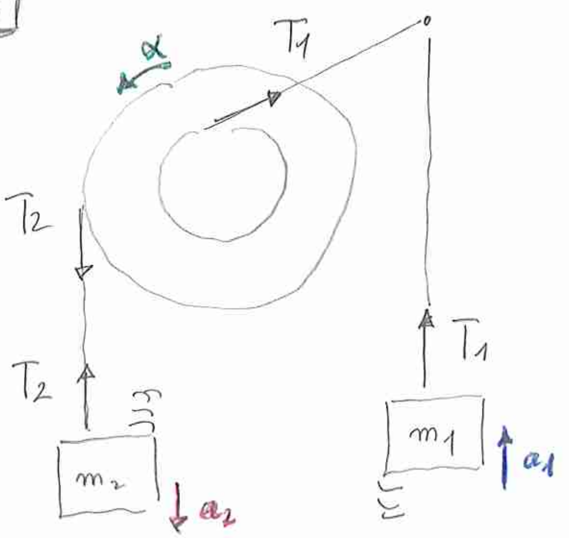
79



$$\boxed{I\alpha = \sum M}$$

3 CORPS

1 POULIE

$$I\alpha = R_2 T_2 - R_1 T_1 \quad (1)$$

2 BLOCS

$$m_2 a_2 = m_2 g - T_2 \quad (2)$$

$$m_1 a_1 = T_1 - m_1 g \quad (3)$$

$$\boxed{m\vec{a} = \sum \vec{F}}$$

A PRIORI
ON IGNORE
QUEL BLOC MONTE
OU DESCEND !

LE SIGNE
FOURNIRA LA SOLUTION :-)

CINEMATIQUE

$$\alpha R_1 = a_1 \quad (4)$$

$$\alpha R_2 = a_2 \quad (5)$$

5 INCONNUES $\alpha, a_1, a_2, T_1, T_2$

5 EQUATIONS

- (5) DANS (2)
- (4) DANS (3)

$$m_2 R_2 \alpha = m_2 g - T_2$$

$$m_1 R_1 \alpha = T_1 - m_1 g$$

$$T_2 = m_2 (g - R_2 \alpha)$$

$$T_1 = m_1 (g + R_1 \alpha)$$

- EN INJECTANT TOUT DANS (1)

$$I\alpha = R_2 m_2 (g - R_2 \alpha) - R_1 m_1 (g + R_1 \alpha)$$

$$(I + m_1 R_1^2 + m_2 R_2^2)\, \alpha = (R_2 m_2 - R_1 m_1)\, g$$

$$\boxed{\alpha = \frac{(R_2 m_2 - R_1 m_1)\, g}{(I + m_1 R_1^2 + m_2 R_2^2)}}$$

VALEURS NUMERIQUES

$\alpha = 10{,}548\ \text{rad/s}^2$

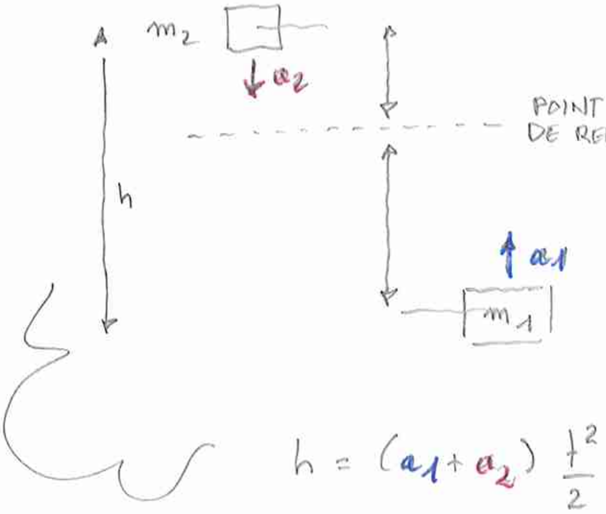
$a_1 = 0{,}52742\ \text{m/s}^2$

$a_2 = 1{,}0548\ \text{m/s}^2$

$T_1 = 10{,}3\ \text{N}$

$T_2 = 26{,}3\ \text{N}$

INSTANT OU LES 2 BLOCS SONT A LA MEME HAUTEUR



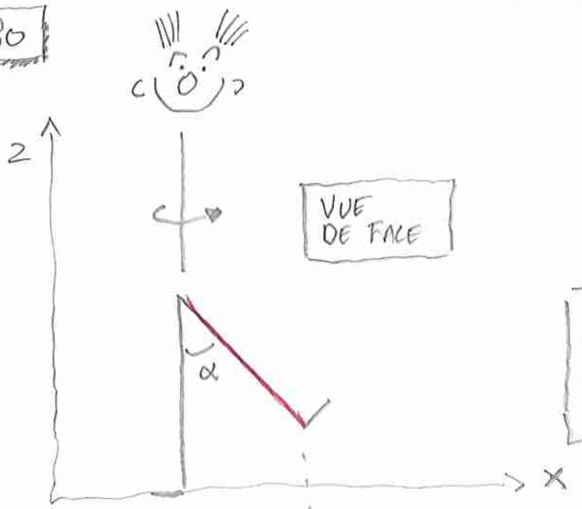
$$h = \underbrace{(a_1 + a_2)\,\frac{t^2}{2}}_{\Sigma \text{ DISTANCES PARCOURUES PAR LES 2 BLOCS}}$$

$$\boxed{t = \sqrt{\frac{2h}{a_1 + a_2}}}$$

VALEUR NUMERIQUE

$t = 1{,}59\ s$

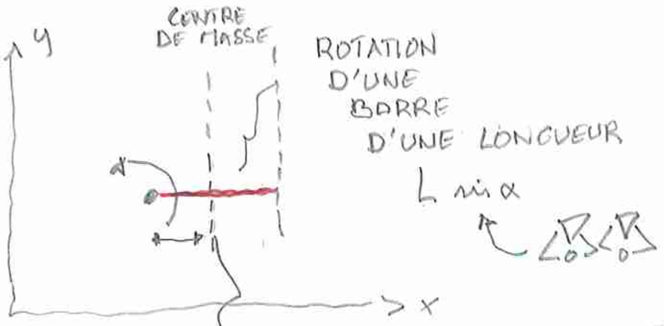
80



VUE DE FACE

$$\boxed{I_h = I + m h^2}$$

$$\boxed{I_h = \underbrace{\frac{1}{12}\, m\,(L \sin\alpha)^2}_{\text{INERTIE BARRE CM}} + m\left(\frac{L \sin\alpha}{2}\right)^2}$$



VUE AERIENNE

UN RARE EXERCICE EN 3D

$$h = \frac{L \sin\alpha}{2}$$

$$I_h = \left(\frac{1}{12} + \frac{1}{4}\right) m L^2 \sin^2\alpha$$

$$= \frac{1+3}{12}\, m L^2 \sin^2\alpha$$

$$\boxed{I_h = \frac{1}{3}\, m L^2 \sin^2\alpha}$$

81

ETAT INITIAL

ETAT FINAL

$K = 0$
$U = 0$

PAS DE FROTTEMENT !

→ CONSERVATION ENERGIE MECANIQUE

INERTIE POULIE

$$K = \frac{1}{2}\overbrace{\left(\frac{1}{2}m_p R_p^2\right)}^{\text{INERTIE POULIE}}\omega^2 + \frac{1}{2}m_b \underbrace{R_p^2\omega^2}_{(\text{VITESSE BLOC})^2}$$

$$U = -m_b g L \sin\theta$$

LE BLOC EST DESCENDU

$$\overbrace{R_p^2\omega^2}^{v^2} = \frac{m_b g L \sin\theta}{\left[\frac{1}{4}m_p + \frac{1}{2}m_b\right]}$$

$$\boxed{v = \sqrt{\frac{4 m_b g L \sin\theta}{m_p + 2 m_b}}}$$

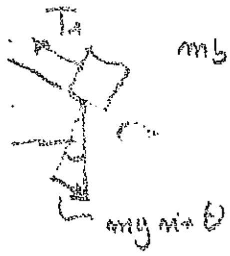
VALEUR NUMERIQUE
$v = 2,8$ m/s

ACCELERATION ?

$$\underbrace{\left(\frac{1}{2}m_p R_p^2\right)}_{I}\alpha = R_p T$$

POULIE

BLOC



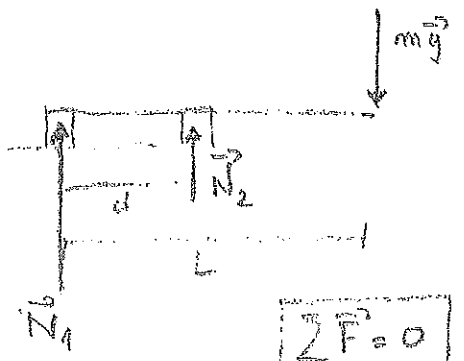
$$m_b R_p \alpha = m_b g \sin\theta - T$$

2 EQUATIONS
2 INCONNUES α, T

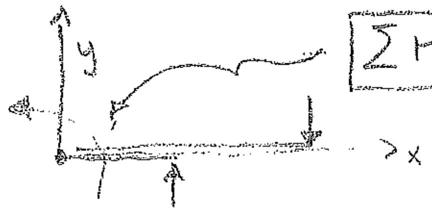
$$\boxed{\alpha = \frac{\sin\theta\, m_b g}{\frac{1}{2}m_p R_p + m_b R_p}}$$

VALEUR NUMERIQUE
$\alpha = 7,83$ rad/s²

82



$\boxed{\sum \vec{F} = 0}$ $\quad N_1 + N_2 = mg$

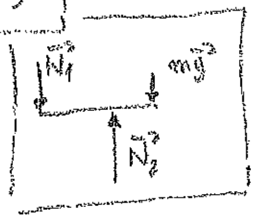


$\boxed{\sum M = 0}$ $\quad N_2 d - mgL = 0$

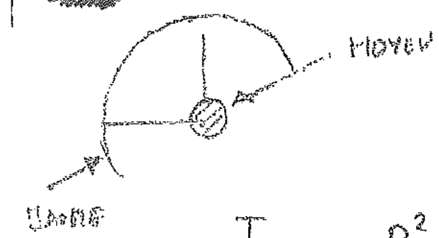
EQUILIBRE
DE ROTATION
A L'ORIGINE (PAR EXEMPLE !)

$$\boxed{\begin{aligned} N_2 &= mgL/d \\ N_1 &= mg\,(1 - L/d) \end{aligned}}$$

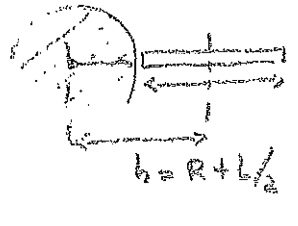
$N_2 = 3652$ N
$N_1 = -2943$ N



83



$$I = \underbrace{\frac{m_1 R_1^2}{2}}_{\text{CYLINDRE PLEIN}} + \underbrace{m_3 R_3^2}_{\text{CYLINDRE CREUX}} + 4\Big(\underbrace{m_2 \frac{L^2}{12}}_{\text{BARRE}} + \underbrace{m_2 (R_1 + L/2)^2}_{\text{THEOREME DES AXES //}}\Big)$$

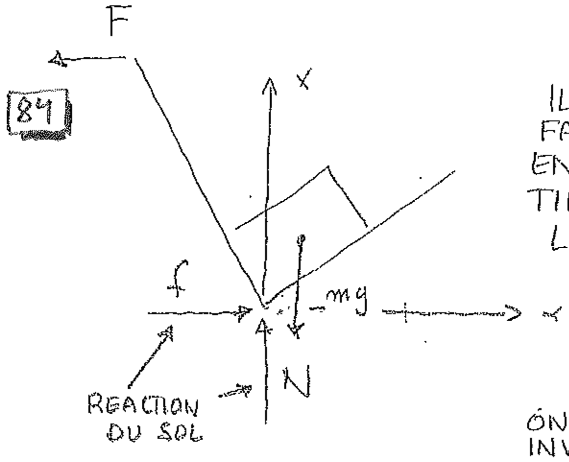


$$= 4 + 72 + 4\left[\frac{16}{12} + \underbrace{(2+2)^2}_{16}\right]$$

$$I = 4 + 72 + 4\left(\frac{4}{3} + 16\right) = 145{,}3$$

$\boxed{I = 145{,}3 \text{ kg m}^2}$ $\quad \boxed{m = 8 \text{ kg}}$ $\quad \boxed{k = \sqrt{\frac{I}{m}} = 4{,}26 \text{ m}}$

[84]

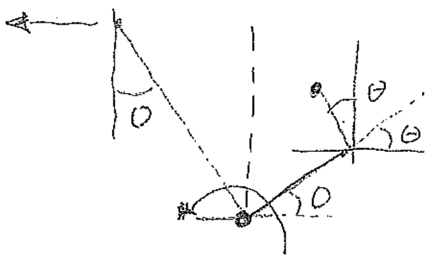


IL
FAUT
EN FAIT
TIRER SUR
LE DIABLE

LE DESSIN
DE L'ENONCE
EST TROMPEUR !

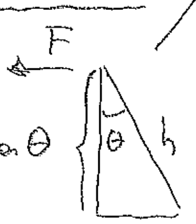
ON PEUT
INVERSER LE
DESSIN ET OBTENIR
UN RESULTAT NEGATIF

$\boxed{\Sigma M = 0}$!



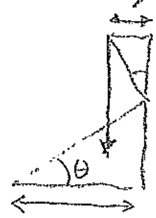
TRIGONOMETRIE !
TRICKY !

$$\boxed{F \underbrace{h \cos\theta} - m g d = 0}$$



$h \cos\theta$

$a \sin\theta/2$



$b \cos\theta/2$

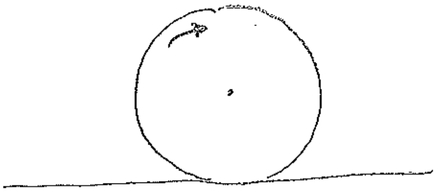
$$\boxed{d = \underbrace{\frac{b\cos\theta - a\sin\theta}{2}}_{0{,}333 \text{ m}}}$$

$$\boxed{F = \frac{m g d}{h \cos\theta}}$$

VALEUR
NUMERIQUE

$F = 68{,}6$ N

85



$$\boxed{\begin{aligned}\Theta(t) &= \omega_0 t + \alpha t^2/2 \quad (1)\\ \omega(t) &= \omega_0 + \alpha t \quad (2)\end{aligned}}$$

- $\omega_0 = 120 \left[\frac{\text{tours}}{\text{minute}}\right] = \overbrace{120 \ \frac{2\Pi}{60}}^{12,6} \left[\frac{\text{rad}}{\text{sec}}\right]$

$$\boxed{\begin{aligned}1 \text{ tour} &= 2\Pi \text{ rad}\\ 1 \text{ min} &= 60 \text{ sec}\end{aligned}}$$

- $t = 60s$
- $\Theta(t) = 90 \, [\text{tours}] = \underbrace{90 \ 2\Pi}_{565,5} \, [\text{rad}]$

$$\boxed{\alpha = 2\frac{\Theta(t)}{t^2} - \frac{2\omega_0}{t}}$$

VALEUR NUMERIQUE

$\alpha = -0,105 \text{ rad}/s^2$

NEGATIF CAR DECELERATION !

ON AURAIT AUSSI PU METTRE LE SIGNE NEGATIF DIRECTEMENT DANS (1) ET ECRIRE : $\Theta(t) = \omega_0 t - \alpha t^2/2$:-)

TEMPS REQUIS POUR S'ARRETER

$$\boxed{t^* = \frac{-\omega_0}{\alpha}}$$

CAR $\underbrace{\omega(t^*)}_{\omega_0 - \alpha t^*} = 0$

VALEUR NUMERIQUE

$t^* = \frac{12,6}{0,105} = 120 \text{ sec}$

APPROXIMATIVEMENT 2. minutes !

$$\boxed{\Theta(t^*) = \omega_0\left(-\frac{\omega_0}{\alpha}\right) + \frac{\alpha}{2}\left(-\frac{\omega_0}{\alpha}\right)^2 = -\frac{1}{2}\frac{\omega_0^2}{\alpha}}$$

ANGLE PARCOURU

VALEUR NUMERIQUE 756 rad, 120 tours

DISTANCE PARCOURUE PAR LA ROUE

$$\boxed{x = \Theta \cdot R}$$

VALEURS NUMERIQUE

$x = 151 \text{ m}$

86

$$\boxed{\begin{aligned}\Theta(t) &= \omega_0 t + \alpha t^2/2\\ \omega(t) &= \omega_0 + \alpha t\end{aligned}}$$

$\omega_0 = 20$

RESOUDRE LE PROBLEME EN TOURS/MINUTES !! EST NETTEMENT PLUS SIMPLE !

$$\begin{cases} 40 = 20t + \alpha t^2/2 \\ 50 = 20 + \alpha t \end{cases}$$

$$\begin{cases} 40 = 20t + \alpha t^2/2 \\ 30/\alpha = t \end{cases}$$

ON DEDUIT QUE

$$40 = \frac{600}{\alpha} + \alpha\left(\frac{900}{\alpha^2}\right)\frac{1}{2}$$

$$4\alpha = 60 + 45$$

$$\alpha = \frac{105}{4} = \underbrace{26{,}25 \left[\frac{\text{tour}}{(\text{minute})^2}\right]}_{0{,}046\ [\text{rad}/s^2]}$$

$\alpha = 26{,}25$

$$\boxed{\begin{aligned}\Theta &= \alpha t^2/2\\ 20 &= \alpha t\end{aligned}}$$

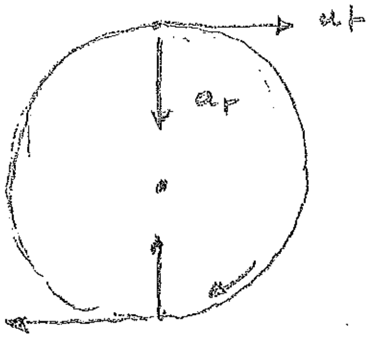
ON DEDUIT QUE

$$t = \frac{20}{\alpha}$$

$$\Theta = \alpha\left(\frac{400}{\alpha^2}\right)\frac{1}{2} = \frac{200}{26{,}25} = 7{,}62\ [\text{tour}]$$

C'EST JUSTE COMME DES EXERCICES ELEMENTAIRES DE MRUA !!

87



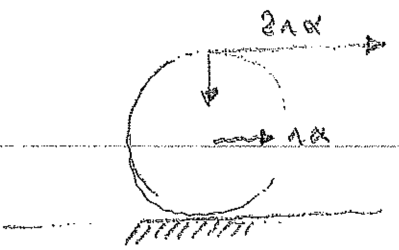
PAR RAPPORT AU CENTRE

$$a_t = \underbrace{r\alpha}_{3\ m/s^2}$$

$$a_c = \underbrace{\frac{v^2}{r}}_{\frac{2^2}{0,25}} = 16\ m/s^2$$

$$a = \sqrt{a_t^2 + a_c^2} = 16,3\ m/s^2$$

PAR RAPPORT AU SOL



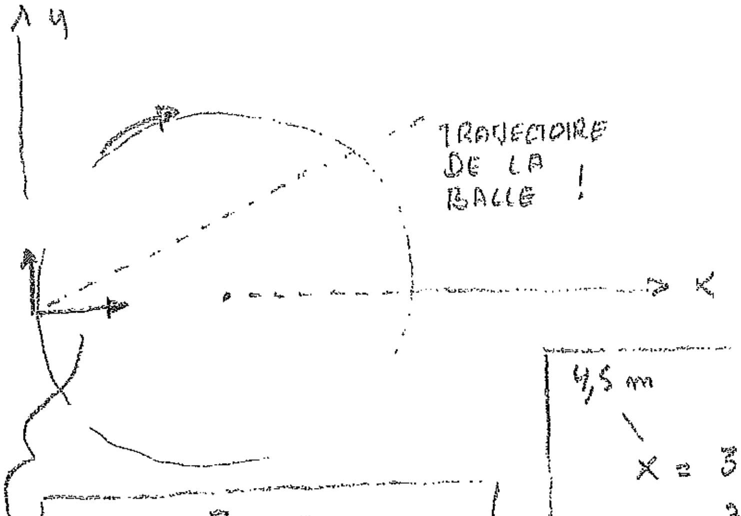
⚠

$$a_t = 2r\alpha = 6\ m/s^2$$

$$a_c = \frac{v^2}{r} = 16\ m/s^2$$

$$a = \sqrt{a_t^2 + a_c^2} = 17,1\ m/s^2$$

88

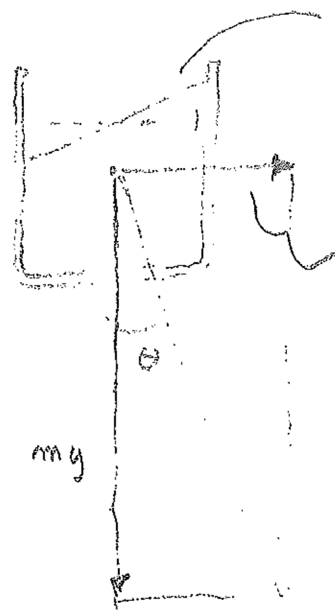


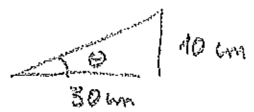
$$\vec{v} = \begin{bmatrix} 30 \\ \underbrace{R\omega}_{3,6} \end{bmatrix}\ m/s$$

$$\underbrace{x}_{4,5\ m} = 30\,t$$
$$y = 3,6\,t$$

$$y = 3,6\ \frac{4,5}{30} = 0,54\ m$$

89





$\underbrace{\dfrac{mv^2}{R}}$

PSEUDO-FORCE CENTRIFUGE

= − ACCELERATION CENTRIPETE × MASSE

LA SURFACE DE L'EAU EST PERPENDICULAIRE A LA RESULTANTE DES 2 FORCES !

$$\frac{1}{3} = \operatorname{tg}\theta = \frac{\;\frac{mv^2}{R}\;}{mg}$$

$$\downarrow$$

$$v^2 = \frac{Rg}{3}$$

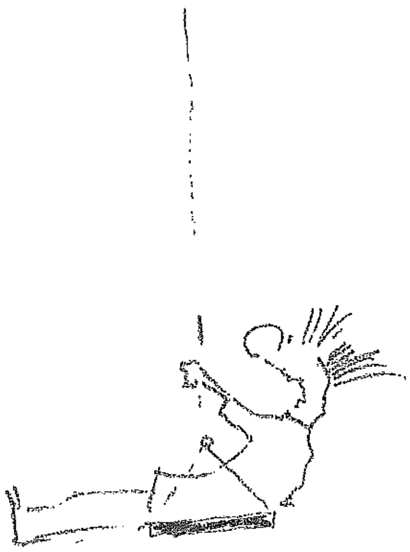
$$\downarrow$$

$$\boxed{v = \sqrt{\frac{Rg}{3}}}$$

VALEUR NUMERIQUE

$v = 11{,}5$ m/s

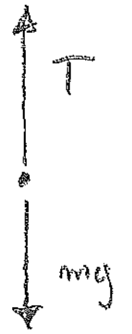
90



TERME DÛ A L'ACCELERATION CENTRIPETE

$$\frac{mv^2}{R} = T - mg$$

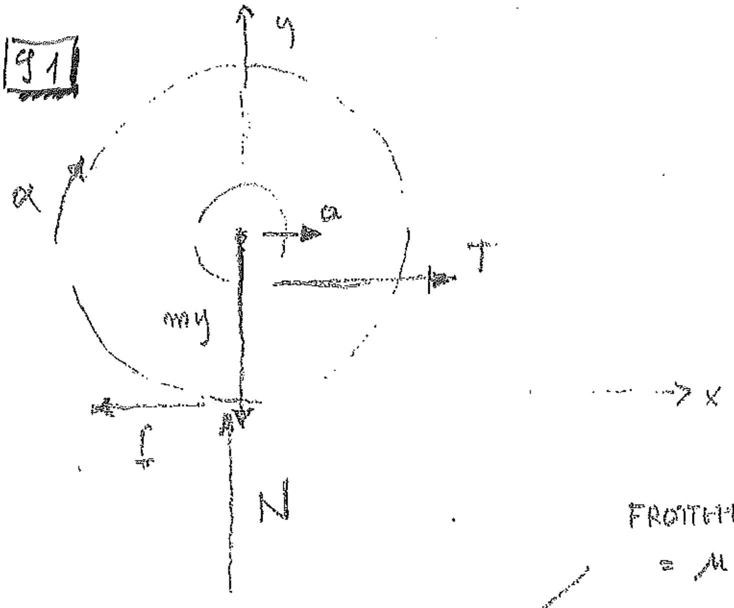
$$\boxed{T = m\left(g + \frac{v^2}{R}\right)}$$



453,75 N
avec $g = 10$ m/s²

445,77 N
avec $g = 9{,}81$ m/s²

$\boxed{91}$



car $N = mg$:-)

FROTTEMENT $= \mu . N = \mu\, mg$

$\boxed{\sum F_x = ma}$ $\quad ma = T - \mu mg \quad (1)$

$\boxed{\sum M = I\alpha}$ $\quad \frac{mR^2}{2}\alpha = -rT + R\mu mg \quad (2)$

$\boxed{\text{CONDITION DE ROULEMENT SANS GLISSEMENT} \quad +\alpha R = a}$

LA ROUE PROGRESSE DANS LE SENS DU FROTTEMENT !

ATTENTION !!

$$\begin{cases} ma = T - \mu mg \\ -\frac{mRa}{2} = rT - R\mu mg \end{cases}$$

$\downarrow$

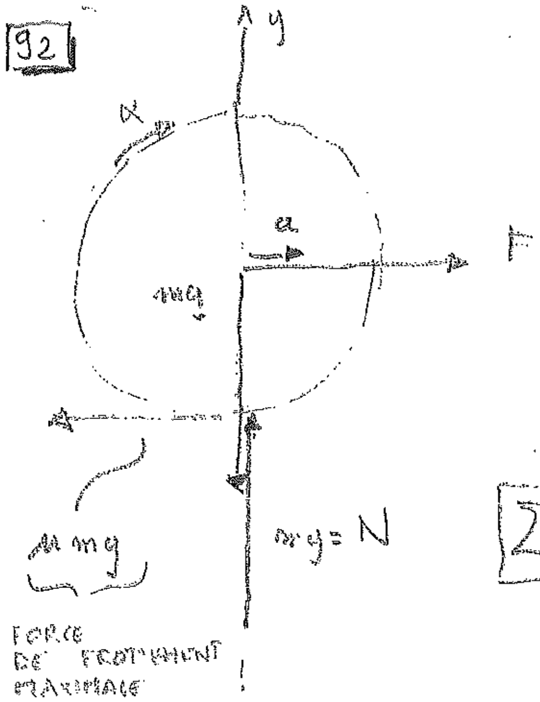
$$\begin{cases} \frac{1}{2} mRa = \frac{R}{2}T - \mu \frac{R}{2} mg \\ -\frac{1}{2} mRa = rT - R\mu mg \end{cases}$$

EN ADDITIONNANT LES 2 ÉQUATIONS

$$\left(\tfrac{1}{2}R + r\right)T = mg\mu\left(\frac{3R}{2}\right)$$

$$\boxed{T = \frac{3\mu mg R}{(R + 2r)}}$$

92



ROULEMENT
SANS
GLISSEMENT

$$\alpha R = a$$

$$\boxed{\sum F_x = ma}$$

$$\hookrightarrow ma = F - \mu mg$$

$$\boxed{\sum M = I\alpha}$$

$$\hookrightarrow \underbrace{\frac{1}{2} m R^2 \alpha}_{= \frac{1}{2} m R a} = R \mu mg$$

$$\mu = \frac{a}{2g}$$

3 EQUATIONS
3 INCONNUES
μ, α, a

$$F = ma + \frac{a}{2g} mg$$

$$F = \frac{3}{2} ma$$

$$\boxed{a = \frac{2F}{3m}}$$

$$f = \underbrace{\frac{2F}{3m} \frac{1}{2g} mg}_{= F/3}$$

IL FAUT

$$\mu_s > \frac{F}{3mg}$$

COEFFICIENT
MINIMAL DE
FROTTEMENT POUR
ROULEMENT SANS GLISSEMENT