9

$$\vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3t^2 - 2t \\ -t^3 \end{bmatrix}$$

$$\vec{v}(t) = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 6t - 2 \\ -3t^2 \end{bmatrix}$$

$$\vec{a}(t) = \begin{bmatrix} x''(t) \\ y''(t) \end{bmatrix} = \begin{bmatrix} 6 \\ -6t \end{bmatrix}$$

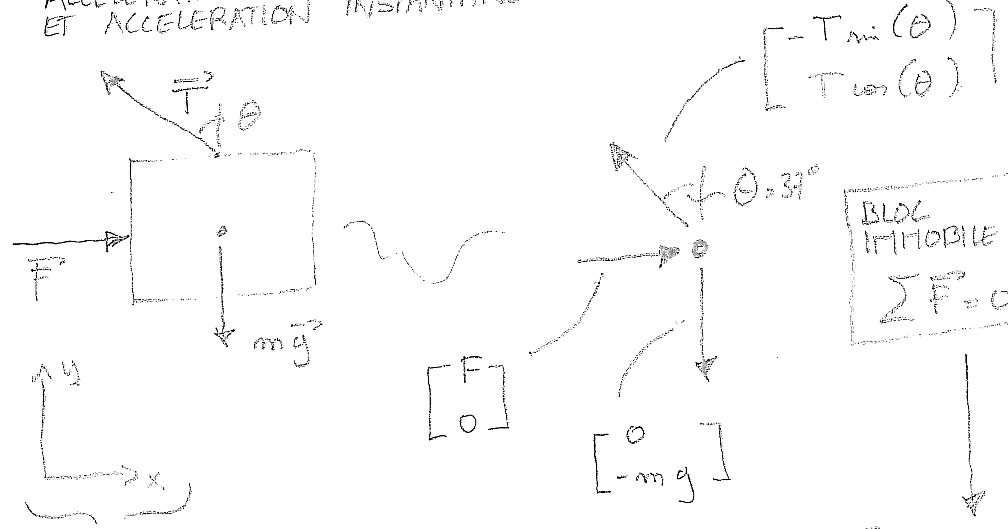
- $\boxed{\vec{v}(2) = \begin{bmatrix} 12-2 \\ -12 \end{bmatrix} = \begin{bmatrix} 10 \\ -12 \end{bmatrix}}$
- $\boxed{\vec{a}(4) = \begin{bmatrix} 6 \\ -24 \end{bmatrix}}$

EN REMPLAÇANT t PAR 2 ET 4 RESPECTIVEMENT

- $$\vec{a}_M = \frac{\vec{v}(3) - \vec{v}(1)}{3-1} = \frac{1}{2}\left(\begin{bmatrix} 18-2 \\ -27 \end{bmatrix} - \begin{bmatrix} 4 \\ -3 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 12 \\ -24 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \end{bmatrix}$$

⚠ NE PAS CONFONDRE ACCÉLÉRATION MOYENNE ET ACCÉLÉRATION INSTANTANÉE

10



REPÈRE À INDIQUER OBLIGATOIREMENT !

BLOC IMMOBILE
$\sum \vec{F} = 0$

$$\boxed{\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -mg \end{bmatrix} + \begin{bmatrix} -T\sin\theta \\ T\cos\theta \end{bmatrix}}$$

$$\begin{cases} F = T\sin\theta \\ mg = T\cos\theta \end{cases}$$

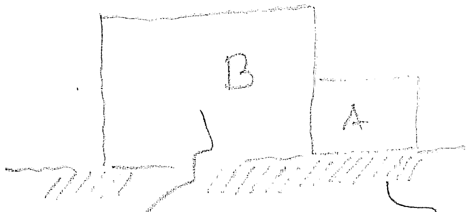
$$\boxed{T = \frac{mg}{\cos\theta}} \rightarrow \boxed{F = \frac{mg\sin\theta}{\cos\theta}}$$

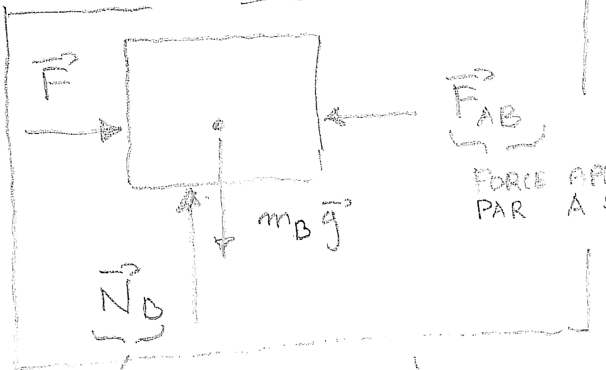
⚠ IL FAUT SUBSTITUER LES VALEURS NUMÉRIQUES UNIQUEMENT À LA FIN DU CALCUL !

VALEURS NUMÉRIQUES

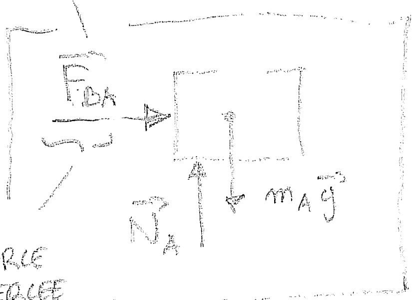
$$T = \frac{20}{\cos(37°)} = 25\ [N] \qquad F = 25\sin(37°) = 15\ N$$

$\boxed{11}$



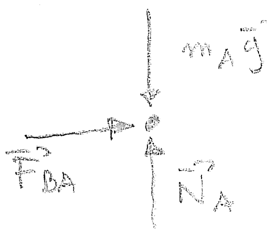


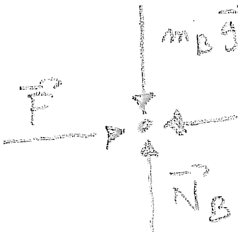
FORCE EXERCEE PAR LE SOL SUR B



FORCE EXERCEE PAR B SUR A

$\boxed{\text{ACTION REACTION :-)}}$





$\boxed{1}$ $\boxed{\sum \vec{F} = m\vec{a}}$

POUR LES 2 BLOCS

COMPOSANTE HORIZONTALE

$\boxed{F = (m_A + m_B)\, a_x}$

$a_x = 20/5 = 4 \text{ m/s}^2$

$\boxed{2}$ $\boxed{\sum \vec{F} = m\vec{a}}$

POUR LE BLOC B

$F - F_{AB} = m_B a_x$

$a_x = \dfrac{F}{(m_B + m_A)}$

$$\boxed{F\Big(1 - \underbrace{\frac{m_B}{(m_B + m_A)}}_{3/5}\Big) = F_{AB}}$$

$F_{AB} = \frac{2}{5}\, 20 = 8 \text{ N}$

$\boxed{3}$

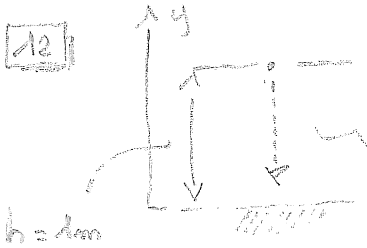
$\underbrace{\sum \vec{F}}_{\text{RESULTANTE DES FORCES}} = m_B \vec{a}$

$= 3 \begin{bmatrix} 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$

$\boxed{4}$ $F\left(1 - \dfrac{m_A}{m_B + m_A}\right) = F_{AB}$

$F_{AB} = \frac{3}{5}\, 20 = 12 \text{ N}$:-)

12

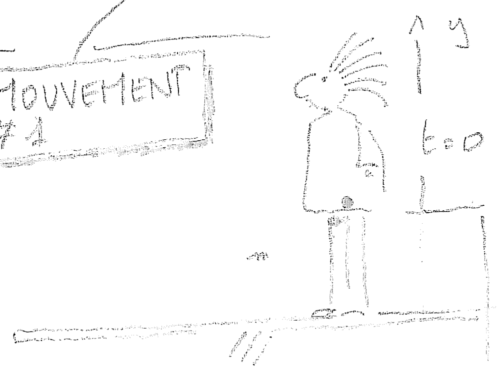


PARTIE 1

CINÉMATIQUE

$y(t) = h - gt^2/2$

MOUVEMENT #1

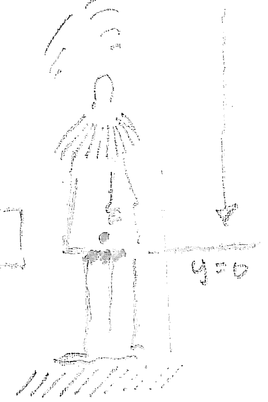


TEMPS CHUTE COMPLET $= t_1 + t_2$

CHUTE LIBRE DE $h = 1\,m$

$y(t) = h - gt^2/2$

$t \in [0\ t_1]$

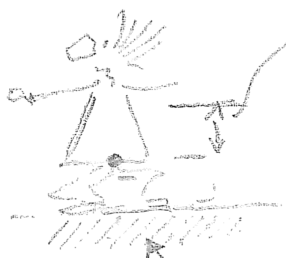


$\boxed{t_1}$

LES PIEDS TOUCHENT LE SOL

$$t_1 = \sqrt{\frac{2h}{g}}$$

$$v(t_1) = -g\sqrt{\frac{2h}{g}} = -\sqrt{2hg}$$



$t' = 0$

ON RALENTIT EN PLIANT LES GENOUX

LA PETITE FILLE EST IMMOBILE APRÈS AVOIR PLIÉ LES GENOUX

t_2

ELLE EFFECTUE UNE DÉCÉLÉRATION EN EFFECTUANT CE MOUVEMENT

ON SUPPOSE L'ACCÉLÉRATION CONSTANTE ICI : C'EST UN PEU ABUSIF !

$t \in [0, t_2]$

$$y(t) = \underbrace{-\sqrt{2hg}}_{\text{VITESSE INITIALE = CELLE FIN CHUTE LIBRE}}\, t + a\,t^2/2$$

MOUVEMENT #2

IMMOBILE EN t_2

$$y'(t_2) = 0$$

$$-\sqrt{2hg} + a t_2 = 0$$

$$t_2 = \frac{\sqrt{2hg}}{a}$$

$d = 0.2$ EN t_2 LORSQU'ON EST IMMOBILE

$$\underbrace{y(t_1)}_{-d} = -\frac{2hg}{a} + \frac{2hg}{2a} = -\frac{hg}{a}$$

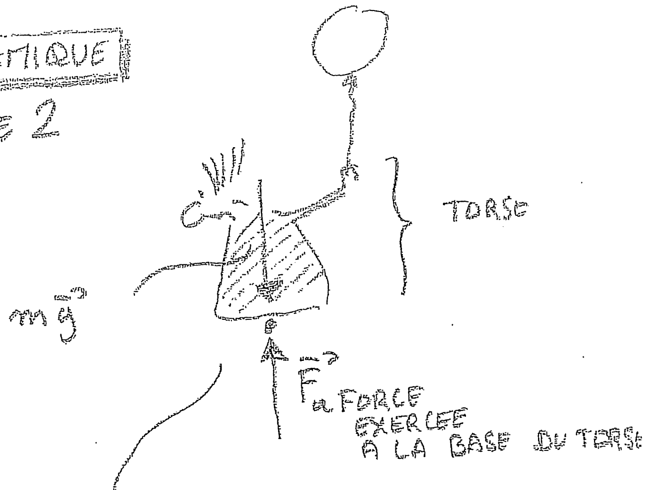
$$\longrightarrow \boxed{a = \frac{hg}{d}}$$

[m] [m/s²] [m]

d PETIT ! a GRAND !! LOGIQUE. IL FAUT PLIER LES GENOUX !

DYNAMIQUE

PARTIE 2



$$\boxed{\sum \vec{F} = m\vec{a}}$$

COMPOSANTE VERTICALE

$$-mg + F_a = m\frac{h g}{d}$$

$$F_a = \underbrace{m}_{40}\underbrace{\left(\frac{d+h}{d}\right)}\overbrace{g}^{10}$$

$d = 0{,}3$	$\frac{d+h}{d} = \frac{1{,}3}{0{,}3}$	$= 4{,}33$
$d = 0{,}04$	$\frac{d+h}{d} = \frac{1{,}04}{0{,}04}$	$= 26$

$F_a = 1733$ [N] si $d = 30$ cm

$F_a = 10400$ [N] si $d = 4$ cm

CONCLUSION : IL FAUT PLIER LES GENOUX !!

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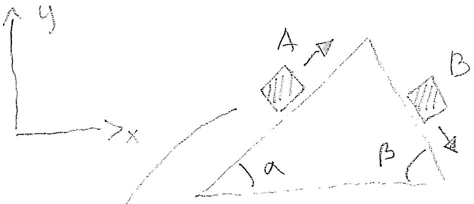
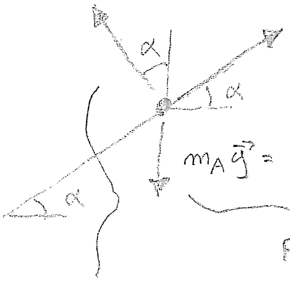




$\vec{T} = \begin{bmatrix} T\cos\alpha \\ T\sin\alpha \end{bmatrix}$ CORDE

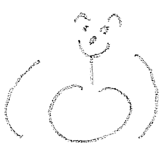
$\underbrace{m_A\vec{g} = \begin{bmatrix} 0 \\ -m_A g \end{bmatrix}}_{\text{POIDS}}$

$\boxed{\sum \vec{F} = m\vec{a}}$ BLOC A

$\underbrace{\vec{N} = \begin{bmatrix} -N\sin\alpha \\ N\cos\alpha \end{bmatrix}}_{\text{REACTION DU SOL}}$

$\vec{a} = \begin{bmatrix} a\cos\alpha \\ a\sin\alpha \end{bmatrix}$

$$\begin{cases} -N\sin\alpha + T\cos\alpha = m_A\, a\cos\alpha & (1) \\ N\cos\alpha + T\sin\alpha - m_A g = m_A\, a\sin\alpha & (2) \end{cases}$$

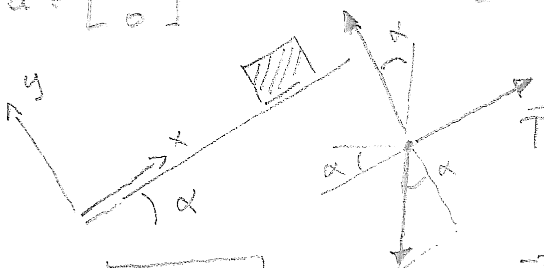


OOOUUUPPPSSS !!
COMPLIQUÉ, COMPLIQUÉ !

PLUS MALIN !
CHOISIR LE BON SYSTEME D'AXES !

$\vec{a} = \begin{bmatrix} a \\ 0 \end{bmatrix}$

$\vec{N} = \begin{bmatrix} 0 \\ N \end{bmatrix}$



$\vec{T} = \begin{bmatrix} T \\ 0 \end{bmatrix}$

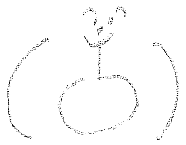
BLOC A
$\sum \vec{F} = m\vec{a}$

$m_A\vec{g} = \begin{bmatrix} -m_A g\sin\alpha \\ -m_A g\cos\alpha \end{bmatrix}$

$$\begin{cases} T - m_A g\sin\alpha = m_A a & (3) \\ N - m_A g\cos\alpha = 0 & (4) \end{cases}$$

ON RELIE DIRECTEMENT T ET a SANS CALCULER N !

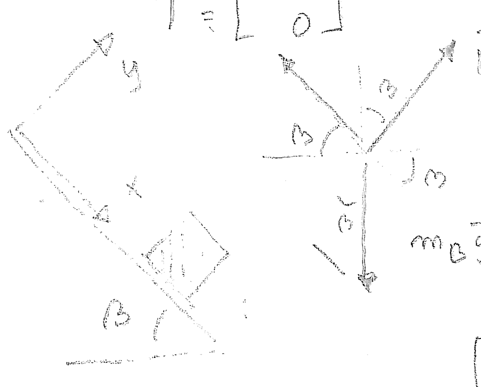
$$\boxed{a = \frac{T}{m_A} - g\sin\alpha} \quad (5)$$

 ET 

SONT <u>STRICTEMENT</u> EQUIVALENTES

C'EST LA MÊME EGALITE VECTORIELLE
EXPRIMEE DANS DEUX SYSTEMES D'AXES DIFFERENTS

<u>VERIFIER</u> ! $\boxed{\begin{aligned} (1)\cos\alpha + (2)\sin\alpha &= (3) \\ -(1)\sin\alpha + (2)\cos\alpha &= (4) \end{aligned}}$!



$\vec{T} = \begin{bmatrix} -T \\ 0 \end{bmatrix}$ $\quad \vec{N_B} = \begin{bmatrix} 0 \\ N_B \end{bmatrix}$ $\quad \vec{a} = \begin{bmatrix} a \\ 0 \end{bmatrix}$

$m_B\vec{g} = \begin{bmatrix} m_B g \sin\beta \\ -m_B g \cos\beta \end{bmatrix}$

BLOC B

$\sum \vec{F} = m\vec{a}$

$m_B a = -T + m_B g \sin\beta$

$\boxed{a = g \sin\beta - \frac{T}{m_B}}$ (6)

- EN COMPARANT (5) ET (6)
MEME VALEUR DE a

$g\sin\beta - \frac{T}{m_B} = \frac{T}{m_A} - g\sin\alpha$

$g(\sin\beta + \sin\alpha) = T\left(\frac{1}{m_A} + \frac{1}{m_B}\right)$

$\boxed{T = \frac{m_B m_A}{(m_B + m_A)} g(\sin\beta + \sin\alpha)}$

- EN COMPARANT (5) ET (6)
MEME VALEUR DE T

$m_A(a + g\sin\alpha) = m_B(g\sin\beta - a)$

$\boxed{a = \frac{m_B g \sin\beta - m_A g \sin\alpha}{(m_A + m_B)}}$

VALEURS NUMERIQUES

$\alpha = 30$
$\beta = 60$

$T = \frac{30}{11} * 9{,}8 * \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 36{,}5$ [N]

$a = \frac{1}{11} * 9{,}8 * \left(6\frac{\sqrt{3}}{2} - 5\,\frac{1}{2}\right) = 2{,}4$ [m/s²]

IL <u>FAUT</u> CONNAITRE LES VALEURS REMARQUABLES DES sin ET COS

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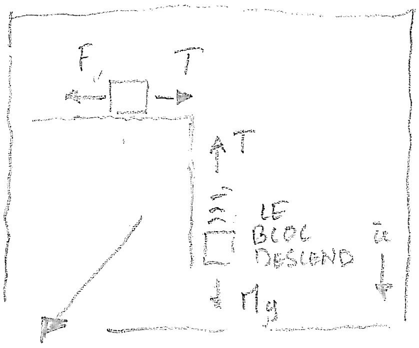

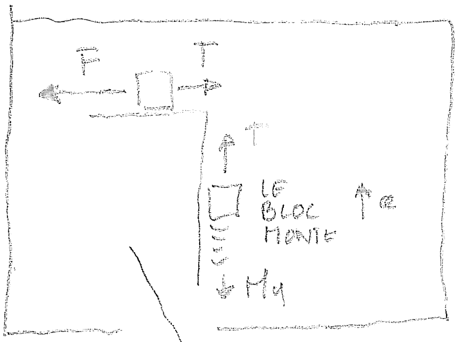

$$\begin{cases} m a = T - F \\ M a = M g - T \end{cases}$$

$F = 22$ [N]
$a = 1$ [m/s²]

$$\boxed{(m+M)\,\underset{1}{a} = -\underset{22}{F} + Mg}$$

$$\begin{cases} m a = F - T \\ M a = T - M g \end{cases}$$

$F = 44$ [N]
$a = 1{,}75$ [m/s²]

$$\boxed{(m+M)\,\underset{1{,}75}{a} = \underset{44}{F} - Mg}$$

2 EQUATIONS
2 INCONNUES m ET M

$$\begin{cases} M(1 - 9{,}8) + m = -22 \\ M(1{,}75 + 9{,}8) + m\,1{,}75 = 44 \end{cases}$$

$$\begin{cases} -8{,}8\,M + m = -22 & (1) \\ 11{,}55\,M + 1{,}75\,m = 44 & (2) \end{cases}$$

$-1{,}75\,(1) + (2) \rightarrow 1{,}75 * 8{,}8\,M + 11{,}55\,M = 1{,}75 * 22 + 44$

$$\boxed{\begin{aligned} M &= 3{,}06 \text{ kg} \\ m &= 4{,}94 \text{ kg} \end{aligned}}$$

$m = -22 + 8{,}8 * 3{,}06$
EN REPRENANT L'EQUATION (1)

NOTE

LE BLOC LE PLUS MASSIF EST m ET NON M :-)

LES NOTATIONS PEUVENT ETRE CONTRE-INTUITIVES !

BE CAREFUL !

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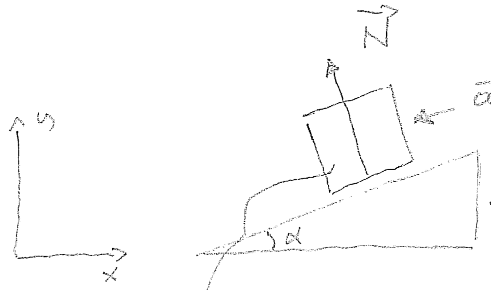
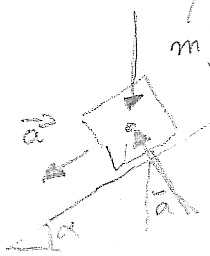


$\vec{a}$ ACCELERATION DU BLOC PAR RAPPORT AU COIN

$\vec{A} = \begin{bmatrix} A \\ 0 \end{bmatrix}$

ACCELERATION DU COIN PAR RAPPORT AU SOL

BLOC

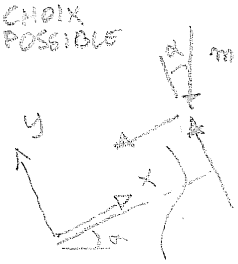


$m\vec{g} = \begin{bmatrix} 0 \\ -mg \end{bmatrix}$

$\vec{N} = \begin{bmatrix} -N\sin\alpha \\ N\cos\alpha \end{bmatrix}$

ACCELERATION BLOC % COIN $\vec{a} = \begin{bmatrix} -a\cos\alpha \\ -a\sin\alpha \end{bmatrix}$

AUTRE CHOIX POSSIBLE



$m\vec{g} = \begin{bmatrix} -mg\sin\alpha \\ -mg\cos\alpha \end{bmatrix}$

$\vec{N} = \begin{bmatrix} 0 \\ N \end{bmatrix}$

$\vec{a} = \begin{bmatrix} -a \\ 0 \end{bmatrix}$

$\vec{A} = \begin{bmatrix} A\cos\alpha \\ A\sin\alpha \end{bmatrix}$

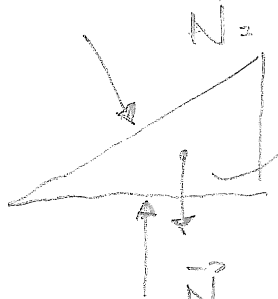
$\sum \vec{F} = m\vec{a}$

REPERE INERTIEL → SOL !

$$\begin{cases} -N\sin\alpha = m(A - a\cos\alpha) \\ N\cos\alpha - mg = m(\quad - a\sin\alpha) \end{cases}$$

IL FAUT METTRE L'ACCELERATION DU COIN !

COIN



$\vec{N} = \begin{bmatrix} N\sin\alpha \\ -N\cos\alpha \end{bmatrix}$

$M\vec{g} = \begin{bmatrix} 0 \\ -Mg \end{bmatrix}$

$\vec{N}_{SOL} = \begin{bmatrix} 0 \\ N_{SOL} \end{bmatrix}$

$$\begin{cases} N\sin\alpha = MA \\ -N\cos\alpha - Mg + N_{sol} = 0 \end{cases}$$

(PAS VRAIMENT UTILE SAUF POUR CONNAITRE N_{SOL} NON DEMANDE ICI !)

3 EQUATIONS
A 3 INCONNUES
N, A et a

$$
\begin{cases}
-N \sin\alpha = m(A - a\cos\alpha) & (1)\\
N\cos\alpha - mg = -m\,a\sin\alpha & (2)\\
N\sin\alpha = MA & (3)
\end{cases}
$$

$$(1)+(3) \longrightarrow (M+m)A = m\,a\cos\alpha \quad \Big\}$$ EGALITE HORIZ POUR ENSEMBLE BLOC + COIN !

$$\cos(\alpha)(1) + \sin(\alpha)(2) \longrightarrow \underbrace{-mg\sin\alpha = mA\cos\alpha - m\,a}$$

EGALITE
LE LONG DE LA
DIRECTION OBLIQUE
POUR LE COIN !

$$-mg\sin\alpha = mA\cos\alpha - \frac{(M+m)}{\cos\alpha}A$$

$$mg\sin\alpha\cos\alpha = \underbrace{-mA\cos^2\alpha + mA}_{mA(\underbrace{1-\cos^2\alpha}_{\sin^2\alpha})} + MA$$

$$\boxed{A = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}}$$

$$\boxed{N = \frac{Mmg\cos\alpha}{M + m\sin^2\alpha}}$$ CAR $N\sin\alpha = MA$:-)

$$\boxed{a = \frac{(M+m)\,g\sin\alpha}{M + m\sin^2\alpha}}$$ CAR $m\,a\cos\alpha = (M+m)A$:-)

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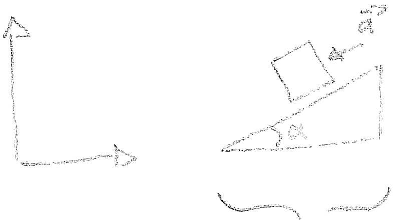
PIECE OF CAKE ! :-)

THE FAST WAY



$\vec{a}$ ACCELERATION DU BLOC PAR RAPPORT AU COIN $\qquad \vec{a} = \begin{bmatrix} -a\cos\alpha \\ -a\sin\alpha \end{bmatrix}$

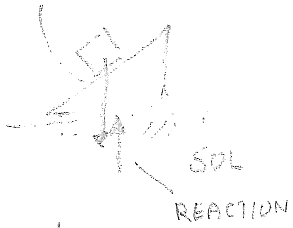
$\vec{A} = \begin{bmatrix} A \\ 0 \end{bmatrix}$

ACCELERATION DU COIN PAR RAPPORT AU SOL

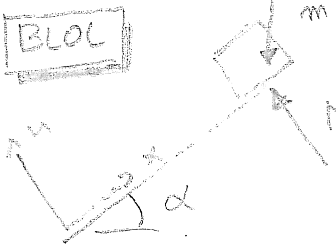
ENSEMBLE BLOC + COIN

$m\vec{a} = \sum \vec{F}$ COMPOSANTE HORIZONTALE

$$\boxed{(m+M)A - ma\cos\alpha = \underbrace{0}_{\text{PAS DE FORCE HORIZ !}}}$$



BLOC



$mg = \begin{bmatrix} -mg\sin\alpha \\ -mg\cos\alpha \end{bmatrix}$

$N = \begin{bmatrix} 0 \\ N \end{bmatrix}$

$m(\vec{A} + \vec{a}) = \sum \vec{F}$ COMPOSANTE HORIZONTALE

$$\boxed{m\underbrace{(A\cos\alpha - a)}_{\text{ACCELERATION PAR RAPPORT AU SOL !}} = -mg\sin\alpha}$$

$$mA\cos\alpha - \frac{(M+m)A}{\cos\alpha} = -mg\sin\alpha$$

$$-mA\cos^2\alpha + mA + MA = mg\sin\alpha\cos\alpha$$

$$mA\underbrace{(1-\cos^2\alpha)}_{\sin^2\alpha} + MA = mg\sin\alpha\cos\alpha$$

$$\boxed{A = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}}$$

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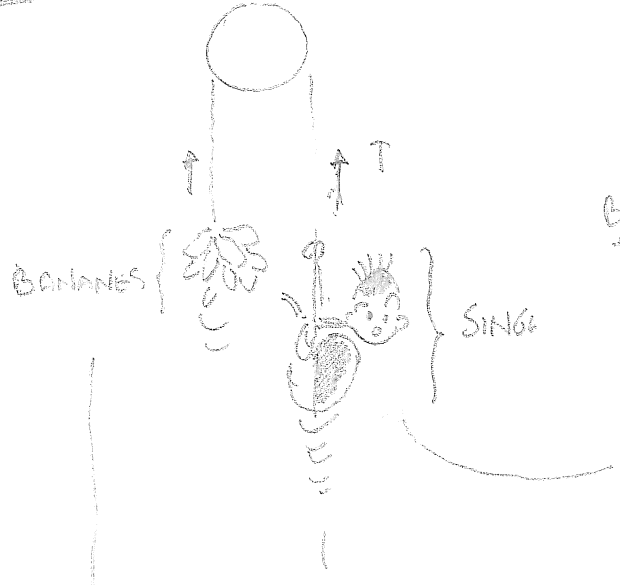

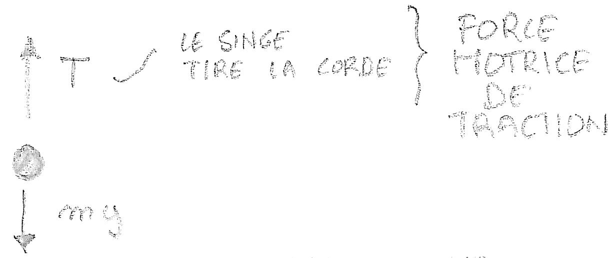

LES BANANES MONTENT SI $T - \underbrace{12g}_{\text{POIDS DES BANANES}} > 0$

$$\boxed{T > 12g}$$

DYNAMIQUE DU SINGE

$$T - 10g = 10\,a$$

ACCÉLÉRATION DU SINGE PAR RAPPORT À LA CORDE

CONCLUSION

IL FAUT $a > \underbrace{\dfrac{12g - 10g}{10}}_{1{,}96\ m/s^2}$

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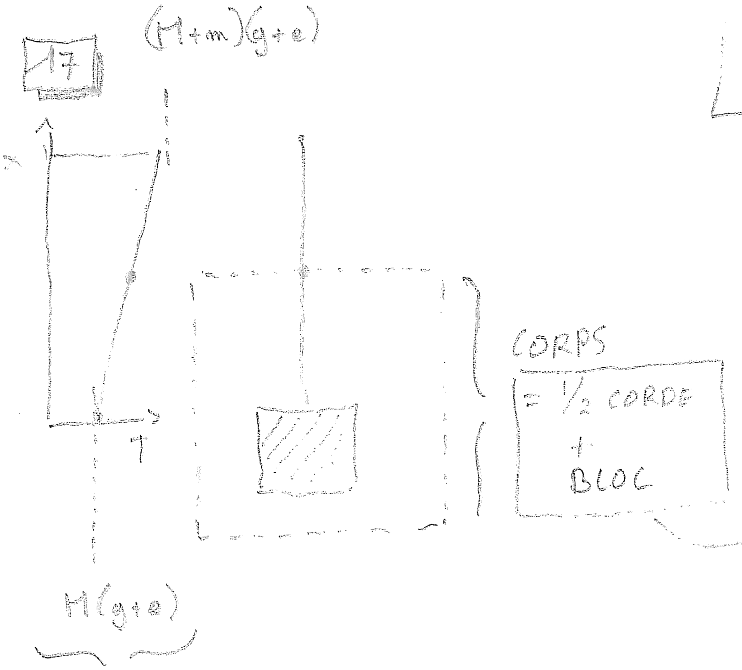
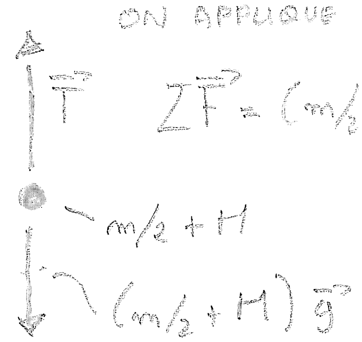


ON APPLIQUE NEWTON !



$$\Sigma \vec{F} = (m/2 + M)\,\vec{a}$$

SI ON TIENT COMPTE DU POIDS DE LA CORDE, EN HAUT, LA TENSION REPREND LE POIDS DU BLOC ET DE LA CORDE !

$$T = \underbrace{(m/2 + M)}_{0{,}215}\ \underbrace{(g + e)}_{13{,}8}$$

$$= 2{,}97\ N$$