	1	2		
0	3	1	3		
0	4	1	4		

Dim	Tag	Nom
1	1	Top
1	2	Lateral
1	3	Bottom
1	4	Lateral

Fixer les coordonnées des points
Définir la topologie ou connectivité des éléments
Associer des domaines physiques à des éléments

Trois tableaux décrivent le maillage final



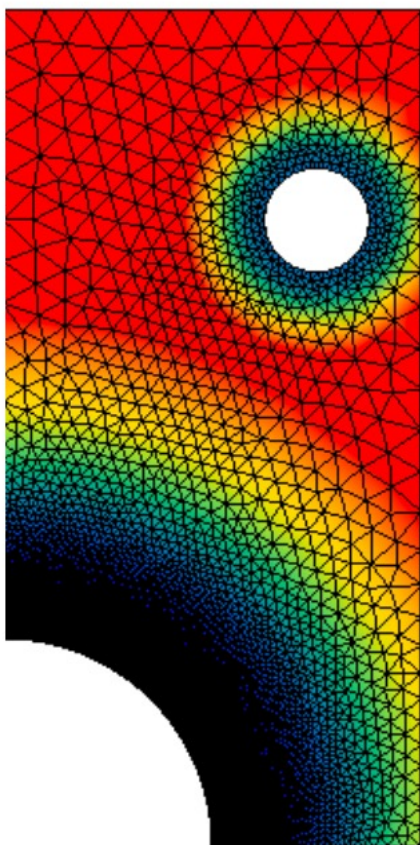
```
typedef struct {  
    int nNodes;  
    double *X;  
    double *Y;  
} femNodes;
```

```
typedef struct {  
    int nLocalNode;  
    int nElem;  
    int *elem;  
    femNodes *nodes;  
} femMesh;
```

```
typedef struct {  
    femMesh *mesh;  
    int nElem;  
    int *elem;  
    char name[MAXNAME];  
} femDomain;
```

Fixer les coordonnées des points
Définir la topologie ou connectivité des éléments
Associer des domaines physiques à des éléments

Et voilà
le maillage !



Number of nodes 2520

```

0 :  3.7500000e-01  5.0000000e-01
1 :  0.0000000e+00 -1.0000000e+00
2 : -5.0000000e-01 -5.0000000e-01
3 :  5.0000000e-01 -1.0000000e+00
4 : -5.0000000e-01  1.0000000e+00
5 :  5.0000000e-01  1.0000000e+00

```

```

2517 : -1.5626551e-01 -6.2635267e-01
2518 : -3.6669678e-02 -7.9420329e-01
2519 : -4.8014465e-03 -8.1397000e-01

```

Number of edges 276

```

0 :      0      6
1 :      6      7

```

```

274 :    274    275
275 :    275     4

```

Number of triangles 4764

```

0 :    513    514    484
1 :   1291   2281   1589

```

```

4762 :    2081    2212    1466
4763 :    1675    1913    1668

```

Number of domains 6

Domain : 0

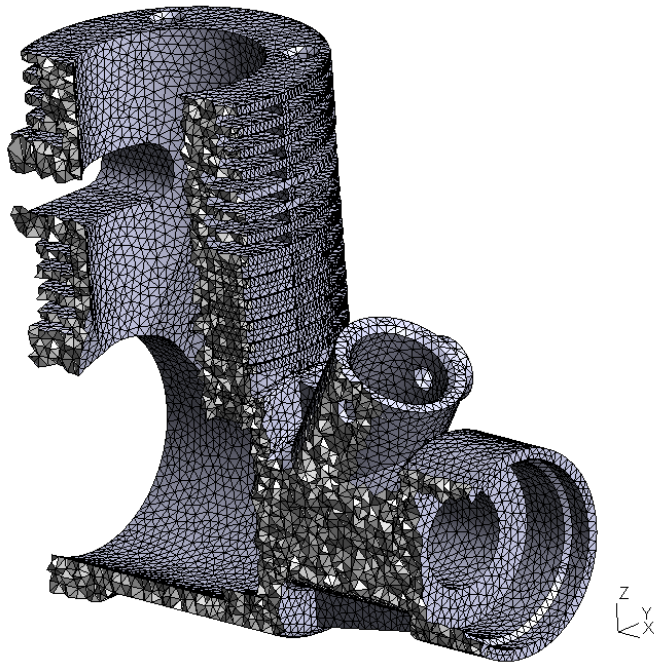
Name : Outer Disk

Number of elements : 40

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39

Domain : 1

Définir localement une fonction sur un maillage



Fonctions de base
spécifiées a priori

$$u(\mathbf{x}) \approx u^h(\mathbf{x}) = \sum_{j=1}^n U_j \tau_j(\mathbf{x})$$

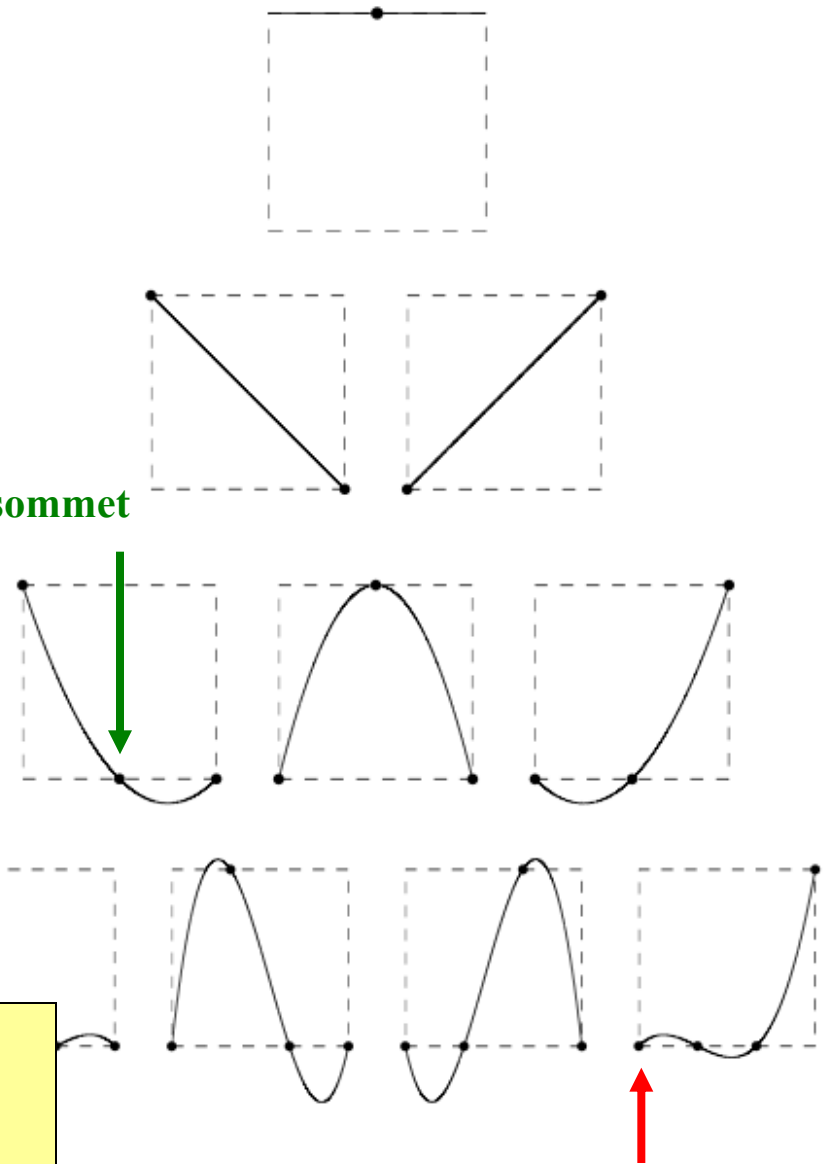
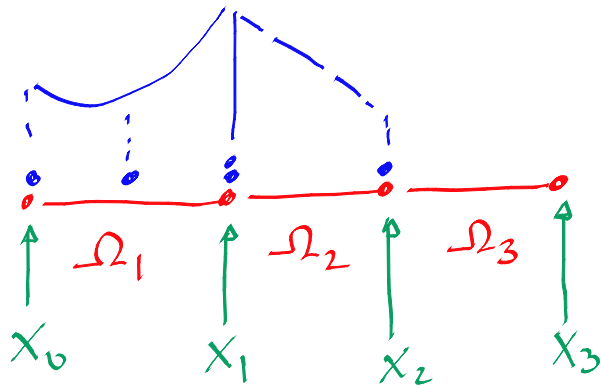
Valeurs nodales inconnues

The problem geometry is divided in small finite elements.

On each element, the solution is approximated
by means of unknown nodal values and given polynomials

Fonctions de forme unidimensionnelles de Lagrange

Ce nœud n'est pas un sommet



Sommet

Nodes and other nodes :-)

Nodes of the meshes = vertices

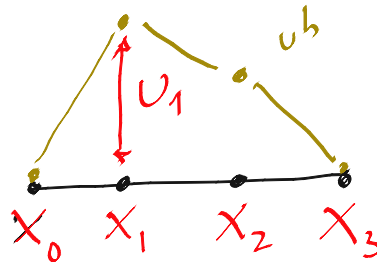
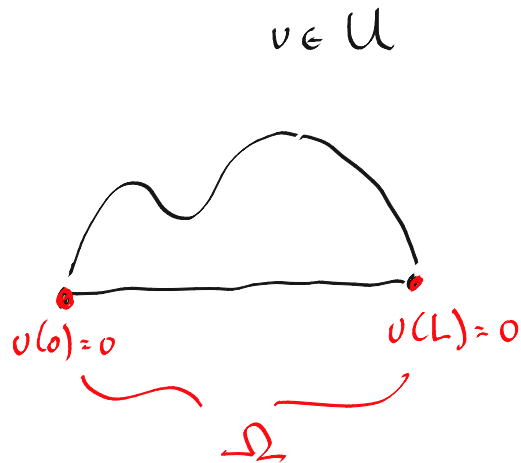
Location of nodal values = nodes

For linear piecewise interpolation, all nodes are vertices

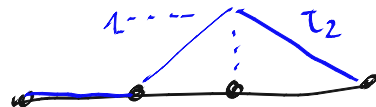
Interpolation linéaire par morceaux : so easy !

$$\tau_i(X_j) = S_{ij}$$

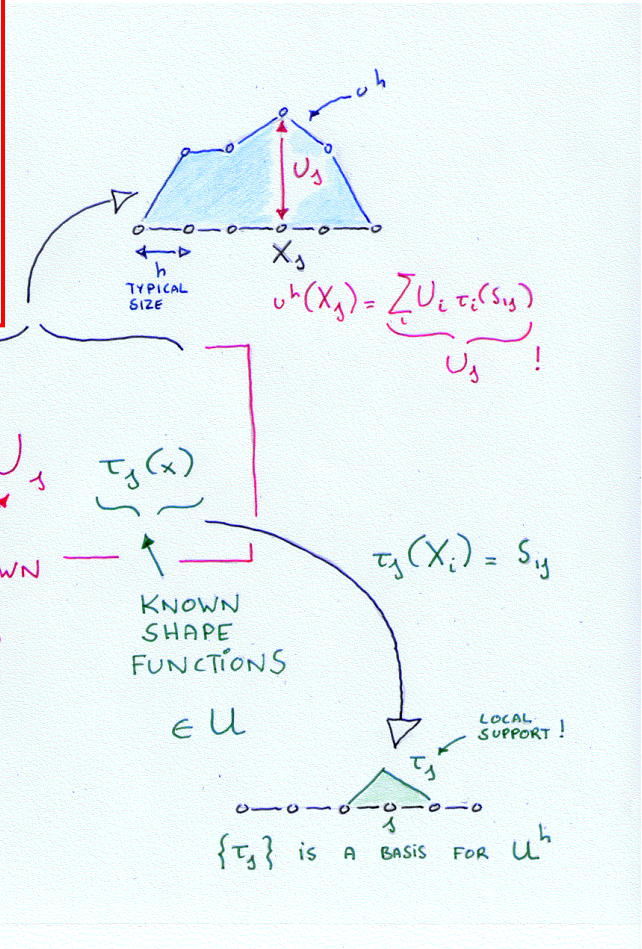
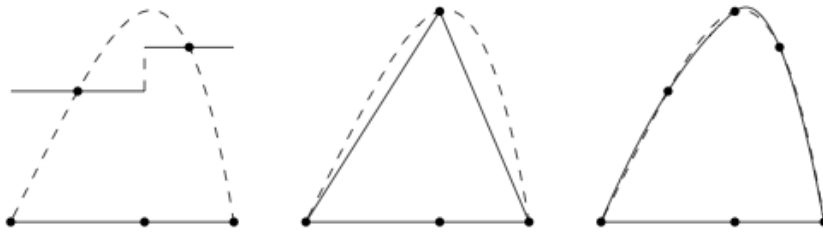
$$\begin{aligned} i=j &\rightarrow S_{ij}=1 \\ i \neq j &\rightarrow S_{ij}=0 \end{aligned}$$



$$u_h = U_1 \tau_1 + U_2 \tau_2$$

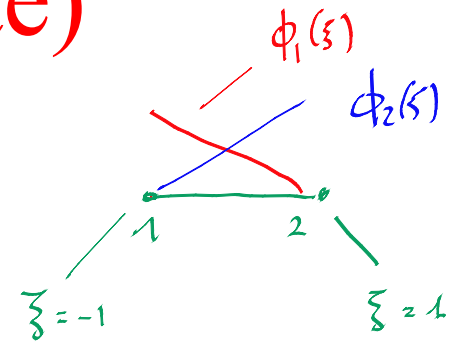


En une dimension...



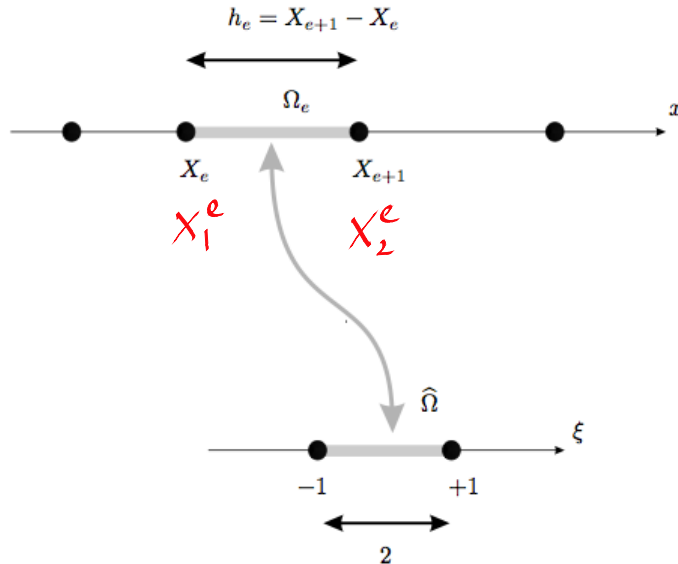
Un élément parent (template)
pour définir
les fonctions
de forme

$$x(\xi) = X_1^e \phi_1(\xi) + X_2^e \phi_2(\xi)$$



$$\phi_1(\xi) = (1 - \xi)/2$$

$$\phi_2(\xi) = (1 + \xi)/2$$

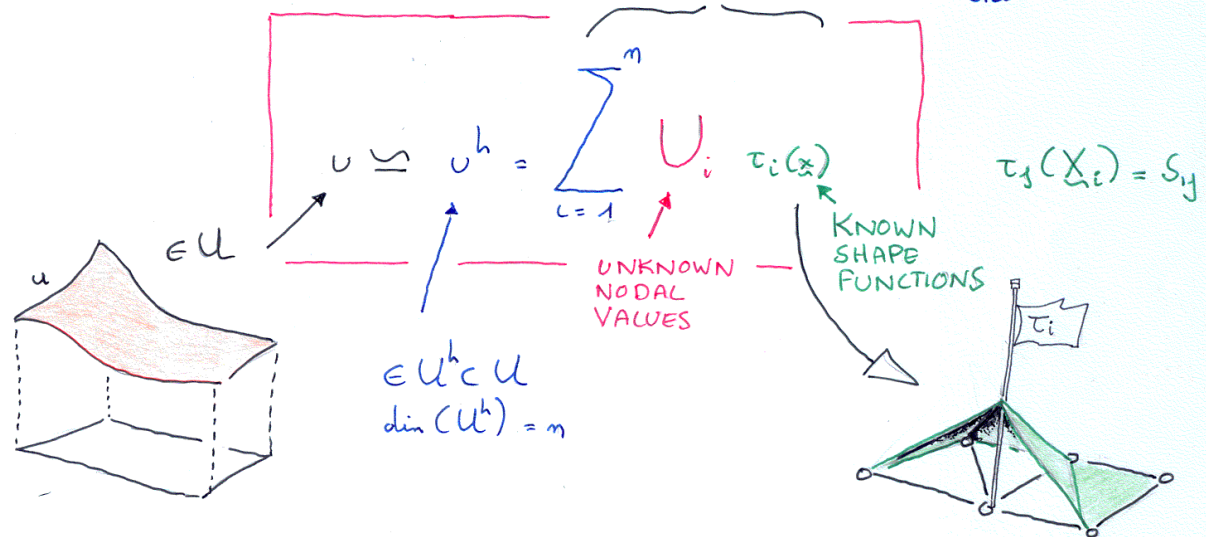
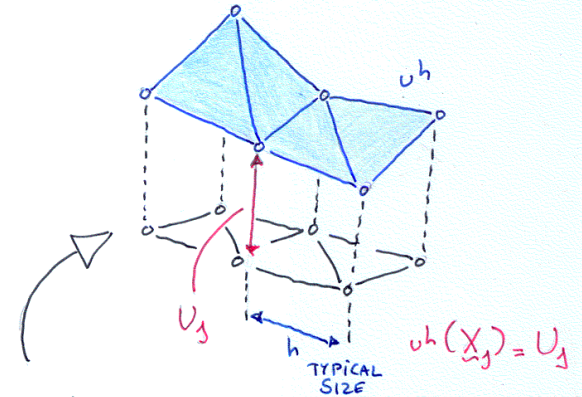
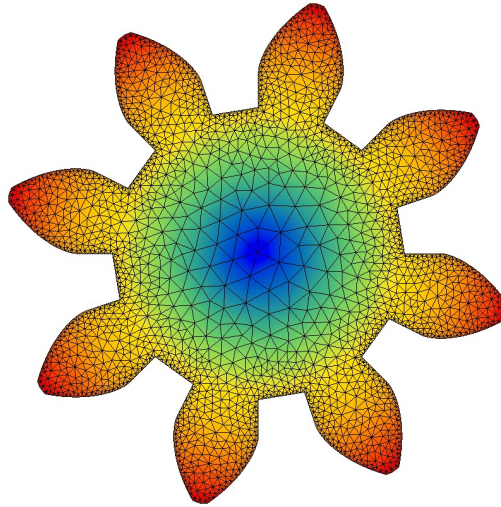


$$x(\xi) = \xi \frac{(X_{e+1} - X_e)}{2} + \frac{(X_{e+1} + X_e)}{2},$$

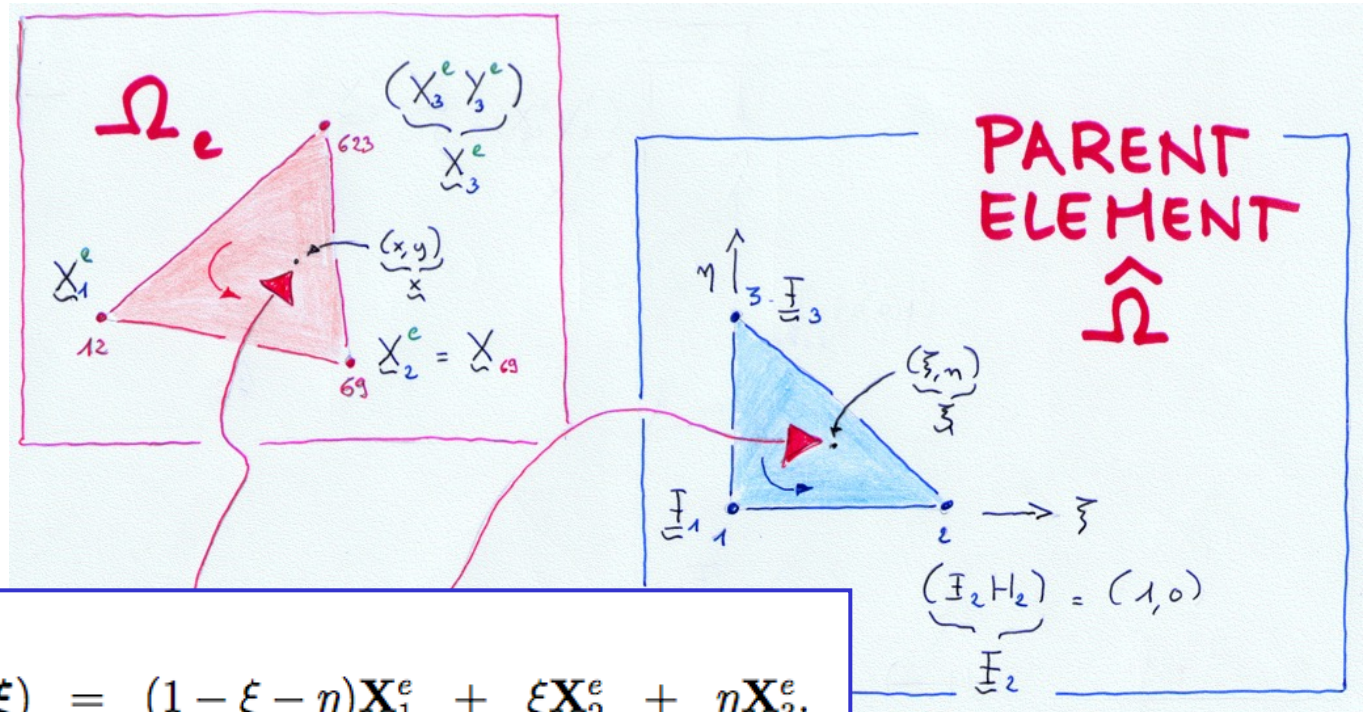
$$\xi(x) = \frac{2x - (X_{e+1} + X_e)}{(X_{e+1} - X_e)}.$$

Isomorphisme linéaire entre l'élément parent
et tous les autres éléments...

En deux dimensions

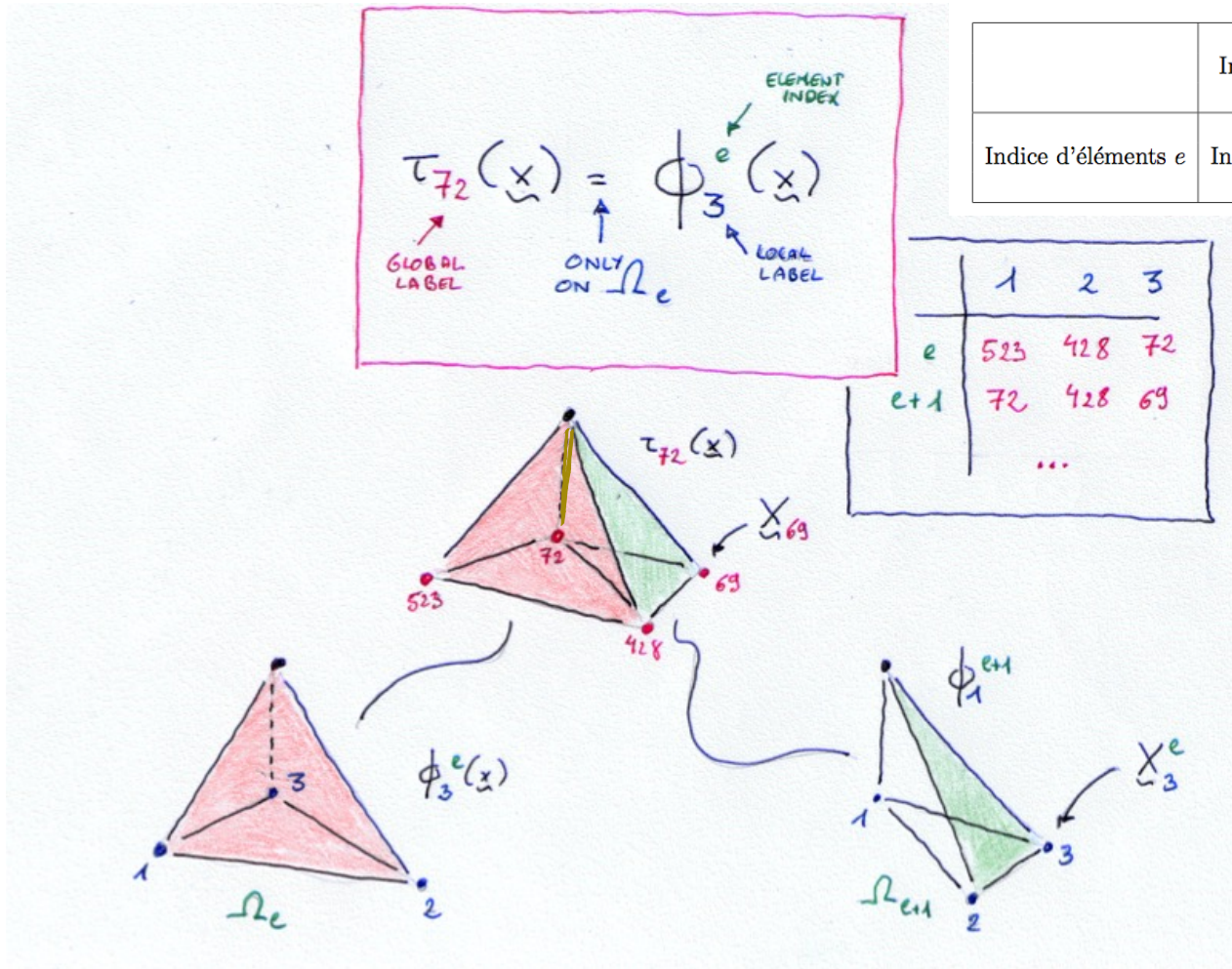


Un triangle parent pour définir toutes les fonctions de forme



Isomorphisme linéaire entre le triangle parent
et tous les autres triangles...

Global shape functions...



... and
local
shape
functions