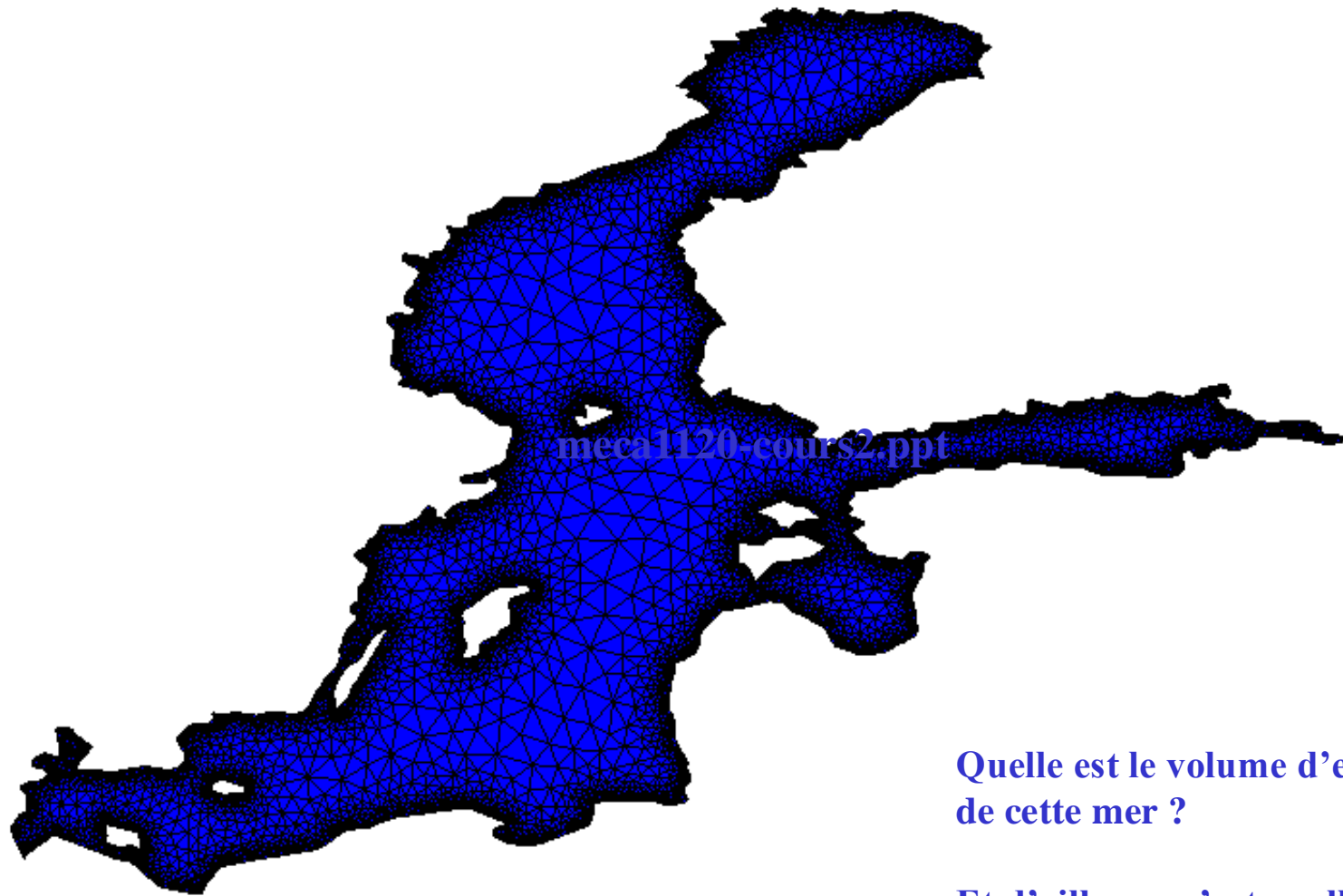


# Ne pas oublier de trouver votre Valentin(e) !



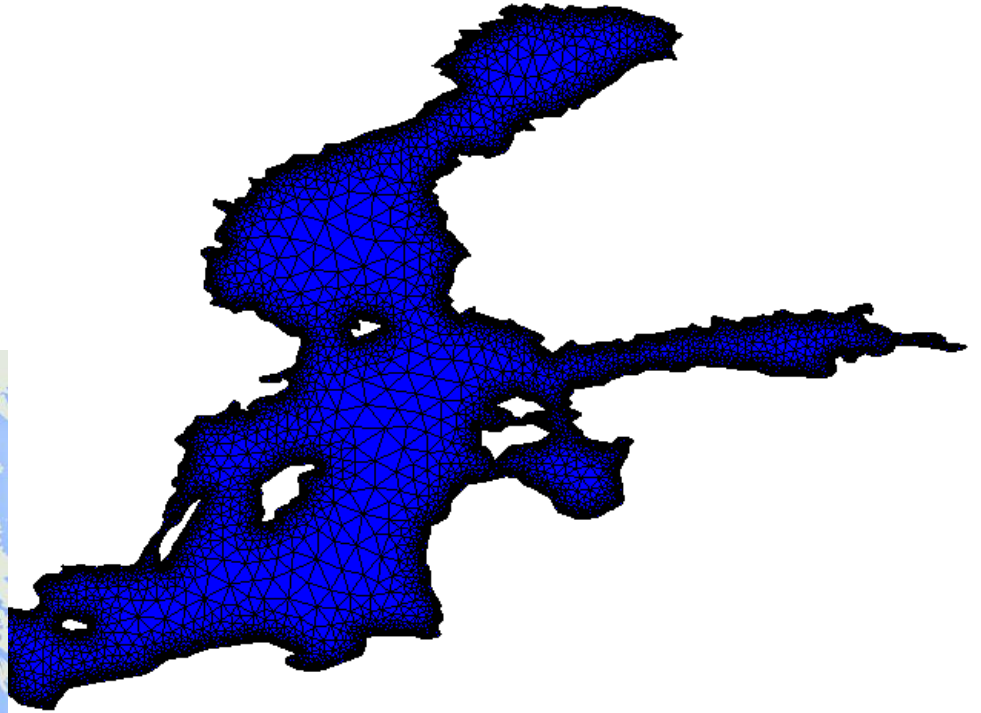
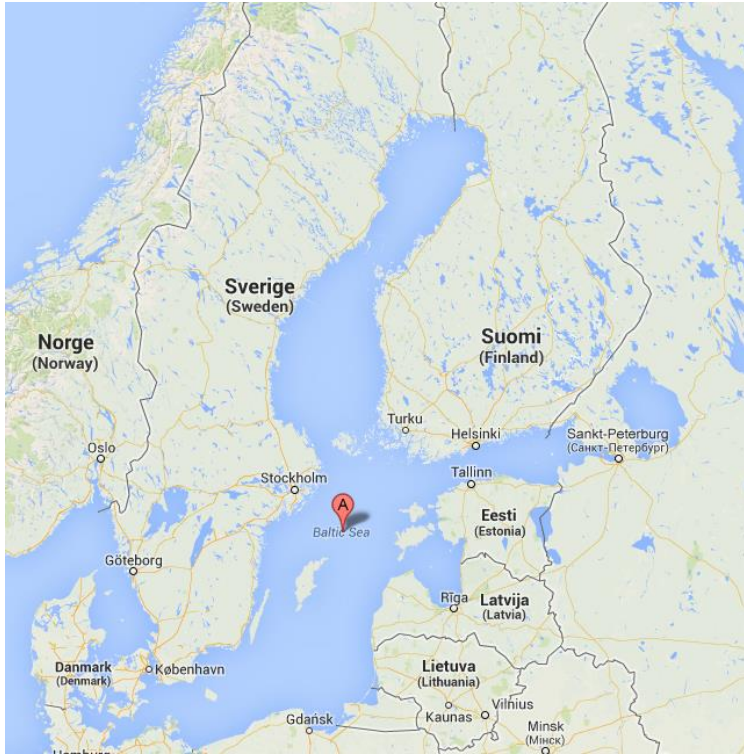
1 :	Abarca, Milton	FSA 1 BA	(Mardi-BA91)	(pas de groupe :-)
2 :	Abeloos, Joachim	FSA 1 BA	(Mardi-BA91)	(groupe : 11)
3 :	Accarain, Jean-Lou	FSA 1 BA	(Mercredi-BA92)	(groupe : 112)
4 :	Adda, Samy	FSA 1 BA	(pas de séance :-)	(groupe : 41)
5 :	Ahou, Lucas	FSA 1 BA	(Mardi-BA91)	(groupe : 121)
6 :	Alexandre, Simon	FSA 1 BA	(Mardi-BA91)	(groupe : 40)
7 :	Amory, Geoffroy	FSA 1 BA	(Mardi-BA91)	(groupe : 139)
8 :	André-Dumont, Aloïs	FSA 1 BA	(Mercredi-BA92)	(groupe : 48)
9 :	Arts, Emilien	FSA 1 BA	(Mercredi-BA92)	(groupe : 99)
10 :	Asqiriba, Ilias	FSA 1 BA	(pas de séance :-)	(pas de groupe :-)
11 :	Aubecq, Bastien	FSA 1 BA	(Mardi-BA91)	(groupe : 121)
12 :	Baijot, Laura	FSA 1 BA	(Mardi-BA91)	(groupe : 46)
13 :	Balon, Audren	MAP 2 MS/G	(pas de séance :-)	(pas de groupe :-)
14 :	Bapack, Simon	FSA 1 BA	(Mardi-BA91)	(pas de groupe :-)
15 :	Bauvir, Matéo	FSA 1 BA	(Mardi-BA91)	(groupe : 46)
16 :	Bellens, Romain	FSA 1 BA	(Mardi-BA91)	(groupe : 106)
17 :	Benjeloul, Ibrahim	FSA 1 BA	(Mercredi-BA92)	(groupe : 41)
18 :	Berthet, Raphael	FSA 1 BA	(pas de séance :-)	(pas de groupe :-)
19 :	Bertrand, Laura	FSA 1 BA	(Mercredi-BA92)	(groupe : 133)
20 :	Bien, Jonathan	FSA 1 BA	(pas de séance :-)	(groupe : 126)
21 :	Blase, Arnaud	FSA 1 BA	(Mardi-BA91)	(groupe : 100)
22 :	Blaude, Raphaël	FSA 1 BA	(Jeudi-BA91)	(groupe : 55)
23 :	Blockx, Alix	FSA 1 BA	(Jeudi-BA91)	(groupe : 35)
24 :	Bognár, Bence	FSA 1 BA	(Jeudi-BA91)	(groupe : 92)
25 :	Boland, Claire	FSA 1 BA	(Jeudi-BA91)	(groupe : 78)
26 :	Boreux, Pierre-Jean	FSA 1 BA	(Mercredi-BA92)	(groupe : 105)
27 :	Boujabout, Sofia	FSA 1 BA	(pas de séance :-)	(groupe : 138)
28 :	Bragard, Charlotte	PHYS2 MA	(Mercredi-BA92)	(groupe : 101)
29 :	Bruyndonckx, Amandine	FSA 1 BA	(Jeudi-BA91)	(groupe : 63)
30 :	Buch Kornreich, Eden	FSA 1 BA	(Mardi-BA91)	(groupe : 59)
31 :	Buckinx, Louan	FSA 1 BA	(Mercredi-BA92)	(groupe : 43)
32 :	Canon, Léa	FSA 1 BA	(Jeudi-BA91)	(groupe : 63)
33 :	Cantaert, Loïc	FSA 1 BA	(Mardi-BA91)	(groupe : 54)
34 :	Capart, Aodren	FSA 1 BA	(Jeudi-BA91)	(pas de groupe :-)
35 :	Capart, Célestine	FSA 1 BA	(Mardi-BA91)	(groupe : 54)
36 :	Carbonez, Simon	FSA 1 BA	(Mardi-BA91)	(groupe : 18)
37 :	Carpiaux, Théo	FSA 1 BA	(Mardi-BA91)	(groupe : 5)
38 :	Carton, Nathan	FSA 1 BA	(Mardi-BA91)	(pas de groupe :-)
39 :	Cavalier, Simon	FSA 1 BA	(Mercredi-BA92)	(groupe : 120)
40 :	Cayphas, Louis	FSA 1 BA	(Mardi-BA91)	(groupe : 100)
41 :	Chougrani, Sami	FSA 1 BA	(pas de séance :-)	(groupe : 133)
42 :	Chrissantakis, Ioannis	FSA 1 BA	(Mercredi-BA92)	(pas de groupe :-)
43 :	Chugh, Aditya	FSA 1 BA	(Jeudi-BA91)	(groupe : 97)
44 :	Clément, Yvan	FSA 1 BA	(pas de séance :-)	(groupe : 32)
45 :	Cloos, Rémi	FSA 1 BA	(Mercredi-BA92)	(pas de groupe :-)

**Plus sérieusement,  
A partir de ce soir, il sera possible de rester célibataire !  
Mais, ce n'est pas la meilleure idée !**

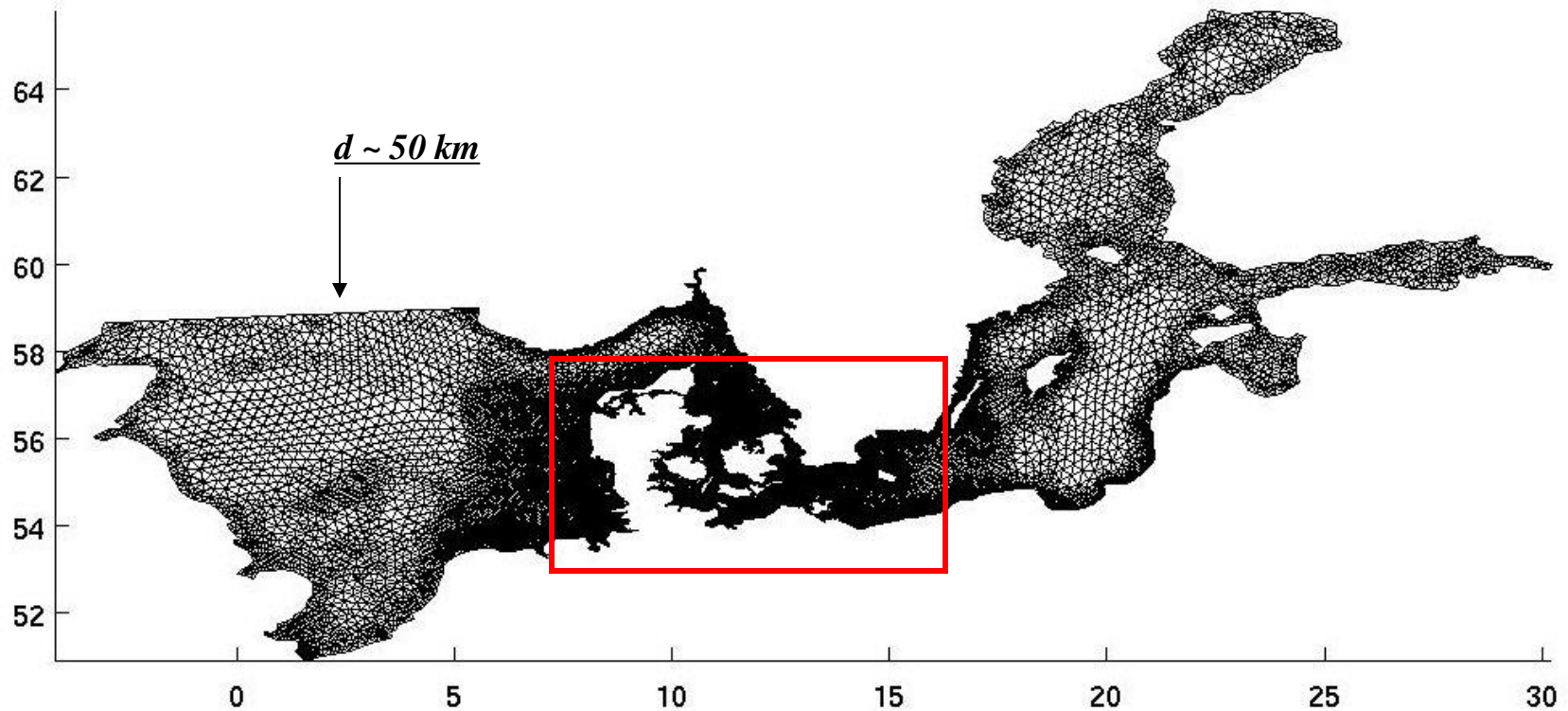


Quelle est le volume d'eau  
de cette mer ?

Et d'ailleurs, c'est quelle mer ?

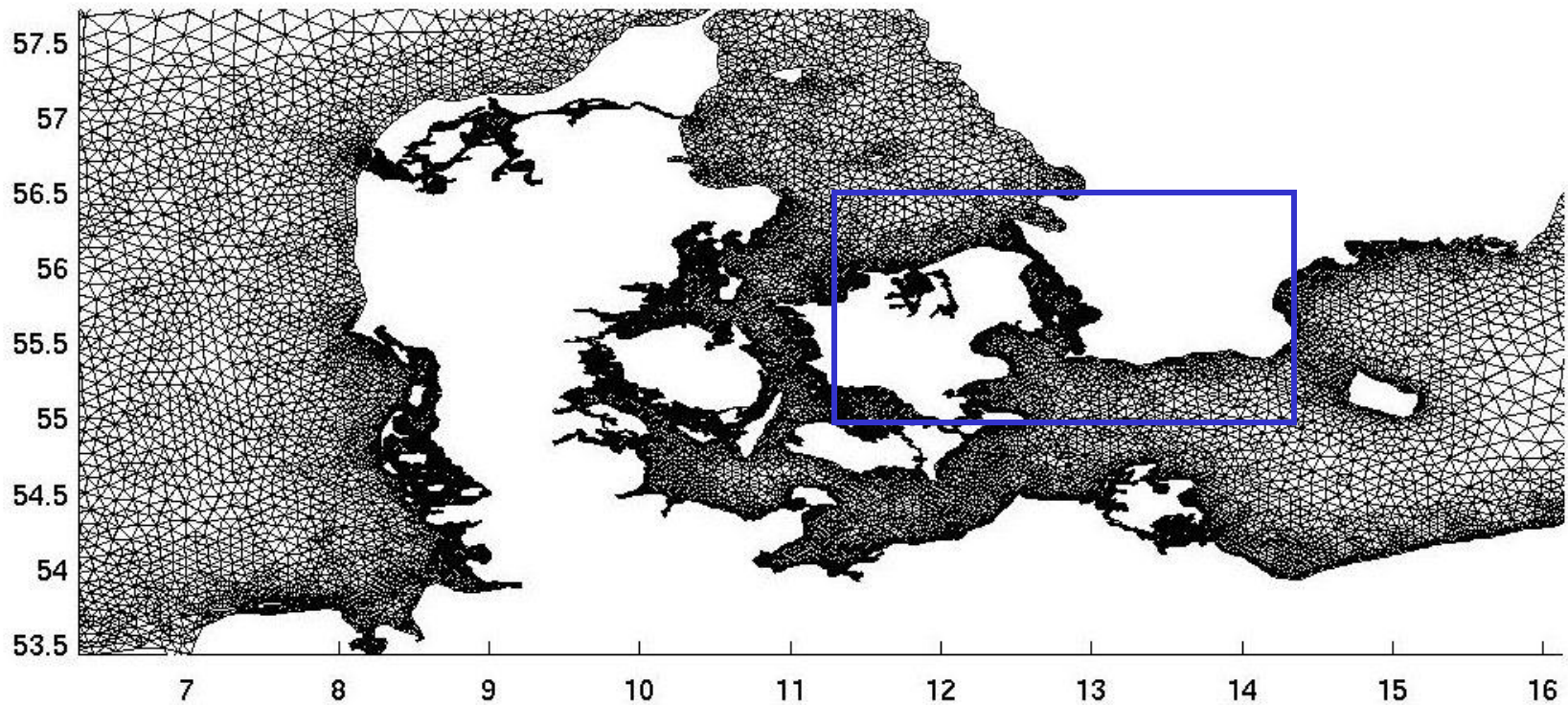


# Pourquoi des maillages non structurés ?



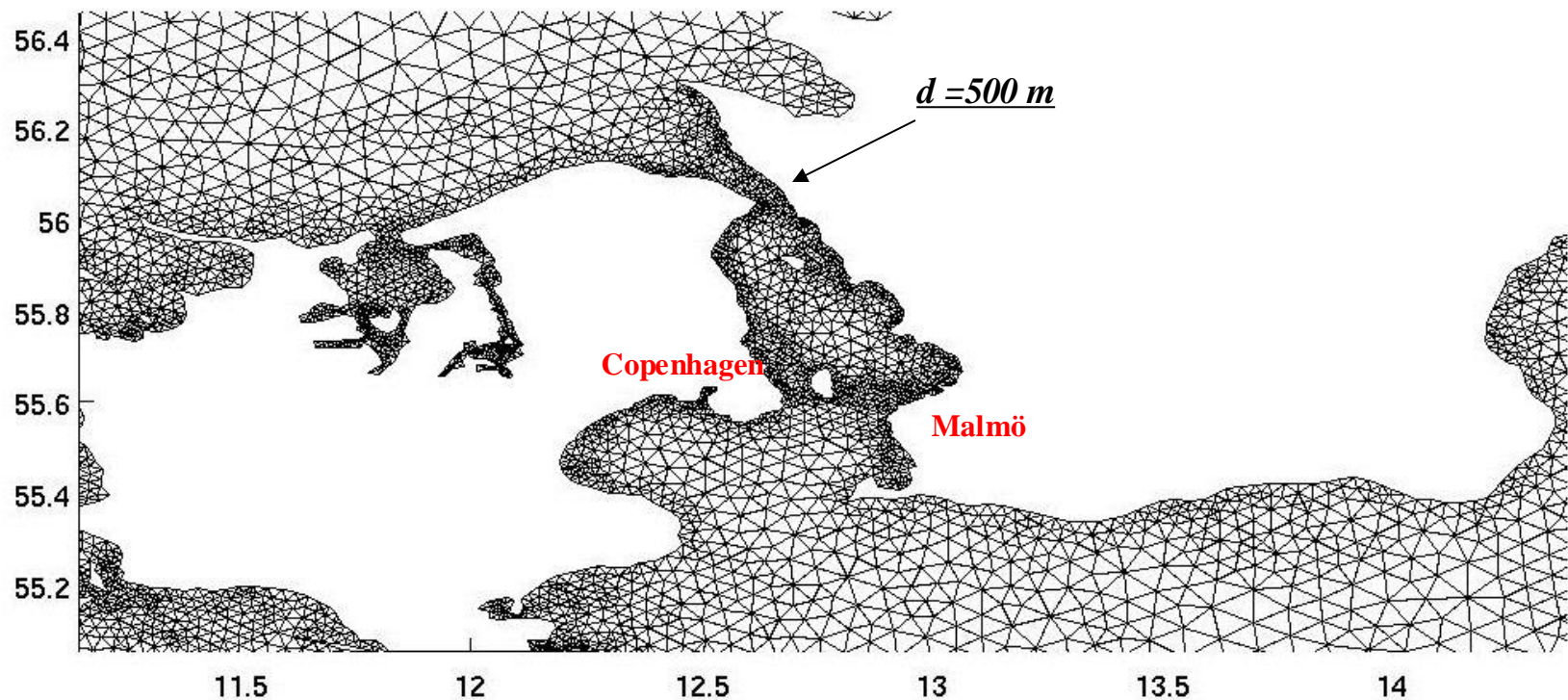
*Courtesy of Dr. N. Kliem*

# Pourquoi des maillages non structurés ?

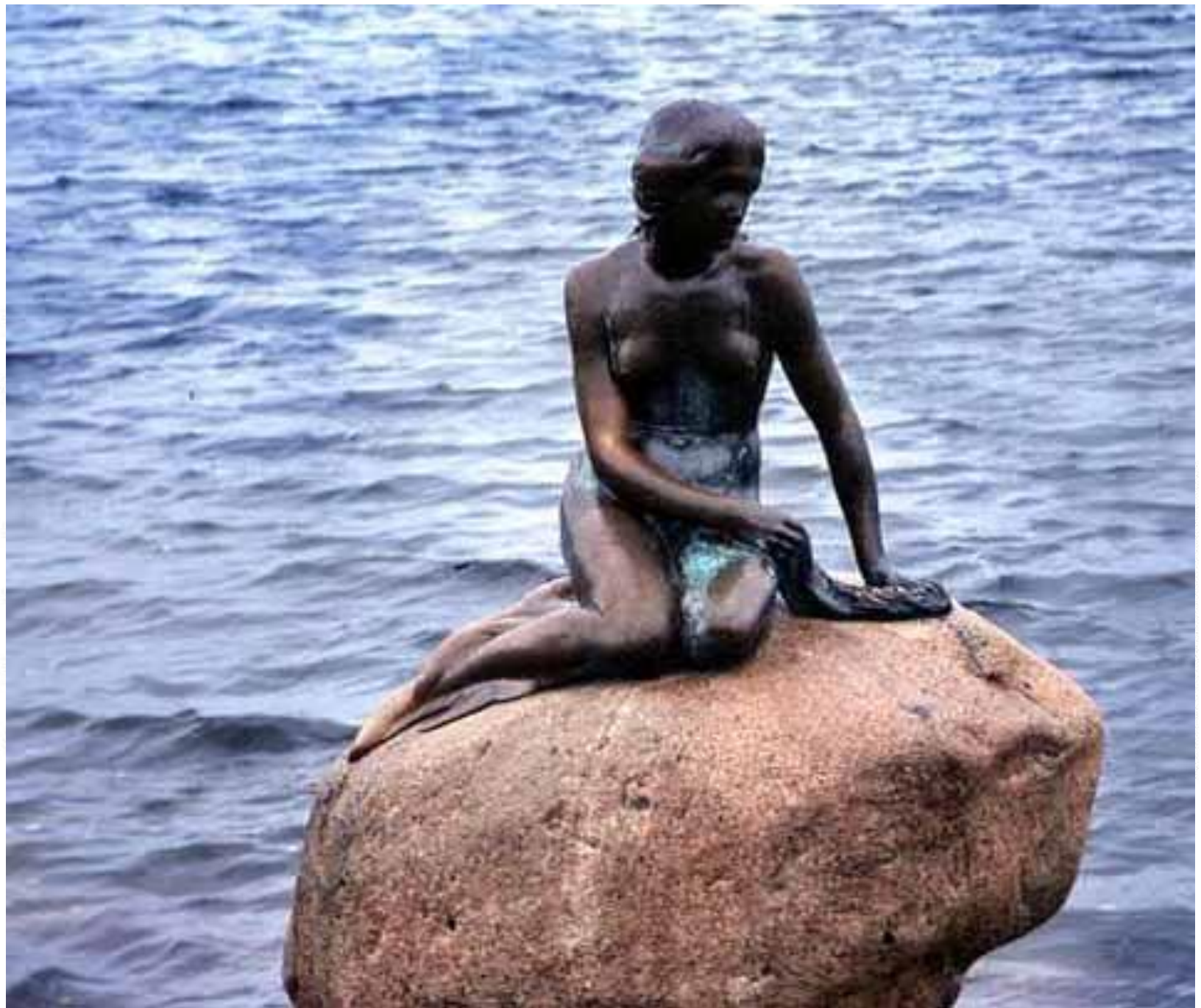


*Courtesy of Dr. N. Kliem*

# Pourquoi des maillages non structurés ?

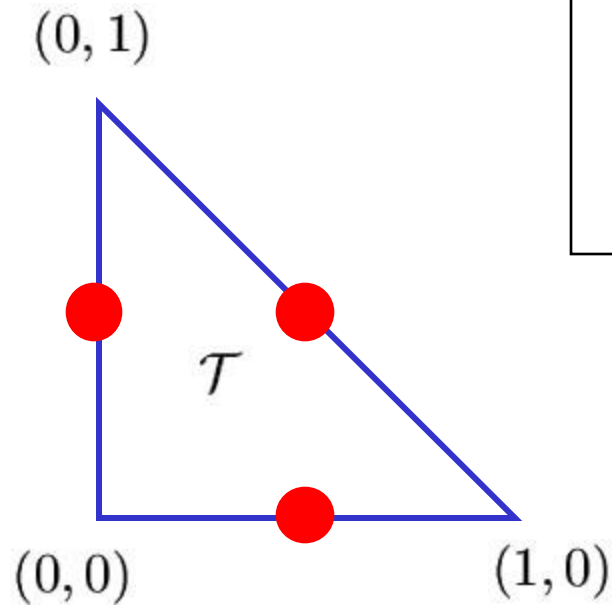


*Courtesy of Dr. N. Kliem*





# Intégration sur un triangle : Règle de Hammer à 3 points

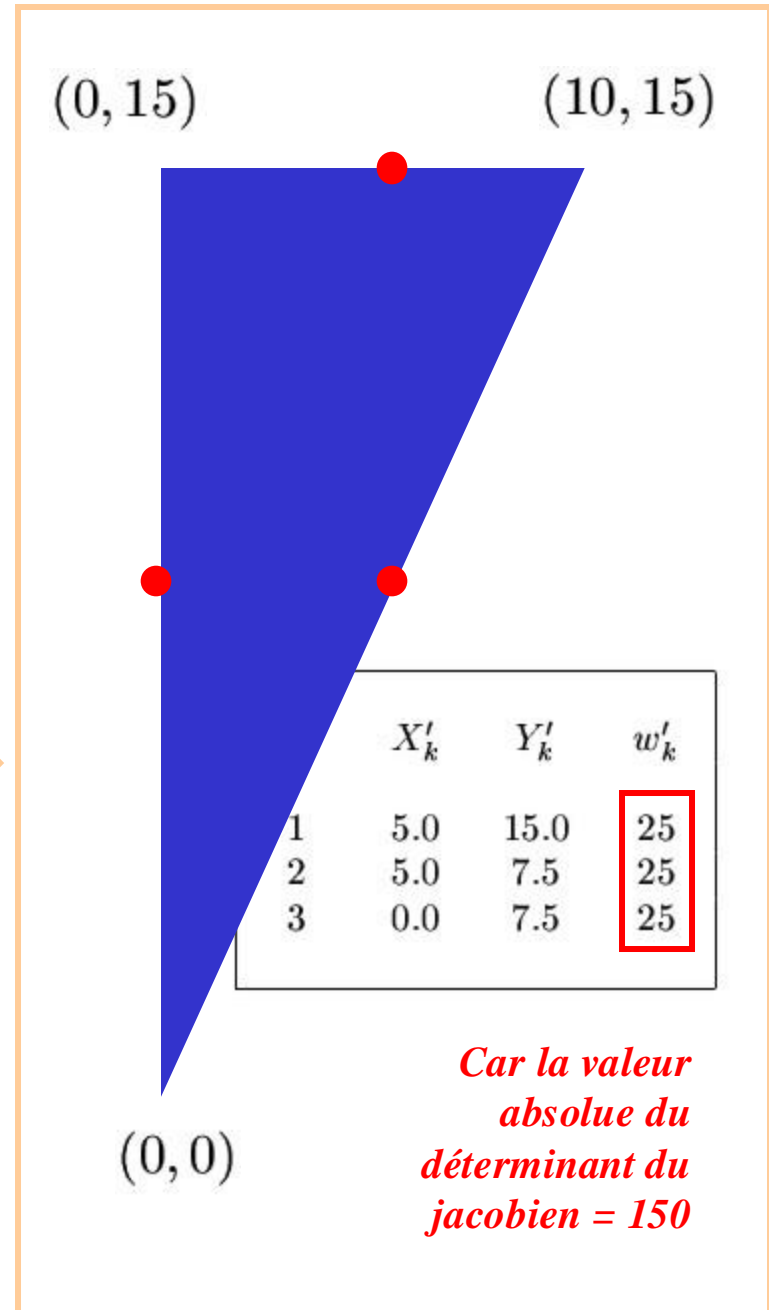
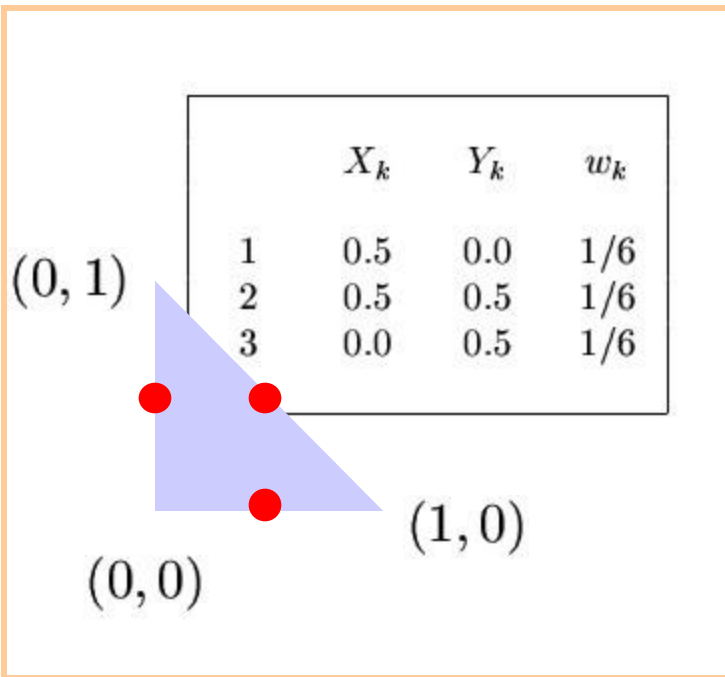


$$\underbrace{\int_{\mathcal{T}} f(x,y) \, dx \, dy}_{I} \approx \underbrace{\sum_{k=1}^3 w_k f(X_k, Y_k)}_{I^h}$$

	$X_k$	$Y_k$	$w_k$
1	0.5	0.0	1/6
2	0.5	0.5	1/6
3	0.0	0.5	1/6

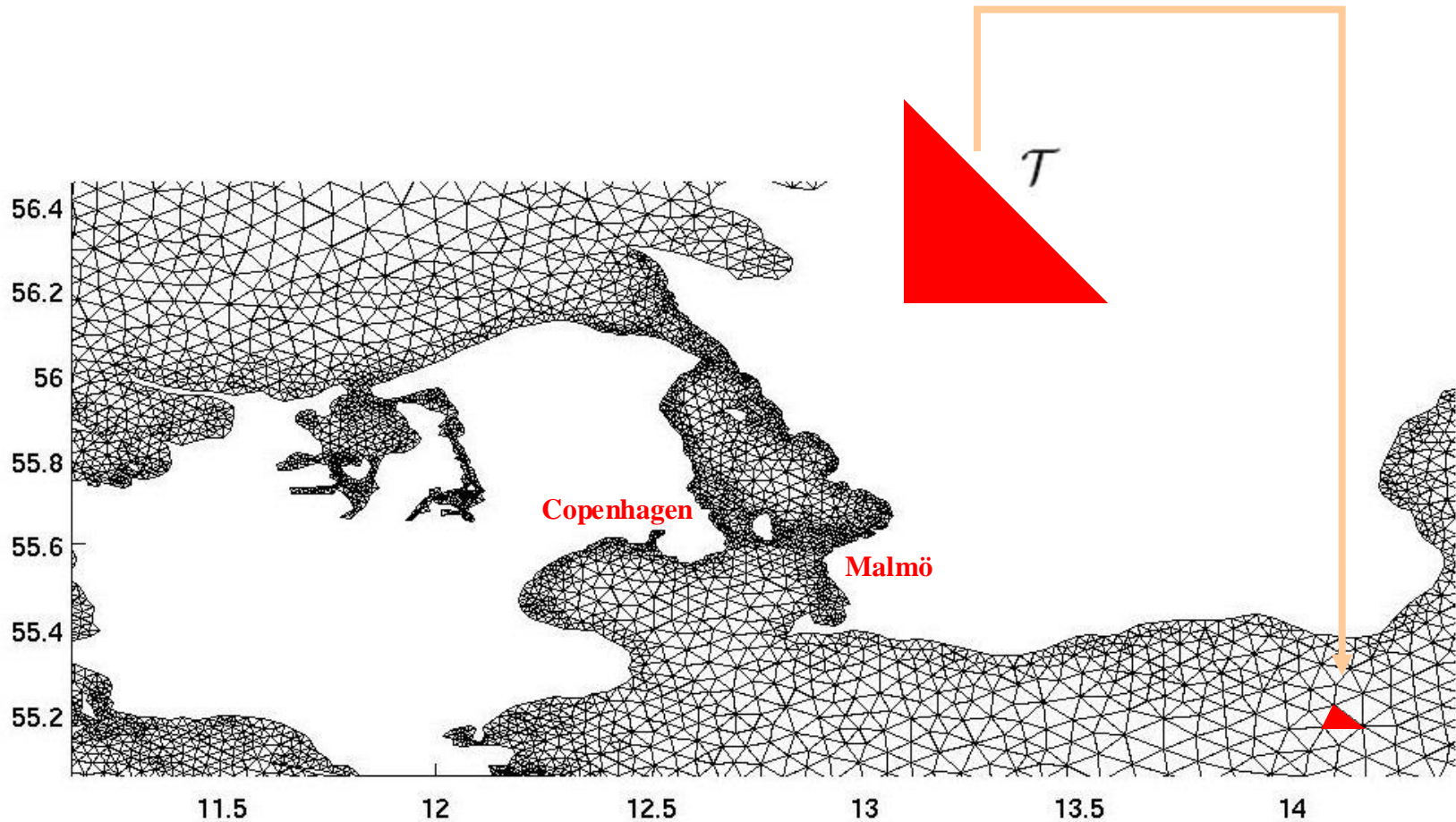
# Et un autre triangle ?

$$\begin{aligned}x' &= 10x \\ y' &= 15 - 15y\end{aligned}$$

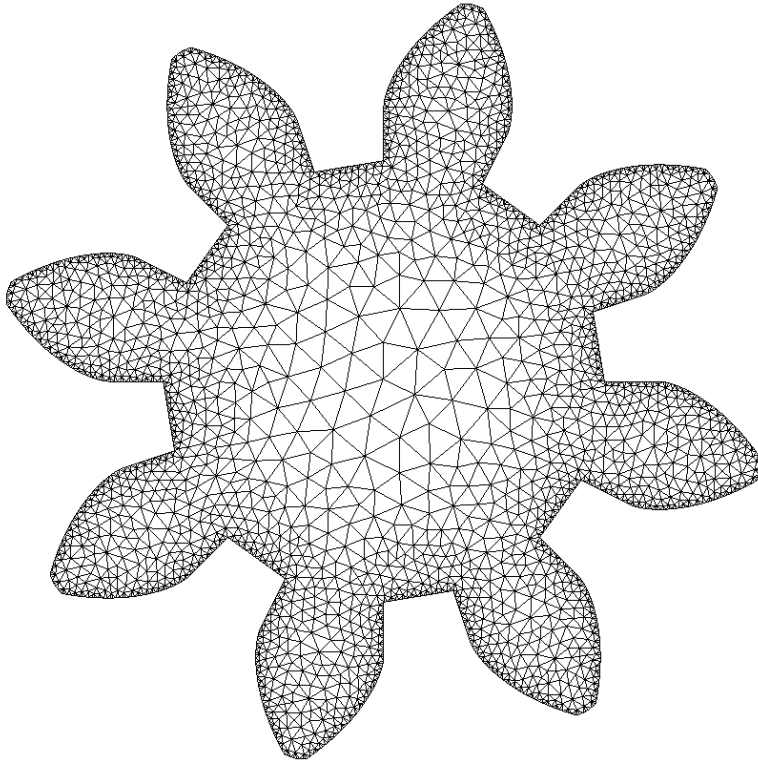


# Application to Finite Elements

*Each triangle can be transformed in  
the parent element through a linear  
transformation.*



# Définir un maillage

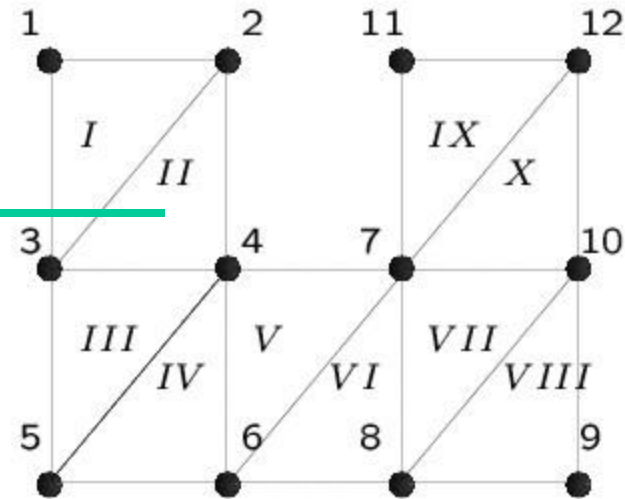


**Coordonnées de noeuds**  
**Tableau d'appartenance des triangles**  
**Nombre de noeuds : nNode**  
**Nombre de triangles : nElem**

```
typedef struct  
{  
    int *elem;  
    double *X;  
    double *Y;  
    int nElem;  
    int nNode;  
} femMesh;
```

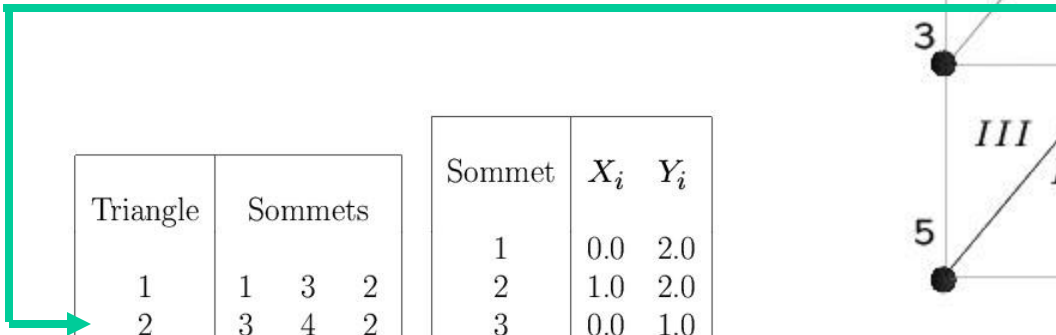
$$\bar{\Omega} = \bigcup_{e=1}^{N_1} \{\bar{\Omega}_e\}, \quad \Omega_e \cap \Omega_f = \emptyset, \quad \text{si } e \neq f.$$

Et concrètement  
c'est quoi  
un maillage ?



Triangle	Sommets		
1	1	3	2
2	3	4	2
3	5	4	3
4	5	6	4
5	6	7	4
6	6	8	7
7	8	10	7
8	8	9	10
9	7	12	11
10	10	12	7

Sommet	$X_i$	$Y_i$
1	0.0	2.0
2	1.0	2.0
3	0.0	1.0
4	1.0	1.0
5	0.0	0.0
6	1.0	0.0
7	2.0	1.0
8	2.0	0.0
9	3.0	0.0
10	3.0	1.0
11	2.0	2.0
12	3.0	2.0



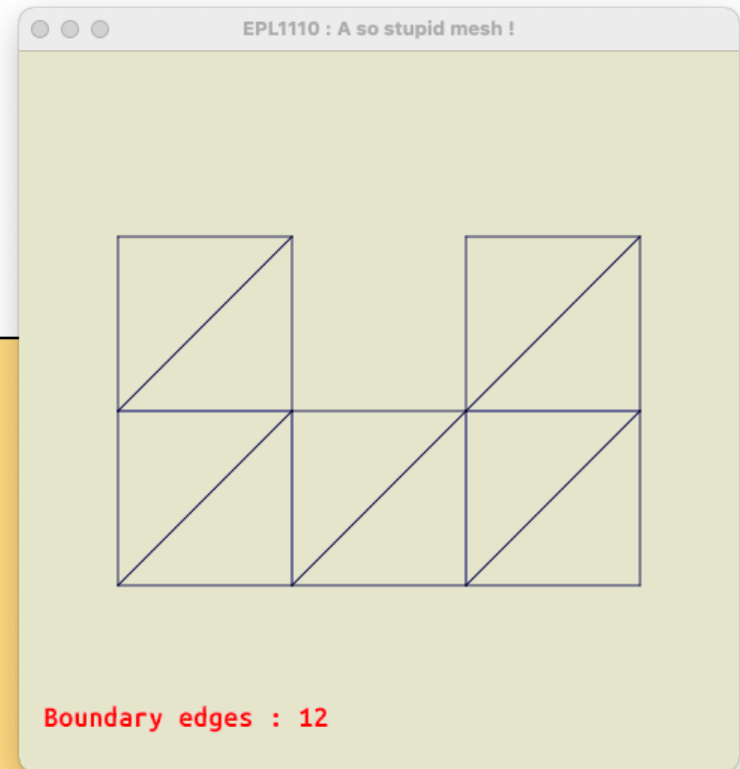
# Et vraiment concrètement ?

Number of nodes 12

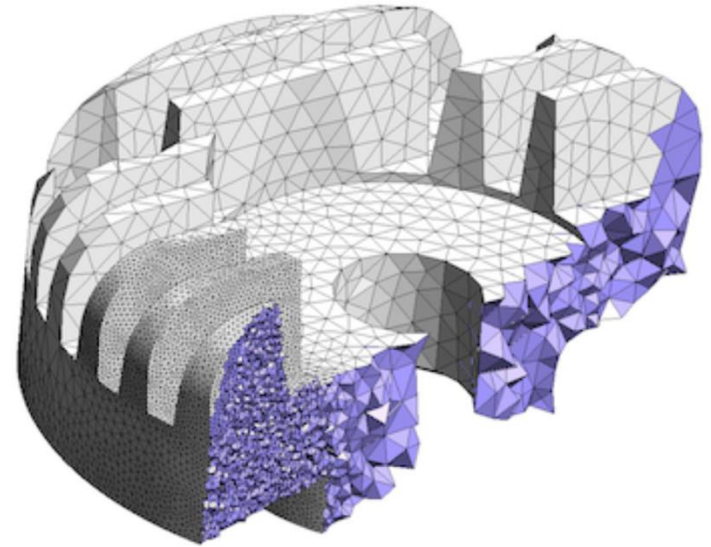
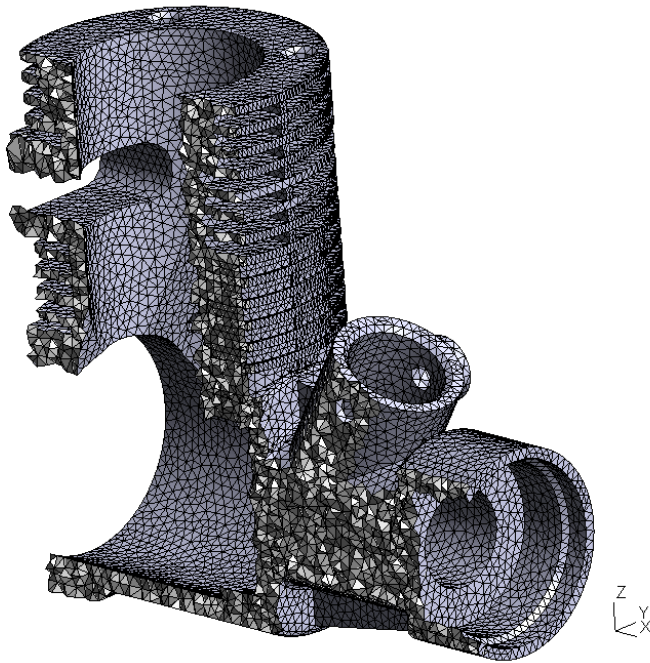
0 :	0.0000000e+00	2.0000000e+00
1 :	1.0000000e+00	2.0000000e+00
2 :	0.0000000e+00	1.0000000e+00
3 :	1.0000000e+00	1.0000000e+00
4 :	0.0000000e+00	0.0000000e+00
5 :	1.0000000e+00	0.0000000e+00
6 :	2.0000000e+00	1.0000000e+00
7 :	2.0000000e+00	0.0000000e+00
8 :	3.0000000e+00	0.0000000e+00
9 :	3.0000000e+00	1.0000000e+00
10 :	2.0000000e+00	2.0000000e+00
11 :	3.0000000e+00	2.0000000e+00

Number of triangles 10

0 :	0	2	1
1 :	2	3	1
2 :	4	3	2
3 :	4	5	3
4 :	5	6	3
5 :	5	7	6
6 :	7	9	6
7 :	7	8	9
8 :	6	11	10
9 :	9	11	6



Et vraiment très très très concrètement ?

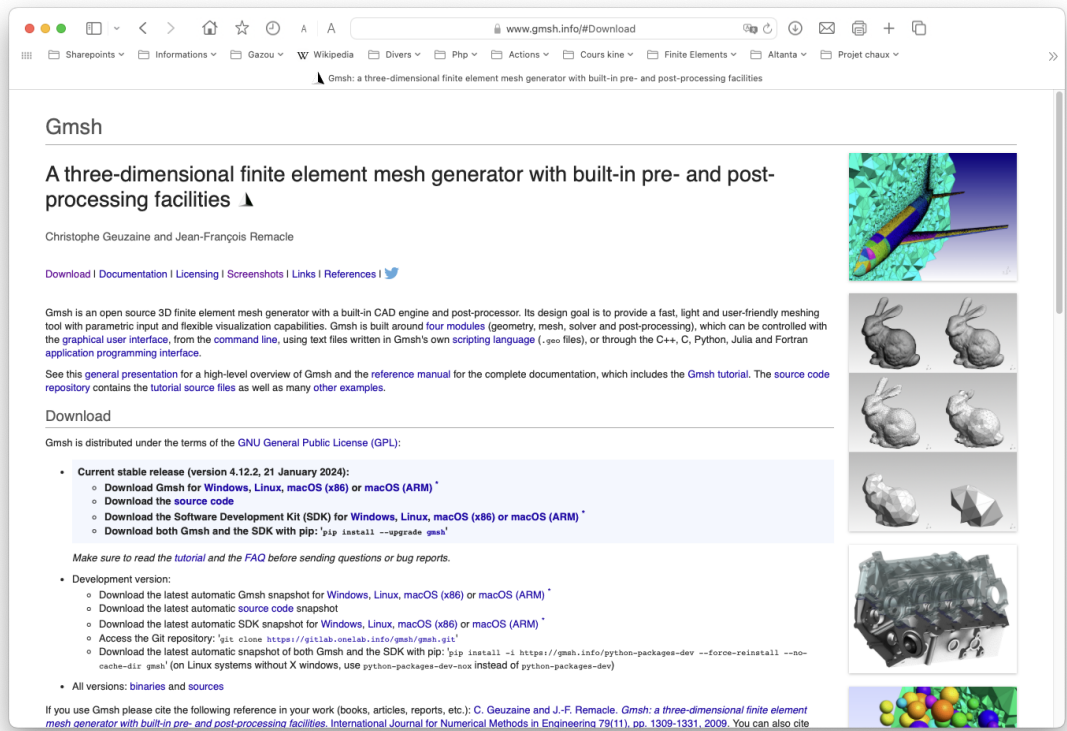


**Il existe des algorithmes pour générer automatiquement les maillages.**

**Cours de géométrie numérique LMECA2170**

<http://www.geuz.org/gmsh/>

<https://www.gmsh.info>



The screenshot shows the Gmsh website with the following content:

- Gmsh**
- A three-dimensional finite element mesh generator with built-in pre- and post-processing facilities**
- Christophe Geuzaine and Jean-François Remacle
- Download | Documentation | Licensing | Screenshots | Links | References | Twitter
- Introduction text: "Gmsh is an open source 3D finite element mesh generator with a built-in CAD engine and post-processor. Its design goal is to provide a fast, light and user-friendly meshing tool with parametric input and flexible visualization capabilities. Gmsh is built around four modules (geometry, mesh, solver and post-processing), which can be controlled with the graphical user interface, from the command line, using text files written in Gmsh's own scripting language (.geo files), or through the C++, C, Python, Julia and Fortran application programming interface." It also mentions a general presentation and reference manual.
- Download**
- Gmsh is distributed under the terms of the GNU General Public License (GPL):
- Current stable release (version 4.12.2, 21 January 2024):**
  - Download Gmsh for Windows, Linux, macOS (x86) or macOS (ARM)
  - Download the source code
  - Download the Software Development Kit (SDK) for Windows, Linux, macOS (x86) or macOS (ARM)
  - Download both Gmsh and the SDK with pip: `pip install --upgrade gmsh`
- Make sure to read the tutorial and the FAQ before sending questions or bug reports.
- Development version:**
  - Download the latest automatic Gmsh snapshot for Windows, Linux, macOS (x86) or macOS (ARM)
  - Download the latest automatic source code snapshot
  - Download the latest automatic SDK snapshot for Windows, Linux, macOS (x86) or macOS (ARM)
  - Access the Git repository: `git clone https://gitlab.onelab.info/gmsh/gmsh.git`
  - Download the latest automatic snapshot of both Gmsh and the SDK with pip: `pip install -i https://gmsh.info/python-packages-dev --force-reinstall --no-cache-dir gmsh` (on Linux systems without X windows, use `python-packages-dev-sox` instead of `python-packages-dev`)
- All versions: binaries and sources
- Footer: "If you use Gmsh please cite the following reference in your work (books, articles, reports, etc.): C. Geuzaine and J.-F. Remacle, *Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities*. International Journal for Numerical Methods in Engineering 79(11), pp. 1309-1331, 2009. You can also cite

On the right side of the page, there is a vertical column of images showing various 3D meshes: a wing, two rabbits, a car engine, and a cluster of spheres.

Installer  
le Software Development Kit  
de gmsh

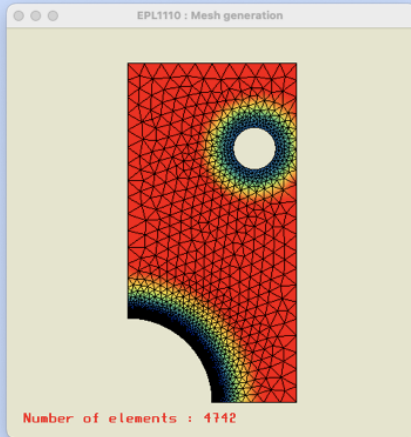


<https://www.gmsh.info>

- **Current stable release (version 4.12.2, 21 January 2024):**
  - Download Gmsh for **Windows, Linux, macOS (x86) or macOS (ARM)** \*
  - Download the **source code**
  - Download the Software Development Kit (SDK) for **Windows, Linux, macOS (x86) or macOS (ARM)** \*
  - Download both Gmsh and the SDK with pip: `'pip install --upgrade gmsh'`

*Make sure to read the [tutorial](#) and the [FAQ](#) before sending questions or bug reports.*

**Il faut choisir la bonne version !**

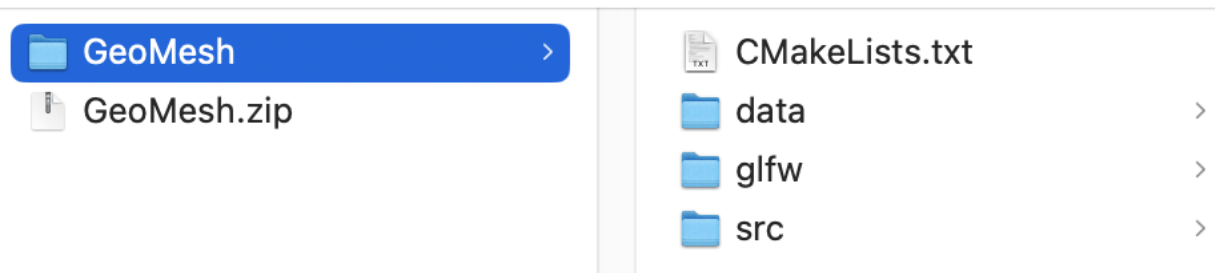


**Devoir 2 : GeoMesh (12/02/2024)**

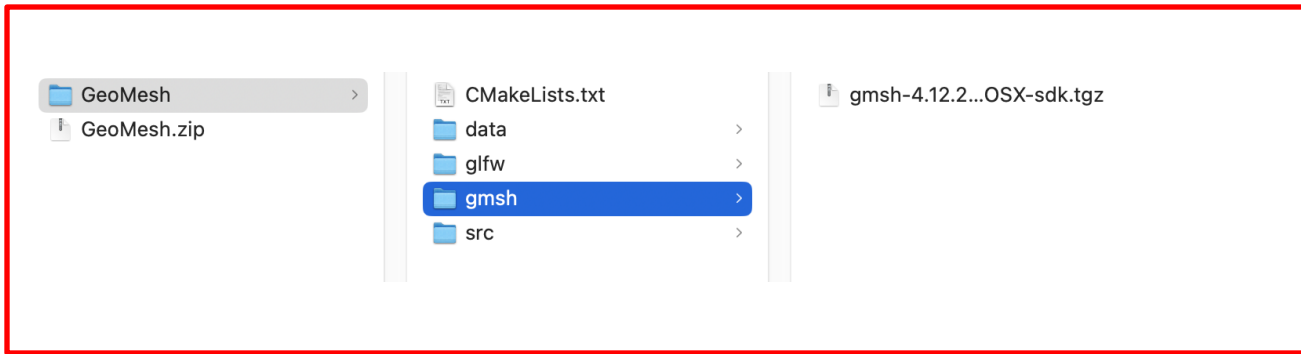
Projet à télécharger : [GeoMesh.zip](#)

Enoncé du devoir : [GeoMesh.pdf](#)

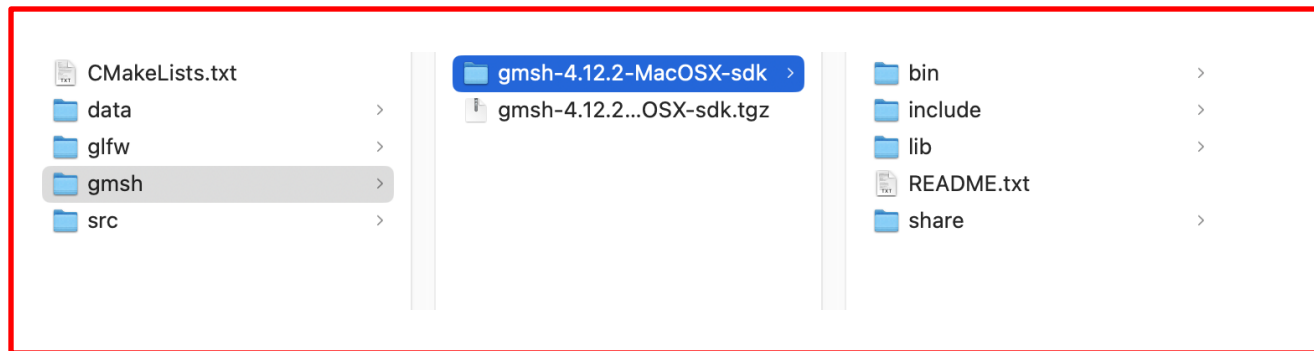
Deadline : **Lundi 26 février 2024 à 23h59**



**Comment compiler  
le second devoir ?**

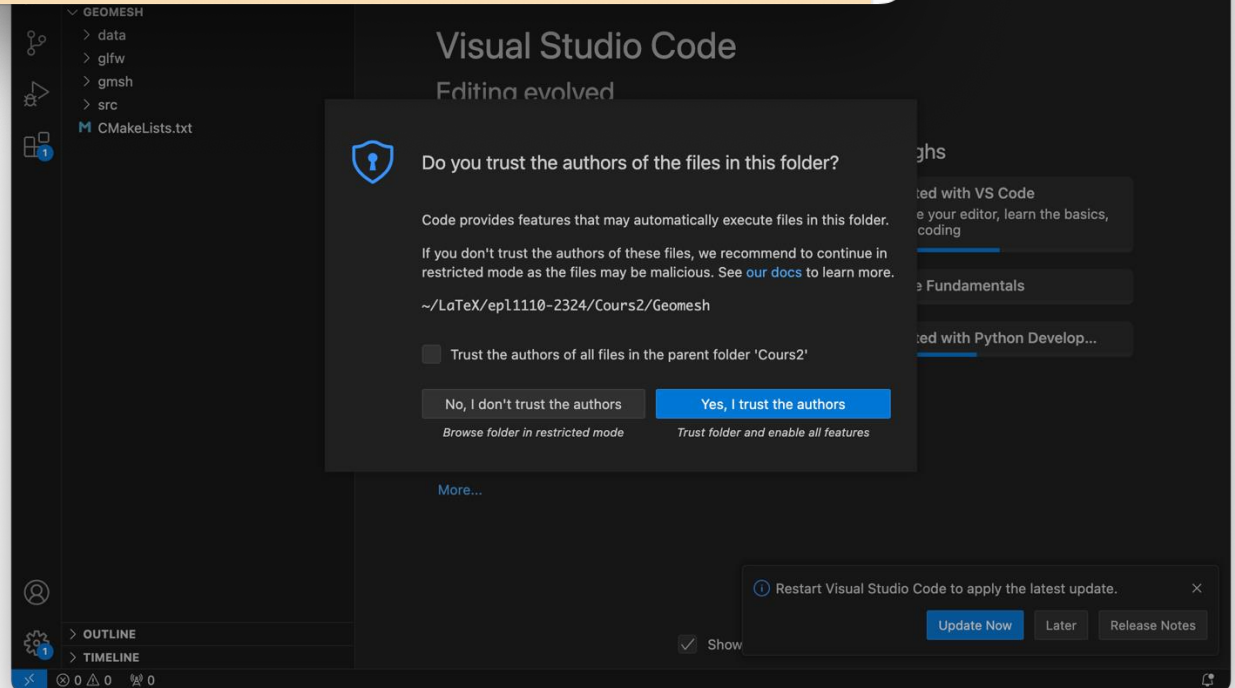


Ajouter la librairie gms  
dans un répertoire gms

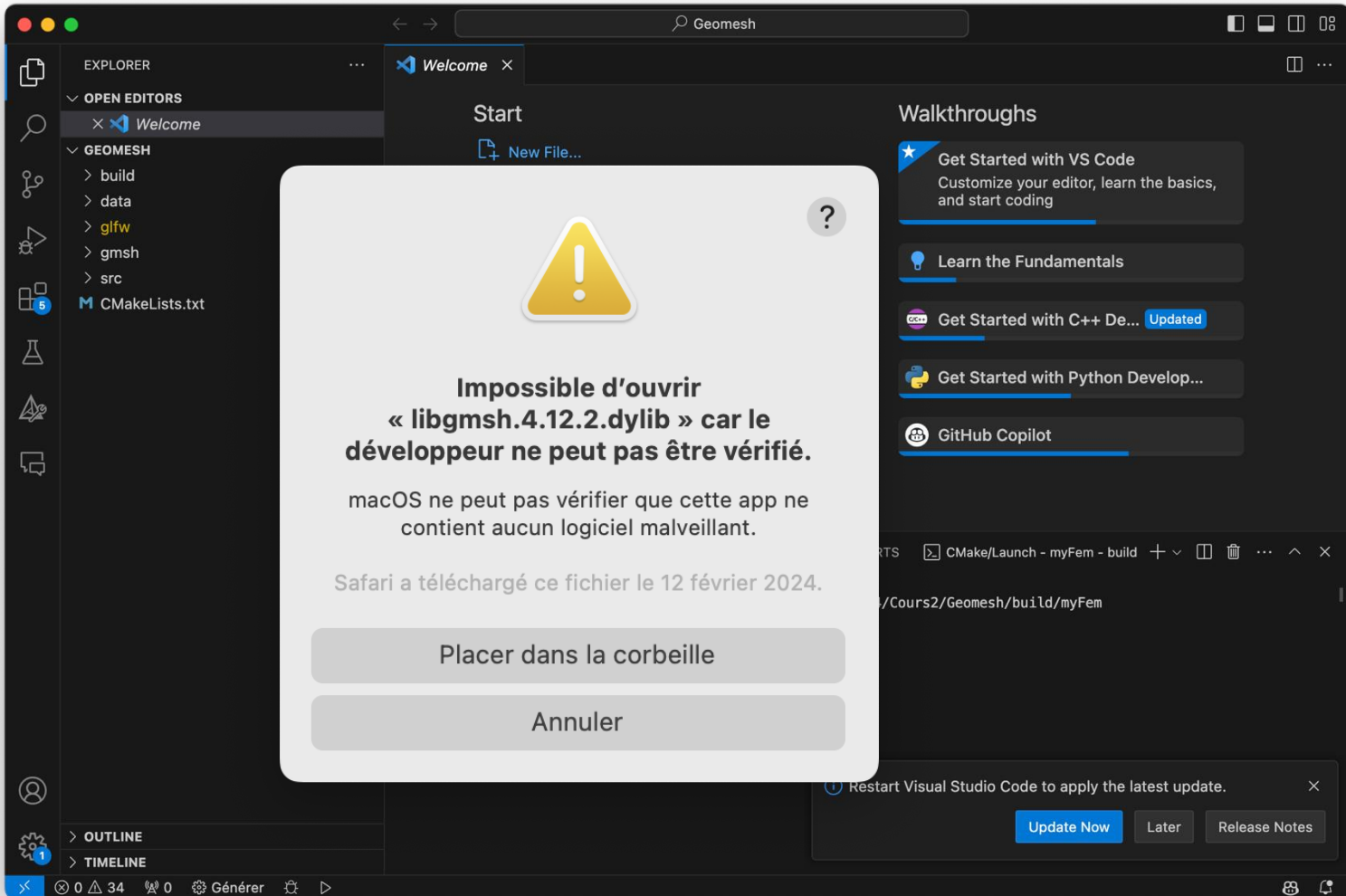


# Et lancer vscode

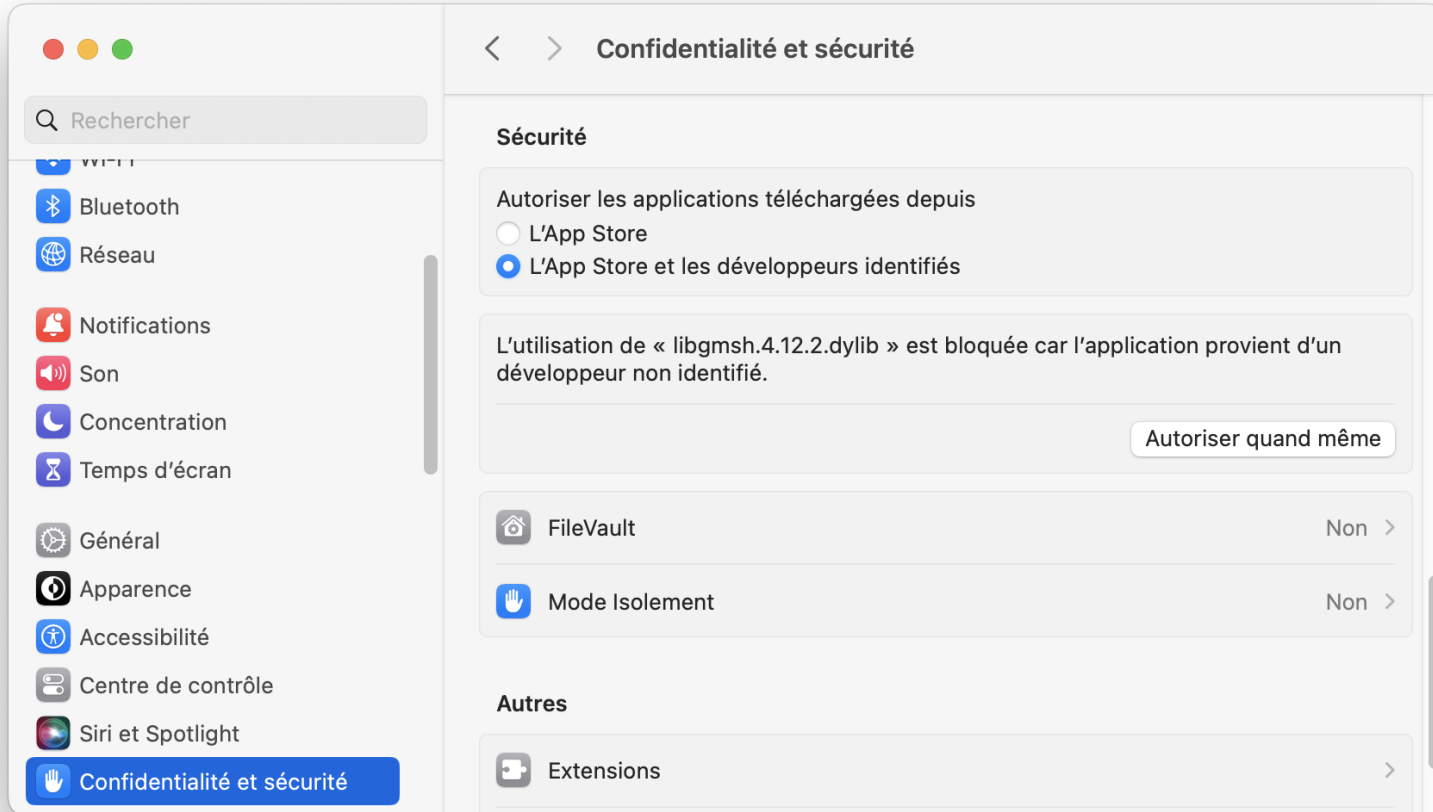
```
Cours2 — -bash — 74x7  
[ (base) mac-1U0-352:Cours2 v1$ ls ]  
GeoMesh          GeoMesh.zip  
[ (base) mac-1U0-352:Cours2 v1$ ls Geomesh ]  
CMakeLists.txt  glfw          src  
data            gmsch  
[ (base) mac-1U0-352:Cours2 v1$ code Geomesh ]  
(base) mac-1U0-352:Cours2 v1$
```



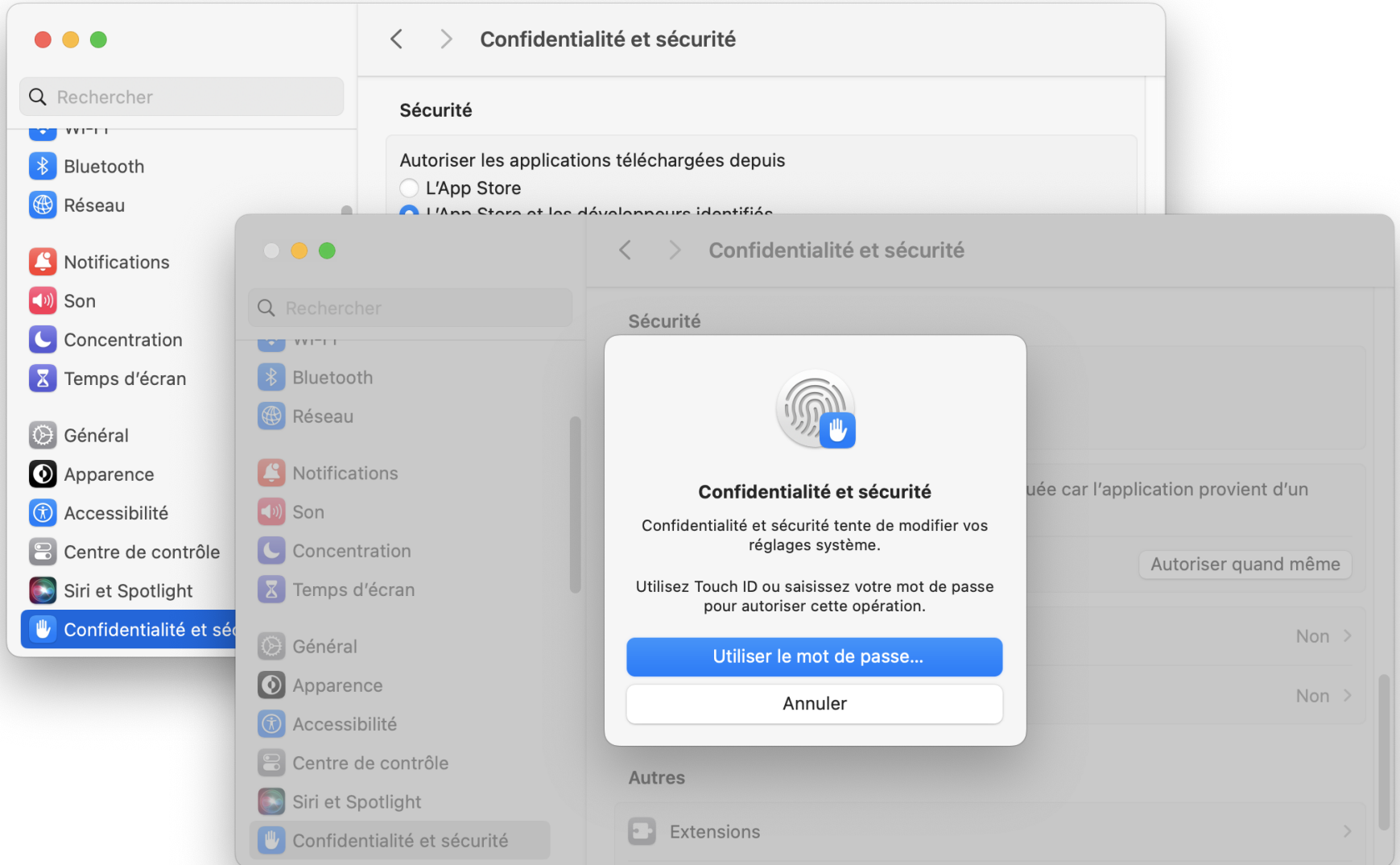
# Et zou !



Non, non :  
Remacle n'est pas malveillant !



Non, non :  
Remacle n'est pas malveillant !









# Et après avoir fait le devoir !

The image displays a development environment with a code editor and a mesh generation window. The code editor shows the following C++ code in `homework.c`:

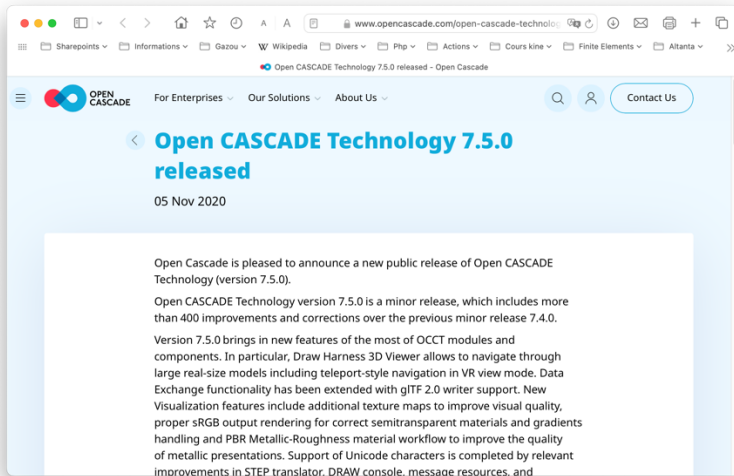
```
src > C homework.c > ...
1  #include "fem.h"
2
3
4
5
6  double geoSize(double x, double y){
7
8      femGeo* theGeometry = geoGetGeometry();
9
10     double h = theGeometry->h;
11     double x0 = theGeometry->xNotch;
12     double y0 = theGeometry->yNotch;
13     double r0 = theGeometry->rNotch;
14     double h0 = theGeometry->hNotch;
15     double d0 = theGeometry->dNotch;
16
17
18     double x1 = theGeometry->xHole;
19     double y1 = theGeometry->yHole;
20     double r1 = theGeometry->rHole;
```

The terminal window shows the following output:

```
PROBLEMS 1 OUTPUT DEBUG CONSOLE TERMINAL
Info : Meshing 2D...
Info : Meshing surface 1 (Plane, Frontal-Delaunay)
Info : Done meshing 2D (Wall 0.0933071s, CPU 0.0866s)
Info : 2520 nodes 5046 elements
Geo : Importing 2520 nodes
Geo : Importing 276 edges
Geo : Importing 4764 elements
Geo : Importing 6 entities
Geo : Entity 0 : 40 elements
Geo : Entity 1 : 158 elements
Geo : Entity 2 : 19 elements
Geo : Entity 3 : 29 elements
Geo : Entity 4 : 20 elements
Geo : Entity 5 : 10 elements
=== Global requested h : 1.0000000e-01
=== Minimum h : 5.0000000e-03
=== Maximum h : 1.0000000e-01
```

The mesh generation window, titled "EPL1110 : Mesh generation", displays a 2D mesh of a rectangular domain with a circular hole and a notch. The mesh is colored with a gradient from red (high stress) to blue (low stress). The number of elements is 4764.

Number of elements : 4764

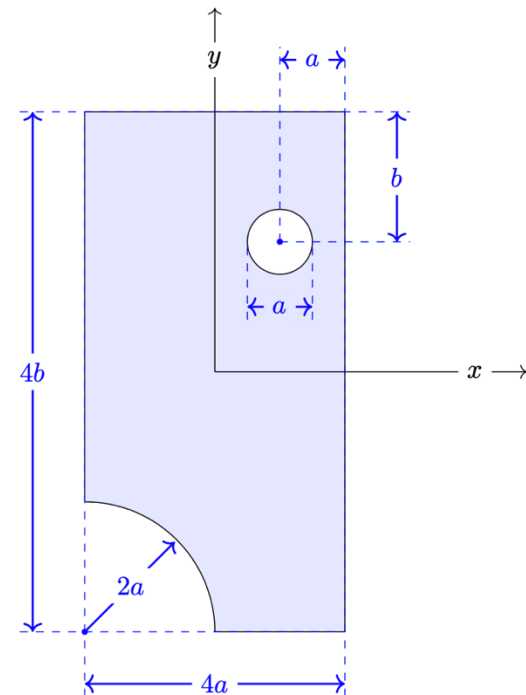


# Définir la géométrie avec OpenCascade !

```

double Lx = 1.0;
double Ly = 2.0;
theGeometry->LxPlate = Lx;
theGeometry->LyPlate = Ly;
theGeometry->xPlate = 0.0;
theGeometry->yPlate = 0.0;
theGeometry->xHole = Lx / 4.0;
theGeometry->yHole = Ly / 4.0;
theGeometry->rHole = Lx / 8.0;
theGeometry->xNotch = -Lx / 2.0;
theGeometry->yNotch = -Ly / 2.0;
theGeometry->rNotch = Lx / 2.0;

```



# Utiliser la géométrie constructive !

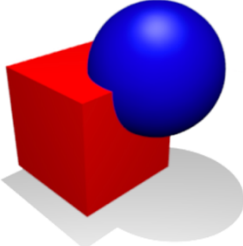
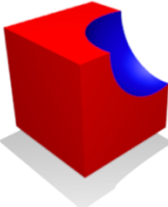

fr.wikipedia.org/wiki/Géométrie\_de

Sharepoints Informations Gazou Wikipedia Divers Php Actions Cours kine

W Géométrie de construction de solides – Wikipédia

- Notation
- Géométrie

## Opérations booléennes [\[ modifier \]](#) [\[ modifier le code \]](#)

Union (ou addition)	Différence (ou soustraction)	Intersection
		
L'assemblage des deux objets.	La soustraction d'un objet de l'autre.	La partie commune aux deux objets.

### Union (ou addition) [\[ modifier \]](#) [\[ modifier le code \]](#)

Le résultat est l'assemblage des deux objets. Il y a parfois la possibilité de réaliser cette opération sur plus de deux objets.

### Différence (ou soustraction) [\[ modifier \]](#) [\[ modifier le code \]](#)

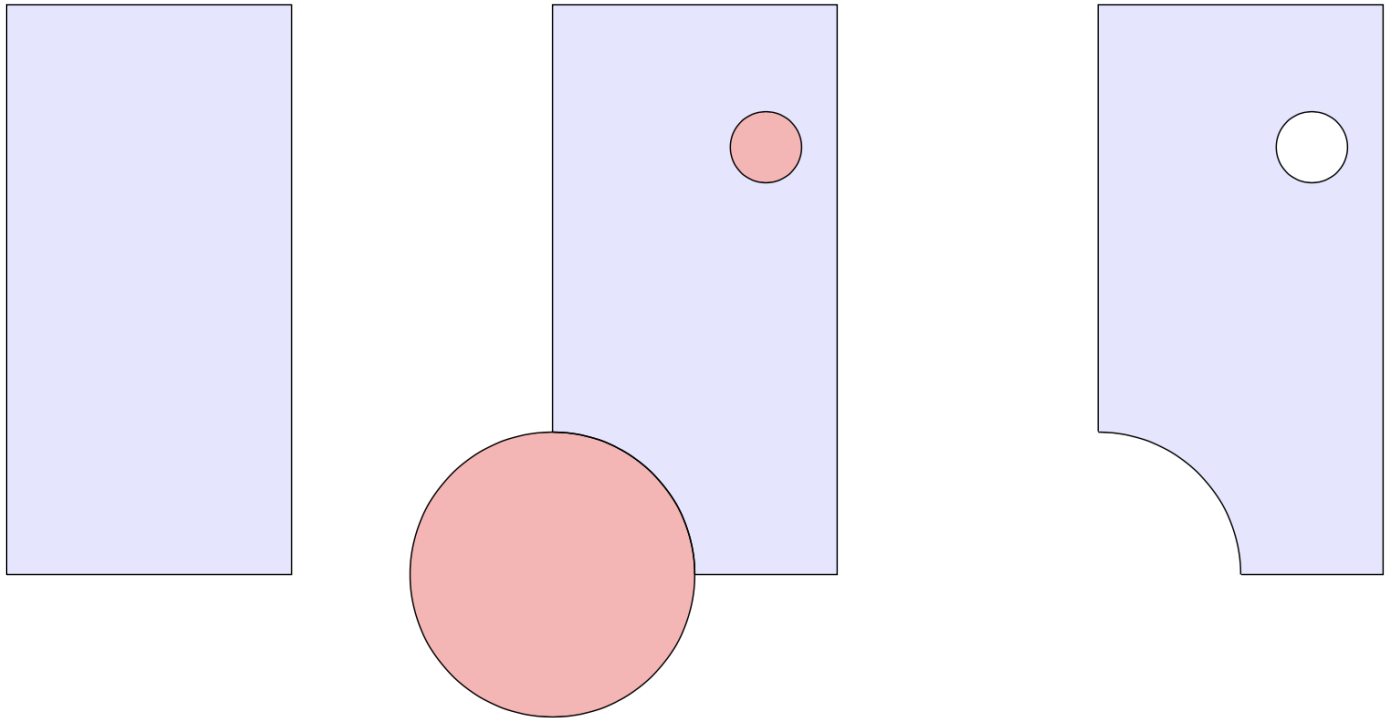
Le résultat est le premier objet moins la partie commune avec le second. Avec certains logiciels ([POV-Ray](#) par exemple), il est possible d'inverser un objet (ce qui revient à faire la soustraction inverse)...

### Intersection [\[ modifier \]](#) [\[ modifier le code \]](#)

Le résultat est la partie commune aux deux objets.

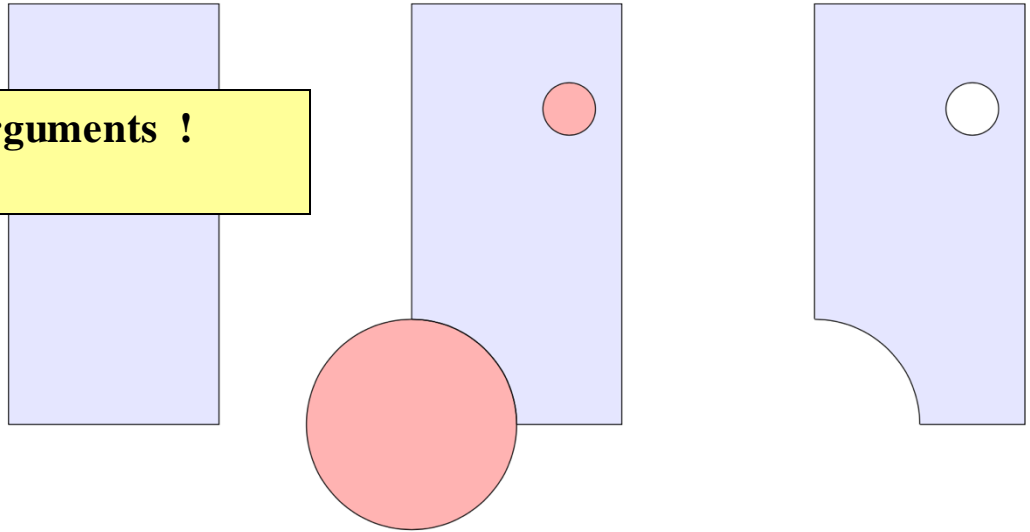
# Utiliser la géométrie constructive !

- Créer le rectangle avec la fonction `gmshModelOccAddRectangle`.
- Créer les deux disques avec la fonction `gmshModelOccAddDisk`.
- Retirer chaque disque du rectangle avec la fonction `gmshModelOccCut`.



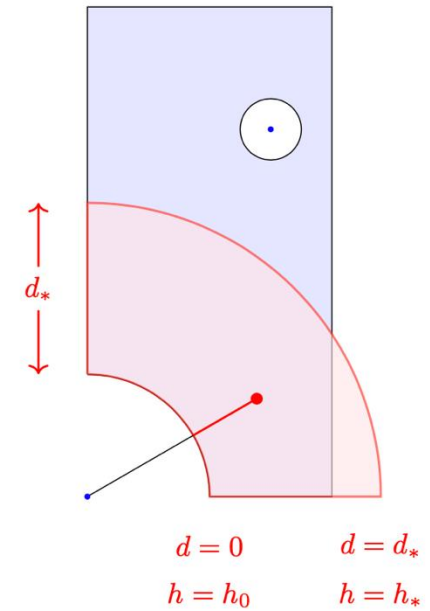
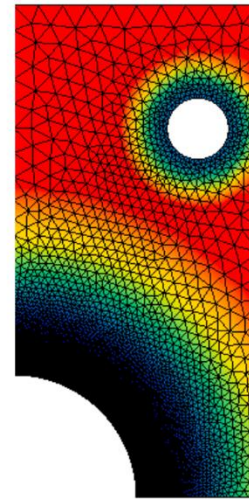
# Utiliser l'API de Gmsh pour accéder à OpenCascade !

Il faut juste trouver les arguments !  
Avec l'aide de copilote ?



```
int ierr;  
int idPlate = gmshModelOccAddRectangle(____, ____', ____', ____', ____', ____', ____', &ierr);  
int idNotch = gmshModelOccAddDisk(____, ____', ____', ____', ____', ____', NULL, 0, NULL, 0, &ierr);  
int idHole = gmshModelOccAddDisk(____, ____', ____', ____', ____', ____', NULL, 0, NULL, 0, &ierr);  
  
int plate[] = {____, ____};  
int notch[] = {____, ____};  
int hole[] = {____, ____};  
gmshModelOccCut(____, ____', ____', ____', NULL, NULL, NULL, NULL, NULL, -1, 1, 1, &ierr);  
gmshModelOccCut(____, ____', ____', ____', NULL, NULL, NULL, NULL, NULL, -1, 1, 1, &ierr);
```

# Construire le maillage avec une carte de taille !

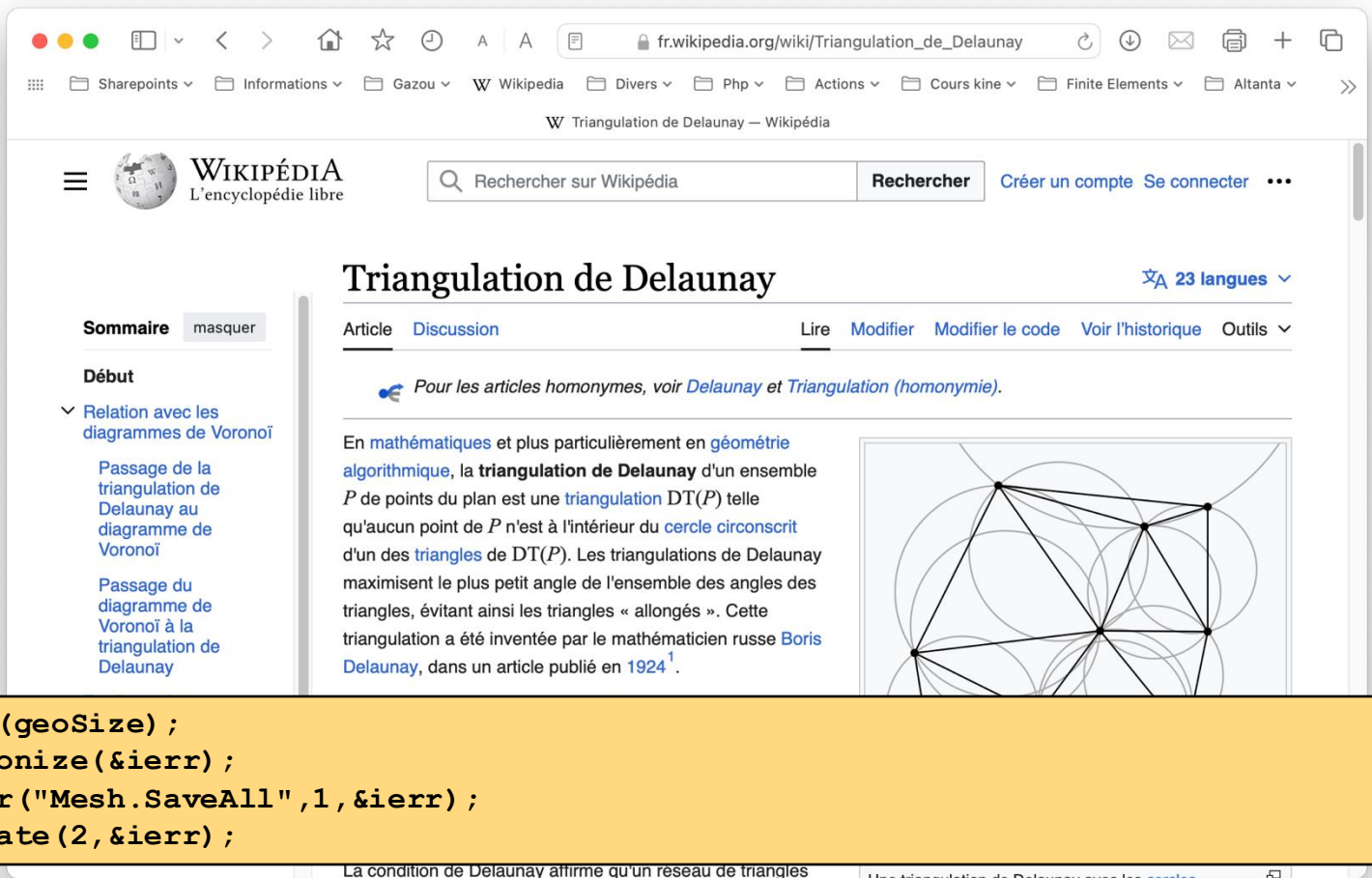


Les données pour construire la carte de taille se résument en cinq paramètres.

```
double h = theGeometry->h;  
double h0 = theGeometry->hNotch;  
double d0 = theGeometry->dNotch;  
double h1 = theGeometry->hHole;  
double d1 = theGeometry->dHole;
```

```
geoSetSizeCallback (geoSize) ;  
gmshModelOccSynchronize (&ierr) ;  
gmshOptionSetNumber ("Mesh.SaveAll", 1, &ierr) ;  
gmshModelMeshGenerate (2, &ierr) ;
```

Ensuite, on construit le plus joli maillage possible avec la densité spécifiée par la carte de taille avec une triangulation de Delaunay !



The screenshot shows the French Wikipedia page for 'Triangulation de Delaunay'. The page title is 'Triangulation de Delaunay' and it is in French. The main text explains that in mathematics, particularly in geometry, a Delaunay triangulation of a set of points  $P$  in the plane is a triangulation  $DT(P)$  such that no point of  $P$  is inside the circumcircle of any triangle in  $DT(P)$ . It mentions that this triangulation maximizes the minimum angle of the triangles, avoiding 'elongated' triangles, and was invented by the Russian mathematician Boris Delaunay in 1924. A diagram on the right shows a set of points with their circumcircles and the resulting Delaunay triangulation.

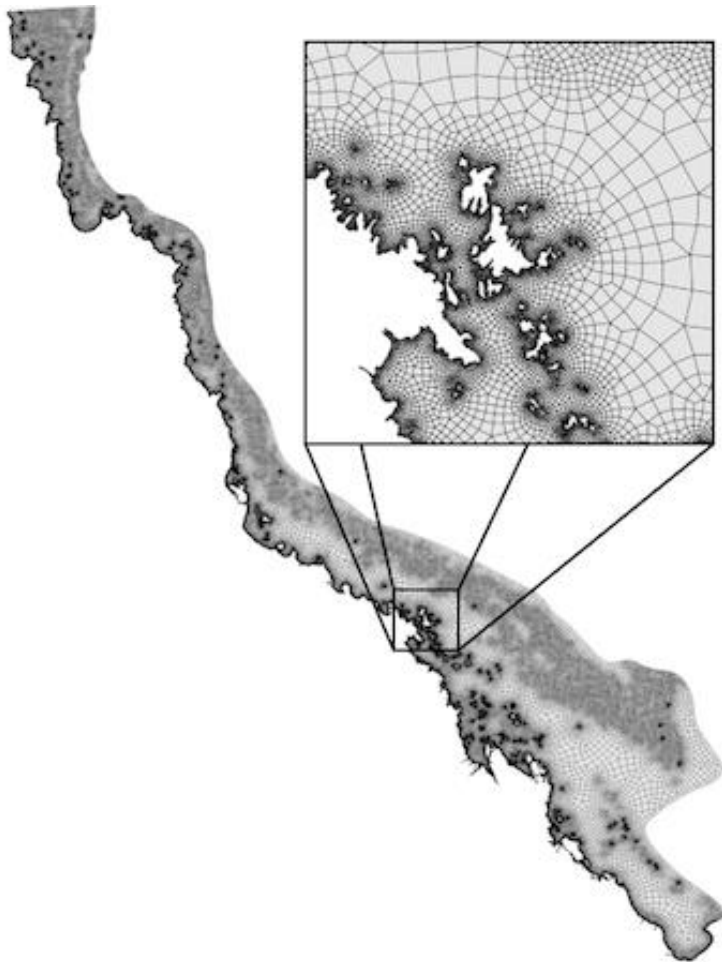
```
geoSetSizeCallback (geoSize) ;  
gmshModelOccSynchronize (&ierr) ;  
gmshOptionSetNumber ("Mesh.SaveAll", 1, &ierr) ;  
gmshModelMeshGenerate (2, &ierr) ;
```

La condition de Delaunay affirme qu'un reseau de triangles

Une triangulation de Delaunay avec les cercles



# Triangles, quadrilatères, tétraèdres, hexaèdres ?



erc  
European Research Council  
Established by the European Commission

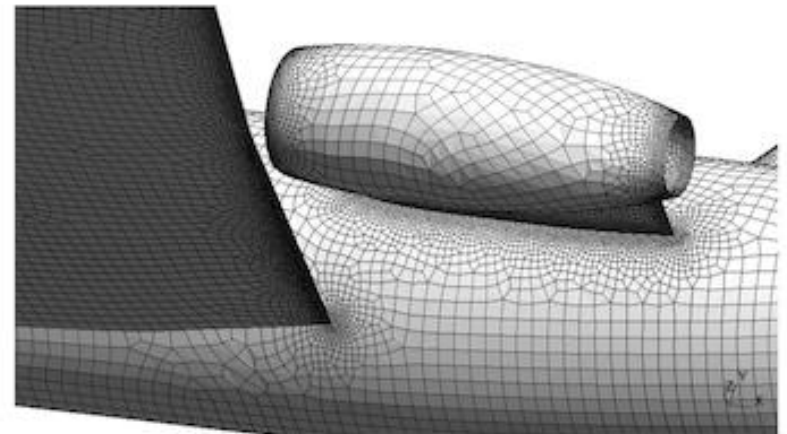
**Hextreme**  
Hexahedral Mesh Generation in Real Time

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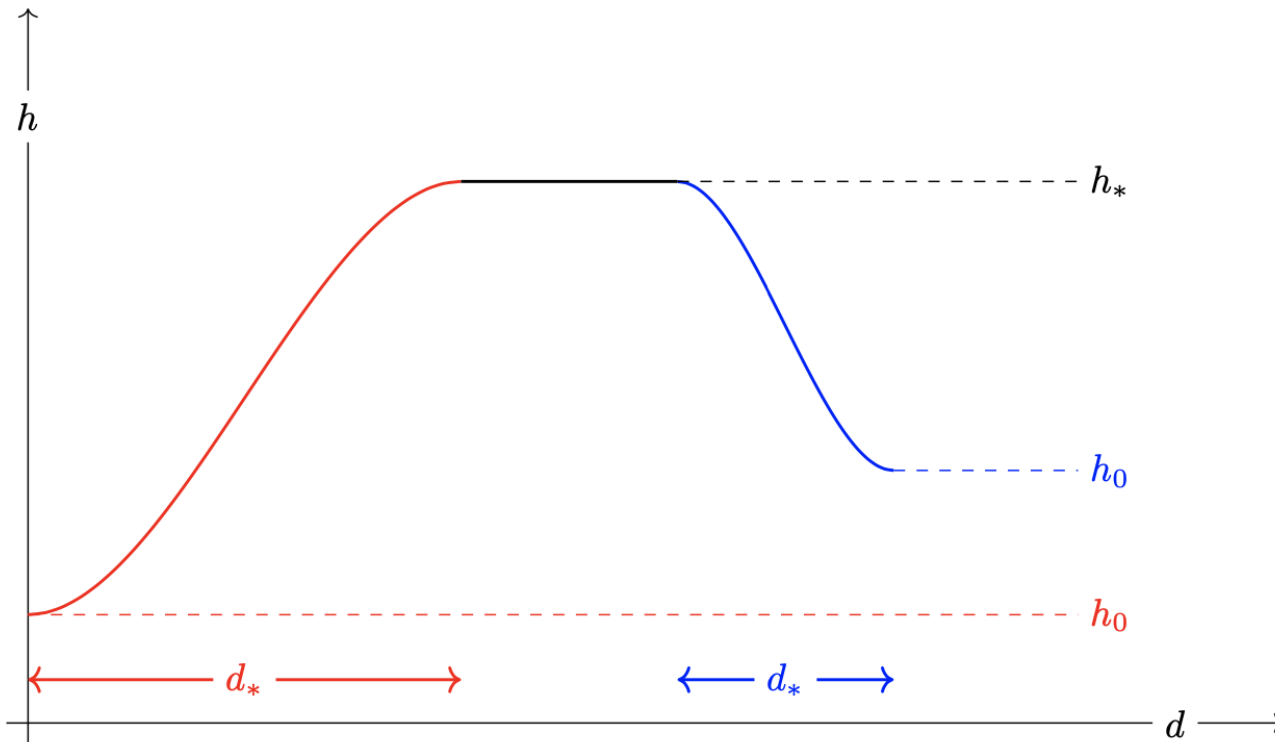
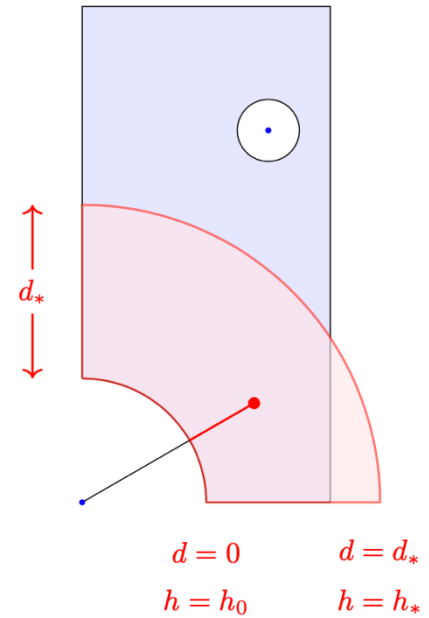
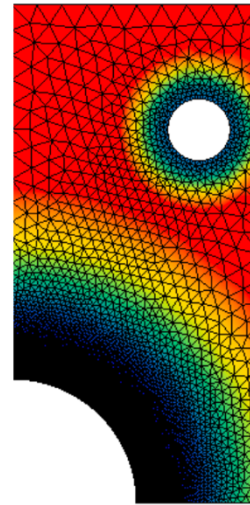
**Staff**

**Jean-François Remacle**  
*Professor*  
[jean-francois.remacle@uclouvain.be](mailto:jean-francois.remacle@uclouvain.be)

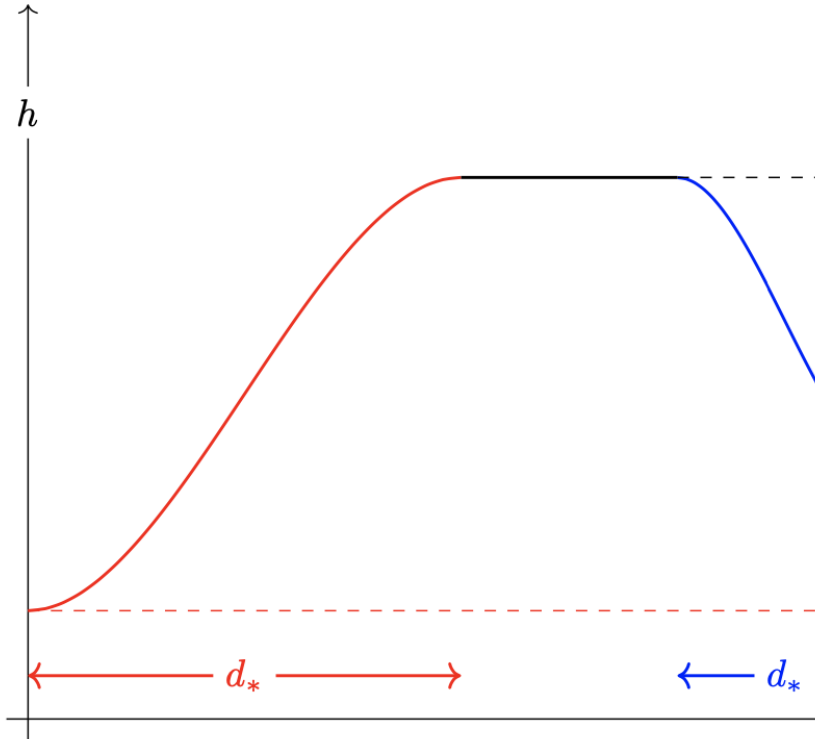
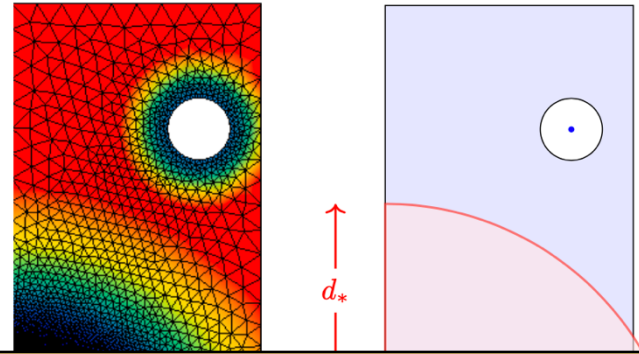
After his Engineering Degree at the University of Liège in Belgium in



Définir une carte de taille !



# Définir une carte de taille !



```
double geoSize(double x,double y)
{
    femGeo* theGeometry = geoGetGeometry();

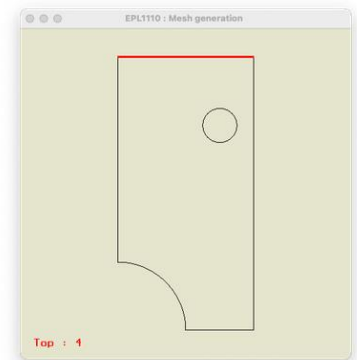
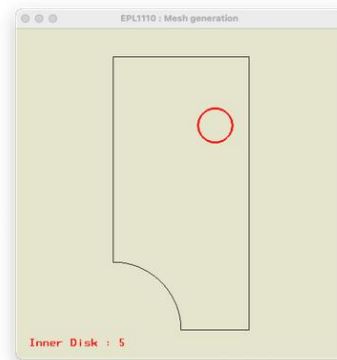
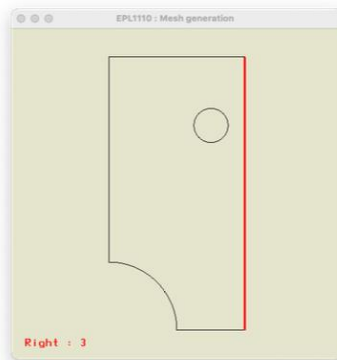
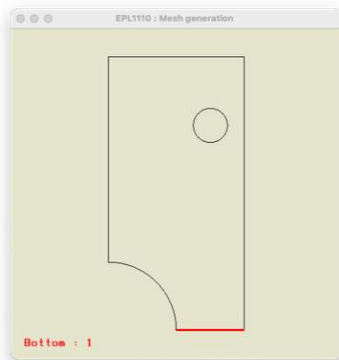
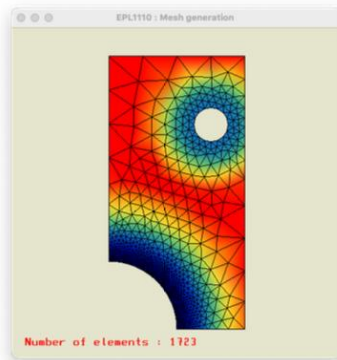
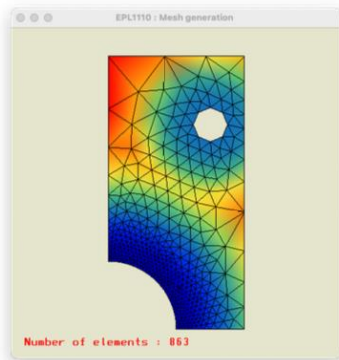
    double h = theGeometry->h;
    double x0 = theGeometry->xNotch;
    double y0 = theGeometry->yNotch;
    double r0 = theGeometry->rNotch;
    double h0 = theGeometry->hNotch;
    double d0 = theGeometry->dNotch;

    double x1 = theGeometry->xHole;
    double y1 = theGeometry->yHole;
    double r1 = theGeometry->rHole;
    double h1 = theGeometry->hHole;
    double d1 = theGeometry->dHole;

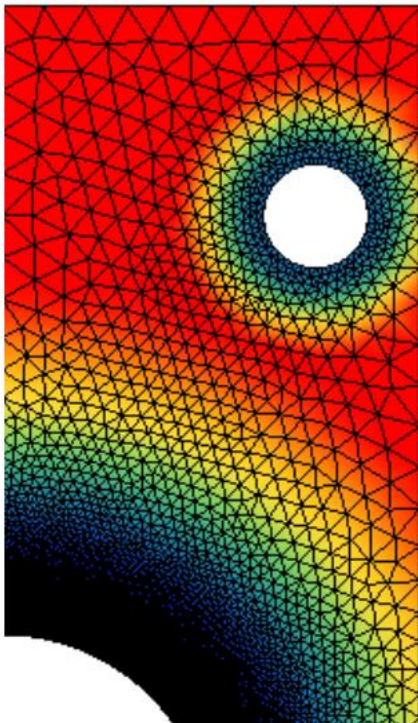
    //
    //     A modifier !
    //

    return h;
}
```

Et tout cela pourquoi ?



# Trois tableaux décrivent le maillage final



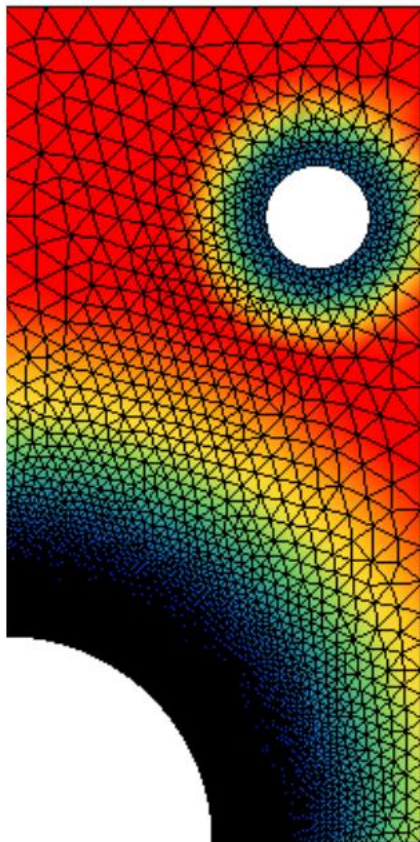
```
typedef struct {  
    int nNodes;  
    double *X;  
    double *Y;  
} femNodes;
```

```
typedef struct {  
    int nLocalNode;  
    int nElem;  
    int *elem;  
    femNodes *nodes;  
} femMesh;
```

```
typedef struct {  
    femMesh *mesh;  
    int nElem;  
    int *elem;  
    char name[MAXNAME];  
} femDomain;
```

**Fixer les coordonnées des points**  
**Définir la topologie ou connectivité des éléments**  
**Associer des domaines physiques à des éléments**

Et voilà  
le maillage !



Number of nodes 2520

```
0 : 3.7500000e-01 5.0000000e-01
1 : 0.0000000e+00 -1.0000000e+00
2 : -5.0000000e-01 -5.0000000e-01
3 : 5.0000000e-01 -1.0000000e+00
4 : -5.0000000e-01 1.0000000e+00
5 : 5.0000000e-01 1.0000000e+00
```

```
2517 : -1.5626551e-01 -6.2635267e-01
```

```
2518 : -3.6669678e-02 -7.9420329e-01
```

```
2519 : -4.8014465e-03 -8.1397000e-01
```

Number of edges 276

```
0 : 0 6
1 : 6 7
```

```
274 : 274 275
```

```
275 : 275 4
```

Number of triangles 4764

```
0 : 513 514 484
```

```
1 : 1291 2281 1589
```

```
4762 : 2081 2212 1466
```

```
4763 : 1675 1913 1668
```

Number of domains 6

```
Domain : 0
```

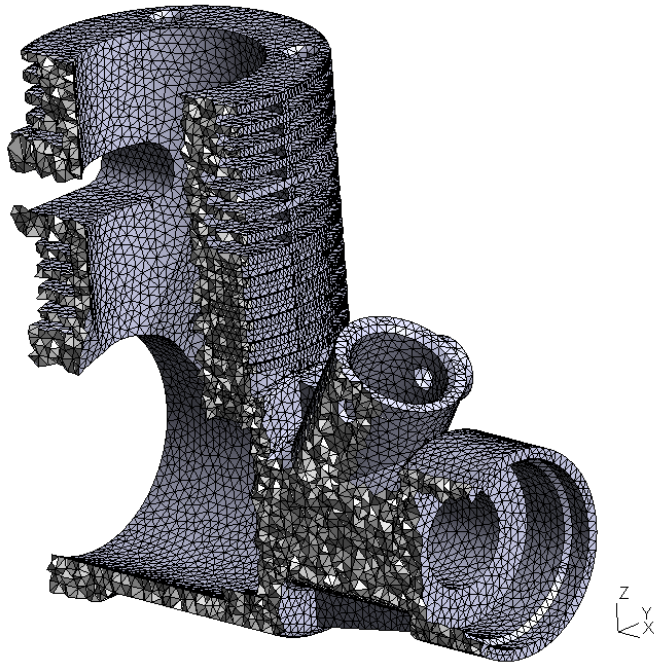
```
Name : Outer Disk
```

```
Number of elements : 40
```

```
0 1 2 3 4 5 6 7 8 9
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29
30 31 32 33 34 35 36 37 38 39
```

```
Domain : 1
```

# Définir localement une fonction sur un maillage



Fonctions de base  
spécifiées a priori

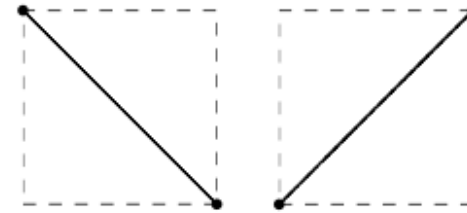
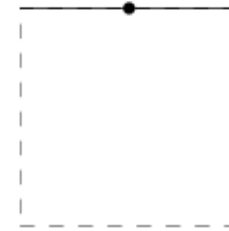
$$u(\mathbf{x}) \approx u^h(\mathbf{x}) = \sum_{j=1}^n U_j \tau_j(\mathbf{x})$$

Valeurs nodales inconnues

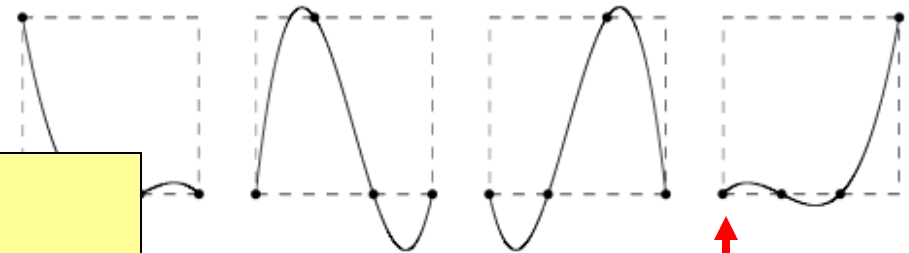
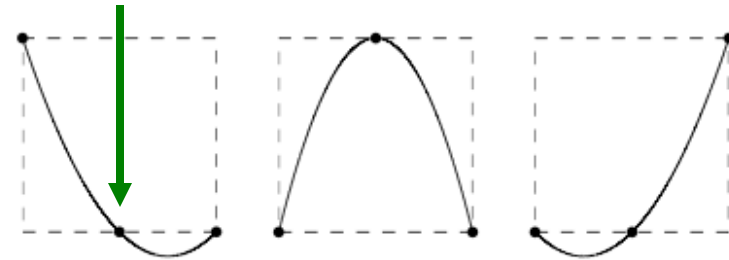
The problem geometry is divided in small finite elements.

On each element, the solution is approximated  
by means of unknown nodal values and given polynomials

# Fonctions de forme unidimensionnelles de Lagrange



Ce nœud n'est pas un sommet



Sommet

Nodes and other nodes :-)

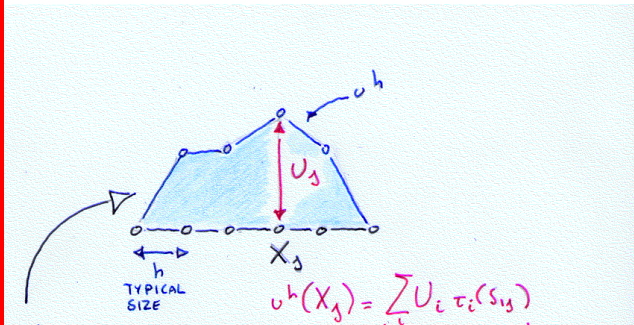
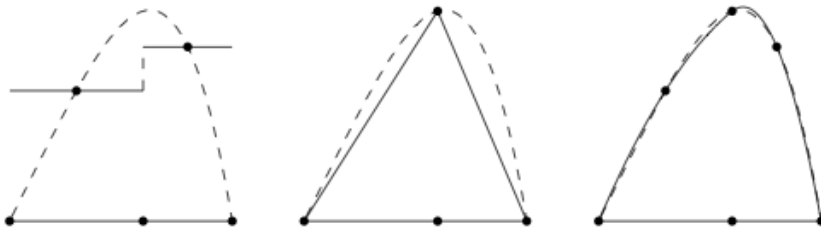
Nodes of the meshes = vertices

Location of nodal values = nodes

For linear piecewise interpolation, all nodes are vertices



# En une dimension...



$u \approx u^h = \sum_{j=1}^m U_j \tau_j(x)$

$U_j$  : UNKNOWN NODAL VALUES  
 $\tau_j(x)$  : KNOWN SHAPE FUNCTIONS

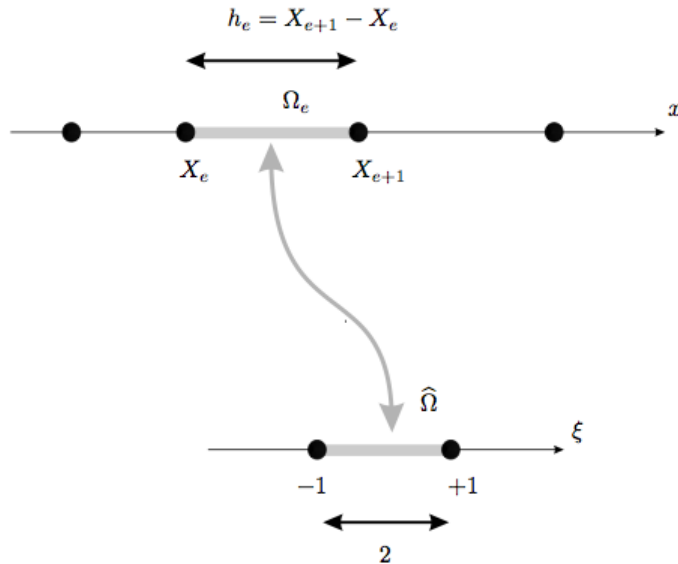
$\tau_j(X_i) = \delta_{ij}$

$\tau_j$  : LOCAL SUPPORT !

$\{ \tau_j \}$  IS A BASIS FOR  $U^h$

$U \in U$   
 $U^h \in U^h \subset U$   
 $\dim(U^h) = m$

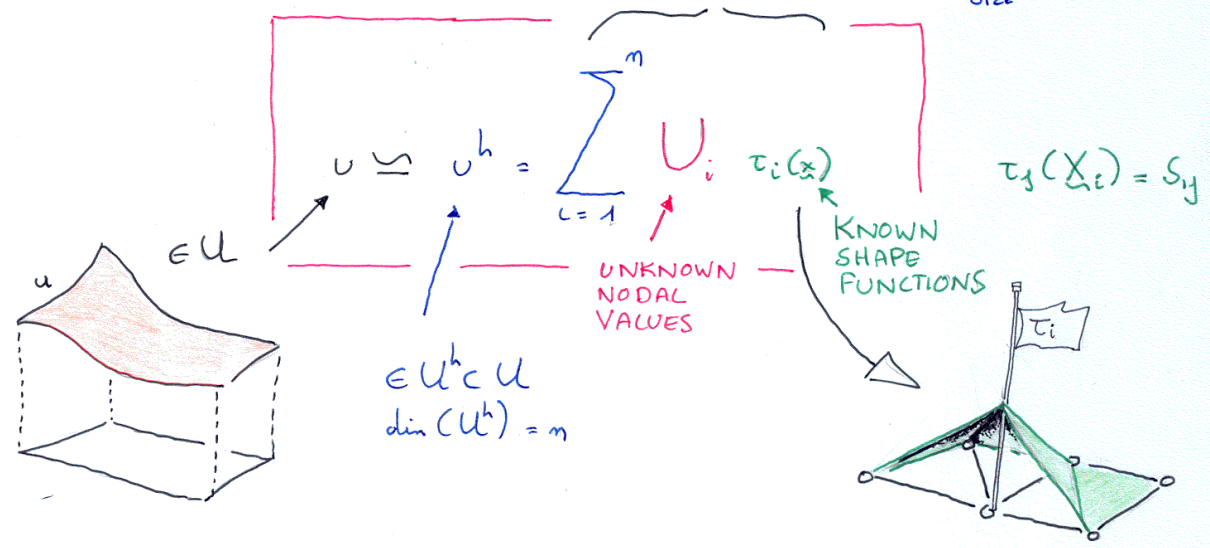
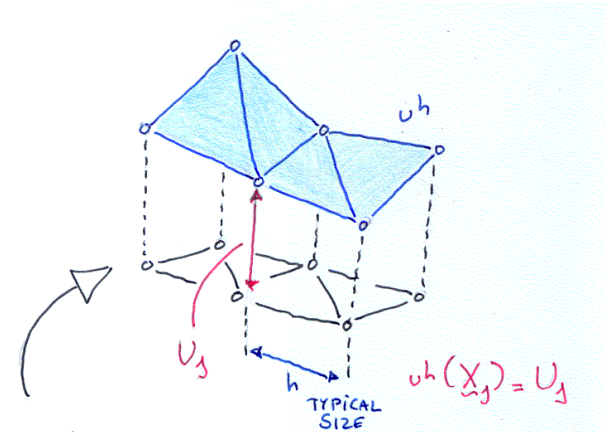
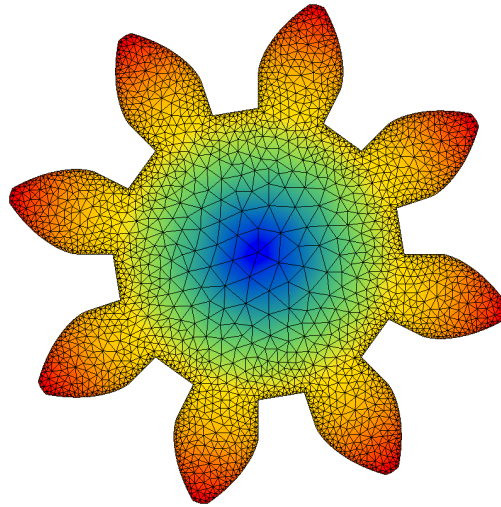
# Un élément parent (template) pour définir les fonctions de forme



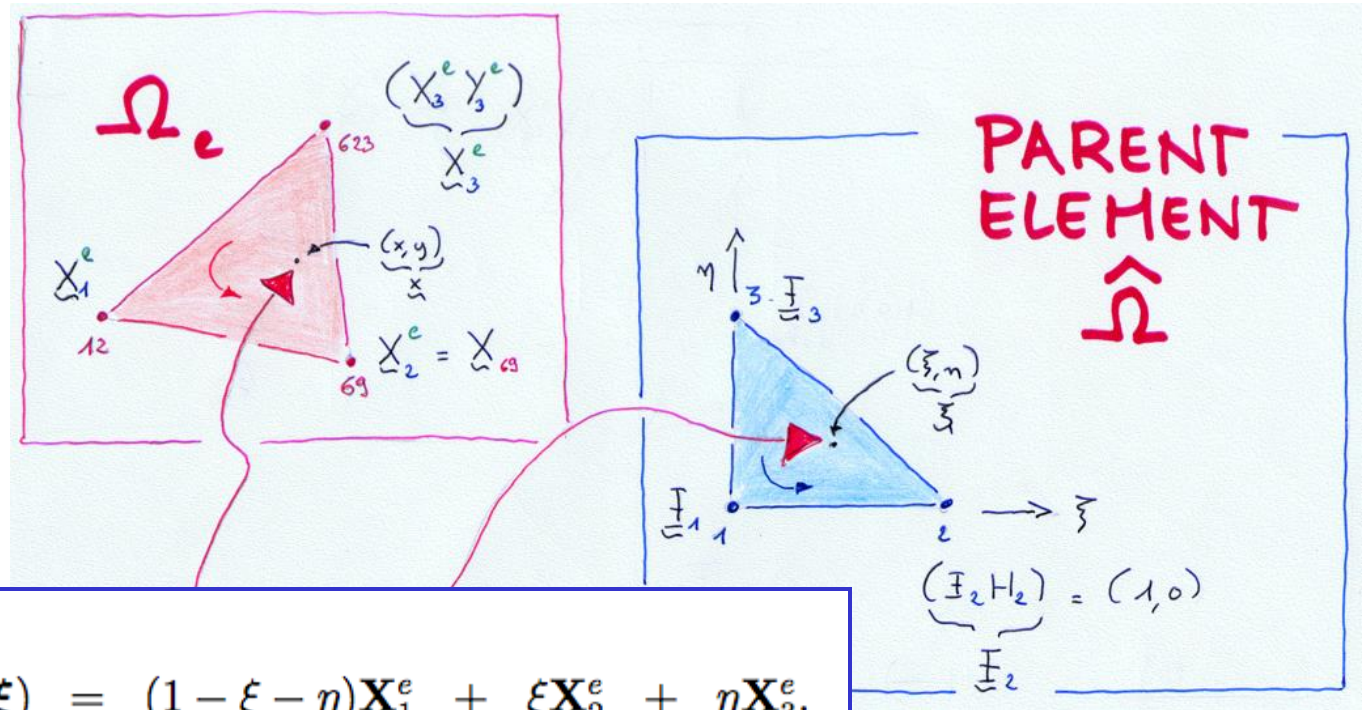
$$x(\xi) = \xi \frac{(X_{e+1} - X_e)}{2} + \frac{(X_{e+1} + X_e)}{2},$$
$$\xi(x) = \frac{2x - (X_{e+1} + X_e)}{(X_{e+1} - X_e)}.$$

**Isomorphisme linéaire entre l'élément parent  
et tous les autres éléments...**

# En deux dimensions



# Un triangle parent pour définir toutes les fonctions de forme



Isomorphisme linéaire entre le triangle parent et tous les autres triangles...