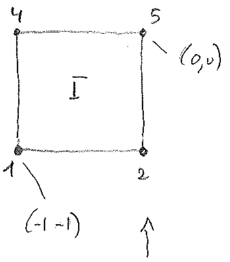
# Equation de Poisson : solutions

**10** Une solution provisoire est disponible :-)

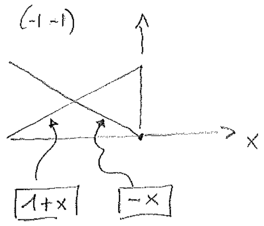
**15**

$$\sum_j \langle \nabla \phi_i \cdot \nabla \phi_j \rangle U_j = \langle \phi_i \rangle$$




$$\boxed{\begin{aligned} \phi_1(x,y) &= xy \\ \phi_2(x,y) &= -(1+x)\,y \\ \phi_5(x,y) &= (1+x)(1+y) \\ \phi_4(x,y) &= -x\,(1+y) \end{aligned}}$$

$\sum \phi_i = 1$ :-)




| $\phi_{i,x}$ | $\phi_{i,y}$ |
|---|---|
| $y$ | $x$ |
| $-y$ | $-(1+x)$ |
| $(1+y)$ | $(1+x)$ |
| $-(1+y)$ | $-x$ |

$\sum \phi_{i,x} = 0$

$\sum \phi_{i,y} = 0$

IL FAUT JUSTE CALCULER

$$\underbrace{4\left[\int_{\Omega_I} \frac{\partial \phi_5}{\partial x}\frac{\partial \phi_5}{\partial x} + \frac{\partial \phi_5}{\partial y}\frac{\partial \phi_5}{\partial y}\right]}_{A^e_{ss}} U_5 = \underbrace{4\int_{\Omega_I} \phi_5}_{B^c_s}$$

LES 4 CONTRIBUTIONS DES QUATRE ELEMENTS POUR LE NOEUD CENTRAL

$$\begin{bmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & 1 & & & & & & \\ & & & 1 & & & & & \\ & & & & \boxed{A_{ss}} & & & & \\ & & & & & 1 & & & \\ & & & & & & 1 & & \\ & & & & & & & 1 & \\ & & & & & & & & 1 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ \boxed{U_5} \\ U_6 \\ U_7 \\ U_8 \\ U_9 \end{bmatrix} = \begin{bmatrix} 0 \\ \\ \\ \\ \boxed{B_s} \\ \\ \\ \\ \\ \end{bmatrix}$$

RESOUDRE
$\nabla^2 v + 1 = 0$ SUR $\Omega$
$v = 0$ SUR $\partial\Omega$

$\rightarrow$ PERMET D'OBTENIR $u$ CAR $u = v + 4$ :-)

- INTEGRER $A^e_{ss}$ EST EQUIVALENT A INTEGRER $A^e_{11}$ !

SYMETRIE DE $A^e_{ij}$

$$\begin{bmatrix} 0 & \times & \blacksquare & \times \\ \times & 0 & \times & \blacksquare \\ \blacksquare & \times & 0 & \times \\ \times & \blacksquare & \times & 0 \end{bmatrix} = \begin{bmatrix} \frac{4}{6} & -\frac{1}{6} & -\frac{2}{6} & -\frac{1}{6} \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

- TOUS LES TERMES DIAGONAUX SONT EGAUX
- LA SOMME D'UNE LIGNE VAUT ZERO (si ! si !)

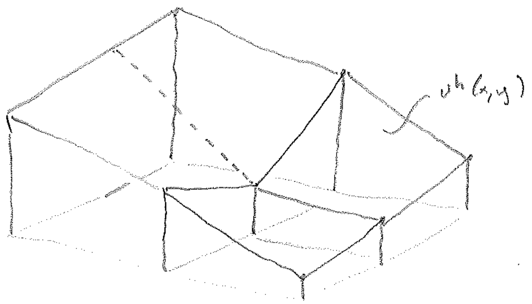
$$A^e_{11} = A^e_{ss} = \underbrace{\int_{-1}^{0}\int_{-1}^{0} x^2 \, dx \, dy}_{\left[\frac{x^3}{3}\right]_{-1}^{0} = \frac{1}{3}} + \int_{-1}^{0}\int_{-1}^{0} y^2 \, dx \, dy = \frac{2}{3}$$

$$B^e_1 = \int_{-1}^{0}\int_{-1}^{0} x y \, dx \, dy = \underbrace{\left[\frac{x^2}{2}\right]_{-1}^{0}}_{-\frac{1}{2}} \underbrace{\left[\frac{y^2}{2}\right]_{-1}^{0}}_{-\frac{1}{2}} = \frac{1}{4}$$

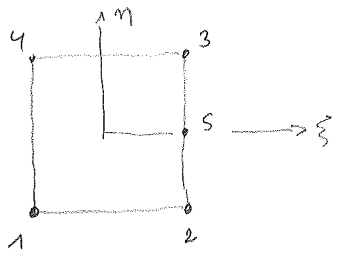
$$\frac{2}{3} U_s = \frac{1}{4}$$

$$\boxed{U_s = \frac{3}{8}}$$

$\boxed{16}$



11 Une solution provisoire est disponible :-)



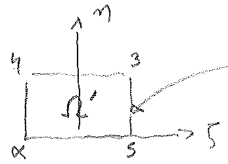

$$\phi_1(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta)$$

$$\phi_2(\xi,\eta) = \begin{cases} 0 & \eta \geq 0 \\ -\frac{1}{2}(1+\xi)\eta & \eta \leq 0 \end{cases}$$

$$\phi_3(\xi,\eta) = \begin{cases} \frac{1}{2}(1+\xi)\eta & \eta \geq 0 \\ 0 & \eta \leq 0 \end{cases}$$

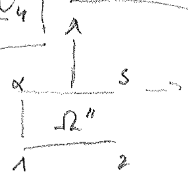
$$\phi_4(\xi,\eta) = \frac{1}{4}(1-\xi)(1+\eta)$$

$$\phi_5(\xi,\eta) = \begin{cases} \frac{1}{2}(1+\xi)(1-\eta) & \eta \geq 0 \\ \frac{1}{2}(1+\xi)(1+\eta) & \eta \leq 0 \end{cases}$$




$$u^h(\xi,\eta) = U_4 \overbrace{\frac{(1-\xi)\eta}{2}}^{\phi'_4} + U_\alpha \overbrace{\frac{(1-\xi)(1-\eta)}{2}}^{\phi'_\alpha} + U_3 \underbrace{\frac{(1+\xi)\eta}{2}}_{\phi'_3} + U_5 \underbrace{\frac{(1+\xi)(1-\eta)}{2}}_{\phi'_5}$$

$$\boxed{U_\alpha = \frac{U_1 + U_4}{2}}$$



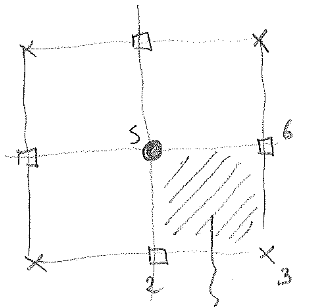

$$= U_4 \underbrace{\left[\frac{(1-\xi)\eta}{2} + \frac{(1-\xi)(1-\eta)}{4}\right]}_{\frac{(1-\xi)}{4}\underbrace{\left[1-\eta+2\eta\right]}_{(1+\eta)}}$$

:-)

**12** Une solution provisoire est disponible :-)

[18]

$$\sum_j \langle \nabla \phi_i \, \nabla \phi_j \rangle \, U_j = \langle \phi_i \rangle + \sum_j \langle\!\langle \phi_i \, \phi_j \rangle\!\rangle \, U_j$$



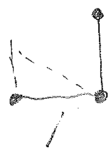

3 VALEURS DISTINCTES

RESOUDRE LE PROBLEME SUR 1/4 PAR SYMETRIE

$$\frac{1}{6}\begin{bmatrix} 4 & -1 & 2 & -1 \\ -1 & 4 & -1 & 2 \\ 2 & -1 & 4 & -1 \\ -1 & 2 & -1 & 4 \end{bmatrix}\begin{bmatrix} U_2 \\ U_3 \\ U_6 \\ U_5 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix} +$$

LES EQUATIONS POUR $U_2$ ET $U_6$ SONT IDENTIQUES

$$\frac{1}{6}\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} U_2 \\ U_3 \\ U_6 \\ U_5 \end{bmatrix}$$



$$\langle\!\langle \phi_2 \, \phi_2 \rangle\!\rangle = \int_0^1 x^2 = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\langle\!\langle \phi_2 \, \phi_3 \rangle\!\rangle = \int_0^1 x(1-x) = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$
\begin{cases}
\cancel{2U_2} - U_3 - U_5 = \frac{3}{2} + \cancel{2U_2} + U_3 \\
-2U_2 + \cancel{4U_3} + 2U_5 = \frac{3}{2} + \cancel{4U_3} + 2U_2 \\
-2U_2 - 2U_3 + 4U_5 = \frac{3}{2}
\end{cases}
$$

$$\boxed{2U_3 = -U_5 - \frac{3}{2}}$$

$$\boxed{4U_2 = -\frac{3}{2} + 2U_5}$$

$$\left[\cancel{U_5} + \cancel{\frac{3}{2}}\right] + \left[\frac{3}{4} - \cancel{U_5}\right] + 4U_5 = \cancel{\frac{3}{2}}$$

$$\boxed{U_5 = -3}$$

A VERIFIER :-)